CCE PF
CCE PR

ಕರ್ನಾಟಕ ಪ್ರೌಢ ಶಿಕ್ಷಣ ಪರೀಕ್ಷಾ ಮಂಡಳಿ, ಮಲ್ಲೇಶ್ವರಂ, ಬೆಂಗಳೂರು – 560 003

KARNATAKA SECONDARY EDUCATION EXAMINATION BOARD, MALLESWARAM, BANGALORE – 560 003

ಎಸ್.ಎಸ್.ಎಲ್.ಸಿ. ಪರೀಕ್ಷೆ, ಮಾರ್ಚ್ / ಏಪ್ರಿಲ್ — 2018

S. S. L. C. EXAMINATION, MARCH/APRIL, 2018

ಮಾದರಿ ಉತ್ತರಗಳು

MODEL ANSWERS

ದಿನಾಂಕ : 26. 03. 2018] **ಸಂಕೇತ ಸಂಖ್ಯೆ : 81-E**

Date : 26. 03. 2018] **CODE NO. : 81-E**

ವಿಷಯ : ಗಣಿತ

Subject : MATHEMATICS

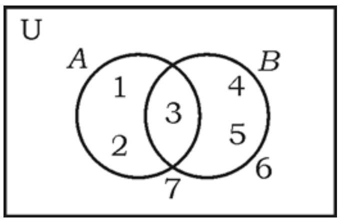
(ಹೊಸ ಪಠ್ಯಕ್ರಮ / New Syllabus)

(ಖಾಸಗಿ ಅಭ್ಯರ್ಥಿ & ಪುನರಾವರ್ತಿತ ಖಾಸಗಿ ಅಭ್ಯರ್ಥಿ / Private Fresh & Private Repeater)

(ಇಂಗ್ಲಿಷ್ ಭಾಷಾಂತರ / English Version)

[ಗರಿಷ್ಠ ಅಂಕಗಳು : **100**

[Max. Marks : 100

Qn. Nos.	Ans. Key	Value Points	Marks allotted
I. 1.		In the given Venn diagram $n(A)$ is 	
		Ans. :	
	A	3	1
2.		Sum of all the first '*n*' terms of even natural number is	
		Ans. :	
	A	$n(n+1)$	1

Qn. Nos.	Ans. Key	Value Points	Marks allotted
3.		A boy has 3 shirts and 2 coats. How many different pairs, a shirt and a coat can he dress up with ?	
		Ans. :	
	C	6	1
4.		In a random experiment, if the occurrence of one event prevents the occurrence of other event is	
		Ans. :	
	D	mutually exclusive event	1
5.		The polynomial $p(x) = x^2 - x + 1$ is divided by $(x - 2)$ then the remainder is	
		Ans. :	
	B	3	1
6.		The distance between the co-ordinates of a point (p, q) from the origin is	
		Ans. :	
	C	$\sqrt{p^2 + q^2}$	1
7.		The equation of a line having slope 3 and y-intercept 5 is	
		Ans. :	
	D	$y = 3x + 5$	1
8.		The surface area of a sphere of radius 7 cm is	
	B	616 cm^2.	1

Qn. Nos.	Value Points		Marks allotted
II.	Answer the following :	$6 \times 1 = 6$	
9.	Find the HCF of 14 and 21.		
	Ans. :		
	$14 = 2 \times 7$		
	$21 = 3 \times 7$	½	
	HCF = 7	½	
	[Direct Answer full marks]		1
10.	The average runs scored by a batsman in 15 cricket matches is 60 and standard deviation of the runs is 15. Find the coefficient of variation of the runs scored by him.		
	Ans. :		
	$\overline{X} = 60$		
	$\sigma = 15$		
	C.V. $= \frac{\sigma}{\overline{X}} \times 100$ C.V. $= \frac{\text{Standard deviation}}{\text{Average}} \times 100$	½	
	$= \frac{15}{60} \times 100$ OR $= \frac{15}{60} \times 100$		
	$= 25.$ $= 25$	½	1
11.	Write the degree of the polynomial $f(x) = x^2 - 3x^3 + 2$.		
	Ans. :		
	Degree 3		1
12.	What are congruent circles ?		
	Ans. :		
	Circles having same radii but different centres. OR Different centres but same radii	½ ½	1
13.	If $\sin \theta = \frac{5}{13}$ then write the value of cosec θ.		
	Ans. :		
	cosec $\theta = \frac{13}{5}$		1

Qn. Nos.	Value Points		Marks allotted
14.	Write the formula used to find the total surface area of a right circular cylinder.		
	Ans. :		
	$TSA = 2\pi r(r + h)$ sq.units		1
III. 15.	If $U = \{0, 1, 2, 3, 4\}$ and $A = \{1, 4\}$, $B = \{1, 3\}$ show that $(A \cup B)' = A' \cap B'$.		
	Ans. :		
	$LHS = (A \cup B)'$		
	$A \cup B = \{1, 3, 4\}$		
	$(A \cup B)' = \{0, 2\}$ ... (i)	½	
	$RHS = A' \cap B'$		
	$\left.\begin{array}{l} A' = \{0, 2, 3\} \\ B' = \{0, 2, 4\} \end{array}\right\}$	½	
	$A' \cap B' = \{0, 2\}$ ... (ii)	½	
	From (i) and (ii)		
	$(A \cup B)' = A' \cap B'$	½	2
16.	Find the sum of the series 3 + 7 + 11 + ... to 10 terms.		
	Ans. :		
	3 + 7 + 11 .... 10 terms		
	$a = 3$		
	$d = 4$		
	$S_n = \frac{n}{2}[2a + (n-1)d]$	½	

Qn. Nos.	Value Points		Marks allotted
	$S_{10} = \frac{10}{2}[2(3)+(10-1)4]$	½	
	$= \frac{10}{2}[6+9(4)]$ $= \frac{10}{2}[6+36]$ $= 5 \times 42.$	½	2
	$S_{10} = 210$	½	
17.	At constant pressure certain quantity of water at 24°C is heated. It was observed that the rise of temperature was found to be 4°C per minute. Calculate the time required to rise the temperature of water to 100°C at sea level by using formula.		
	Ans. :		
	$a = 24$		
	$d = 4$		
	$T_n = 100$		
	$n = ?$		
	$T_n = a + (n-1)d$	½	
	$100 = 24 + (n-1)4$	½	
	$100 = 24 + 4n - 4$	½	
	$100 = 20 + 4n$		
	$n = \frac{80}{4}$		
	$n = 20.$ $(20-1) = 19$ minutes or 20th minute	½	2
	Alternate Method :		
	By taking $a = 28$ and $n = 19$		
	OR		
	Any other correct alternate method give marks.		

