DEAR STUDENTS, PLEASE NOTE NCERT AND NCERT EXEMPLAR SHOULD BE GIVEN THE FIRST PREFERENCE AND THE GIVEN ASSIGNMENTS.

CONTENTS:

ASSIGNMENT FOR CLASS XII 2018-19:-

TOPIC : RELATION AND FUNCTION:-

1. Let $R = \{(a, a^3) : a$ is a prime number less than 5$\}$ be are relation. Find the range of $\mathbb{R}$.

2. State the reason for the relation R in the set $\{1, 2, 3\}$ given by $\mathbb{R} = \{(1, 2), (2, 1)\}$ is not to be transitive.

3. Let $A = \{0, 1, 2, 3\}$ and defined a relation R on A as follows: $\mathbb{R} = \{(0, 0), (0, 1), (0, 3), (1, 1), (2, 2), (3, 0), (3, 3)\}$
   Is $\mathbb{R}$ reflexive, symmetric and transitive?

4. For real numbers $'x'$ and $'y'$, define xRy , if and only if $x - y + \sqrt{2}$ is an irrational number. Is R transitive? Explain your answer.

5. Let $A = \{a, b, c\}$ and the relation R be defined on A as follows.
   $R = \{(a, a), (b, c), (a, b)\}$.
   Then, write minimum number of ordered pairs to be added in R to make reflexive and transitive.

6. Show that the relation S in set $A = \{x \in \mathbb{Z} : 0 \le x \le 12\}$ given by $S = \{(a, b) : a, b \in A, |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1.

7. Show that the relation R in the set A of real numbers defined as $R = \{(a, b) a \le b\}$ is reflexive and transitive but not symmetric.

8. Show that the relation S in the set $\mathbb{R}$ of real numbers defined as $S = \{(a, b) : a, b \in R \text{ and } a, b \in A\}$ is neither reflexive nor transitive.

9. Let a relation R on the set A of real numbers be defined as $(a, b) \mathbb{R} \Rightarrow 1 + ab > 0, \forall a, b \in A$. Show that R is reflexive and symmetric but not transitive.

10. Let $\mathbb{N}$ be the set of all natural numbers and let $\mathbb{R}$ be a relation on $\mathbb{N}$, define by
    $R = \{(a, b) : a \text{ is a multiple of } 'b'\}$. Show that R is reflexive and transitive but not symmetric.

11. Let A be the set of all points in a plane and R be a relation on A defined as
    $R = \{(P, Q) : \text{distrance between } P \text{ and } Q \text{ is less than } 2 \text{ units}\}$. Show that R is reflective and symmetric but not transitive.

12. Let $A = \{x \in \mathbb{R} : 0 \le x < 1\}$ .If $f : A \to A$ is defined by $f(x) = \begin{cases} x & ,if \ x \in \mathbb{Q} \\ 1 - x & ,if \ x \notin \mathbb{Q} \end{cases}$ then prove that

    $fof(x) = x$ for all $x \in A$.

13. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two function such that $fog\,(x) = \sin x^2$ and $gof\,(x) = \sin^2 x$. Then, find $f(x)$ and $g(x)$.

14. If $f : \mathbb{R} \to \mathbb{R}$ be given by for all $x \in \mathbb{R}$, and $g : \mathbb{R} \to \mathbb{R}$ be such that $g(5/4) = 1$, then prove that $gof : \mathbb{R} \to \mathbb{R}$ its a constant function.

15. Let $f : \mathbb{Z} \to \mathbb{Z}$ be defined by $f(n) = 3n$ for all $n \in \mathbb{Z}$ and $g : \mathbb{Z} \to \mathbb{Z}$ be defined by

$$g(n) = \begin{cases} \dfrac{n}{3} & \text{,if } n \text{ is a multiple of } 3 \\ 0 & \text{,if } n \text{ is not a multiple of } 3 \end{cases} \quad \text{for all } n \in \mathbb{Z}. \text{ Find } fog \text{ and } gof.$$

16. Let $f : \mathbb{R} \to \mathbb{R}$ be a function given by $f(x) = ax + b, \ \forall x \in \mathbb{R}$. Find the constants a and b such that $fof = I_R$.

17. Let $f : A \to A$ be a function such that $fof = f$ show that f is into onto if f is one-one. Describe f in this case.

18. Let $f, g : \mathbb{R} \to \mathbb{R}$ be a two function defined as $f(x) = |x| + x$ and $g(x) = |x| - x$ for all $x \in \mathbb{R}$. Then, find fog and gof.

