SRI BHAGAWAN MAHAVEER JAIN COLLEGE

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Mock Examination Question Paper-2 (January 2019)

Course:	II PUC	**Subject:**	Mathematics
Max. Marks:	100	**Duration:**	3:15 hrs.

Instructions:
The question paper has five parts, Answer ALL
Use graph sheet for the question on LPP in PART-E

PART-A

I. Answer ALL the questions. **10 x 1 = 10**

1. A relation R on A = {1,2,3} defend by R = {(1,1) , (1, 2) (3, 3)} is not symmetric, why?
2. Find the value of $\cos\ (\sin^{-1} x + \cos^{-1} x)\ |x| \geq 1$
3. Write the condition for the matrix $A = \left[a_{ij} \right]_{m \times n}$ to be a square matrix.
4. Find the number of all possible matrices of order 3x3 with each entry 0 or 1
5. If $y = e^{3\log x}$ show that $\frac{dy}{dx} = 3x^2$
6. Evaluate $\int \sec x\left(\sec x + tan\ x\right) dx.$
7. If a line makes angle $90^0, 60^0$ and 30^0 with positive direction of x, y and z axis respectively. Find its direction cosines.
8. Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{Z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.
9. Define Skew lines.
10. If $P(\mathrm{A}) = \frac{7}{13}$ P(B)= $\frac{9}{13}$ and P(A $\cap$ B)= $\frac{4}{13}$. Find P(A/B)

PART-B

II. Answer any TEN questions. **10x2=20**

11. Verity whether the operation $*$ defined on Q, by $a * b = \frac{ab}{4}$ is associative or not.
12. Prove that $2\ \sin^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)\ -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$
13. Find the value of $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$
14. Prove that the value of a determinant remains unchanged of its rows or columns are interchanged by considering a third order determinant.
15. If $y = \sin\left(\log_e x\right),$ prove that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{x}$
16. If $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), \frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$ find $\frac{dy}{dx}$.
17. If the radius of a sphere is measured as 9 cms with an error of 0.03 cms, then find the approximate error in calculating its volume.

18. Evaluate $\int \frac{\sin x}{\sin(x-a)}dx$

19. Evaluate $\int e^x\left(\frac{x-1}{x^2}\right)dx$

20. Find the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0.$

21. Let $|\vec{a}|=3, |\vec{b}|=\frac{\sqrt{2}}{3}$ and $|\vec{a}\times\vec{b}|=1$. Find the angle between $\vec{a}$ and $\vec{b}$

22. Find the value of $i.\left(\hat{j}\times\hat{k}\right)+\hat{j}.\left(\hat{i}\times\hat{k}\right)+\hat{k}.\left(\hat{i}\times\hat{j}\right)$

23. Find the equation of the plane through the intersection of planes $3x-y+2Z-4=0$ and $x+y+z-2=0$ and the point (2,2,1)

24. If $P(A)=0.8, P(B)=0.5$ and $P\left(\frac{B}{A}\right)=0.4$ find $P(A\cap B)$

PART-C

III. Answer any TEN questions. **10x3=30**

25. Show that the relation R in the set of integers given by R= {(a,b)/5 divides (a-b)} is an equivalence relation.

26. Prove that $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy<1$

27. If A and B are symmetric matrices, prove that AB=BA is skew symmetric.

28. If $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ prove that $\frac{dy}{dx}=\frac{1}{2(1+x^2)}$

29. Find two positive numbers x and y such that $x+y=60$ and xy^3 is maximum.

30. Find $\frac{dy}{dx}$ if $x=a(\cos\theta+\theta\sin\theta), y=a(\sin\theta-\theta\cos\theta)$

31. Evaluate $\int\frac{1}{x+x\log x}dx$

32. Evaluate $\int_a^b x\,dx$ as limit of a sum

33. Find the area of the region bounded by the curve $x^2=4y$ and the lines $y=2, y=4$ and the y-axis.

34. Form the differential equation of the family of parabolas having vertex at origin..

35. Show that the four points with position vectors $4\hat{i}+8\hat{j}+12\hat{k}$, $2\hat{i}+4\hat{j}+6\hat{k}$, $3\hat{i}+5\hat{j}+4\hat{k}$ and $5\hat{i}+8\hat{j}+5\hat{k}$ are coplanar.

36. Prove that $\left[\vec{a}+\vec{b}\ ,\ \vec{b}+\vec{c}\ ,\ \vec{c}+\vec{a}\right]=2\left[\vec{a},\vec{b},\vec{c}\right]$

37 Find the shortest distance between the lines l_1 and l_2 whose vector equation are $\vec{r}=\hat{l}+\hat{j}+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right)$ and $\vec{r}=2\hat{i}+\hat{j}-\hat{k}+\mu\left(3\hat{i}-5\hat{j}+2\hat{k}\right)$

38. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

PART-D

IV. Answer any SIX of the following questions. **6x5=30**

39. Let $f:R\to R$ defined by $f(x)=4x+3$ show that f is invertible, find the inverse of f.

40. If $A=\begin{bmatrix}1&1&-1\\2&0&3\\3&-1&2\end{bmatrix}, B=\begin{bmatrix}1&3\\0&2\\-1&4\end{bmatrix}$ and $C=\begin{bmatrix}1&2&3&-4\\2&0&-2&1\end{bmatrix}$ prove that $(AB)C=A(BC)$

41. Solve the following system of equations by Matrix method.
$3x-2y+3z=8$
$2x+y-z=1$
$4x-3y+2z=4$

42. If $y=\left(\tan^{-1}x\right)^2$, show that $\left(x^2+1\right)^2\frac{d^2y}{dx^2}+2x\left(x^2+1\right)\frac{dy}{dx}=2$

43. A ladder 5m long is leaning against a wall, the bottom of the ladder is pulled along the ground, away from the wall at the rate of 2 cms/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall?

44. Find $\int\frac{1}{x^2-a^2}dx$ and hence evaluate $\int\frac{1}{3x^2+13x-10}dx$

45. Find the area of the curve $y=\cos x,$ between $x=0$ and $x=2\pi$

46. Find the particular solution of the differential equation
$(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}$ when $y=0$ and $x=1$.

47. Derive the equation of a plane passing through a given point and perpendicular to a given vector.

48. A die is thrown 6 times, if 'getting an odd number is success' what is the probabrity of
(i) 5 success (ii) atleast 5 success (iii) atmost 5 success.

PART-E

V. Answer any ONE question. **1x10=10**

49. (a) Prove that $\int_{-a}^{a}f(x)\,ax=\begin{cases}2\int_0^a f(x)dx, & \text{if } f(x) \text{ is even}\\ 0 & \text{if } f(x) \text{ is odd.}\end{cases}$

and hence evaluate $\int_{\frac{-\pi}{2}}^{\pi/2}\left(x^3+x\cos x\right)dx$

(b) Prove that $\begin{vmatrix}x+y+2z & x & y\\ z & y+z+2x & y\\ z & x & z+x+2y\end{vmatrix}=2\left(x+y+z\right)^3$

50. (a) Maximize and minimize $Z=x+2y$
subject to the constraints $x+2y\geq 100$, $2x-y\leq 0$, $2x+y\leq 200$, $x\geq 0, y\geq 0$ by graphical method.

(b) Determine the value of K if $f(x)=\begin{cases}\frac{K\cos x}{\pi-2x} & \text{if } x\neq\frac{\pi}{2}\\ 3 & \text{if } x=\frac{\pi}{2}.\end{cases}$ is continous at $x=\frac{\pi}{2}$

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