Qn. Nos.	Value Points		Marks allotted
18.	Prove that $2+\sqrt{5}$ is an irrational number.		
	Ans. :		
	Let us assume $2+\sqrt{5}$ is rational		
	$2+\sqrt{5} = \frac{p}{q}$, $p, q \in z$, $q \neq 0$	½	
	$\left.\begin{aligned}\sqrt{5} &= \frac{p}{q}-2 \\ \sqrt{5} &= \frac{p-2q}{q}\end{aligned}\right\}$	½	
	$\Rightarrow$ $\sqrt{5}$ is rational		
	but $\sqrt{5}$ is not a rational number	½	
	This is against our assumption		
	$\therefore$ $2+\sqrt{5}$ is an irrational number.	½	2
19.	If ${}^nP_4 = 20\,({}^nP_2)$ then find the value of *n*.		
	Ans. :		
	${}^nP_4 = 20\,{}^nP_2$		
	$n(n-1)(n-2)(n-3) = 20\,n(n-1)$	½	
	$(n-2)(n-3) = 20$ OR $(n-2)(n-3) = 5 \times 4$		
	$n^2-3n-2n+6 = 20$ $\Rightarrow n-2=5$		
	$n^2-5n-14 = 0$ $n = 5+2$	1½	
	$n^2-7n+2n-14 = 0$ $\therefore n=7$		
	$n(n-7)+2(n-7) = 0$		
	$(n-7)(n+2) = 0$		
	$n-7=0$ or $n+2=0$		
	$n = 7$ $n=-2$		2
	(*Any alternate method to be considered*)		

Qn. Nos.	Value Points	Marks allotted
20.	A die numbered 1 to 6 on its faces is rolled once. Find the probability of getting either an even number or multiple of '3' on its top face.	
	Ans. :	
	$S = \{1, 2, 3, 4, 5, 6\}$ ½	
	$n(S) = 6$	
	$A = \{2, 3, 4, 6\}$	
	$n(A) = 4$	
	$p(A) = \frac{n(A)}{n(S)}$	
	$= \frac{4}{6}$ OR $\frac{2}{3}$ ½	
	This can also be considered	
	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ½	
	$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6}$	
	$= \frac{4}{6}$ ½	2
	(Any other alternate methods give marks)	
21.	What are like surds and unlike surds ?	
	Ans. :	
	A group of surds having same order and same radicand in their simplest form. ½ + ½	
	Group of surds having different orders or different radicands or both in their simplest form. ½ + ½	2
22.	Rationalise the denominator and simplify : $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$.	
	Ans. :	
	$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ ½	
	$= \frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{(\sqrt{5})^2 - (\sqrt{3})^2}$ ½	
	$= \frac{5+3+2\sqrt{15}}{2}$ ½	
	$= \frac{8+2\sqrt{15}}{2}$	
	$= 4 + \sqrt{15}$. ½	2

Qn. Nos.	Value Points		Marks allotted
23.	Find the quotient and the remainder when $f(x) = 2x^3 - 3x^2 + 5x - 7$ is divided by $g(x) = (x-3)$ using synthetic division.		
	OR		
	Find the zeros of the polynomial $p(x) = x^2 - 15x + 50$.		
	Ans. :		
	$f(x) = 2x^3 - 3x^2 + 5x - 7$		
	$g(x) = x - 3$		

3	2	−3	5	−7
		6	9	42
	2	3	14	35

Marks: ½ (second row), ½ (third row)

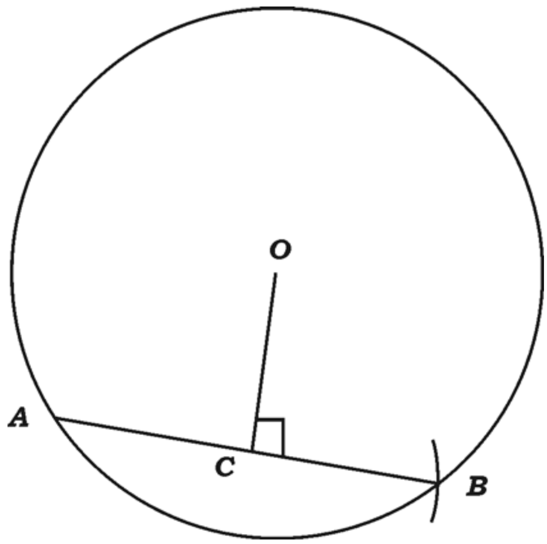
	$q(x) = 2x^2 + 3x + 14$	½	
	$r(x) = 35$.	½	2
	OR		
	$f(x) = x^2 - 15x + 50$		
	At zeroes of the polynomial		
	$f(x) = 0$		
	$x^2 - 15x + 50 = 0$		

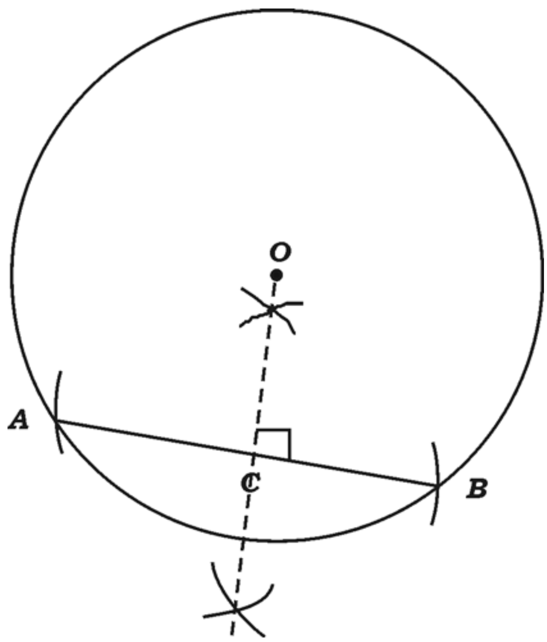
Qn. Nos.	Value Points		Marks allotted
	$x^2 - 10x - 5x + 50 = 0$	½	
	$x(x - 10) - 5(x - 10) = 0$	½	
	$(x - 10)(x - 5) = 0$	½	
	$x - 10 = 0$ or $x - 5 = 0$		
	$x = 10$ $x = 5$		
	∴ The zeroes of the polynomial are 10 and 5.	½	2
24.	Solve the equation $x^2 - 12x + 27 = 0$ by using formula.		
	Ans. :		
	$a = 1, \quad b = -12, \quad c = 27$		
	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	½	
	$x = \dfrac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(27)}}{2(1)}$		
	$x = \dfrac{12 \pm \sqrt{144 - 108}}{2}$	½	
	$x = \dfrac{12 \pm \sqrt{36}}{2}$		
	$x = \dfrac{12 \pm 6}{2}$	½	
	$x = \dfrac{12 + 6}{2}$ or $x = \dfrac{12 - 6}{2}$		
	$x = \dfrac{18}{2}$ or $x = \dfrac{6}{2}$		
	$x = 9$ or $x = 3$	½	2

Qn. Nos.	Value Points	Marks allotted

25. Draw a chord of length 6 cm in a circle of radius 5 cm. Measure and write the distance of the chord from the centre of the circle.

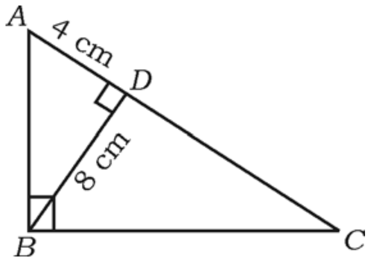
Ans.





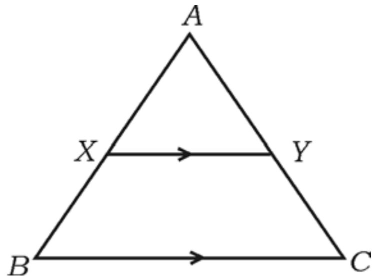
Circle	½	
Chord	½	
Mid-point marking	½	
By measuring $OC = 4$ cm.	½	2

Qn. Nos.	Value Points	Marks allotted
26.	In $\triangle ABC$ $\angle ABC = 90°$, $BD \perp AC$. If $BD = 8$ cm, $AD = 4$ cm, find CD and AB.	