19. Let f and g be real function defined by $f(x) = \dfrac{x}{x+1}$ and $g(x) = \dfrac{x}{x+3}$. Describe the functions gof and fog (if they exist).

20. If $f(x) = \dfrac{3x-2}{2x-3}$, prove that $f(f(x)) = x$ for all $x \in \mathbb{R} - \left\{ \dfrac{3}{2} \right\}$.

21. Let $f : \mathbb{N} \cup \{0\} \to \mathbb{N} \cup \{0\}$ be defined by

$$f(n) = \begin{cases} n+1 & \text{,if } n \text{ is even} \\ n-1 & \text{, if } n \text{ is odd} \end{cases} \quad \text{Show that } f \text{ is invertible and } f = f^{-1}.$$

22. Let $f(x) = x^3$ be a function with domain $\{0,1,2,3\}$. Then show that f invertible and then write the domain of $f^{-1}(x)$.

23. Consider $f : \mathbb{R}_+ \to [3, \infty)$ given by $f(x) = x^2 + 3$. Show that f is invertible with the inverse $'f^{-1}'$ given by $f^{-1}(y) = \sqrt{y-3}$, where $\mathbb{R}_+$ is the set of all non-negative real numbers.

24. Find the inverse of the function $f : \mathbb{R} \to \{x \in \mathbb{R} : x < 1\}$ given by $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

25. If $f : \mathbb{R} \to (-1, 1)$ is defined by $f(x) = \dfrac{-x|x|}{1+x^2}$, then show that $f^{-1}(x)$ equals to $-Sgn(x)\sqrt{\dfrac{|x|}{1-|x|}}$

Q1. Find the matrix X such that $\quad X.\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$

Q2. For what value of k, is the following matrix singular? $\begin{bmatrix} 3 - 2k & k + 1 \\ 2 & 4 \end{bmatrix}$

Q3. Find x , if $\quad (x \quad 4 \quad 1)\begin{pmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{pmatrix}\begin{pmatrix} x \\ 4 \\ -1 \end{pmatrix} = 0$

Q4. Using the properties of determinant prove that

1. $\begin{vmatrix} x & x^2 & y+z \\ y & y^2 & z+x \\ z & z^2 & x+y \end{vmatrix} = $ (x-y)(y-z)(z-x)(x+y+z).

2. $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \text{abc } (1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c})$

3. $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$

4. $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$

5. $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$, solve for x

6. *If* a, b, c are all positive and are pth , qth and rth elements of G. P. then

   Show that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0.$

7. $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$

8. $\begin{vmatrix} x & x^2 & 1+cx^3 \\ y & y^2 & 1+cy^3 \\ z & z^2 & 1+cz^3 \end{vmatrix} = (1+cxyz)(x-y)(y-z)(z-x)$

9. Evaluate: $\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$

10. $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

11. Evaluate: $\begin{vmatrix} 1+p & 1 & 1+p+q \\ 3+2p & 2 & 4+3p+2q \\ 6+3p & 3 & 10+6p+3q \end{vmatrix}$

12. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

13. $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

Q14. Solve the following using the matrix method:

(i) $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4, \quad \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1, \quad \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$

(ii) $x-y+z=4 \;, \; 2x+y-3z=0 \; , \; x+y+z=2$

Q15. If A= $\begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{pmatrix}$,find A^{-1} Using A^{-1} solve the system of equations:

$$x+y-z = 3$$
$$2x+3y+z = 10$$
$$3x-y-7z = -1$$

1. If the function $f(x) = \begin{cases} 3ax+b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases}$ is continuous at $x=1$, then find the values of a and b

2. Test the continuity of the function $f(x)$ at the origin $f(x) = \begin{cases} \dfrac{x}{|x|}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

3. For what value of k is the function $f(x)$ continuous at $x=0$

   $f(x) = \begin{cases} \dfrac{\sin x + x \cos x}{x}, & \text{when } x \neq 0 \\ k, & \text{when } x = 0 \end{cases}$

4. Examine the continuity of the following function $f(x) = \begin{cases} \dfrac{x}{2|x|}, & x \neq 0 \\ \dfrac{1}{2}, & x = 0 \end{cases}$

5. If $f(x) = \begin{cases} \dfrac{5x+|x|}{3x}, & \text{if } x \neq 0 \\ 2, & \text{if } x = 0 \end{cases}$ then show that $f(x)$ is discontinuous at $x=0$.

