SHRI VIDHYABHARATHI MATRIC HR.SEC.SCHOOL

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PUBLIC EXAMINATION 2019

XI - BUSINESS MATHEMATICS **TENTATIVE ANSWER KEY** **18.03.2019**

I

1. b $\frac{16}{5}$
2. a $|A|^{n-1}$
3. c 1128
4. a $nC_2 - n$
5. b $(1, -1)$
6. d Latus Rectum
7. b $\frac{-1}{\sqrt{3}}$
8. a 3
9. c 0
10. d. $\frac{x-1}{x^2}$
11. c. $\eta_d = \frac{AR}{AR-MR}$
12. d. Decreasing function
13. d 8.75 %
14. b. Annuity due
15. c. Arithmetic Mean
16. a $\frac{1}{51}$
17. d -1 to $+1$
18. c. Dependent Variable
19. a. an optimal Solution
20. d. 15

II

21. $nC_4 = 495$

$$\frac{n!}{(n-4)!\,4!} = 495$$

$$n(n-1)(n-2)(n-3) = 495 \times 4!$$

$$n(n-1)(n-2)(n-3) = 12 \times 11 \times 10 \times 9$$

$$\boxed{n = 12}$$

22. Length of Tangent

$$= \sqrt{x_1^2 + y_1^2 - 2x_1 + 4y_1 + 9}$$

$$= \sqrt{1+4-2+8+9} = \sqrt{20}$$

$$= 2\sqrt{5} \text{ Units.}$$

23. $\cos 510^\circ = -\frac{\sqrt{3}}{2}$ | $\sin 390^\circ = \frac{1}{2}$

$\cos 330^\circ = \frac{\sqrt{3}}{2}$ | $\cos 120^\circ = -\frac{1}{2}$

$$L.H.S = \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)$$

$$= -1$$

24. $f(x) = \log \frac{1+x}{1-x}$

$$f\left(\frac{2x}{1+x^2}\right) = \log \frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}$$

$$= \log\left(\frac{1+x}{1-x}\right)^2$$

$$= 2\log\left(\frac{1+x}{1-x}\right)$$

$$= 2f(x).$$

25. $P = 10e^{-x/2}$

$$\frac{dp}{dx} = -5e^{-x/2}$$

$$\frac{dx}{dp} = \frac{-1}{5e^{-x/2}}$$

Elasticity of demand

$$\eta_d = \frac{-p}{x}\frac{dx}{dp} = \frac{2}{x}$$

26. $u = x^2y - x^3 + xy^2 - y^3$

$$\frac{\partial u}{\partial x} = 2xy - 3x^2 + y^2$$

$$\frac{\partial u}{\partial y} = x^2 + 2xy - 3y^2$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = -2(x^2+y^2-2xy)$$

$$= -2(x-y)^2.$$

27. $\left.\begin{array}{l}\text{Market value of}\\ \text{1 share}\end{array}\right\} = ₹.\ 132$

$\therefore \left.\begin{array}{l}\text{Market value}\\ \text{of 62 Shares}\end{array}\right\} = 62 \times 132$

$= ₹.\ 8184$

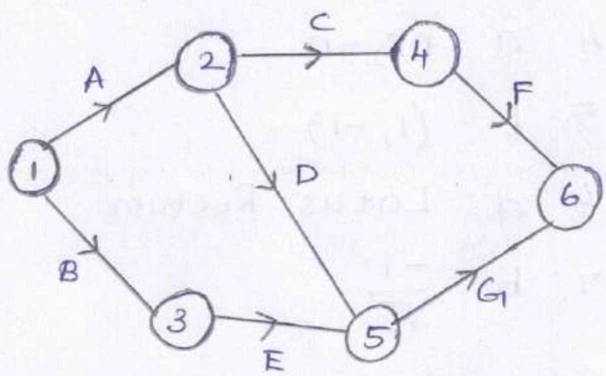
28.

x (km)	f (hrs)	f/x
48	10	0.2083
40	12	0.3
32	15	0.4688
	$\Sigma f = 37$	$\Sigma \frac{f}{x} = 0.9771$

$$\text{Harmonic Mean} = \frac{N}{\Sigma \frac{f}{x}}$$

$$= 37.86 \text{ km/hr.}$$

29.