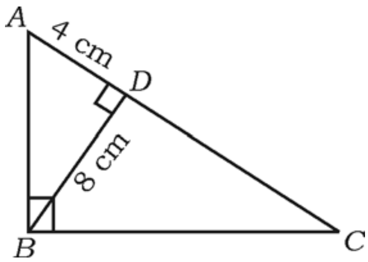


OR

In $\Delta\ ABC$, $XY \parallel BC$ and $XY = \frac{1}{2}\ BC$. If the area of $\Delta\ AXY = 10\ cm^2$, find the area of trapezium $XYCB$.



Ans. :



$BD^2 = AD\,.\,CD$ ½

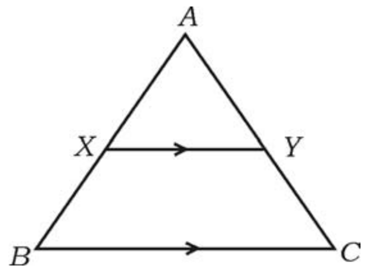
$8^2 = 4\ .\ CD$

$\frac{64}{4} = CD$

$CD = 16$ cm ½

$\therefore \quad AC = CD + AD = 16 + 4 = 20$ cm

Qn. Nos.	Value Points	Marks allotted
	$AB^2 = AD \cdot AC$ ½	
	$= 4 \times 20$	
	$AB^2 = 80$	
	$AB = \sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$ cm ½	2
	(Any other alternate methods give marks)	
	OR	



Since $XY \parallel BC$

$\Delta AXY \sim \Delta ABC$ $\dfrac{ar(\Delta AXY)}{ar(\Delta ABC)} = \dfrac{XY^2}{BC^2}$ ½

$\dfrac{ar(\Delta AXY)}{ar(\Delta ABC)} = \dfrac{XY^2}{4XY^2}$ $\left[\because XY = \dfrac{1}{2} BC, \quad 2XY = BC \right]$ ½

$\dfrac{10}{ar(\Delta ABC)} = \dfrac{1}{4}$

$40 = ar\,\Delta ABC$ ½

ar ⏢ $XYCB = 40 - 10$

$= 30 \text{ cm}^2$. ½ — **2**

27. Show that, $\cot\theta \cdot \cos\theta + \sin\theta = \operatorname{cosec}\theta$.

Ans. :

$\cot\theta \cdot \cos\theta + \sin\theta = \operatorname{cosec}\theta$

LHS $= \cot\theta \cdot \cos\theta + \sin\theta$

$= \dfrac{\cos\theta}{\sin\theta} \cdot \cos\theta + \sin\theta$ ½

$= \dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta}$ ½

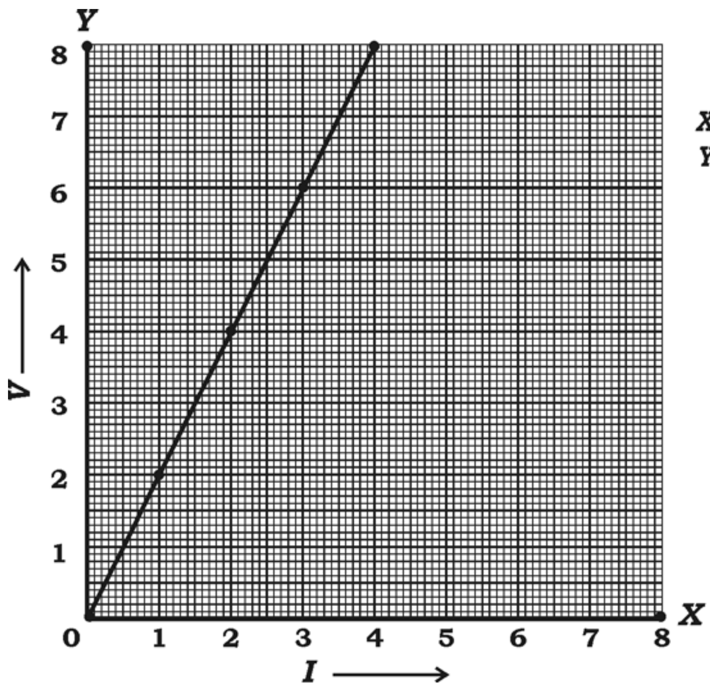
$= \dfrac{1}{\sin\theta}$ ½

$= \operatorname{cosec}\theta$. ½ — **2**

(Any other alternate methods give marks)

Qn. Nos.	Value Points	Marks allotted
28.	A student while conducting an experiment on Ohm's law, plotted the graph according to the given data. Find the slope of the line obtained.	

X-axis I	1	2	3	4
Y-axis V	2	4	6	8



Scale :
X-axis : 1 unit = 1 cm
Y-axis : 1 unit = 1 cm

Ans. :

$(x_1, y_1) = (1, 2)$ — Alternate method may be given full marks.

$(x_2, y_2) = (2, 4)$ — ½

Slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$ — ½

Slope $= m = \dfrac{4-2}{2-1} = \dfrac{2}{1} = 2$ — 1

Or $(x_1, y_1) = (2, 4)$ $(x_2, y_2) = 3, 6$

Or $(x_1, y_1) = (3, 6)$ $(x_2, y_2) = 4, 8$

Or any two points may be taken to find the slope. — 2

Qn. Nos.	Value Points		Marks allotted
29.	Draw the plan for the information given below :		
	(Scale 20 m = 1 cm)		

	Metre To C	
	140	
To D 50	100	
	60	40 to B
To E 30	40	
	From A	

Ans. :

$40 \text{ m} = \frac{1}{20} \times 40 = 2 \text{ cm}$

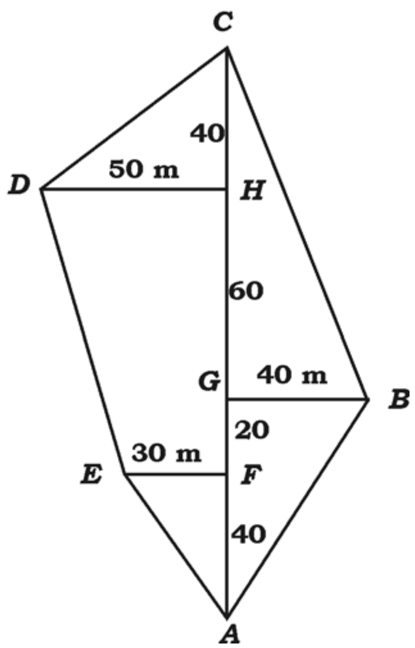
$60 \text{ m} = \frac{1}{20} \times 60 = 3 \text{ cm}$

$100 \text{ m} = \frac{1}{20} \times 100 = 5 \text{ cm}$

$140 \text{ m} = \frac{1}{20} \times 140 = 7 \text{ cm}$ ½

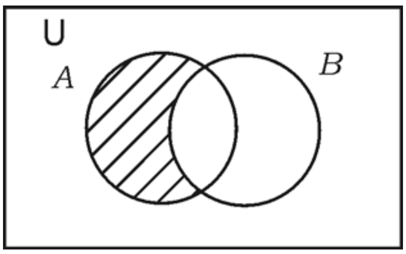
$30 \text{ m} = \frac{1}{20} \times 30 = 1 \cdot 5 \text{ cm}$

