

ICSE Paper 2007

MATHEMATICS

SECTION A [40 MARKS]

(Answer all questions from this Section.)

Question 1.

- (a) Show that $(x - 1)$ is a factor of $x^3 - 7x^2 + 14x - 8$. Hence, completely factorise the above expression. [3]
- (b) Dinesh bought an article for ₹ 374, which included a discount of 15% on the marked price and a sales tax of 10% on the reduced price. Find the marked price of the article. [3]
- (c) Ramesh invests ₹ 12,800 for three years at the rate of 10% per annum compound interest. Find :
- The sum due to Ramesh at the end of the first year.
 - The interest he earns for the second year.
 - The total amount due to him at the end of the third year. [4]

Solution.

- (a) If $(x - 1)$ is a factor \Rightarrow

$$f(1) = 0$$

$$f(1) = 1 - 7 + 14 - 8 = 15 - 15 = 0$$

Hence $(x - 1)$ is a factor.

$$\begin{array}{r} x^2 - 6x + 8 \\ x - 1 \overline{) x^3 - 7x^2 + 14x - 8} \\ \underline{x^3 - x^2} \\ -6x^2 + 14x - 8 \\ \underline{-6x^2 + 6x} \\ 8x - 8 \\ \underline{8x - 8} \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{array}$$

$$\begin{aligned} x^2 - 6x + 8 &= x^2 - 4x - 2x + 8 \\ &= (x - 2)(x - 4) \end{aligned}$$

Factors are $(x - 1)(x - 2)(x - 4)$.

- (b) Let marked price be x .

Ans.

$$\text{Discount} = 15\%$$

$$\therefore \text{Discount on ₹ } x = \frac{15}{100} x$$

$$\therefore \text{Price discount} = x - \frac{15x}{100} = \frac{85x}{100}$$

$$\text{Rate of Sales tax} = 10\%$$

$$\begin{aligned} \text{Sales Tax} &= \frac{85x}{100} \times \frac{10}{100} \\ &= \frac{85x}{1000} \end{aligned}$$

$$\begin{aligned} \therefore \text{Final price, } & \left(\frac{85x}{100}\right) + \frac{1}{10} \left(\frac{85x}{100}\right) = 374 \\ \Rightarrow & \frac{85x}{100} \left(1 + \frac{1}{10}\right) = 374 \\ \Rightarrow & \frac{85x}{100} \left(\frac{11}{10}\right) = 374 \\ \Rightarrow & x = \frac{374 \times 10 \times 100}{85 \times 11} = ₹ 400 \end{aligned}$$

Hence marked price is ₹ 400.

Ans.

(c) Given : P = 12800, R = 10%

$$\begin{aligned} \text{(i) Interest for first year} &= \frac{PRT}{100} = \frac{12800 \times 10 \times 1}{100} \\ &= ₹ 1280 \end{aligned}$$

$$\begin{aligned} \text{Amount for first year} &= ₹ 1280 + ₹ 12800 \\ &= ₹ 14080 \end{aligned}$$

Ans.

$$\begin{aligned} \text{(ii) Interest for second year} &= \frac{PRT}{100} = \frac{14080 \times 10 \times 1}{100} \\ &= ₹ 1408 \end{aligned}$$

Ans.

$$\begin{aligned} \text{(iii) Amount of second year} &= ₹ 1408 + ₹ 14080 \\ &= ₹ 15488 \end{aligned}$$

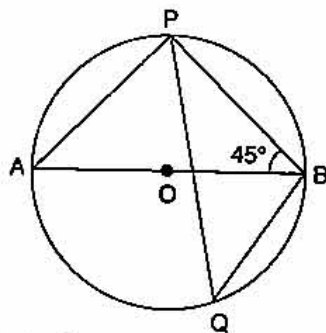
$$\begin{aligned} \text{Interest for third year} &= \frac{PRT}{100} = \frac{15488 \times 10 \times 1}{100} \\ &= ₹ 1548.80 \end{aligned}$$

$$\begin{aligned} \text{Amount of end of third year} &= ₹ 15488 + ₹ 1548.80 \\ &= ₹ 17036.80. \end{aligned}$$

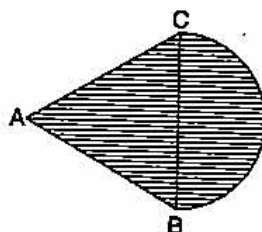
Ans.

Question 2.