6. If $f(x) = \begin{cases} x \sin \dfrac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ then discuss the continuity of the $f(x)$ at $x=0$

7. Determine the constants a and b such that the function $f(x) = \begin{cases} ax^2 + b, & \text{if } x > 2 \\ 2, & \text{if } x = 2 \\ 2ax - b, & \text{if } x < 2 \end{cases}$ is continuous.

8. Let $f(x) = \begin{cases} \dfrac{1 - \sin^3 x}{3 \cos^2 x}, & \text{if } x < \dfrac{\pi}{2} \\ a, & \text{if } x = \dfrac{\pi}{2} \\ \dfrac{b(1 - \sin x)}{(\pi - 2x)^2}, & \text{if } x > \dfrac{\pi}{2} \end{cases}$. If $f(x)$ be a continuous function at $x = \dfrac{\pi}{2}$. Find a and b.

9. Determine the values of a, b and c, for which the function $f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \dfrac{\sqrt{x + bx^2} - \sqrt{x}}{b\sqrt{x^3}}, & x > 0 \end{cases}$

   may be continuous at $x=0$

10. Let $f(x) = \begin{cases} \dfrac{1 - \cos 4x}{x^2}, & x < 0 \\ a, & x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4}, & x > 0 \end{cases}$. Determine the value of a if possible so that the function is continuous at $x=0$.

11. Is the function $f(x) = \begin{cases} \dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$

12. The function $f(x) = \dfrac{\log_e(1 + ax) - \log_e(1 - bx)}{x}$ is not defined at $x = 0$. Find the value of $f(0)$, so that $f(x)$ is continuous at $x = 0$

13. Prove that $\lim\limits_{x \to \alpha} \dfrac{x^n g(x) + h(x)}{x^n + 1} = \begin{cases} h(x) & \text{, when } 0 < x < 1 \\ \dfrac{1}{2}\{h(x) + g(x)\}, & \text{when } x = 1 \\ g(x) & \text{, when } x > 1 \end{cases}$

14. Evaluate $\lim\limits_{x \to 4} \dfrac{(\cos\alpha)^x - (\sin\alpha)^x - \cos 2\alpha}{x - 4}$, $\left(0 < \alpha < \dfrac{\pi}{4}\right)$

15. Given that $f(x) = \dfrac{x^3 + x^2 - 16x + 20}{(x - 2)^2}$, when $x \neq 2$

$= \quad k \quad$, when $x = 2$ \qquad If $f(x)$ is continuous at the origin then find k.

16. If $f(x) = \dfrac{x}{1 + e^{\frac{1}{x}}}$, when $x \neq 0$ and $f(0) = 0$. Show that $f(x)$ is continuous at $x = 0$

17. Discuss the continuity of the function $f(x) = x \cdot \dfrac{e^{\frac{1}{x}}}{1 + e^{\frac{1}{x}}}$, when $x \neq 0$ and $f(0) = 0$

18. A function $f(x)$ is defined in the following way $f(x) = \begin{cases} -2\sin x, & \text{when } -\pi \leq x \leq -\dfrac{\pi}{2} \\ a\sin x + b, & \text{when } -\dfrac{\pi}{2} < x < \dfrac{\pi}{2} \\ \cos x, & \text{when } \dfrac{\pi}{2} \leq x \leq \pi \end{cases}$

19. If $f(x)$ is continuous in $-\pi \leq x \leq \pi$, then find a and b.

20. Find the values of a and b such that the function $f(x) = \begin{cases} x + a\sqrt{2}\sin x, & \text{when } 0 \leq x < \dfrac{\pi}{4} \\ 2x\cot x + b, & \text{when } \dfrac{\pi}{4} \leq x \leq \dfrac{\pi}{2} \\ a\cos 2x - b\sin x, & \text{when } \dfrac{\pi}{2} < x \leq \pi \end{cases}$

is continuous for $0 \leq x \leq \pi$.

21. Show that the function $\lim\limits_{n \to \infty} \dfrac{\cos\pi x - x^{2n}\sin(x - 1)}{1 + x^{2n+1} - x^{2n}}$ is continuous at $x = 1$.

22. The value of $f(0)$, so that the function $f(x) = \sqrt{a^2 - ax + x^2} - f(x) = \sqrt{a^2 + ax + x^2}$ becomes continuous for all x is given by $f(0) = k$, then find k.

23. Show that the function $f(x) = \begin{cases} |x - a|\sin\dfrac{1}{x - a}, & \text{if } x \neq a \\ 0, & \text{if } x = a \end{cases}$ is continuous at $x = a$.

24. For what value of 'k', the function $f(x) = \begin{cases} \dfrac{\sin 2x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$?