$50 \text{ m} = \frac{1}{20} \times 50 = 2 \cdot 5 \text{ cm}$



1½

2

Qn. Nos.	Value Points		Marks allotted
30.	Out of 8 different bicycle companies, a student likes to choose bicycle from three companies. Find out in how many ways he can choose the companies to buy bicycle. *Ans.* : From 8 different bicycle companies he chooses 3 bicycle companies. $^8C_3$ $^8C_3 = \frac{8P_3}{3!}$ $= \frac{8 \times 7 \times 6}{3 \times 2 \times 1}$ $= 56.$ *Alternate Method* : $^nC_r = \frac{n!}{(n-r)!.r!}$ $^8C_3 = \frac{8!}{(8-3)!.3!}$ $= \frac{8 \times 7 \times 6 \times \cancel{5 \times 4 \times 3 \times 2 \times 1}}{\cancel{5!} \times 3 \times 2 \times 1} = 56$	½ ½ ½ ½	2
31.	If A and B are two non-disjoint sets, draw Venn diagram to represent $A \setminus B$. *Ans.* :  Writing set A & B Correct shading	 1 1	2
32.	What is an Arithmetic progression ? Write its general form. *Ans.* : A sequence in which the consecutive terms either increase or decrease by a fixed number. OR An arithmetic progression is a sequence in which each term is obtained by adding a fixed number to the preceeding term. $a, \ a+d, \ a+2d, \ a+3d \ ....$	1 1	2

Qn. Nos.	Value Points		Marks allotted
33.	There are 10 points in a plane such that no three of them are collinear. Find out how many triangles can be formed by joining these points.		
	Ans. :		
	$n = 10$		
	$r = 3$		
	$^{n}C_{r} = \dfrac{n!}{(n-r)!\,r!}$	½	
	$^{10}C_{3} = \dfrac{10!}{(10-3)!\,3!}$	½	
	$= \dfrac{10!}{7!\;3!}$	½	
	$= 120.$	½	2
	Alternate method :		
	$^{10}C_{3} = \dfrac{^{10}P_{3}}{3!}$		
	$= \dfrac{10\times9\times8}{3\times2\times1}$		
	$= 120.$		
34.	A student reads the books according to the given data. Draw a pie chart to represent it.		

Name of the books	*Novels*	*Short stories*	*Magazines*	*Journals*
No. of books	10	60	20	30

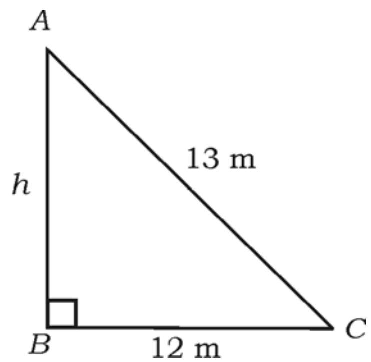
Ans. :

Name of the books	*No. of books*	*Central angle*
1. Novels	10	$\dfrac{10}{120}\times360 = 30^\circ$
2. Short stories	60	$\dfrac{60}{120}\times360 = 180^\circ$
3. Magazines	20	$\dfrac{20}{120}\times360 = 60^\circ$
4. Journals	30	$\dfrac{30}{120}\times360 = 90^\circ$

1

Sum of books = 120

Qn. Nos.	Value Points		Marks allotted
		1	2
35.	Simplify : $\sqrt{75} + \sqrt{108} - \sqrt{192}$.		
	Ans. :		
	$\sqrt{75} + \sqrt{108} - \sqrt{192} = \sqrt{25 \times 3} + \sqrt{36 \times 3} - \sqrt{64 \times 3}$	1	
	$= 5\sqrt{3} + 6\sqrt{3} - 8\sqrt{3}$	½	
	$= 3\sqrt{3}$.	½	2
36.	A polynomial $p(x) = x^2 + 4x + 2$ is divided by $g(x) = (x + 2)$. Find the quotient by using division algorithm.		
	Ans. :		
	$P(x) = x^2 + 4x + 2$ $\quad g(x) = (x + 2)$		
	$P(x) = [g(x) * q(x)] + r(x)$ $x^2 + 4x + 2 = [(x + 2)(ax + b)] + r(x)$	1	
	$= ax^2 + bx + 2ax + 2b + r(x)$		
	$x^2 + 4x + 2 = ax^2 + x(b + 2a) + 2b + r(x)$		
	$\boxed{a = 1}$ $\quad b + 2a = 4$ $\quad 2b + r(x) = 2$		
	$b = 4 - 2$ $\quad \boxed{b = 2}$ $\quad r(x) = 2 - 4$	½	
	$\boxed{r(x) = -2}$		
	Quotient $= (x + 2)$		
	Remainder $= -2$	½	2

Qn. Nos.	Value Points		Marks allotted
37.	If $v^2 = u^2 + 2as$, solve for v and find the value of v, if $u = 0$, $a = 2$ and $s = 100$.		
	Ans. :		
	$v^2 = u^2 + 2as$		
	$v = \sqrt{u^2 + 2as}$	½	
	if $u = 0$, $a = 2$, $s = 100$, $v = ?$		
	$v = \pm\sqrt{0 + 2(2)100}$	½	
	$v = \pm\sqrt{400}$	½	
	$v = \pm 20$.	½	2
38.	A vertical building casts a shadow of length 12 m. If the distance between the top of the building to the tip of the shadow at a particular time of the day is 13 m. Find the height of the building.		
	Ans. :		
			
	Height of the building = h		
	Length of the shadow = 12 m	½	
	Distance between top of the building to tip of the shadow = 13 m		
	$AC^2 = AB^2 + BC^2$	½	
	$13^2 = h^2 + 12^2$		
	$169 = h^2 + 144$		
	$169 - 144 = h^2$	½	
	$25 = h^2$		
	$h = \sqrt{25} = 5$ m	½	2

Qn. Nos.	Value Points		Marks allotted
39.	Show that $(\sin\theta + \cos\theta)^2 = 1 + 2 \sin\theta \cos\theta$.		
	Ans. :		
	L.H.S. $= (\sin\theta + \cos\theta)^2$		
	$= \sin^2\theta + \cos^2\theta + 2 \sin\theta . \cos\theta$	1	
	$= 1 + 2 \sin\theta . \cos\theta$	1	2
40.	Find the co-ordinates of the mid-point of the line segment joining the points (14, 12) and (8, 6).		
	Ans. :		
	$x_1 = 14$ $x_2 = 8$		
	$y_1 = 12$ $y_2 = 6$		
	$d = \left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right)$	½	
	$d = \left(\frac{14 + 8}{2} , \frac{12 + 6}{2} \right)$	½	
	$= \left(\frac{22}{2} , \frac{18}{2} \right)$	½	
	$= (11 , 9)$	½	2
IV. 41.	In a Geometric progression the sum of first three terms is 14 and the sum of next three terms of it is 112. Find the Geometric progression.		
	OR		
	If '*a*' is the Arithmetic mean of *b* and *c*, '*b*' is the Geometric mean of *c* and *a*, then prove that '*c*' is the Harmonic mean of *a* and *b*.		
	Ans. :		
	Let the terms be $a, ar, ar^2, ar^3, ar^4, ar^5$.		
	$a + ar + ar^2 = 14$		
	$a (1 + r + r^2) = 14$ ... (i)	½	
	$ar^3 + ar^4 + ar^5 = 112$		
	$ar^3 (1 + r + r^2) = 112$ ... (ii)	½	