(a) In the given figure, O is the centre of the circle and $\angle PBA = 45^\circ$. Calculate the value of $\angle PQB$. [3]



(b) In an equilateral ΔABC of side 14 cm, side BC is the diameter of a semi-circle as shown in the figure below. Find the area of the shaded region. [3]
(Take $\pi = 22/7$ and $\sqrt{3} = 1.732$)



(c) If $x = \frac{\sqrt{a+3b} + \sqrt{a-3b}}{\sqrt{a+3b} - \sqrt{a-3b}}$, prove that $3bx^2 - 2ax + 3b = 0$

[4]

Solution.

(a)

$$\begin{aligned} \Rightarrow \quad \angle AOB &= 180^\circ \\ \Rightarrow \quad \angle APB &= 90^\circ && \text{(angle of diameter)} \\ \Rightarrow \quad \angle PAB &= 90^\circ - 45^\circ = 45^\circ \\ \Rightarrow \quad \angle PQB &= 45^\circ && \text{(angle for same arc) Ans.} \end{aligned}$$

(b) Area of equilateral triangle ABC = $\frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times (14)^2 = \frac{\sqrt{3}}{4} \times 196$

$$= 49\sqrt{3} \text{ cm}^2$$

$$= 84.868 \text{ cm}^2$$

$$\begin{aligned} \text{Area of semi-circle} &= \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7^2 \\ &= 77 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Total area of shaded region} &= 84.868 + 77 \\ &= 161.868 \text{ cm}^2. \end{aligned}$$

Ans.

(c) Given :

$$\frac{x}{1} = \frac{\sqrt{a+3b} + \sqrt{a-3b}}{\sqrt{a+3b} - \sqrt{a-3b}}$$

Applying componendo and dividendo

$$\Rightarrow \frac{x+1}{x-1} = \frac{\sqrt{a+3b} + \sqrt{a-3b} + \sqrt{a+3b} - \sqrt{a-3b}}{\sqrt{a+3b} + \sqrt{a-3b} - \sqrt{a+3b} + \sqrt{a-3b}}$$

$$\Rightarrow \frac{x+1}{x-1} = \frac{2\sqrt{a+3b}}{2\sqrt{a-3b}} = \frac{\sqrt{a+3b}}{\sqrt{a-3b}}$$

Squaring both sides,

$$\Rightarrow \frac{x^2 + 2x + 1}{x^2 - 2x + 1} = \frac{a + 3b}{a - 3b}$$

Applying componendo and dividendo

$$\Rightarrow \frac{2x^2 + 2}{4x} = \frac{2a}{6b} = \frac{a}{3b}$$

$$\Rightarrow \frac{x^2 + 1}{2x} = \frac{a}{3b}$$

$$\Rightarrow 3bx^2 + 3b = 2ax$$

$$\Rightarrow 3bx^2 - 2ax + 3b = 0$$

Proved**Question 3.**

(a) If $2 \begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$ Find the values of x and y [3]

(b) Solve the following inequation and graph the solution on the number line :

$$-2\frac{2}{3} \leq x + \frac{1}{3} < 3\frac{1}{3}; x \in \mathbb{R} \quad [3]$$

(c) Use a graph paper for this question.

(i) The point $P(2, -4)$ is reflected about the line $x = 0$ to get the image Q . Find the co-ordinates of Q .

- (ii) Point Q is reflected about the line $y = 0$ to get the image R. Find the coordinates of R.
 (iii) Name the figure PQR.
 (iv) Find the area of figure PQR. [4]

Solution.

$$(a) \Rightarrow \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$$

$$\Rightarrow 8+y = 0 \Rightarrow y = -8$$

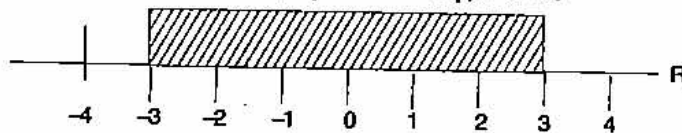
$$\Rightarrow 2x+1 = 5 \Rightarrow x = 2$$

$$x = 2 \text{ and } y = -8$$

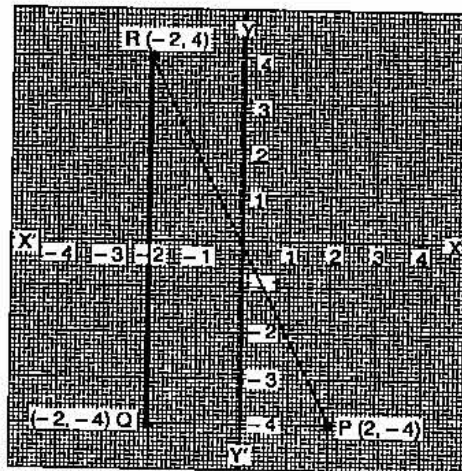
Ans.

