ST. XAVIER'S SENIOR SECONDARY SCHOOL, DELHI - 110054
Pre-Board Examination 2018 in **MATHEMATICS**

Set 1

Std. 12 **Max. Marks : 100**
18-01-2018 **Time : 3 hrs.**

GENERAL INSTRUCTIONS:

i) Attempt all the questions.
ii) Section - A consists of 4 questions of 1 mark each.
iii) Section - B consists of 8 questions of 2 marks each.
iv) Section - C consists of 11 questions of 4 marks each.
v) Section - D consists of 6 questions of 6 mark each.

SECTION - A

1. State the reason why the Relation R = $\{(a, b): a \le b^2\}$ on the set R of real numbers is not reflexive.

2. For what value of 'a' the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear?

3. If $2\begin{pmatrix} 3 & 4 \\ 5 & x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$, find x - y.

4. If '*' is defined on the set R of real numbers by $a * b = \frac{4ab}{9}$, find the identity element in R for the binary operation '*'.

SECTION - B

5. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$, $xy < 1$, then find the value of x + y + xy.

6. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are coplanar, show that $c^2 = ab$.

7. If A is a square matrix of order 3 such that $|\text{adj}\,A| = 225$, find $|A'|$.

8. Using differentials, find the approximate value of $(0.007)^{\frac{1}{3}}$.

9. Find the differential equation of all circles touching y-axis at the origin.

10. Simplify : $\sin^{-1}\left(\frac{x}{\sqrt{a^2 + x^2}}\right)$.

11. Evaluate : $\int x \tan^{-1} x \, dx$.

12. Two dice are rolled once. Find the probability that the total number on the two dice is atleast 4.

SECTION - C

13. Discuss the differentiability of the function $f(x)=\begin{cases}2x-1, & x<\frac{1}{2}\\ 3-6x, & x\geq\frac{1}{2}\end{cases}$ at $x=\frac{1}{2}$.

(OR)

For what value of k is the following function continuous at $x=\frac{\pi}{6}$?

$$f(x)=\begin{cases}\dfrac{\sqrt{3}\sin x+\cos x}{x+\frac{\pi}{6}}, & x\neq-\frac{\pi}{6}\\ k, & x=-\frac{\pi}{6}\end{cases}$$

14. If $y = x^x$, show that $\frac{d^2y}{dx^2}-\frac{1}{y}\left(\frac{dy}{dx}\right)^2-\frac{y}{x}=0$.

15. Solve the following differential equation : $\frac{dy}{dx}=\frac{y}{x}+\frac{\sqrt{x^2+y^2}}{x}$, $x>0$.

(OR)

Solve the following differential equation : $(1 + y^2)dx = (\tan^{-1} y - x)dy$.

16. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

(OR)

Find the intervals in which $f(x) = \sin 3x - \cos 3x$, $0 < x < \pi$ is strictly increasing or strictly decreasing.

17. Evaluate : $\int\frac{3x+1}{\sqrt{5-2x-x^2}}dx$

18. Show that the four points A(4, 5, 1) , B(0, -1, -1) , C(3, 9, 4) and D(-4, 4, 4) are coplanar.

19. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6
P (x)	C	2C	2C	3C	C^2	$2C^2$	$7C^2+C$

Find the value of C and also calculate mean of the distribution.

20. Bag I contains 5 red and 4 white balls and Bag II contains 3 red and 3 white balls. Two balls are transferred from the Bag I to the Bag II and then one ball is drawn from the Bag II . If the ball drawn from the Bag II is red , then find the probability that one red and one white ball are transferred from Bag I to Bag II.

21. If $A=\begin{pmatrix}1 & -1\\ 2 & -1\end{pmatrix}$ and $B=\begin{pmatrix}a & 1\\ b & -1\end{pmatrix}$ and $(A+B)^2=A^2+B^2$ then find the values of a and b.

22. Find the distance of the point P(3, 4, 4) from the point, where the line joining the points A(3, -4, -5) and B(2, -3, 1) intersects the plane $2x + y + z = 7$.

23. If the sum of lengths of hypotenuse and a side of a right angled triangle is given, show that the area of triangle is maximum when the angle between them is $\frac{\pi}{3}$.

SECTION - D

24. Let A = {1, 2, 3........9) and R be the relation in A x A defined by (a, b) R (c, d) if $a + d = b + c$ for a, b, c, d $\in$ A. Prove that R is an equivalence relation. Also obtain the equivalence class [(2,5)].

(OR)

Let $f : N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$ is invertible, where S is the range of f. Hence find the inverse of f.

25. Using the properties of determinants, prove that

$$\begin{vmatrix} \frac{(a+b)^2}{c} & c & c \\ a & \frac{(b+c)^2}{a} & a \\ b & b & \frac{(c+a)^2}{b} \end{vmatrix} = 2(a+b+c)^3$$

(OR

If $p \neq 0, q \neq 0$ and $\begin{vmatrix} p & q & p\alpha + q \\ q & r & q\alpha + r \\ p\alpha + q & q\alpha + r & 0 \end{vmatrix} = 0$, then using the properties of determinants, prove that atleast one of the following statements is true:

a) p, q, r are in G.P.

b) α is a root of the equation $px^2 + 2qx + r = 0$.

26. Using integration, find the area of the region : $\{(x, y): x^2 + y^2 \leq 2ax,\ y^2 \geq ax,\ x, y \geq 0\}$

27. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$ (OR) $\int_2^4 (|x-2| + |x-3| + |x-4|)dx$

28. Find the equation of the plane containing two parallel lines $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x}{4} = \frac{y-2}{-2} = \frac{z+1}{6}$. Also, find if the plane thus obtained contains the line $\frac{x-2}{5} = \frac{y-1}{1} = \frac{z-2}{5}$.

29. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at profit of Rs. 7 and that of B at a profit of Rs.4. Find the production level per day for the maximum profit using LPP.

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