Qn. Nos.	Value Points		Marks allotted
	Substitute (i) in (ii) $r^3\ (14) = 112$ $r^3 = \frac{112}{14} = 8$ $r = \sqrt[3]{8} = 2$ OR Divide equation (2) by (1) $\frac{ar^3\ (1+r+r^2)}{a\ (1+r+r^2)} = \frac{112}{14}$ $r^3 = 8$ $\therefore\ r = 2$	1	
	Substitute $r = 2$ in (i) $a\ (1 + 2 + 2^2) = 14$ $a\ (7) = 14$	½	
	$a = 2$ $\therefore$ The terms are 2, 4, 8, 16, 32, 64.	½	
	Any other alternate methods can also be considered.		3
	OR		
	$a = \frac{b+c}{2}$ $b = \sqrt{ac}$ $b^2 = ac$	½	
	$a = \frac{b+c}{2}$	½	
	$2a = b + c$ $\frac{2ab}{b} = b + c$ [dividing & multiplying by b in the LHS]	½	
	OR $2ab = b\ (b + c)$ Multiply RHS & LHS by 'b' $2ab = b^2 + bc$	½	
	$2ab = ac + bc$ $2ab = c\ (a + b)$	½	
	$\frac{2ab}{a+b} = c$ $\therefore$ c is the harmonic mean between a and b.	½	3

Qn. Nos.	Value Points	Marks allotted

Alternate method :

$a = \frac{b+c}{2}$ ... (i) $b = \sqrt{ac}$

$b^2 = ac$

$b = \frac{ac}{b}$ 1

Substitute $b = \frac{ac}{b}$ in (i)

$a = \frac{\frac{ac}{b} + c}{2}$ ½

$2a = \frac{ac + bc}{b}$ ½

$2ab = c(a + b)$ ½

$\frac{2ab}{a+b} = c.$ ½ 3

42. Marks scored by 30 students of 10th standard in a unit test of mathematics is given below. Find the variance of the scores :

Marks (x)	4	8	10	12	16
No. of students (f)	13	6	4	3	4

Ans. :

Assumed mean method :

X	f	$d = X - A$	fd	d^2	fd^2
4	13	– 6	– 78	36	468
8	6	– 2	– 12	4	24
10	4	0	0	0	0
12	3	2	6	4	12
16	4	6	24	36	144

$n = 30$ $A = 10$ $\sum fd = +60$ $\sum fd^2 = 648$ 1½

Qn. Nos.	Value Points		Marks allotted
	Variance $= \frac{\sum f d^2}{n} - \left(\frac{\sum f d}{n} \right)^2$	½	
	$= \frac{648}{30} - \left(\frac{60}{30} \right)^2$	½	
	$= 21 \cdot 6 - 2^2$		
	$= 17 \cdot 6.$	½	3

Direct Method :

X	X^2	f	fX	$f X^2$
4	16	13	52	208
8	64	6	48	384
10	100	4	40	400
12	144	3	36	432
16	256	4	64	1024

$n = 30$ $\sum fX = 240$ $\sum f X^2 = 2448$ 1½

Variance $= \frac{\sum f X^2}{n} - \left(\frac{\sum f X}{n} \right)^2$ ½

$= \frac{2448}{30} - \left(\frac{240}{30} \right)^2$ ½

$= 81 \cdot 6 - 8^2$

$= 17 \cdot 6.$ ½ 3

Actual mean method :

X	f	$f X$	$d = X - \overline{X}$	d^2	$f d^2$
4	13	52	– 4	16	208
8	6	48	0	0	0
10	4	40	2	4	16
12	3	36	4	16	48
16	4	64	8	64	256

$n = 30$ $\sum fX = 240$ $\sum f d^2 = 528$ ½

Qn. Nos.	Value Points		Marks allotted
	$\overline{X} = \frac{\Sigma f X}{n}$		
	$= \frac{240}{30} = 8$	1	
	Variance $= \frac{\Sigma f d^2}{n} = \frac{528}{30}$	½	
	$= 17{\cdot}6$	1	3

Step deviation Method :

X	f	$d = \frac{X - A}{C}$	fd	d^2	fd^2
4	13	– 3	– 39	9	117
8	6	– 1	– 6	1	6
10	4	0	0	0	0
12	3	1	3	1	3
16	4	3	12	9	36

$n = 30$ $\Sigma f d^2 = 162$ 1

$A = 10$ $C = 2$

S.D. $= \sqrt{\frac{\Sigma f d^2}{n} - \left(\frac{\Sigma f d}{n}\right)^2} \times C$ ½

$= \sqrt{\frac{162}{30} - \left(\frac{30}{30}\right)^2} \times 2$

$= \sqrt{5{\cdot}4 - 1} \times 2$

$= \sqrt{4{\cdot}4} \times 2$

$= 2{\cdot}1 \times 2$

$= 4{\cdot}2$

OR

Variance $= \frac{\Sigma f d^2}{n} - \left(\frac{\Sigma f d}{n}\right)^2 \times C^2$ ½

$= \frac{162}{30} - \left(\frac{30}{30}\right)^2 \times 4$ ½

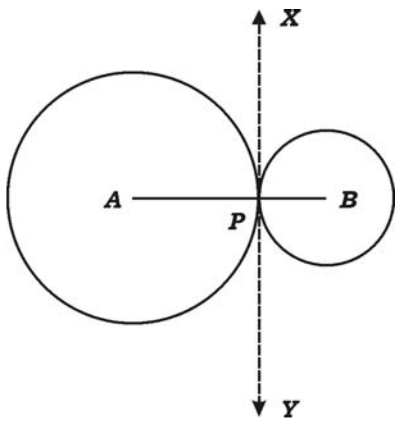
$= (5{\cdot}4 - 1)\,4$

$= 4{\cdot}4 \times 4$

$= 17{\cdot}6$

$\therefore$ Variance $\sigma^2 = (4{\cdot}2)^2 = 17{\cdot}6$. ½ 3

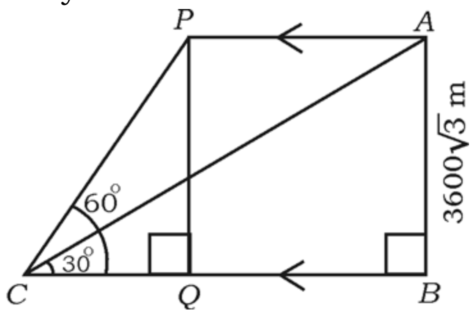
Qn. Nos.	Value Points	Marks allotted
43.	If p and q are the roots of the equation $x^2 - 3x + 2 = 0$, find the value of $\frac{1}{p} - \frac{1}{q}$.	
	OR	
	A dealer sells an article for Rs. 16 and loses as much per cent as the cost price of the article. Find the cost price of the article.	
	Ans. :	
	$a = 1 \qquad b = -3 \qquad c = 2$	
	$p + q = \frac{-b}{a} = \frac{-(-3)}{1} = 3$ ½	
	$pq = \frac{c}{a} = \frac{2}{1} = 2$ ½	
	$\frac{1}{p} - \frac{1}{q} = \frac{q-p}{pq}$ ½	
	$= \pm \frac{\sqrt{(p+q)^2 - 4pq}}{pq}$ ½	
	$= \pm \frac{\sqrt{3^2 - 4(2)}}{2}$	
	$= \pm \frac{\sqrt{9-8}}{2}$ ½	
	$= \pm \frac{1}{2}$ ½	3
	$\frac{1}{p} - \frac{1}{q} = +\frac{1}{2}$ or $-\frac{1}{2}$	
	OR	
	C.P. $= x$	
	S.P. $= 16$	
	Loss $= x\,\% = \frac{x}{100} \times x = \frac{x^2}{100}$ } OR $\frac{x-16}{x} = \frac{x}{100}$ ½	
	S.P. = C.P. − loss $100x - 1600 = x^2$ ½	
	$16 = x - \frac{x^2}{100}$	
	$1600 = 100x - x^2$	