30. $y = x^{\log x}$

$$\log y = \log x \cdot \log x$$

$$\frac{1}{y}\frac{dy}{dx} = 2\log x \cdot 1/x$$

$$\frac{dy}{dx} = x^{\log x}\left[\frac{2\log x}{x}\right].$$

III

31. Total letters = 11.

$\left.\begin{array}{l}\text{No. of}\\ \text{permutations}\end{array}\right\} = \frac{11!}{2!\,2!\,2!}$

32. No. of ways $= 10C_8 \times 10C_5$
$= 10C_2 \times 10C_5$
$= 11340$.

33. $L.H.S = \tan^{-1}\left(\dfrac{\frac{3}{4}+\frac{1}{7}}{1-\left(\frac{3}{4}\right)\left(\frac{1}{7}\right)}\right)$

$= \tan^{-1}(1)$

$= \pi/4$

34. $\lim\limits_{x\to\infty} \dfrac{6-5x^2}{4x+15x^2} = \lim\limits_{x\to\infty} \dfrac{\frac{6}{x^2}-5}{\frac{4}{5}+15}$

$= \dfrac{0-5}{0+15}$

$= -\dfrac{1}{3}$

35. $A = (a_{ij})_{2\times2}$, $a_{ij} = 2i-j$

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$

$|A| = 2 \quad \operatorname{adj} A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$

$A^{-1} = \dfrac{1}{2}\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$

$AA^{-1} = \dfrac{1}{2}\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$

$= \dfrac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$AA^{-1} = I$

36. $f(x) = 2x^3+9x^2+12x+1$

$f'(x) = 6x^2+18x+12$

$f'(x) = 0 \Rightarrow x=-2,\ x=-1$

when $x=-2 \Rightarrow f(-2) = -3$

when $x=-1 \Rightarrow f(-1) = -4$

The stationary points are $(-2,-3)$ and $(-1,-4)$

37. $a = ₹.\ 2000 \quad i = 10\% = 0.1$

$n = 14$

$P = \dfrac{a}{i}\left[1-(1+i)^{-n}\right]$

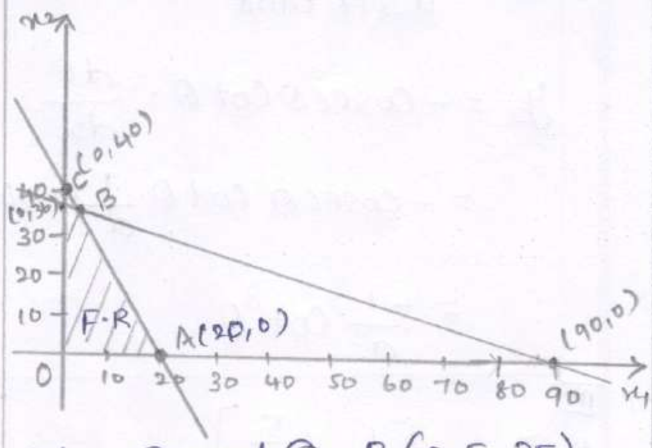
$= \dfrac{2000}{0.1}\left[1-(1.1)^{-14}\right]$

$= ₹.\ 14,736$.

38. $2x_1+x_2 = 40$ —① $\quad 2x_1+5x_2 = 180$ —②

x_1	0	20
x_2	40	0

x_1	0	90
x_2	36	0



Solve ① and ② $B(2.5, 35)$

Extreme point	Coordinates	$z=3x_1+4x_2$
O	(0,0)	0
A	(20,0)	60
B	(2.5, 35)	147.5
C	(0, 36)	144

The optimum value 147.5 occurs at $B(2.5, 35)$.

$\therefore$ Solution is $x_1 = 2.5$, $x_2 = 35$ and $\max z = 147.5$

39. $u(x,y) = \dfrac{x^2+y^2}{\sqrt{x+y}}$

$u(tx,ty) = t^{3/2}\, u(x,y)$

u is a homogeneous function of degree $n = 3/2$.

By Euler's theorem,

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$$

40. $x = a\sec\theta \qquad y = a\tan\theta$

$\dfrac{dx}{d\theta} = a\sec\theta\tan\theta \qquad \dfrac{dy}{d\theta} = a\sec^2\theta$

$$y_1 = \frac{a\sec^2\theta}{a\sec\theta\tan\theta} = \operatorname{cosec}\theta$$

$$y_2 = -\operatorname{cosec}\theta\cot\theta\cdot\frac{d\theta}{dx}$$

$$= -\operatorname{cosec}\theta\cot\theta\cdot\frac{1}{a}\cos\theta\cot\theta$$

$$= \frac{-1}{a}\cot^3\theta$$

III

41. a. $B = \begin{bmatrix} \frac{1}{5} & \frac{5}{12} \\ \frac{2}{5} & \frac{1}{2} \end{bmatrix}$

$I-B = \begin{bmatrix} \frac{4}{5} & -\frac{5}{12} \\ -\frac{2}{5} & \frac{1}{2} \end{bmatrix}$ elements of main diagonal are positive

$|I-B| = \frac{7}{30} > 0$

$|I-B|$ is positive

The problem has a solution.