(b) $-\frac{8}{3} \leq x + \frac{1}{3}$ and $x + \frac{1}{3} < \frac{10}{3}$
 $-\frac{8}{3} - \frac{1}{3} \leq x$ and $x < \frac{10}{3} - \frac{1}{3} = 3$
 $-3 \leq x$ and $x < 3$

So $|x| - 3 \leq x < 3, x \in \mathbb{R}$



- (c) (i) $Q(-2, -4)$



- (ii) $R(-2, 4)$

(iii) Right angle triangle.

(iv) Area of $\Delta PQR = \frac{1}{2} \times 4 \times 8$

$= 16 \text{ sq. unit}$

Ans.

Question 4.

(a) Evaluate: $\frac{\sin 80^\circ}{\cos 10^\circ} + \sin 59^\circ \sec 31^\circ$

[3]

(b) Saloni deposited ₹ 150 per month in her bank for eight months under the Recurring Deposit Scheme. What will be the maturity value of her deposit, if the rate of interest is 8% per annum and the interest is calculate at the end of every month?

[3]

(c) Find the mean of the following distribution :

| Class Interval | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
|----------------|------|-------|-------|-------|-------|
| Frequency | 10 | 6 | 8 | 12 | 5 |

[4]

Solution :

(a) Given :

$$\begin{aligned} & \frac{\sin 80^\circ}{\cos 10^\circ} + \sin 59^\circ \sec 31^\circ \\ &= \frac{\sin (90^\circ - 10^\circ)}{\cos 10^\circ} + \frac{\sin (90^\circ - 31^\circ)}{\cos 31^\circ} \\ &= \frac{\cos 10^\circ}{\cos 10^\circ} + \frac{\cos 31^\circ}{\cos 31^\circ} \\ &= 1 + 1 = 2. \end{aligned}$$

Ans.

(b) Equivalent principal for 1 month = $P \left[\frac{n(n+1)}{2} \right]$

$$= ₹ 150 \left[\frac{8(8+1)}{2} \right] = ₹ 150 \times 36$$

Interest on this principal = $\frac{P \times R \times T}{100}$

$$= ₹ \left[\frac{150 \times 36 \times 8}{12 \times 100} \right] = ₹ 36$$

∴ Total money deposited by Amit = ₹ (150 × 8)

$$= ₹ 1200$$

Maturity value = ₹ 1200 + ₹ 36

$$= ₹ 1236.$$

Ans.

(c)

| Class Interval | f | x | d = x - A | fd |
|----------------|-------------|--------|-----------|-----------|
| 0-10 | 10 | 5 | -20 | -200 |
| 10-20 | 6 | 15 | -10 | -60 |
| 20-30 | 8 | 25 = A | 0 | 0 |
| 30-40 | 12 | 35 | 10 | 120 |
| 40-50 | 5 | 45 | 20 | 100 |
| | N = Σf = 41 | | | Σfd = -40 |

$$\begin{aligned} \text{Mean} &= A + \frac{\Sigma fd}{N} \\ &= 25 + \frac{-40}{41} = 25 - 0.97 \\ &= 24.03 \end{aligned}$$

Ans.

SECTION B [40 Marks]

Answer any Four Questions in this Section.

Question 5.

(a) Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$. Find $A^2 + AB + B^2$.

[3]

(b) Ajay owns 560 shares of a company. The face value of each share is ₹ 25. The company declares a dividend of 9%. Calculate :

(i) The dividend that Ajay will get.

(ii) The rate of interest on his investment, if Ajay had paid ₹ 30 for each share.

[3]

(c) The surface area of a solid metallic sphere is 616 cm^2 . It is melted and recast into smaller spheres of diameter 3.5 cm. How many such spheres can be obtained ?

[4]

Solution.

(a)

$$A^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix}$$

$$\begin{aligned} A^2 + AB + B^2 &= \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 9 \\ 5 & 4 \end{bmatrix} \end{aligned}$$

Ans.