25. Examine that $\sin |x|$ is a continuous function.

26. The value of $f(0)$, so that the function $f(x) = \dfrac{(27 - 2x)^{\frac{1}{3}} - 3}{9 - 3(243 + 5x)^{\frac{1}{5}}}$ $(x \neq 0)$ becomes continuous for all x is given by

   $f(0) = k$, then find k.

27. The value of $f(0)$, so that the function $f(x) = \dfrac{2 - (256 - 7x)^{\frac{1}{8}}}{(5x + 32)^{\frac{1}{5}} - 2}$ $(x \neq 0)$ becomes continuous for all x is given by

   $f(0) = k$, then find k.

28. Find the value of the constants a, b and c for which the function $f(x) = \begin{cases} (1 + ax)^{\frac{1}{x}}, & x < 0 \\ b & , \quad x = 0 \\ \dfrac{(x + c)^{\frac{1}{3}} - 1}{(x + 1)^{\frac{1}{2}} - 1}, & x > 0 \end{cases}$

   may be continuous at $x = 0$

29. Find the values of a and b so that $f(x) = \begin{cases} x^2 + 3x + a, & \text{if } x \leq 1 \\ bx + 2 & , \quad \text{if } x > 1 \end{cases}$ is differentiable at each $x \in \mathbb{R}$.

30. For what value of a and b is the function $f(x) = \begin{cases} x^2 & , \text{ when } x \leq c \\ ax + b, & \text{ when } x > c \end{cases}$ is differentiable at $x = c$.

31. Test the differentiability of $f(x) = |\sin x - \cos x|$ at $x = \dfrac{\pi}{4}$.

32. If $f(x) = \begin{cases} ax^2 + 1, & \text{if } |x| < 1 \\ \dfrac{1}{|x|}, & \text{if } |x| \geq 1 \end{cases}$ is differentiable at $x = 1$, then find a and b.

33. If $f(x) = \sqrt{x^2 + 9}$, evaluate $\displaystyle\lim_{x \to 4} \dfrac{f(x) - f(4)}{x - 4}$

34. Discuss the differentiability of the following functions in their domain
   (i) $f(x) = |\cos x|$,
   (ii) $f(x) = \sin|x|$

35. Discuss the continuity and differentiability of the function $f(x) = [x]$ at $x = 1, 2.5$

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1. If $x \sin y = 3 \sin y + 4 \cos y$, then find $\dfrac{dy}{dx}$.

2. Find the derivative of $\sec^{-1}\left(\dfrac{1}{2x^2-1}\right)$ with respect to $\sqrt{1+3x}$ at $x = -\dfrac{1}{3}$

3. Let f be twice differential such that $f''(x) = -f(x)$ and $f'(x) = g(x)$. If $h(x) = \left(f(x)\right)^2 + \left(g(x)\right)^2$ and $h(5) = 7$, find $h(10)$.

4. If $x = e^t \sin t$ and $y = e^t \cos t$, then show that $(x+y)^2 \dfrac{d^2y}{dx^2} = 2\left(x\dfrac{dy}{dx} - y\right)$

5. If $y = \sin^{-1}\left(\dfrac{x^2-1}{x^2+1}\right) + \sec^{-1}\left(\dfrac{x^2+1}{x^2-1}\right)$, $|x| > 1$, then find $\dfrac{dy}{dx}$.

6. Find the derivative of $\sin^{-1}\left(\dfrac{x}{\sqrt{1+x^2}}\right)$ w.r.t. $\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$, $(x > 0)$

7. If $y = 1 + \dfrac{p}{x-p} + \dfrac{qx}{(x-p)(x-q)} + \dfrac{rx^2}{(x-p)(x-q)(x-r)}$, then show that $\dfrac{dy}{dx} = \dfrac{y}{x}\left(\dfrac{p}{p-x} + \dfrac{q}{q-x} + \dfrac{r}{r-x}\right)$.

8. If $y = \cos^{-1}\left[x^{4/3} - \sqrt{\left(1-x^2\right)\left(1-x^{2/3}\right)}\right]$, $0 \le x \le 1$, show that $\dfrac{dy}{dx} = -\dfrac{1}{\sqrt{1+x^2}} - \dfrac{1}{3x^{2/3}\sqrt{1-x^{2/3}}}$.

9. If $y = \sin^{-1}\dfrac{a + b\cos x}{b + a\cos x}$, prove that $\dfrac{dy}{dx} = \mp\dfrac{\sqrt{b^2-a^2}}{b + a\cos x}$.