Qn. Nos.	Value Points	Marks allotted
	$x^2 - 100x + 1600 = 0$ ½	
	$x^2 - 80x - 20x + 1600 = 0$	
	$x(x - 80) - 20(x - 80) = 0$	
	$(x - 80)(x - 20) = 0$	
	$x - 80 = 0$ or $x - 20 = 0$	
	$x = 80$ $x = 20$ 1	
	$\therefore$ Cost price is Rs. 80 or Rs. 20. ½	3
44.	Prove that, "If two circles touch each other externally, their centres and the point of contact are collinear."	
	Ans. :	
	 ½	
	Data : A and B are the centres of touching circles, P is the point of contact. ½	
	To prove : A, P and B are collinear. ½	
	Construction : Draw the tangent XY at P. ½	
	Proof : In the figure,	
	$\angle APX = 90°$ ... (i) $\angle BPX = 90°$ ... (ii) } Radius drawn at the point of contact is perpendicular to the tangent ½	
	$\angle APX + \angle BPX = 90 + 90$ by adding (i) and (ii)	
	$\angle APB = 180°$ APB is a straight line ½	
	A, P and B are collinear.	3

Qn. Nos.	Value Points	Marks allotted
45.	If $7\sin^2\theta + 3\cos^2\theta = 4$ and 'θ' is acute then show that $\cot\theta = \sqrt{3}$.	

OR

The angle of elevation of an aircraft from a point on horizontal ground is found to be 30°. The angle of elevation of same aircraft after 24 seconds which is moving horizontally to the ground is found to be 60°. If the height of the aircraft from the ground is $3600\sqrt{3}$ metre. Find the velocity of the aircraft.



Ans. :

$4\sin^2\theta + 3\sin^2\theta + 3\cos^2\theta = 4$

$4\sin^2\theta + 3(\sin^2\theta + \cos^2\theta) = 4$

$4\sin^2\theta + 3(1) = 4$

$4\sin^2\theta = 4 - 3$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \frac{1}{2}$

$\therefore \quad \theta = 30°$

$\therefore \quad \cot\theta = \sqrt{3}$.

Alternate methods can also be considered.

Alternate Method :

$7\sin^2\theta + 3\cos^2\theta = 4$ — ½

$7\sin^2\theta + 3[1 - \sin^2\theta] = 4$

$7\sin^2\theta + 3 - 3\sin^2\theta = 4$ — ½

$4\sin^2\theta = 1$ — ½

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \frac{1}{2}$ — ½

$\cos^2\theta = 1 - \sin^2\theta$ — ½

$\cos\theta = \sqrt{1 - \sin^2\theta}$ — ½

$= \sqrt{1 - \frac{1}{4}}$

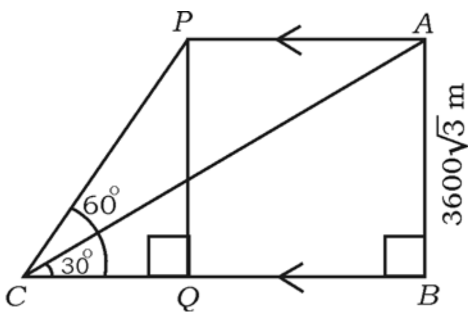
$= \frac{\sqrt{3}}{2}$

$\therefore \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= \sqrt{3}$

3

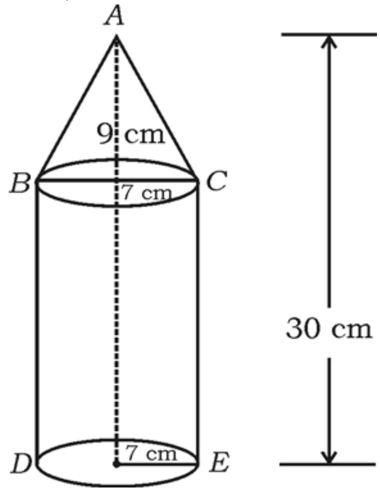
OR



Qn. Nos.	Value Points		Marks allotted
	In ⊿ ABC, $\angle ABC = 90°$		
	$\tan\theta = \frac{AB}{BC}$		
	$\tan 30° = \frac{3600\sqrt{3}}{BC}$	½	
	$\frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{BC}$		
	$BC = 3600\sqrt{3} \,.\, \sqrt{3}$	½	
	$BC = 10800$ m		
	In ⊿ PCQ, $\angle PQC = 90°$		
	$\tan\theta = \frac{PQ}{CQ}$		
	$\tan 60° = \frac{3600\sqrt{3}}{CQ}$	½	
	$\sqrt{3} = \frac{3600\sqrt{3}}{CQ}$		
	$CQ = 3600$ m	½	
	$\therefore \quad BQ = BC - CQ = 10800 - 3600$		
	$BQ = 7200$ m	½	
	$\therefore \quad$ Velocity $= \frac{\text{distance}}{\text{time}} = \frac{d}{t}$		
	$= \frac{7200}{24}$		
	$= 300$ m/s	½	3
	OR		
	(*Any Alternate method*)		

Qn. Nos.	Value Points	Marks allotted

46. A solid is in the form of a cone mounted on a right circular cylinder, both having same radii as shown in the figure. The radius of the base and height of the cone are 7 cm and 9 cm respectively. If the total height of the solid is 30 cm, find the volume of the solid.



OR

The slant height of the frustum of a cone is 4 cm and the perimeters of its circular bases are 18 cm and 6 cm respectively. Find the curved surface area of the frustum.

Ans. :

$r = 7$ cm   Let $h_1 = 21$ cm for cylinder

$r = 7$ cm $h_2 = 9$ cm for cone

Volume of solid = Volume of cylinder + Volume of cone ½

$= \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$ ½

$= \pi r^2 \left(h_1 + \frac{1}{3} h_2 \right)$ ½

$= \frac{22}{7} \times 7^2 \left(21 + \frac{1}{\cancel{3}} \times \cancel{9}^{\,3} \right)$ ½

$= \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \; (24)$ ½

$= 3696$ c.c. ½ **3**

Direct substitution of h_1 and h_2 value can also be considered.