(b) Given :

$$\text{Number of shares} = 560$$

$$\text{Dividend on 1 share} = \frac{9}{100} \times 25 = ₹ \frac{9}{4}$$

(i) Dividend on 560 shares = $\frac{9}{4} \times 560$

$$= ₹ 1260$$

Ans.

(ii) Total investment = 560×30

$$= ₹ 16800$$

$$\text{Rate of interest} = \frac{1260}{16800} \times 100$$

$$= \frac{15}{2} = 7\frac{1}{2} \%$$

Ans.

(c)

⇒

$$\text{Surface area of solid sphere} = 4\pi R^2 = 616$$

$$4\pi R^2 = 616$$

$$R^2 = \frac{616 \times 7}{4 \times 22} = 49$$

$$R = 7 \text{ cm.}$$

$$\text{Volume of original sphere} = \frac{4}{3} \times \pi \times 7^3 \text{ cm}^3.$$

$$\text{Radius of small sphere} = \frac{1}{2} \times 3.5 = \frac{7}{4} \text{ cm.}$$

$$\text{Volume of small sphere} = \frac{4}{3} \times \pi \times \left(\frac{7}{4}\right)^3$$

$$\begin{aligned} \text{Number of small sphere} &= \frac{\frac{4}{3}\pi \times 7^3}{\frac{4}{3}\pi \times \left(\frac{7}{4}\right)^3} \\ &= 64. \end{aligned}$$

Ans.

Question 6.

(a) Mr. Ashok Sharma's income from his salary in the year 2005-06 was ₹ 2,96,000.**

Savings :

Contribution towards Provident Fund : ₹ 1,500 per month

Contribution towards L.I.C. premium : ₹ 10,000 per year

National Saving Certificates : ₹ 15,000

Donations :

To Prime Minister's Relief Fund : ₹ 12,000 (eligible for 100% tax exemption)

If a sum of ₹ 4,000 was deducted every month towards Income Tax from his salary for the first 11 months of the year, calculate Mr. Sharma's income tax liability in the last month of the year.

Tax Slab :

Upto ₹ 50,000 : No tax

₹ 50,001 to ₹ 60,000 : 10% of income exceeding ₹ 50,000

₹ 60,001 to ₹ 1,50,000 : ₹ 1,000 + 20% of the income exceeding ₹ 60,000

Above ₹ 1,50,000 : ₹ 19,000 + 30% of the income exceeding ₹ 1,50,000

Standard Deduction : ₹ 20,000

Rebate in tax : 20% of the total savings or ₹ 14,000 whichever is less

Surcharge : 10% of the total tax payable after rebate. [6]

(b) Five years ago, a woman's age was the square of her son's age. Ten years hence her age will be twice that of her son's age. Find :

(i) The age of the son five years ago.

(ii) The present age of the women. [4]

Solution.

(b) Let the present age of women be x years and her son be y years.

Five years ago :

$$(x - 5) = (y - 5)^2 \quad \dots(1)$$

Ten years hence :

$$(x + 10) = 2(y + 10)$$

$$\Rightarrow x + 10 = 2y + 20$$

$$\Rightarrow x - 2y = 10$$

$$\Rightarrow x = 2y + 10 \quad \dots(2)$$

From (1),

$$x - 5 = y^2 + 25 - 10y$$

$$x = y^2 - 10y + 30 \quad \dots(3)$$

** Solution has not given due to out of present syllabus.

From (2) and (3)

$$2y + 10 = y^2 - 10y + 30$$

$$\Rightarrow y^2 - 12y + 20 = 0$$

$$\Rightarrow y(y - 10) - 2(y - 10) = 0$$

$$y = 2 \text{ or } y = 10$$

$y = 2$ years is not possible.

i.e.,

$$y = 10$$

(i) Age of the son five years ago = $y - 5 = 10 - 5$
= 5 years

(ii) Present age of women = $2y + 10 = 2 \times 10 + 10$
= 30 years.

Ans.

Question 7.

(a) Solve the following quadratic equation for x and give your answer correct to two decimal places :

$$x^2 - 3x - 9 = 0 \quad [3]$$

(b) Using a ruler, construct a triangle ABC with $BC = 6.4$ cm, $CA = 5.8$ cm and $\angle ABC = 60^\circ$. Draw its incircle. Measure and record the radius of the incircle.