10. If $y = \dfrac{x^2 + x - 1}{x^3 + x^2 - 6x}$, show that $y_1 = -\dfrac{1}{6x^2} - \dfrac{1}{2(x-2)^2} - \dfrac{1}{3(x+3)^2}$.

TOPIC : INTEGRATION:-

1. Evaluate : $\int \sqrt{\dfrac{x-1}{x^5}}\,dx$

2. Evaluate: $\int \sin(\log_e x)\,dx$

3. Show that $\displaystyle\int_0^{\alpha} \dfrac{dx}{1-\cos\alpha\cos x} = \dfrac{\pi}{2\sin\alpha}$.

4. Using definition of definite integral (limit of sum): show that $\displaystyle\int_0^1 x\sqrt{x}\,dx = \dfrac{2}{5}$.

5. Evaluate: $\int \dfrac{\left(x^4 - x\right)^{\frac{1}{4}}}{x^5}\,dx$.

6. Integrate $\int \left(\dfrac{\tan^{-1}x}{x} + \dfrac{\ln x}{1+x^2}\right)dx$

7. Integrate $\int \dfrac{2x^3\left(x^2-1\right)}{x^{10}+1}\,dx$.

8. Integrate $\int \dfrac{x^2-1}{x} \cdot \dfrac{dx}{\sqrt{\left(x^2+ax+1\right)\left(x^2+bx+1\right)}}$

9. Integrate $\int \sqrt{\dfrac{3-x}{3+x}}\,\sin^{-1}\sqrt{\dfrac{3-x}{6}}\,dx$

10. Integrate $\int \dfrac{\cos^2 x}{\sin^2 x\left(\sin^2 x - \sin^2\alpha\right)}\,dx$

11. Integrate $\int \sqrt{2+\tan^2 x}\,dx$.

12. Evaluate $\displaystyle\int_{-\pi}^{\pi} \dfrac{2x(1+\sin x)}{1+\cos^2 x}\,dx$

13. Evaluate the following $\displaystyle\int_0^{\pi/2} \dfrac{x\sin x\cos x}{\sin^4 x + \cos^4 x}$

14. Evaluate $\displaystyle\int_0^4 \left(x+e^{2x}\right)dx$ as the limit of a sum.

15. Evaluate: $\int \dfrac{\log|x|}{(x+1)^2}\,dx$.

1. Find the area , bounded by the curves : $y^2 = 2x + 1$ and $x = y + 1$

2. Find the area lying above the x- axis and under the parabola $y = 2x - x^2$.

3. Using integration , find the area of triangle bounded by the lines $x + 3y = 8$, $5x - y = 8$, and $x - y + 4 = 0$.

4. Find the area bounded by $y = \sin x$ and $y = \cos x$, between two consecutive point of their intersection .

5. Find the area bounded by $y = x^2 - 6x + 10$ and the straight lines $x = 6$ and $y = 2$.

6. Using integration , find the area of $\triangle ABC$, where $A(-1 , 1)$, $B(0, 5)$, $C(3 , 3)$.

7. Find the area of the region : $\{ (x, y) : y^2 \le 4x , \ 4x^2 + 4y^2 \le 9 \}$.

8. Find the area of the region : $\{ (x, y) : 0 \le y \le x^2 + 1 , 0 \le y \le x + 1 , 0 \le x \le 2 \}$

9. Find the area of the region : $\{ (x , y) : x^2 + y^2 \le 1 \le x + y \}$.

10. Find the area bounded by $y = \sqrt{(5 - x^2)}$ and $y = | x - 1 |$.

11. If the area enclosed by $x^2 = 72y$ and the line $y = k$ be $64\sqrt{2}$ sq unit , then prove that the given line touches the curve $x^2 + y^2 - 4y = 0$.

12. If $y = mx$ bisects the area bounded by the lines $x = 0$, $y = 0$, $x = 3/2$ and $y = 1 + 4x - x^2$, find m .

13. Find the area bounded by $y = \log x$ and $y = (\log x)^2$.

14. Find the area enclosed by $y = 9/x$, $y = x$, $x = 9$ and $y = 0$.

15. Find the area enclosed by $y = x^2 - 2x - 3$ and $x + y = 9$.

16. Find the area between $y = x (x - 1)^2$, $x = 0$ and $y = 2$.

17. Find the area bounded by $y = \cos x$, $y = 1 + 2x/ \pi$, and $x = \pi/2$.

18. Find the area enclosed by $y = | 4 - x^2 | / 4$ and $y = 7 - | x |$.

LEVEL 1

1.Find the shortest distance between the lines $\vec{r}= \hat{i} + \hat{j} + \mu (2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r}=2 \hat{i} + \hat{j} - \hat{k} + \rho(3 \hat{i} - 5 \hat{j} + 2\hat{k})$.

