S.S.L.C -2019

Mathematics

Question Paper

With

Key Answer

Key Ans: YK

KARNATAKA STATE SECONDARY EDUCATION EXAMINATION BOARD - 2019

Subject : MATHEMATICS

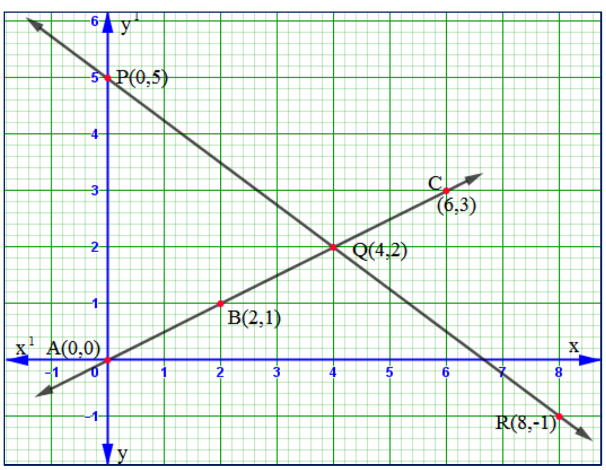
Total No. of questions : 40 **Subject Code : 81E**
Time : 3 Hrs. **Max. marks: 80**

I. Four alternatives are given for each of the following questions/incomplete statements. Only one of them is correct or most appropriate. Choose the correct alternative and write the complete answer along with its letter of alphabet. **$8 \times 1 = 8$**

1. If n-th term of an arithmetic progression $a_n=24-3n$, then its 2^{nd} term is
   (A) 18 (B) 15 (C) 0 (D) 2
2. The lines represented by $2x+3y-9=0$ and $4x+6y-18=0$ are
   (A) Intersecting lines (B) Perpendicular lines to each other
   (C) Parallel lines (D) Coincident line.
3. A straight line which passes through two points on a circle is
   (A) A chord (B) A secant (C) A tangent (D) the radius
4. If the area of a circle is 49 ∏ sq. units then its perimeter is
   (A) 7π units (B) 9π units (C) 14π units (D) 49π units
5. " The product of two consecutive positive integers is 30. " this can be expressed algebraically as
   (A) $x(x+2)=30$ (A) $x(x-2)=30$ (A) $x(x+3)=30$ (A) $x(x+1)=30$
6. If a and b are any two positive integers then HCF (a, b) $\times$ LCM (a, b) is equal to
   (A) $a + b$ (B) $a - b$ (C) $a \times b$ (D) $a \div b$
7. The value of $\cos 48^0 - \sin 42^0$ is
   (A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1
8. If P(A)= 0.05 then $P(\overline{A})$ is
   (A) 0.59 (B) 0.95 (C) 1 (D) 1.05

II. Answer the following : **$6 \times 1 = 6$**

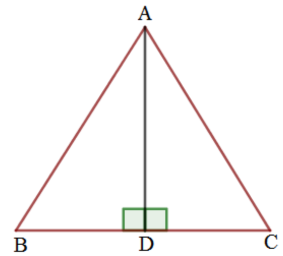
9. The given graph represents a pair of linear equations in two variables. Write how may solutions these pair of equations have.



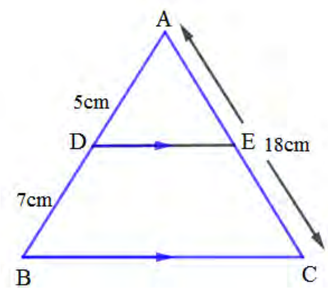
10. 17 = 6 x 2 + 5 is compared with Euclid's Division lemma a = bq + r, then which number is representing the remainder?
11. Find the zeros of the polynomial $p(x) = x^2 - 3$
12. Write the degree of the polynomial $p(x) = 2x^2 - x^3 + 5$
13. Find the value of the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$
14. Write the formula to calculate the curved surface area of the frustum of a cone.

III. Answer the following : $16 \times 2 = 32$

15. Find the sum of first twenty terms of Arithmetic series 2 + 7 + 12 + ….. using suitable formula.
16. In Δ ABC, AD $\perp$ BC and $AD^2 = BD \times CD$. Prove that $AB^2 + AC^2 = (BD + CD)^2$

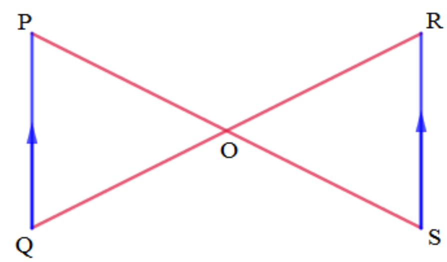


17. In Δ ABC, DE ∥ BC, If AD = 5cm, BD = 7cm and AC = 18cm, find the length of AE.

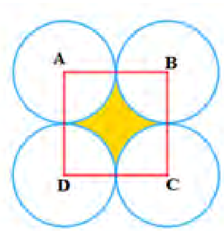


OR

In the given figure if PQ ∥ RS, prove that $\Delta POQ \sim \Delta SOR$.