[3]

(c) Mrs. Kumar has an account with The Bank of India. The following entries are from her pass book :

| Date | Particulars | Withdrawals | | Deposits | | Balance | |
|----------|-------------|-------------|---|----------|---|---------|---|
| | | ₹ | P | ₹ | P | ₹ | P |
| 08.02.06 | B/F | — | — | — | — | 8500.00 | — |
| 18.02.06 | To self | 4000.00 | — | — | — | — | — |
| 12.04.06 | By cash | — | — | 2238.00 | — | — | — |
| 15.06.06 | To self | 5000.00 | — | — | — | — | — |
| 08.07.06 | By cash | — | — | 6000.00 | — | — | — |

Complete the above page of her pass book and calculate the interest for the six months, February to July 2006, at 4.5% per annum. [4]

Solution :

(a) Given : $x^2 - 3x - 9 = 0$

Comparing with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -3, c = -9$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{+3 \pm \sqrt{9 + 36}}{2} = \frac{3 \pm \sqrt{45}}{2}$$

$$= \frac{3 \pm 6.70}{2}$$

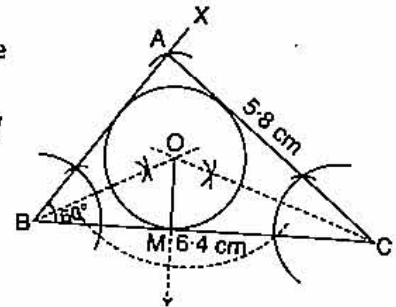
$$= \frac{9.7}{2} \text{ or } \frac{-3.70}{2}$$

$$= 4.85 \text{ or } -1.85.$$

Ans.

(b) Steps of Constructions : www.10yearsquestionpaper.com

1. Draw $BC = 6.4$ and at B angle 60° .
2. Taking C as centre cut $AC = 5.8$ cm on the arm XB .
3. Draw angle bisector of $\angle ACB$ and $\angle ABC$ which intersect at O .
4. Taking O as centre. Draw a incircle.
5. Draw \perp from O on BC which cut at M .
6. Required radius of incircle $OM = 1.5$ cm.



(c)

| Date | Particulars | Withdrawals ₹ | Deposits ₹ | Balance ₹ |
|----------|-------------|------------------|---------------|--------------|
| 08.02.06 | B/F | — | — | 8500.00 |
| 18.02.06 | To self | 4000.00 | — | 4500.00 |
| 12.04.06 | By cash | — | 2238.00 | 6738.00 |
| 15.06.06 | To self | 5000.00 | — | 1738.00 |
| 08.07.06 | By cash | — | 6000.00 | 7738.00 |

Minimum balance for February = ₹ 4,500.00
 March = ₹ 4,500.00
 April = ₹ 4,500.00
 May = ₹ 6,738.00
 June = ₹ 1,738.00
 July = ₹ 7,738.00

Principal for 1 month = ₹ 29,714.00

$$\text{Interest} = \frac{PRT}{100} = ₹ \frac{29,714 \times 4.5 \times 1}{100 \times 12}$$

$$= ₹ 111.43$$

Question 8.

(a) Prove the identity : $\frac{\sec A - 1}{\sec A + 1} = \frac{1 - \cos A}{1 + \cos A}$

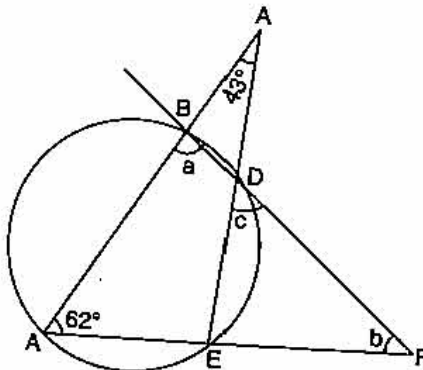
Ans.

(b) The mid point of the line segment joining $(2a, 4)$ and $(-2, 2b)$ is $(1, 2a + 1)$. Find the values of a and b .

[3]

(c) In the given figure, if $\angle ACE = 43^\circ$ and $\angle CAF = 62^\circ$ find the values of a , b and c .

