**Pre Board Examination 2015**
**Std. 12**
**16-01-2015**

**Set 2**

# MATHEMATICS

**Max. Marks : 100**
**Time : 3 hrs.**

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General Instructions:

i) All questions are compulsory.

ii) The question paper comprises of 26 questions divided into 3 sections A, B and C.

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| Section A | 6 questions | 1 mark each |
| Section B | 13 questions | 4 marks each |
| Section C | 7 questions | 6 marks each |

iii) Use of calculator is not permitted.

## SECTION A

1. Find $\lambda$ if $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ , $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ , $\vec{c} = 3\hat{\imath} + \lambda\hat{\jmath} + 2\hat{k}$ are coplanar.

2. If a unit vector $\vec{a}$ makes angles $\pi/3$ with $\hat{i}$, $\pi/4$ with $\hat{j}$ and an acute angle $\theta$ with $\hat{k}$ find the value of $\theta$.

3. Write the vector equation of line which passes through (-2,4,-5) and parallel to line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$

4. Write the differential equation of all straight lines passing through origin.

5. Find $x$ if A = $\begin{bmatrix} -3 & 2(x+1) \\ -5 & 7 \end{bmatrix}$ is singular?

6. If m and n are order and degree , respectively of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 2\sin x$ then write the value of m+n.

## SECTION - B

7. Using properties of determinants, prove the following:

$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} = 3(x+y+z)(xy+yz+zx).$$

8. If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$.

(OR)

If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, $0 < t < \frac{\pi}{2}$, find $\frac{d^2y}{dt^2}, \frac{d^2x}{dt^2}, \frac{d^2y}{dx^2}$.

9. Evaluate : $\int_0^{\pi} \frac{4x\sin x}{1+\cos^2 x} dx$.

10. Evaluate : $\int (x-3)\sqrt{x^2+3x-18}\, dx$ (OR) Evaluate : $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x . \cos^2 x} dx$.

11. A store in a mall has three dozen shirts with 'SAVE ENVIRONMENT' printed, two dozen shirts 'SAVE TIGER' printed and five dozen shirts with 'GROW PLANTS' printed. The cost of each shirt is Rs. 595/-, Rs. 610/- and Rs. 795/- respectively. All these items were sold in a day. Find total collection of the store using matrix method.

12. How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80%.

13. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $|\vec{a}| = 5$ , $|\vec{b}| = 12$ and $|\vec{c}| = 13$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ , Find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ .

14. Find the image of the point (1,6,3) in the line $\frac{x}{1} = \frac{y-2}{2} = \frac{z-2}{3}$

(OR)

Find the distance of a point A(1,-2,3) from the plane x - y + z = 5 measured along the line parallel to $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ .

15. If $y=\frac{\log(\sqrt{x^2+1}-x)}{\sqrt{x^2+1}}$, show that $(x^2+1)\frac{dy}{dx}+xy+1=0$

16. Evaluate : $\int \frac{x^2}{(x^2+4)(x^2+9)}dx$.

17. Find the value of K, for which $f(x)=\begin{cases}\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & if\ -1\le x<0\\ \frac{2x+1}{x-1}, & if\ \ 0\le x<1\end{cases}$ is continuous at $x=0$.

18. Prove that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{x}{2}, x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$.

(OR)

Solve for : $3\sin^{-1}\left(\frac{2x}{1+x^2}\right)-4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)+2\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\frac{\pi}{3}$.

19. If A = $\begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}$ satisfies the given equation $A^2-aA+bI=0$, find a and b and hence find $A^{-1}$.

## SECTION - C

20. A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and atmost 300 units of cholesterol. How many packets of each food should be used to minimize the amount of vitamin A in the diet?
What is the minimum amount of vitamin A?

21. Find the area of the region in the first quadrant enclosed by the x-axis, the line y = x and the curve $x^2 + y^2 = 32$.

22. In answering a question on a MCQ test, a student knows the answer, guesses or copies the answer. Let ½ be the probability that he knows the answer, ¼ be the probability that he guesses and ¼ that he copies it. Assuming that a student , who copies the answer, will be correct with the probability ¾, what is the probability that the student knows the answer, given that he answered it correctly?
Which value would a student violate if he resorts to unfair means?

23. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$ .

24. Find the equation of plane passing through the point (1,1,1) and containing the line $\vec{r}=(-3\hat{\imath}+\hat{\jmath}+5\hat{k})+\lambda(3\hat{\imath}-\hat{\jmath}-5\hat{k})$

25. Show that the differential equation $2ye^{\frac{x}{y}}dx+(y-2xe^{\frac{x}{y}})dy=0$ is homogeneous and find its particular solution given that y(0) = 1.

(OR)

Find the particular solution of the differential equation : cosx dy = sinx ( cosx - 2y)dx given that y(1) = 0.

26. Let A = $\{1,2,3,.....9\}$ and R be the relation in A x A defined by (a,b)R(c,d) if a+d = b+c for a,b,c,d $\in A$ . Prove that R is an equivalence relation. Also find the equivalence class $[(2,5)]$.

(OR)

Consider f: $R_+\to[-5,\infty)$ where $f(x)=9x^2+6x-5$. Show that f is bijective function. Also find the inverse of f.

-x-x-x-x-x-x-