OR

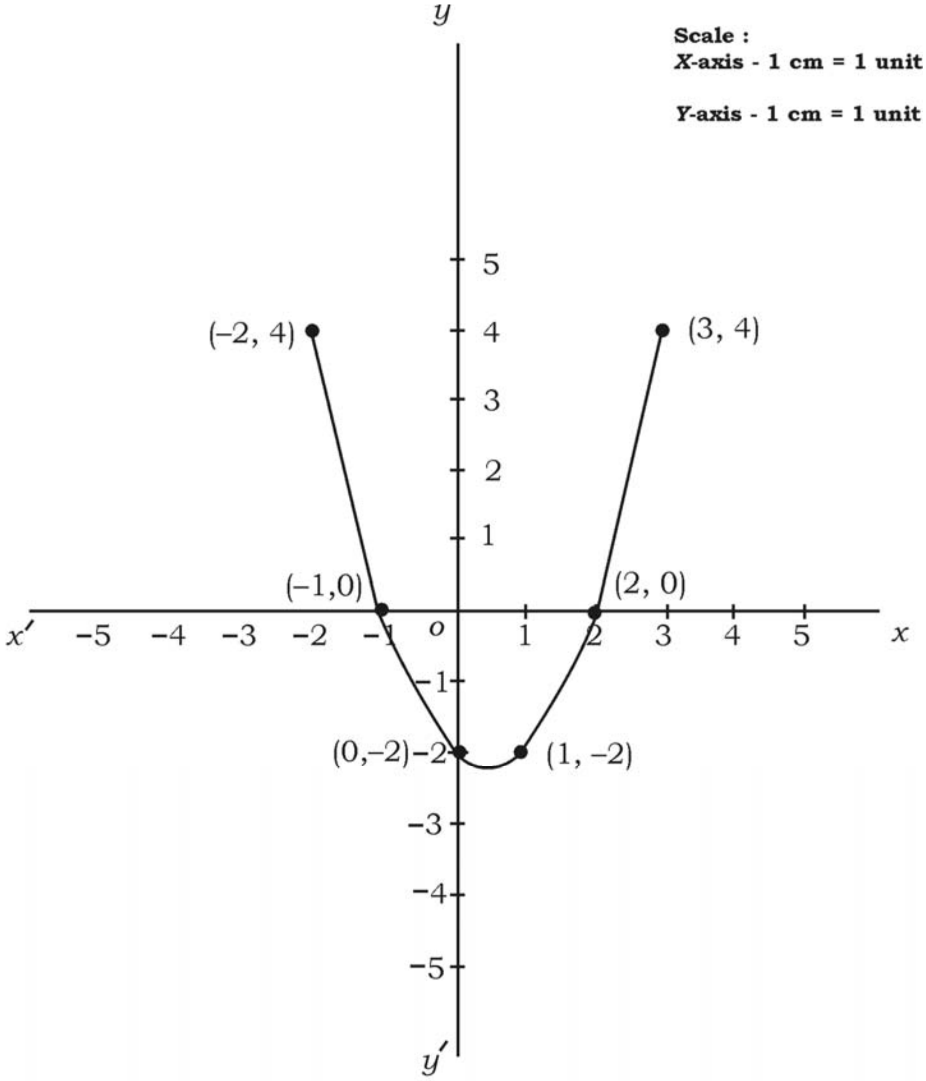
Qn. Nos.	Value Points		Marks allotted
	$2\pi r_1 = 18$ cm $2\pi r_2 = 6$ cm $l = 4$ cm	½	
	$r_1 = \frac{18}{2\pi} = \frac{9}{\pi}$ cm $r_2 = \frac{6}{2\pi} = \frac{3}{\pi}$ cm		
	Curved Surface Area $= \pi(r_1 + r_2)l$	1	
	$= \pi\left(\frac{9}{\pi} + \frac{3}{\pi}\right)4$	1½	
	$= 48\ \text{cm}^2.$		3
	OR		
	CSA $= l\,[\pi r_1 + \pi r_2]$		
	$= 4\,[9 + 3]$		
	$= 4\,[12]$		
	$= 48\ \text{cm}^2$		
V. 47.	Solve the equation $x^2 - x - 2 = 0$ graphically.		
	Ans. :		
	Let $y = 0$		
	$x^2 - x - 2 = 0$ given		
	$\therefore\ y = x^2 - x - 2$		

x	0	1	− 1	2	3	− 2
y	− 2	− 2	0	0	4	4

Graph roots

Table —	2	
Parabola —	1	
Roots —	½ + ½	4

Qn. Nos.	Value Points	Marks allotted



Roots of the equation are – 1 or 2

Alternate Method :