[4]



Solution.

$$\begin{aligned}
 \text{(a)} \quad \text{L.H.S.} &= \frac{\sec A - 1}{\sec A + 1} \\
 &= \frac{1/\cos A - 1}{1/\cos A + 1} \\
 &= \frac{(1 - \cos A) \cos A}{\cos A (1 + \cos A)} \\
 &= \frac{1 - \cos A}{1 + \cos A} \\
 &= \text{R.H.S.}
 \end{aligned}$$

Proved

$$\text{(b) Mid points are } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\begin{aligned}
 \Rightarrow \quad \frac{x_1 + x_2}{2} &= \frac{2a - 2}{2} = 1 \\
 \Rightarrow \quad a - 1 &= 1 \\
 \Rightarrow \quad a &= 2 \\
 \Rightarrow \quad \frac{y_1 + y_2}{2} &= \frac{4 + 2b}{2} = 2a + 1 \\
 \Rightarrow \quad 4 + 2b &= 4a + 2 \\
 \Rightarrow \quad 4 + 2b &= 4 \times 2 + 2 \\
 \Rightarrow \quad 4 + 2b &= 10 \\
 \Rightarrow \quad 2b &= 10 - 4 \\
 \Rightarrow \quad b &= 3
 \end{aligned}$$

(c)

$$a = 2 \text{ and } b = 3.$$

Ans.

$$\begin{aligned}
 \angle AEC &= 180^\circ - (\angle EAC + \angle ACE) \\
 &= 180^\circ - (62^\circ + 43^\circ) = 180^\circ - 105^\circ \\
 &= 75^\circ
 \end{aligned}$$

$$\begin{aligned}
 \angle CEF &= 180^\circ - 75^\circ \quad (\text{cyclic quadrilateral}) \\
 &= 105^\circ.
 \end{aligned}$$

$$\begin{aligned}
 \angle ABD = \angle a &= 180^\circ - 75^\circ \\
 &= 105^\circ
 \end{aligned}$$

Ans.

$$\begin{aligned}
 \angle b = \angle AFD &= 180^\circ - (62^\circ + 105^\circ) \\
 &= 180^\circ - (167^\circ) \\
 &= 13^\circ
 \end{aligned}$$

Ans.

$$\begin{aligned}
 \angle c = \angle EDF &= 180^\circ - (105^\circ + 13^\circ) \\
 &= 180^\circ - (118^\circ) \\
 &= 62^\circ.
 \end{aligned}$$

Ans.

Question 9.

$$\text{(a) A function in } x \text{ is defined as } f(x) = \frac{3x^2 + 2x - 1}{x + 1}, x \in R \text{ and } x \neq -1. **$$

$$\text{Find the value of the expression: } \frac{f(2)}{f(-3)} + 1.$$

[3]

** Solution has not given due to out of present syllabus.

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(b) Find the equation of the line parallel to the line $3x + 2y = 8$ and passing through the point $(0, 1)$. [3]

(c) From the top of a hill, the angles of depression of two consecutive kilometer stones, due east are found to be 30° and 45° respectively. Find the distance of the two stones from the foot of the hill. [4]

Solution

(b) Given :

$$3x + 2y = 8$$

\Rightarrow

$$2y = -3x + 8$$

\Rightarrow

$$y = -\frac{3}{2}x + 4$$

$$\text{Slope of the line} = m_1 = -\frac{3}{2}$$

Equation of the line passing through the point $(0, 1)$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{3}{2}(x - 0)$$

$$2y - 2 = -3x$$

\Rightarrow

$$3x + 2y = 2$$

Ans.

(c) In ΔABC ,

$$\tan 45^\circ = \frac{h}{x} = 1$$

$$h = x$$

...(1)

In ΔABD ,

$$\tan 30^\circ = \frac{h}{x+1} = \frac{1}{\sqrt{3}}$$

\Rightarrow

$$(x+1) = h\sqrt{3} = x\sqrt{3}$$

\Rightarrow

$$x\sqrt{3} - x = 1$$

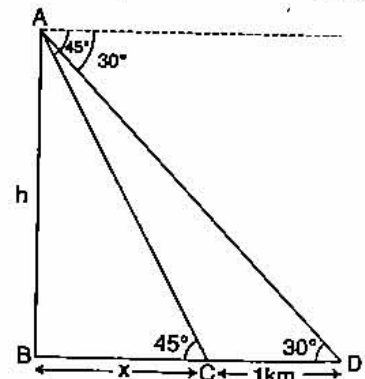
\Rightarrow

$$x = \frac{1}{\sqrt{3} - 1} = 0.732$$

$$= 1.366$$

First stone = 1.366 km,

Second stone = $1 + 1.366 = 2.366$ km. Ans.



Question 10.

