Jain College, Jayanagar
II PUC Mock Paper II Jan 2019
Subject: II PUC Mathematics (35)

Duration: 3 hours 15 minutes **Max. Marks: 100**

PART-A

I. Answer all the TEN questions: **10X1=10**

1. A relation R on A = {1,2,3} defined by R = {(1,1) (1,2) (3,3)} is not symmetric why?
2. Find the value of $\cot(\tan^{-1}\alpha+\cot^{-1}\alpha)$
3. Define a scalar matrix.
4. If $\begin{bmatrix} x & 8 \\ 8 & x \end{bmatrix}=\begin{bmatrix} x & 8 \\ 8 & x \end{bmatrix}$ find value of x
5. Find $\frac{dy}{dx}$, if y = cos $\left(\sqrt{x}\right)$
6. Evaluate $\int\frac{1-x}{\sqrt{x}}dx$.
7. For what value of λ, the vectors $\vec{a}=2\hat{i}-3\lambda\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}-2\hat{k}$ are perpendicular to each other?
8. Find the intercepts cut off by the plane 2x + y – z = 5.
9. Define feasible region
10. If P(A) = 0.6 P(B) = 0.3 and (A$\cap$B) = 0.2 find P(A/B).

PART-B

II. Answer any TEN questions:
10X2=20

11. Prove that the greatest integer function, defined by f(x) =[x] indicates the gretest integer not greater than x, is neither one-one nor outo.
12. Evaluate sin $\left(\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right)$
13. Write $\tan^{-1}\left(\frac{a\cos x-b\,Sinx}{b\cos+a\sin x}\right)$, if $\frac{a}{b}$ tan x > -1 in the simplest form.
14. Find the equation of the line joining (3,1) and (9,3) using determinants.
15. If y = x^a + a^x+a^a for some fixed a > 0 and x > 0 find $\frac{dy}{dx}$
16. Differentiate $x^{\sin x}$, x > 0 w.r.t x
17. Show that the function of given by f(x) = x^3 – $3x^3$ + 4x, xε IR is strictly increasing on IR.
18. Evaluate : $\int\frac{10x^9+10^x\log_e 10}{x^{10}+10^x}dx$.
19. Evaluate : $\int\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$
20. Determine order and degree of the $\left(\frac{d^2y}{dx^2}\right)^2+\cos\left(\frac{dy}{dx}\right)=0$
21. Find the unit vector perpendicular to both $\vec{a}\ and\ \vec{b}$ when $\vec{a}=3\hat{i}+\hat{j}-2\hat{k}$ $\vec{b}=2\hat{i}+3\hat{j}-\hat{k}$.
22. Prove that $[\vec{a}\ \vec{b}\ \ \vec{c}+\vec{d}\ ]=[\vec{a}\ \vec{b}\ \ \vec{c}\ ]+[\vec{a}\ \vec{b}\ \ \vec{c}\ ]$.

23. Show that the lines $\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5}$ and $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$ are coplanar.

24. Find the probability distribution of number of tails in three tosses of a coin.

PART-C

III. Answer any TEN questions: 10X3=30

25. A relation R on the set A = {1, 2, 3…… 13,14} is defined as R = {(x,y) : 3x-y=0}. Determine whether R is reflexive, symmetric and transitive.

26. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x\in\left(0,\frac{\pi}{4}\right)$

27. Find $\frac{1}{2}(A+A')\ and\ \frac{1}{2}(A-A`)\ \ when\ A=\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & o \end{bmatrix}$.

28. Find $\frac{dy}{dx}$, $if\ \ x=a\ (\cos t+\log\tan\frac{t}{2}),\ y=a\sin t$.

29. Verify mean value theorem if f(x) = x^2 – 4x – 3 in the interval [a,b] where a = 1and b=4.

30. Find the points on the curve y = x^3 at which the slope of the tangent is equal to the y – coordinate of the points.

31. Evaluate $\int\frac{x}{(x-1)\ (x-2)}dx$.

32. Evaluate $\int\left[\log(\log x)+\frac{1}{(\log x)^2}\right]dx$

33. Find the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

34. Show that y^1 = $\frac{(x+y)}{x}$ is a homogeneous differential equation and solve it.

35. Derive the formula for position vector of a point which divides the join of two points A and B internally in ratio m : n.

36. If a unit vector $\vec{a}$ makes angle $\frac{\pi}{3}$ $with\ \hat{i}$ $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$ then find θ and hence the components of $\vec{a}$.

37. Find the distance between the point P(6,5,9) and the plane determined by the points A(3,-1,2) B (5,2,5) and C (-1, -1,6).

38. Two dice are thrown simultaneously, If X denotes the number of sixes, find the expectation (mean) of X.

PART-D

IV. Answer any SIX of the following: 6X5=30

39. Consider f : $IR_+ \rightarrow [-5, \infty)$ given by f(x) = $9x^2$ + 6x – 5. Show that f is invertible with $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$.

40. If A = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ Prove that $A^3 - 6A^2 + 7A + 2I = 0$

41. Solve by matrix method:

x-y+2z=7
3x+4y-5z = -5
2x-y+3z=12

42. If y = 500 e^{7x} + 600e ^{7x}.Show that y_2 = 49y.

43. A bubble, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x.

44. Find the integral of $\sqrt{a^2-x^2}$ with respect to x and evaluate $\int\sqrt{1+3x-x^2}\,dx$.

45. Using integration find the area enclosed by the parabola $y^2 = 4ax$ and the chord $y = mx$.

46. Solve the differential equation $(\tan^{-1}y-x)\,dy = (1+y^2)\,dx$

47. Derive the equation of plane passing through the intersection of two planes both in vector form and Cartesian form.

48. If 90% of people are right handed. What is the probability that atmost 6 of a random sample of 10 people are right handed.

PART-E

V. Answer any one of the following: 1X10=10

49. a) Solve the following linear programming problem graphically. Maximum z=4x+y subject to constraints $x + y \le 50$, $3x+y \le 90$. $X \ge 0$, $y \ge 0$.

b) Discuss the continuity of the function $f(x)\begin{cases} -2 & if \quad x \le -1 \\ 2x & if \quad -1 < x \le 1 \\ 2 & if \quad x > 1 \end{cases}$

50) a) Prove that $\int_b^a f(x)\,dx = \int_0^a f(a-x)dx$ *hence* evaluate $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$

b) Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$.

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