$\operatorname{adj}(I-B) = \begin{bmatrix} \frac{1}{2} & \frac{5}{12} \\ \frac{2}{5} & \frac{4}{5} \end{bmatrix}$

$$(I-B)^{-1} = \frac{1}{|I-B|}\operatorname{adj}(I-B)$$

$$= \frac{30}{7}\begin{bmatrix} \frac{1}{2} & \frac{5}{12} \\ \frac{2}{5} & \frac{4}{5} \end{bmatrix}$$

$$X = (I-B)^{-1}D = \begin{bmatrix} 150 \\ 204 \end{bmatrix}$$

The output of Industry

$P_1 = ₹150$ Crores

$P_2 = ₹.\ 204$ Crores.

b. $n = 12 \quad x = 2x^2 \quad a = 1/x$

$$T_{r+1} = nC_r\, x^{n-r} a^r$$

$$= 12C_r\, 2^{12-r}\, x^{24-3r}$$

Since, the term is Independent of x,

$$24 - 3r = 0$$

$$r = 8$$

$$T_9 = 12C_8\, 2^4\, x^0$$

The term Independent of x

$$= 12C_4\, 2^4$$

$$= 7920.$$

42 a. Given $x = \tan A - \tan B$

$y = \cot B - \cot A$

$$R.H.S = \frac{1}{x} + \frac{1}{y}$$

$$= \frac{1}{\tan A - \tan B} + \frac{1}{\cot B - \cot A}$$

$$= \frac{1}{\tan A - \tan B} + \frac{\tan A \tan B}{\tan A - \tan B}$$

$$= \frac{1 + \tan A \tan B}{\tan A - \tan B}$$

$$= \frac{1}{\tan(A-B)}$$

$$= \cot(A-B)$$

43 a. $y^2 - 4x - 4y + 8 = 0$

$(y-2)^2 = 4(x-1)$.

$Y^2 = 4X$, $X = x-1$, $Y = y-2$

$\boxed{a=1}$

It is a parabola open Right ward.

Axis : $y = 2$

Vertex : $V(1,2)$

Focus : $F(2,2)$

Equation of directrix : $x = 0$

Length of Latus Rectum : $4a = 4$

b. $L.H.S = \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$

$$= \frac{1}{2}\cos 2^\circ \left[\cos(60^\circ + 20^\circ)\cos(60^\circ - 20^\circ)\right]$$

$$= \frac{1}{2}\cos 20^\circ \left[\cos^2 60^\circ - \sin^2 20^\circ\right]$$

$$= \frac{1}{2}\cos 20^\circ \left[\frac{1}{4} - \sin^2 20^\circ\right]$$

$$= \frac{1}{2}\cos 20^\circ \left[\frac{1}{4} - (1 - \cos^2 20^\circ)\right]$$

$$= \frac{1}{2}\cos 20^\circ \left[\frac{-3 + 4\cos^2 20^\circ}{4}\right]$$

$$= \frac{1}{8}\left[-3\cos 20^\circ + 4\cos^3 20^\circ\right]$$

$$= \frac{1}{8}\cos 60^\circ$$

$$= \frac{1}{8}(1/2)$$

$$= \frac{1}{16}$$

b. Equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ — (1)

It passes $(1,1)$

$2g + 2f + c = -2$ — (2)

Eqn. (1) passes $(2,-1)$

$4g - 2f + c = -5$ — (3)

(1) passes $(2,3)$

$4g + 6f + c = -13$ — (4)

(3) − (4) $\Rightarrow -8f = 8$

$\Rightarrow \boxed{f = -1}$

(2) − (3) $\Rightarrow -2g + 4f = 3$

$\Rightarrow \boxed{g = -7/2}$

(2) $\Rightarrow \boxed{c = 7}$

∴ The required equation is

$x^2 + y^2 - 7x - 2y + 7 = 0$.

44. a.
$F.V = ₹.100$

$\text{Income} = \frac{20}{100} \times 5000$

$= ₹.1000$

Case (i):

Investment ₹. 5000

Face value $= ₹.100$

$M.V = ₹.100 + 62 - 2 = 160$

$\text{No. of Shares} = \frac{\text{Investment}}{F.V} = \frac{5000}{100}$

$= 50$

Sales proceeds $= 50 \times 160 = 8000$

Case (ii):

$M.V = ₹.100 - 22 + 2 = 80$

$\text{No. of Shares} = \frac{\text{Investment}}{F.V} = \frac{8000}{80}$

$= 100$

$\text{Income} = 100 \times \frac{15}{100} \times 100 = 1500$

Change of Income $= ₹.1500 - 1000$

$= ₹.500$

44. b.
$\Sigma X = 392 \quad \Sigma Y = 328$

$\Sigma X^2 = 22112$

$\Sigma Y^2 = 15792$

$\Sigma XY = 18568$.

