St. Xavier's Sr. Sec. School
Delhi-54

Pre-Board Examination 2016
Std. 12
14-01-2016

MATHEMATICS

Set 2

Max. Marks : 100
Time : 3 hrs.

General Instructions:
1. All questions are compulsory.
2. The question paper comprises of 26 questions divided into 3 sections A, B and C.

Section A	6 questions	1 mark each
Section B	13 questions	4 marks each
Section C	7 questions	6 marks each

3. Use of calculator is not permitted.

SECTION - A

1. The side of a square sheet is increasing at the rate of 4cm per minute. At what rate is the area increasing when the side is 8cm long?

2. Differentiate $2^{cos^2 x}$ with respect to x.

3. Write the element a_{21} of where $a_{ij} = \frac{|j-i|}{2}$ matrix $A = [a_{ij}]_{2\times 2}$.

4. Evaluate $\cos\left[\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$.

5. Find the angle θ between the line $\frac{x-2}{3} = \frac{y-3}{5} = \frac{z-4}{4}$ and the plane $2x - 2y + z - 5 = 0$.

6. Let relation R be defined on $\mathbb{R}$ such that R $=\{(a, b)\ a \leq b^2\}$. State the reason why R is not reflexive.

SECTION - B

7. If $y = 3e^{2x} + 2e^{3x}$, prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.

8. Evaluate: $\int \frac{x-3}{(x-1)^3} e^x dx$ (OR) $\int \frac{5x+1}{\sqrt{7-6x-x^2}} dx$.

9. Find the probability of throwing at most 2 sixes in 6 throws of a single die.

(OR)

Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

10. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ find x and y such that $A^2 + xI = yA$ and hence find A^{-1}.

(OR)

If $A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$ then find A^{-1} using elementary column operations.

11. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}$ (OR) Solve : $\tan^{-1}2x+\tan^{-1}3x={}^{\pi}/_{4}$.

12. Solve the differential equation: $(x - y)(dx + dy) = \mathrm{d}x - \mathrm{d}y$ given that $y = -1$.

When x =0.

13. Find the distance between the point (−1, −5, −10) and the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane $x-y+z=5$.

14. Using properties of determinants, prove $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$.

15. For a positive constant $'a'$, find $\frac{dy}{dx}$ where $y=a^{t+\frac{1}{t}}$ and $x=(t+\frac{1}{t})^a$.

16. Consider $f:[-1,1]\rightarrow\mathbb{R}$ given by $f(x)=\frac{x}{x+2}$. Show that f(x) is one-one and onto. Also find the inverse of f.

17. Find the particular solution of the differential equation $(x^2-y^2)dx+2xydy=0$ given that y = 1 when x = 1.

18. To raise money for an orphanage, students of three schools A, B and C organized an exhibition in their locality where they sold paper bags , scrap books and pastel sheets made by them using recycled paper, at the rate of Rs. 20, Rs. 15 and Rs. 5 per unit respectively. School A sold 25 paper bags, 12 scrap books and 34 pastel sheets. School B sold 22 paper bags, 15 scrap books and 28 pastel sheets while School C sold 26 paper bags, 18 scrap books and 26 pastel sheets. Using matrices, find the total amount raised by each school . By such exhibition, which values are inculcated in the students?

19. Evaluate: $\int_0^1\frac{\log(1+x)}{1+x^2}dx$.

SECTION - C

20. A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each type of food should be used to minimize the amount of Vitamin A in the diet? What is the minimum amount of vitamin A?

21. For three coplanar vectors $\vec{a}, \vec{b}$ and $\vec{c}$, prove that $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar.

(OR)

If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$ then prove: (i) $\vec{a} = \pm 2(\vec{b} \times \vec{c})$ (ii) $[\vec{a}+\vec{b} \;\; \vec{b}+\vec{c} \;\; \vec{c}+\vec{a}] = \pm 1$.

22. In a factory, which manufactures bolts, machines A, B and C manufacture respectively 30%, 50% and 20% of the bolts. Of their outputs 3, 4 and 1 percent respectively are defective bolts. A bolt is drawn at random from the product and is found to be defective. Find the probability that this is not manufactured by machine B.

23. Find the intervals in which $f(x) = x^4 - 2x^2$ is strictly increasing or decreasing. Also, the points of local maximum and minimum if any.

24. Find the area of the region enclosed between the curves $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 4$.

25. Show that altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

26. Find the value of k for which the following lines are perpendicular to each other:
$\frac{x+3}{k-5} = \frac{y-1}{1} = \frac{5-z}{-2k-1}$; and $\frac{x+2}{-1} = \frac{2-y}{-k} = \frac{z}{5}$.
Hence find the equation of the plane containing the above lines.

(OR)

Find the distance of the point $(-2, 2, -4)$ from the line $\frac{x+2}{3} = \frac{y+3}{4} = \frac{z+4}{5}$ measured parallel to plane $4x + 12y - 3z + 1 = 0$.

-x-x-x-x-x-x-x-