2.Find the equation of the plane through the line of intersection of the planes x+y+z=1 and 2x +3y +4z=5 which is perpendicular to the plane x-y+z=0.

3.Find the vector equation of the plane which contains the line of intersection of the planes $\mathbf{r}.(\hat{i} + \hat{j} + 3 \hat{k}) - 4 = 0$ and $\mathbf{r}.(2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\mathbf{r}.(5 \hat{i} + 3 \hat{j} - 6 \hat{k}) + 8 = 0$.

4. Find the vector equation of the line passing through (1,2,3,) and parallel to the planes

$\mathbf{r}.(\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\mathbf{r}.(3\hat{i} + \hat{j} + \hat{k}) = 6$.

5. Find the equation of the plane passing through the points (1,-2,5) , (0,-5,-1) and (-3,5,0).

6. Find the image of the point (3,-2,1) in the plane 3x-y+4z=2.

7. If l,m,nand l',m',n' are the dcs of two mutually perpendicular lines .Show that the dcs of the line perpendicular to both of them are mn'-m'n , nl'-n'l , lm'-l'm .

8. A line makes angles α, β, γ and δ with the diagonals of a cube ,prove that $cos^2\alpha + cos^2\beta + cos^2\gamma + cos^2\delta = \frac{4}{3}$

9.Show that the lines $\vec{r}= \hat{i} + \hat{j} - k + \mu (3\hat{i} - \hat{j})$ and $\vec{r}=4 \hat{i} - \hat{k} + \rho(2 \hat{i} + 3\hat{k})$. Intersect .Find their point of intersection.

10. Show that the lines $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z}{-1}$ and $\frac{x-4}{3} = \frac{y-1}{-2} = \frac{z-1}{1}$ are coplanar . Also find the equation of the plane containing them.

11.13. Find the equation of the perpendicular from the point (3,-1,11) to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also find the foot of the perpendicular and the length of perpendicular.

12.Find the distance of the point (1,-2,3) from the plane x-y+z=5 measured parallel to the line $\frac{x+1}{2} = \frac{y+3}{3} = \frac{z+1}{-6}$

13. Show that the equation of the plane passing through (1,1,1) , (1,-1,1) and (-7,3,-5) is perpendicular to XZ plane .

14.Find α so that the four points with position vectors .$-6\hat{i} + 3 \hat{j} + 2 \hat{k}$, $3\hat{i} + \alpha\hat{j} + 4 \hat{k}$, $5\hat{i} + 7\hat{j} + \hat{k}$ and -$13\hat{i} + 17 \hat{j} - \hat{k}$ are coplanar.

15.Find the distance between the planes $\mathbf{r}.(\hat{i} + 2\hat{j} - 2 \hat{k}) + 5 = 0$ and $\mathbf{r}.(\hat{i} + 2 \hat{j} - 2\hat{k}) - 8 = 0$

16. Find the position vector of the point at a distance of $3\sqrt{11}$ units from $\hat{i} - \hat{j} + 2k$ $on\ a\ line\ passing\ through\ the\ points$ $\hat{i} - \hat{j} + 2k$ $and\ 3\ \iota + \hat{j} + k$

17.Find the distance of the point (3,4,5) from the plane x+y+z=2 measured parallel to the line 2x=y=z

18.Find the distance between the line $\vec{r}=2 \hat{i} - 2\hat{j} + 3 \hat{k} + \rho(\hat{i} - \hat{j} + 4\hat{k})$ and the plane $\mathbf{r}.(\hat{i} + 5 \hat{j} \mp \hat{k}) = 5$.

LEVEL 2

1. The plane lx+my = 0 is rotated through an angle α about its line of intersection with the plane z=0 .Prove that the equation of the plane in its new position is lx+my$\pm(\sqrt{l^2+m^2}\tan\alpha)z = 0$

2. Two systems of rectangular axes have the same origin .If a plane cuts them at a disatance a , b , c and a' , b' , c' respectively prove that $a^{-2} + b^{-2} + c^{-2} = (a')^{-2} + (b')^{-2} + (c')^{-2}$

3. A variable plane which remains at a constant distance 3p from the origin cuts the coordinate axes at A , B , C . Show that the locus of the centroid of ΔABC is $x^{-2} + y^{-2}+z^{-2} = p^{-2}$