Correlation coefficient

$$r = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{N\Sigma X^2 - (\Sigma X)^2}\sqrt{N\Sigma Y^2 - (\Sigma Y)^2}}$$

$r = 0.7689$

45. a.
$A_1, A_2 \rightarrow$ Bag I, Bag II

Let B be ball drawn is red.

$P(A_1) = 1/2 \quad P(B/A_1) = 3/7$

$P(A_2) = 1/2 \quad P(B/A_2) = 5/11$

$$P(A_1/B) = \frac{P(A_1) \cdot P(B/A_1)}{P(A_1) P(B/A_1) + P(A_2) \cdot P(B/A_2)}$$

$$= \frac{33}{68}$$

b. Given $q = 80 - P_1^2 + 5P_2 - P_1P_2$

$\frac{\partial q}{\partial P_1} = -2P_1 - P_2$

$$\frac{Eq}{EP_1} = \frac{-P_1}{q}\left(\frac{\partial q}{\partial P_1}\right) = \frac{2P_1^2 + P_1P_2}{80 - P_1^2 + 5P_2 - P_1P_2}$$

when $P_1 = 2 \quad P_2 = 1$

$\frac{Eq}{EP_1} = \frac{10}{79}$

$\frac{\partial q}{\partial P_2} = 5 - P_1$

$$\frac{Eq}{EP_2} = \frac{-P_2}{q}\left(\frac{\partial q}{\partial P_2}\right) = \frac{-5P_2 + P_1P_2}{80 - P_1^2 + 5P_2 - P_1P_2}$$

when $P_1 = 2, \ P_2 = 1$

$\frac{Eq}{EP_2} = \frac{-3}{79}$

46. a.

C.I	Mid val M	f	fM	|D| = |M-mean|	f|D|
0-5	2.5	3	7.5	10.5	31.5
5-10	7.5	5	37.5	5.5	27.5
10-15	12.5	12	150	0.5	6
15-20	17.5	6	105	4.5	27
20-25	22.5	4	90	9.5	38

$N = \Sigma f = 30$

$\Sigma fM = 390$

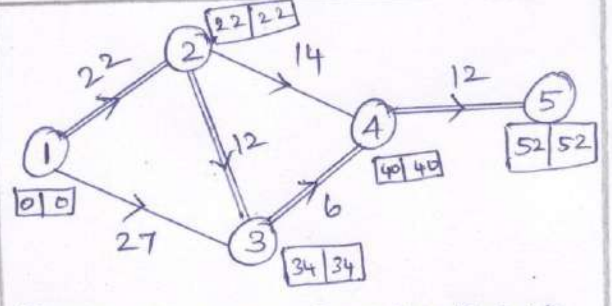
$\Sigma f|D| = 130$

$$\text{Mean} = \frac{\Sigma fm}{N} = 13$$

$$\left.\begin{array}{l}\text{M.D about} \\ \text{Mean}\end{array}\right\} = \frac{\Sigma f|D|}{N}$$

$$= 4.33$$

b.



$E_1 = 0$, $E_2 = 22$, $E_3 = 34$, $E_4 = 40$
$E_5 = 52$
$L_5 = 52$, $L_4 = 40$ $L_3 = 34$ $L_2 = 22$ $L_1 = 0$

Activity	Duration	EST	EFT	LST	LFT.
1-2	22	0	22	0	22
1-3	27	0	27	7	34
2-3	12	22	34	22	34
2-4	14	22	36	36	40
3-4	6	34	40	34	40
4-5	12	40	52	40	52

Critical path is 1-2-3-4-5

Project Completion time $= 22+12+6+12$
$= 52$ days

47. a. Average Revenue $= \frac{R}{x} = 15 + \frac{x}{3} - \frac{x^3}{36}$

$$\frac{d}{dx}(AR) = \frac{1}{3} - \frac{3x^2}{36}$$

$$\frac{d}{dx}(AR) = 0 \Rightarrow x = \pm 2$$

$$\frac{d^2}{dx^2}(AR) = \frac{-x}{6}$$

when $x = 2$, $\frac{d^2(AR)}{dx^2} < 0$

$\therefore$ AR is maximum when $x = 2$

At the highest point, $x = 2$

$AR = 15.444 = 15.45$ (app) —(1)

$$MR = \frac{dR}{dx} = 15 + \frac{2x}{3} - \frac{x^3}{9}$$

when $x = 2$, $MR = 15.45$ —(2)

From (1) and (2),
At the highest point,
Average Revenue = Marginal Revenue

b. $f(x) = |x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

$$L[f'(0)] = \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \to 0^-} \frac{-x}{x} = -1$$

$$R[f'(0)] = \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \to 0^+} \frac{x}{x} = 1$$

$$L[f'(0)] \neq R[f'(0)]$$

$\therefore f(x)$ is not differentiable at $x = 0$.

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