ST. XAVIER'S SENIOR SECONDARY SCHOOL, DELHI - 110054
Pre-Board Examination 2018 in **MATHEMATICS**

Set 2

Std. 12 **Max. Marks : 100**
18-01-2018 **Time : 3 hrs.**

GENERAL INSTRUCTIONS:

i) Attempt all the questions.
ii) Section - A consists of 4 questions of 1 mark each.
iii) Section - B consists of 8 questions of 2 marks each.
iv) Section - C consists of 11 questions of 4 marks each.
v) Section - D consists of 6 questions of 6 mark each.

SECTION - A

1. For what value of 'a' the vectors $2\hat{i}-3\hat{j}+4\hat{k}$ and $a\hat{i}+6\hat{j}-8\hat{k}$ are collinear?

2. If $2\begin{pmatrix}4 & 3\\ x & 5\end{pmatrix}+\begin{pmatrix}y & 1\\ 1 & 0\end{pmatrix}=\begin{pmatrix}0 & 7\\ 5 & 10\end{pmatrix}$, find y - x.

3. If '*' is defined on the set R of real numbers by $a*b=\frac{4ab}{9}$, find the identity element in R for the binary operation '*'.

4. State the reason why the Relation R = $\{(a, b):a \le b^2\}$ on the set R of real numbers is not reflexive.

SECTION - B

5. If A is a square matrix of order 3 such that $|\text{adj}\,A|=225$, find $|A'|$.

6. Using differentials, find the approximate value of $(0.009)^{\frac{1}{3}}$.

7. If $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4}$, $xy<1$, then find the value of x + y + xy.

8. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are coplanar, show that $c^2 = ab$.

9. Evaluate : $\int x\tan^{-1}x\,dx$.

10. Two dice are rolled once. Find the probability that the total number on the two dice is atleast 4.

11. Find the differential equation of all circles touching x-axis at the origin.

12. Simplify : $\sin^{-1}\left(\frac{x}{\sqrt{a^2+x^2}}\right)$.

SECTION - C

13. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

(OR)

Find the intervals in which $f(x) = \sin 3x - \cos 3x$, $0 < x < \pi$ is strictly increasing or strictly decreasing.

14. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, find $(\vec{r} \times \hat{i}).(\vec{r} \times \hat{j}) + xy$.

15. Evaluate : $\int \frac{(3\sin\theta - 2)\cos\theta}{5 - \cos^2\theta - 4\sin\theta} d\theta$.

16. Discuss the differentiability of the function $f(x) = \begin{cases} 2x - 1, & x < \frac{1}{2} \\ 3 - 6x, & x \geq \frac{1}{2} \end{cases}$ at $x = \frac{1}{2}$.

(OR)

For what value of k is the following function continuous at $x = \frac{\pi}{6}$?

$$f(x) = \begin{cases} \dfrac{\sqrt{3}\sin x + \cos x}{x + \frac{\pi}{6}}, & x \neq -\frac{\pi}{6} \\ k, & x = -\frac{\pi}{6} \end{cases}$$

17. If the sum of lengths of hypotenuse and a side of a right angled triangle is given, show that the area of triangle is maximum when the angle between them is $\frac{\pi}{3}$.

18. If $A = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix}$ and $(A + B)^2 = A^2 + B^2$ then find the values of a and b.

19. If $y = x^x$, show that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.

20. Show that the four points A(4, 5, 1) , B(0, -1, -1) , C(3, 9, 4) and D(-4, 4, 4) are coplanar.

21. Solve the following differential equation : $\frac{dy}{dx} = \frac{y}{x} + \frac{\sqrt{x^2 + y^2}}{x}$, $x > 0$.

(OR)

Solve the following differential equation : $(1 + y^2)dx = (\tan^{-1} y - x)dy$.

22. Bag I contains 5 red and 4 white balls and Bag II contains 3 red and 3 white balls. Two balls are transferred from the Bag I to the Bag II and then one ball is drawn from the Bag II.

If the ball drawn from the Bag II is red, then find the probability that one red and one white ball are transferred from Bag I to Bag II.

23. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6
P (x)	C	2C	2C	3C	C^2	$2C^2$	$7C^2+C$

Find the value of C and also calculate mean of the distribution.

SECTION - D

24. Using the properties of determinants, prove that

$$\begin{vmatrix} \frac{(a+b)^2}{c} & c & c \\ a & \frac{(b+c)^2}{a} & a \\ b & b & \frac{(c+a)^2}{b} \end{vmatrix} = 2(a+b+c)^3$$

(OR

If $p \neq 0, q \neq 0$ and $\begin{vmatrix} p & q & p\alpha+q \\ q & r & q\alpha+r \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$, then using the properties of determinants, prove that atleast one of the following statements is true:

a) p, q, r are in G.P.   b) α is a root of the equation $px^2 + 2qx + r = 0$.

25. Find the equation of the plane containing two parallel lines $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{3}$ and $\frac{x}{4} = \frac{y-2}{-2} = \frac{z+1}{6}$. Also, find if the plane thus obtained contains the line $\frac{x-2}{5} = \frac{y-1}{1} = \frac{z-2}{5}$.

26. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at profit of Rs. 7 and that of B at a profit of Rs.4. Find the production level per day for the maximum profit using LPP.

27. Let A = {1, 2, 3........9) and R be the relation in A x A defined by (a, b) R (c, d) if a + d = b + c for a, b, c, d $\in$ A. Prove that R is an equivalence relation. Also obtain the equivalence class [(2,5)].

(OR)

Let $f : N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \to S$ is invertible, where S is then range of f. Hence find the inverse of f.

28. Using integration, find the area of the region : $\{(x, y): y^2 \leq 6ax \text{ and } x^2 + y^2 \leq 16a^2, x, y \geq 0\}$

29. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$ (OR) $\int_2^4 (|x-2| + |x-3| + |x-4|)dx$

-x-x-x-x-x-x-