4. Vertices B and C of ΔABC lie along the line
$\frac{x+2}{2} = \frac{y-1}{1} = \frac{z}{4_{..}}$ Find the area of the triangle given that A has coordinates $(1,-1,2)$ and line BC has length 5

5. Show that the angles between the diagonals of a cube is $\cos^{-1}\frac{1}{3}$

6. Prove that the straight lines whose dc are given by the relations al+bm+cn=0 and fmn+gnl+hlm=0 are perpendicular if $\frac{f}{a}+\frac{g}{b}+\frac{h}{c} = 0$ and parallel if $a^2f^2 + b^2g^2 + c^2h^2 - 2abfg - 2bcgh2achf = 0$

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ASSIGNMENT ON DIFFERENTIAL EQUATION FOR CLASS XII 2018-19:-

1. $\frac{dy}{dx} +$ x sin2y $=$ x^{3}xos^{2}y.
2. $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$.
3. y e^y dx $= (y^3 + 2xe^y)$ dy, given y(0)=1.
4. $\frac{dy}{dx} + x(x+y) + 1 = x^3(x+y)^3$.
5. $x^4\frac{dy}{dx} + x^3y + \operatorname{cosec}(xy) = 0$.
6. A curve is defined by the property that the sum of the x and y intercepts of its tangent is always equal to a. Express this in the form of a differential equation.
7. Form the differential equation by eliminating the parameter λ from the following equation
$\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$.
8. Solve the differential equation $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$

9. Solve the differential equation $(x-y)^2\frac{dy}{dx} = k^2$

10. Solve the differential equation $y^2 + \left(x - \frac{1}{y}\right)\frac{dy}{dx} = 0$

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TOPIC : LPP:-

1. You have ₹ 12,000 to invest, and three different funds from which to choose. The municipal bond fund has a 7% return, the local bank's CDs have an 8% return, and the high-risk account has an expected (hoped-for) 12% return. To minimize risk, you decide not to invest any more than ₹ 2,000 in the high-risk account. For tax reasons, you need to invest at least three times as much in the municipal bonds as in the bank CDs. Assuming the year-end yields are as expected, what are the optimal investment amounts?

2. In order to ensure optimal health (and thus accurate test results), a lab technician needs to feed the rabbits a daily diet containing a minimum of 24 grams (g) of fat, 36 g of carbohydrates, and 4 g of protien. But the rabbits should be fed no more than five ounces of food a day.

3. Rather than order rabbit food that is custom-blended, it is cheaper to order Food X and Food Y, and blend them for an optimal mix. Food X contains 8 g of fat, 12 g of carbohydrates, and 2 g of protein per ounce, and costs ₹ 0.20 per ounce. Food Y contains 12 g of fat, 12 g of carbohydrates, and 1 g of protein per ounce, at a cost of ₹ 0.30 per ounce. What is the optimal blend ?

4. A calculator company produces a scientific calculator and a graphing calculator. Long-term projections indicate an expected demand of at least 100 scientific and 80 graphing calculators each day. Because of limitations on production capacity, no more than 200 scientific and 170graphing calculators can be made daily. To satisfy a shipping contract, a total of at least 200 calculators much be shipped each day.
If each scientific calculator sold results in a ₹ 2 loss, but each graphing calculator produces a ₹ 5 profit, how many of each type should be made daily to maximize net profits?

5. **A** manufacturer considers men and women workers are equally efficient and so he pays them at the same rate . He has 30 and 17 units of workers and capital respectively , which he uses to produce two types of goods A and B. To produce one unit of A 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital is required to produce one unit of B . If A and B are priced at ₹ 100 and ₹ 120 per unit respectively , how should he use his resources to maximize the total revenue ? Form the above as an LPP and solve graphically .Do you agree with this view of the manufacturer that men and women workers are equally efficient and so should be paid at the same rate ?

6. **A**n aeroplane can carry a maximum of 200 passengers. A profit of ₹ 1000 is made on each executive class ticket and a profit of ₹ 600 is made on each economy class ticket . The airline reserves at least 20 seats for executive class .However at least 4 times as many passengers prefer to travel by economy class than by the executive class .Determine how many tickets of each type must be sold in order to maximise the profit for the airline . What is the maximum profit?

7. A company sells two different type of plastic products A and B.The two products in a common production process which has a total capacity of 500 man hours.It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B.The demand in the market shows that the maximum number of units of A that can be sold is 70 and that of B is 125.Profit on each unit of A is ₹ 20 and on B is ₹ 15.How many units of A and B should be produced to maximise the profit. Form an LPP and solve it graphically.
How does a person who is recycling plastic pet bottles impact the environment?