Given $x^2 - x - 2 = 0$

$x^2 = x + 2$

Consider $y = x^2$ and $y = x + 2$

(i) $y = x^2$

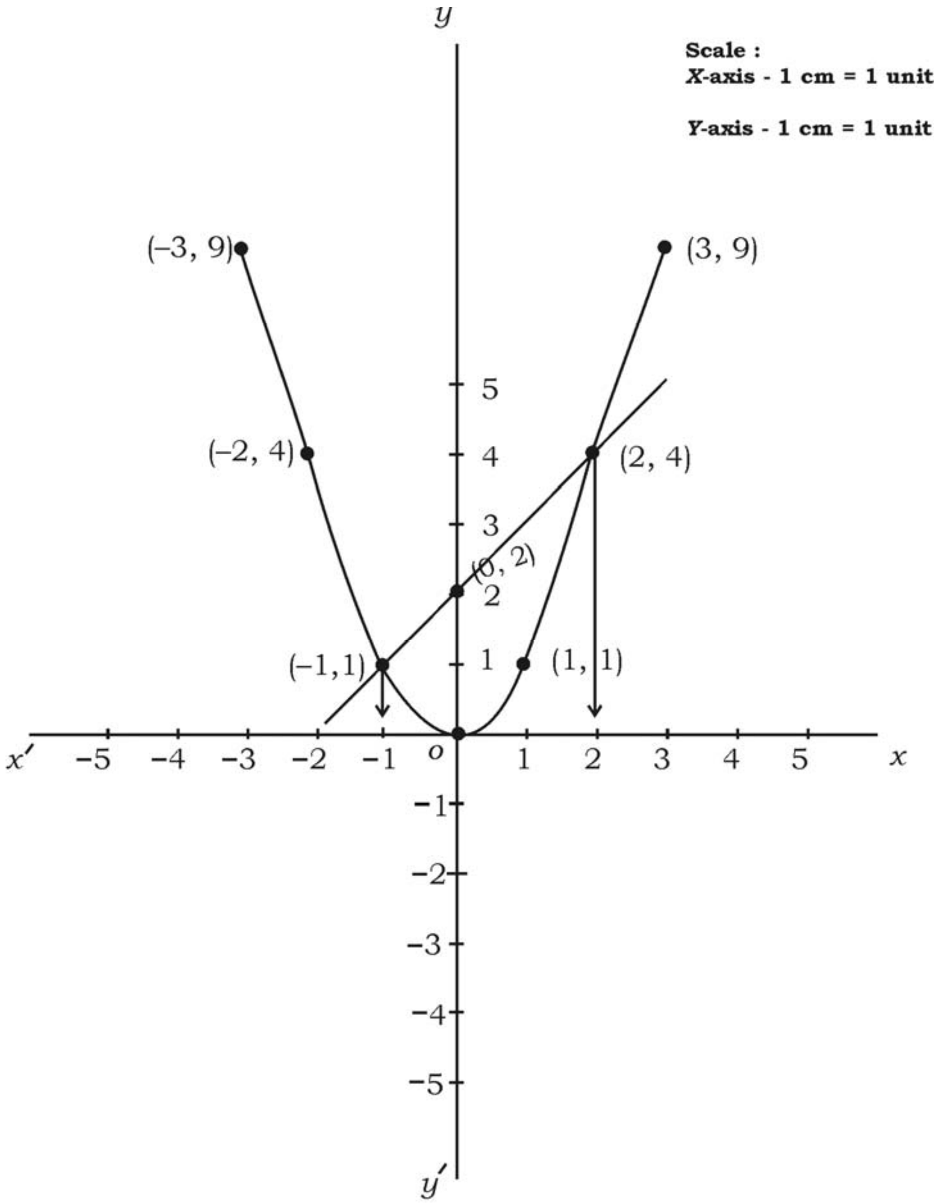
x	0	1	– 1	2	– 2	3	– 3
y	0	1	1	4	4	9	9

Qn. Nos.	Value Points	Marks allotted

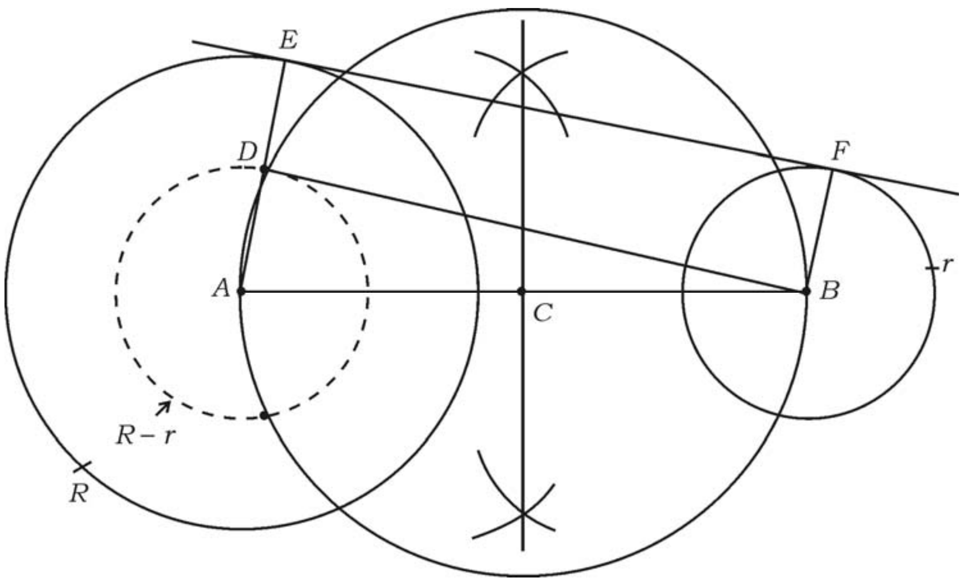
(ii) $y = x + 2$

x	0	1	2	– 1	2
y	2	3	4	1	0

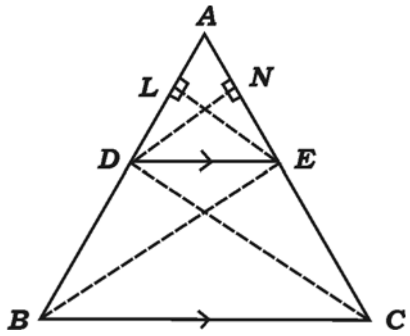
Tables —	2	
Line —	½	
Parabola —	½	
Roots —	½ + ½	4



Roots of the equation are 2 or –1

Qn. Nos.	Value Points	Marks allotted
48.	Construct a direct common tangent to two circles of radii 4 cm and 2 cm whose centres are 9 cm apart. Measure and write the length of the tangent.	
	Ans. :	
	$R = 4$ cm $\quad r = 2$ cm $\quad d = 9$ cm	
	$R - r = 2$ cm	
		
	Length of the tangent = 8·7 cm	
	Drawing four circles — 2	
	Drawing tangents — 1½	
	Finding the length — ½	4
49.	State and prove Basic Proportionality (Thale's) Theorem.	
	Ans. :	
	If a straight line is drawn parallel to a side of a triangle, then it divides the other two sides proportionally. — 1	

Qn. Nos.	Value Points	Marks allotted



½

Data : In $\Delta\, ABC$, $DE \parallel BC$

To prove : $\dfrac{AD}{BD} = \dfrac{AE}{CE}$ ½

Construction : Join DC and EB

Draw $EL \perp AB$ and $DN \perp AC$. ½

Proof :

$$\frac{\text{Area of } \Delta\, ADE}{\text{Area of } \Delta\, BDE} = \frac{\frac{1}{2} \times AD \times EL}{\frac{1}{2} \times BD \times EL} \quad \left[\because \;\; A = \frac{1}{2} bh \right]$$ ½

$$\therefore \quad \frac{\Delta\, ADE}{\Delta\, BDE} = \frac{AD}{BD} \qquad \ldots \text{(i)}$$

$$\frac{\text{Area of } \Delta\, ADE}{\text{Area of } \Delta\, CDE} = \frac{\cancel{\frac{1}{2}} \times AE \times \cancel{DN}}{\cancel{\frac{1}{2}} \times EC \times \cancel{DN}}$$ ½

$$\frac{\Delta\, ADE}{\Delta\, CDE} = \frac{AE}{EC}$$

$$\Rightarrow \quad \frac{AD}{BD} = \frac{AE}{CE} \qquad \left(\because \text{ Area } \Delta\, BDE = \text{area of } \Delta\, CDE \text{ and Axiom-1} \right)$$ ½

4

Qn. Nos.	Value Points	Marks allotted
50.	A vertical tree is broken by the wind at a height of 6 metre from its foot and its top touches the ground at a distance of 8 metre from the foot of the tree. Calculate the distance between the top of the tree before breaking and the point at which tip of the tree touches the ground, after it breaks.	
	OR	
	In ΔABC, AD is drawn perpendicular to BC. If $BD : CD = 3 : 1$, then prove that $BC^2 = 2(AB^2 - AC^2)$.	

Ans. :

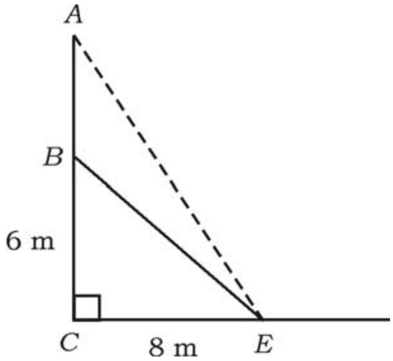


Fig. : 1

In the figure,

Let AC represents the tree h.

B is the point of break $BC = 6$ m

E is the top of the tree touches the ground $CE = 8$ m

AE is the distance between the top of the tree before break and after the break.

In $\triangle BCE$, $\angle BCE = 90°$ — ½

$BE^2 = BC^2 + CE^2$

$BE^2 = 6^2 + 8^2$ — ½

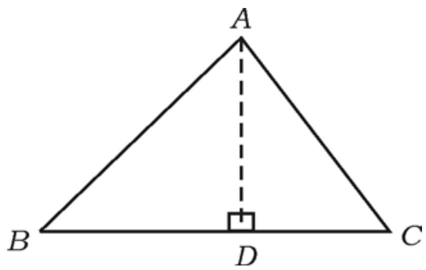
$BE^2 = 36 + 64$

$BE^2 = 100$

$BE = \sqrt{100} = 10$ m — ½

$BE = AB = 10$ m

(Any other alternate methods give marks)

Qn. Nos.	Value Points		Marks allotted
	In $\triangle ACE$, $\angle ACE = 90°$		
	$AE^2 = AC^2 + CE^2$		
	$= 16^2 + 8^2$	½	
	$= 256 + 64$		
	$AE^2 = 320$	½	
	$AE = \sqrt{320}$		
	$= 8\sqrt{5}$ m	½	4
	OR		
		Fig. : ½	
	$AB^2 = \cancel{AD^2} + BD^2$ … (i)	½	
	$\underset{-}{}AC^2 = \underset{-}{}\cancel{AD^2} \underset{-}{+} CD^2$ … (ii)	½	
	By subtracting		
	$AB^2 - AC^2 = BD^2 - CD^2$	½	
	$AB^2 - AC^2 = \left[\frac{3}{4}BC\right]^2 - \left[\frac{1}{4}BC\right]^2$	½	
	$= \frac{9}{16}BC^2 - \frac{1}{16}BC^2$	½	
	$\left(AB^2 - AC^2\right) = \frac{8\,BC^2}{16}$	½	
	$= \frac{BC^2}{2}$	½	
	$\because \quad 2\left(AB^2 - AC^2\right) = BC^2$		4
	[Marks will be given for any alternate method.]		