(a) The table below shows the distribution of the scores obtained by 120 shooters in a shooting competition. Using a graph sheet, draw an ogive for the distribution.

| Scores obtained | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 | 80-90 | 90-100 |
|--------------------|------|-------|-------|-------|-------|-------|-------|-------|-------|--------|
| Number of Shooters | 5 | 9 | 16 | 22 | 26 | 18 | 11 | 6 | 4 | 3 |

Use your ogive to estimate :

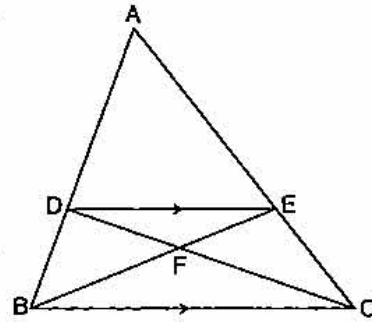
(i) The Median.

(ii) The inter quartile range.

(iii) The number of shooters who obtained more than 75% scores.

[6]

(b) In the given figure, ABC is a triangle. DE is parallel to BC and $\frac{AD}{DB} = \frac{3}{2}$.



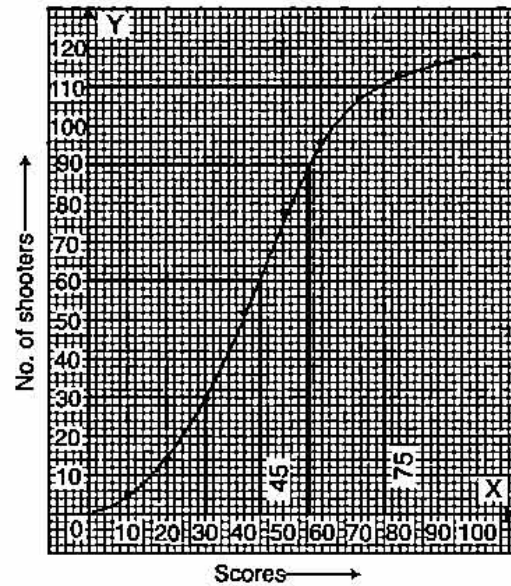
- (i) Determine the ratios $\frac{AD}{AB}$, $\frac{DE}{BC}$
- (ii) Prove that $\triangle DEF$ is similar to $\triangle CBF$. Hence, find $\frac{EF}{FB}$.
- (iii) What is the ratio of the areas of $\triangle DFE$ and $\triangle BFC$?

[4]

Solution.

(a)

| Scores | Number of Shooters | Cumulative Frequency |
|--------|--------------------|----------------------|
| 0-10 | 5 | 5 |
| 10-20 | 9 | 14 |
| 20-30 | 16 | 30 |
| 30-40 | 22 | 52 |
| 40-50 | 26 | 78 |
| 50-60 | 18 | 96 |
| 60-70 | 11 | 107 |
| 70-80 | 6 | 113 |
| 80-90 | 4 | 117 |
| 90-100 | 3 | 120 |



(i) To find the median

$$\begin{aligned} \text{Let A be a point on y axis} &= \frac{1}{2} \left[\frac{n}{2} + \left(\frac{n}{2} + 1 \right) \right] \\ &= \frac{1}{2} [60 + 61] = 60.5 \end{aligned}$$

Median = 45

Ans.

(ii) To find the lower quartile :

$$\text{Let B be the point on y axis} = \frac{n}{4} = \frac{120}{4} = 30$$

The lower quartile = 30

To find the upper quartile :

$$\text{Let C be the point on y axis} = \frac{3n}{4} = \frac{3 \times 120}{4} = 90$$

The upper quartile = 56

Inter quartile range = 56 - 30 = 26.

Ans.

(iii) The number of shooters when obtained more than 75% scores. From E draw a vertical line to meet ogive at F. From F draw horizontal line to D

$$= 120 - 110 = 10$$

Ans.

(b) Given : $\frac{AD}{DB} = \frac{3}{2}$

(i) $\frac{AD}{AD + DB} = \frac{3}{3 + 2} = \frac{3}{5}$

$\Rightarrow \frac{AD}{AB} = \frac{3}{5}$

$\angle ADE = \angle ABC$ and $\angle A = \angle A$

$\Rightarrow \triangle ADE \sim \triangle ABC$

$\Rightarrow \frac{DE}{BC} = \frac{AD}{AB} = \frac{3}{5}$ Ans.