8. A manufacturer make two types of toys A and B .Three machines are required for this purpose and the time (in minutes) required for each toy on the machines is given below :

TYPES	Machine I	Machine II	Machine III
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hrs per day. If the profit on each toy of type A is ₹ 7.50 and that of type B is ₹ 5 , how many of each toy should be manufactured in a day in order to maximize the profit ?

9. A company produces two different products. One of them needs 1/4 of an hour of assembly work per unit, 1/8 of an hour in quality control work and ₹ 1.2 in raw materials. The other product requires 1/3 of an hour of assembly work per unit, 1/3 of an hour in quality control work and ₹ 0.9 in raw materials. Given the current availability of staff in the company, each day there is at most a total of 90 hours available for assembly and 80 hours for quality control. The first product described has a market value (sale price) of ₹ 9 per unit and the second product described has a market value (sale price) of ₹ 8 per unit. In addition, the maximum amount of daily sales for the first product is estimated to be 200 units, without there being a maximum limit of daily sales for the second product. Formulate and solve graphically the LPP and find the maximum profit.

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TOPIC :PROBABITY:-

1. A fair coin and an unbiased die are tossed. Let A be the event head appear on the coin and B be the event 3 on the die. Check weather A and B are independent events or not.

2. A box is filled with candies in different colors. We have 40 white candies, 24 green ones, 12 red ones, 24 yellow ones and 20 blue ones. If we have selected one candy from the box without peeking into it, find the probability of getting a green or red candy.

   A basket has 12 marigolds and 8 carnations. If we had picked two flowers one by one with replacement, find the probability of getting a marigold and a carnation.

3. In a certain day care class, 30% of the children have grey eyes, 50% of them have blue and the other 20%'s eyes are in other colors. One day they play a game together. In the first run, 65% of the grey eye ones, 82% of the blue eyed ones and 50% of the children with other eye color were selected. Now, if a child is selected randomly from the class, and we know that he/she was not in the first game, what is the probability that the child has blue eyes?

4. Suppose two balls are removed from a bag containing three green and two red balls and that the first ball is not replaced before the second is removed. What is the probability that the balls are the same colour?

   Two balls are removed from a bag containing three green and two red balls. The first ball is not replaced before the second is removed. If the second ball is red, what is the probability that the first ball was green?

5. A card is drawn at random from a well shuffled pack of 52 cards. Find the probability that it is neither an ace nor a king.

6. Probability of solving specific problem independently by A and B are and respectively of both try to solve the problem independently, find the probability that (i) the problem is solved (ii) Exactly one of them solves the problem.

7. In a factory which manufactures bulbs, machines X, Y and Z manufacture respectively 1000,2000, 3000 of the bulbs. Of their outputs,1%, 1.5% and 2 % are respectively defective bulbs. A bulb is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine X ?

8. There are three coins . One is two headed coin , another is a biased coin that comes up head 75% of the times and the third one is an unbiased coin . One of the three coins is chosen at random and tossed . If it shows head what is the probabity that it was a two headed coin?

9. Four bad oranges are accidentally mixed with 16 good ones. Find the probability distribution of the number of bad oranges when two oranges are drawn at random from this lot. Find the mean and variance of the distribution.

10. A glazier buys his glass from four different manufacturers - Clearglass (10%), Strongpane (25%), Mirrorglass (30%) and Reflection (35%). In the past, the glazier has found that 1% of Clearglass' product is cracked, 1.5% of Strongpane's product is cracked, and 2% of Mirrorglass' and Reflection's products are cracked. The glazier removes the protective covering from a sheet of glass without looking at the manufacturer's name - in other words, it's a random choice. He finds the glass is cracked. What is the probability it was made by Mirrorglass?

11. A test for a disease gives a correct positive result with a probability of 0.95 when the disease is present, but gives an incorrect positive result (false positive) with a probability of 0.15 when the disease is not present. If 5% of the population has the disease, and Raj tests positive to the test, what is the probability he really has the disease?

12. Rohan is getting married tomorrow, at an outdoor ceremony in the desert. In recent years, it has rained only 5 days each year. Unfortunately, the weatherman has predicted rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 10% of the time. What is the probability that it will rain on the day of Rohan's wedding?

13. An insurance company insured 2000 scooter drivers , 4000 car drivers and 6000 truck drivers .The probability of accidents are   0.01 , 0.03 and 0.15 respectively .One of the insured persons meets with an accident . What is the probability that he is a scooter driver ? What would you do if someone gets hurt or injured in a road accident ?

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