(ii) $\angle FED = \angle FBC$

$\angle DFE = \angle BFC$ (vert. opp.) Proved

$\Rightarrow \triangle DEF \sim \triangle CFB$

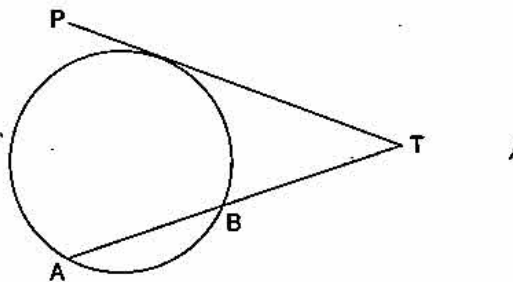
$\Rightarrow \frac{DE}{BC} = \frac{FE}{FB} = \frac{3}{5}$ Ans.

(iii) $\frac{\text{area of } \triangle DFE}{\text{area of } \triangle CFB} = \frac{DE^2}{BC^2} = \left(\frac{DE}{BC}\right)^2$

$= \frac{9}{25}$ Ans.

Question 11.

- (a) If the line joining the points A (4, -5) and B (4, 5) is divided by the point P such that $\frac{AP}{AB} = \frac{2}{5}$, find the co-ordinates of P. [3]
- (b) In the figure given below, PT is a tangent to the circle. Find PT if AT = 16 cm and AB = 12 cm. [3]



- (c) Construct a triangle BCP given BC = 5 cm, BP = 4 cm and $\angle PBC = 45^\circ$.
- (i) Complete the rectangle ABCD such that :
- (1) P is equidistant from AB and BC.
 - (2) P is equidistant from C and D.
- (ii) Measure and record the length of AB. [4]

Solution.

(a) Given : $\frac{AP}{AB} = \frac{2}{5}$

$\Rightarrow \frac{AB}{AP} = \frac{5}{2}$

$\frac{AB}{AP} - 1 = \frac{5}{2} - 1$

$\Rightarrow \frac{PB}{AP} = \frac{3}{2}$

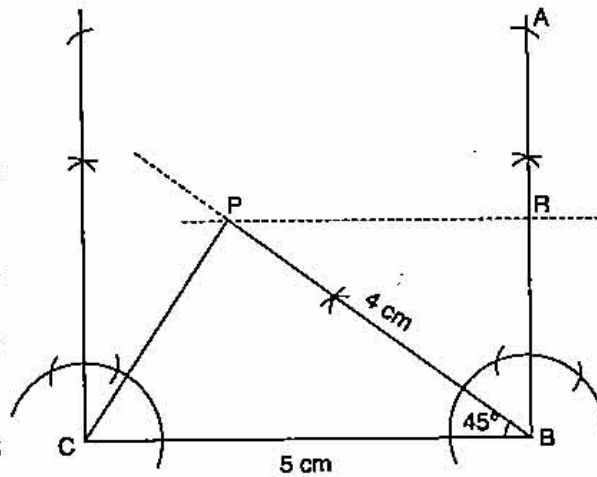
$$\begin{aligned} \text{Coordinates of P} &= \left(\frac{mx_2 + nx_1}{m+x}, \frac{my_2 + ny_1}{m+x} \right) \\ &= \left(\frac{2 \times 4 + 3 \times 4}{2+3}, \frac{2 \times 5 + 3 \times (-5)}{2+3} \right) \\ &= \left(\frac{8+12}{5}, \frac{10-15}{5} \right) \\ &= (4, -1) \end{aligned} \quad \text{Ans.}$$

(b) \therefore PT is a tangent,

$$\begin{aligned} \therefore TA \times TB &= PT^2 && \text{(By theorem)} \\ \Rightarrow 16 \times 4 &= PT^2 \\ \Rightarrow PT^2 &= 64 \\ \Rightarrow PT &= 8 \text{ cm.} \end{aligned} \quad \text{Ans.}$$

(c) **Steps of Constructions :**

- (i) Draw a line BC = 5 cm and make an angle of 45° at B.
- (ii) Cut an arc of 4 cm from B at P.
- (iii) Make an angle of 90° at B and draw a line \parallel to BC from P. Which cuts the line of 90° angle from B at R.
- (iv) Cut RB = RA and Mark D at PC = PD
- (v) Complete the rectangle ABCD of length AB = 6.5 cm.



Ans.