18. Solve the following pair of linear equations by any suitable method :
19. In the figure, ABCD is a square of side 14cm. A, B, C and D are the centers of four congruent circle such that each circle touches externally two of the remaining three circles. Find the area of the shaded region.



20. Draw a circle of radius 4cm and construct pair of tangents such that the angle between them is 60^0
21. Find the co-ordinates of point which divides the line segment joining the points A(4,-3) and B(8,5) in the ratio 3 : 1 internally.
22. Prove that $3 + \sqrt{5}$ is an irrational number.
23. The sum and product of zeroes of quadratic polynomial $p(x) = ax^2 + bx + c$ are -3 and 2 respectively. Show that $b + c = 5a$
24. Find the quotient and remainder when $p(x) = 3x^3 + x^2 + 2x + 5$ is divided by $g(x) = x^2 + 2x + 1$
25. Solve $2x^2 - 5x + 3 = 0$ by using formula.
26. The length of a rectangular field is 3 times its breadth. If the area of the field is 147 sq.m, find its length and breadth.
27. If $\sin\theta = \frac{12}{13}$, find the value of $\cos\theta$ and $\tan\theta$

OR

If $\sqrt{3}\tan\theta = 1$ and θ is acute, find the value of $\sin 3\theta + \cos 2\theta$

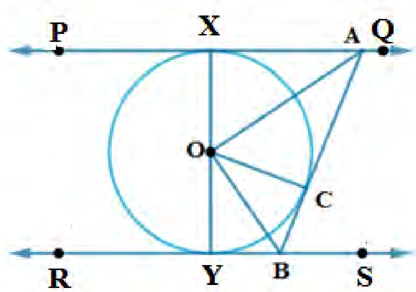
28. Prove that $\left(\frac{1+cos\theta}{1-cos\theta}\right) = (cosec\,\theta + cot\theta)^2$
29. A cubical die numbered from 1 to 6 is rolled twice. Find the probability of getting the sum of numbers on its face is 10.
30. The radii of two circular ends of a frustum of a cone shaped dustbin are 15cm and 8cm. if fits depth is 63cm, find the volume of the dustbin.

IV. Answer the following : **$6 \times 3 = 18$**

31. Prove that " The lengths of tangents drawn from an external point to a circle are equal " .

OR

In the given figure PQ and RS are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting PQ at A and RS at B. prove that ∟AOB = 90^0



32. Calculate the median of the following frequency distribution table.

Class interval	1- 4	4-7	7-10	10-13	13-16	16-19
Frequency(f_i)	6	30	40	6	4	4

$\sum f_i = 100$

OR

Calculate the mode for the following frequency distribution table.

Class interval	10-25	25-40	40-55	55-70	70-85	85-100
Frequency(f_i)	2	3	7	6	6	6

$\sum f_i = 30$

33. During the medical check-up 35 students of a class, their weights were recorded as follows. Draw a less than type of ogive for the given data.

Weight (in kg)	Number of studens
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

34. The seventh term of an Arithmetic progression is four times its second term and twelfth term is 2 more than three times of its fourth term. Find the progression.

OR

A line segment is divided into four parts forming an Arithmetic progression. The sum of the lengths of 3^{rd} and 4^{th} parts is three times the sum of the lengths of first two parts. If the length of fourth part is 14cm, find the total length of the line segment.

35. The vertices of a ΔABC are A(-3,2), B(-1,-4) and C(5,2). If M and N are the mid – points of AB and AC respectively, show that 2MN = BC.

OR

The vertices of a ΔABC are A(-5,-1), B(3,-5) and C(5,2). Show that the area of the ΔABC is four times the area of the triangle formed by joining the mid-points of the sides of the triangle ABC.

36. Construct a triangle with sides 5cm, 6cm, and 7cm and then construct another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.

V. Answer the following : 4 × 4= 16

37. Find the solution of the following pairs of linear equation by the graphical method :
$2x + y = 6$
$2x – y = 2$

38. The angles of elevation of the top of a tower from two points at a distance of 4m and 9m from the base of the tower and in the same straight line with it are complimentary. Find the height of the tower.

39. The bottom of a right cylindrical shaped vessel as shown in the figure. The radius of the circular base of the cylinder and radius of the circular base of the cone are each is equal to 7cm. if the height of the cylinder is 20cm and height of cone is 3cm, calculate the cost of milk to fill completely this vessel at the rate of Rs. 20 per liter.

OR

A hemispherical vessel of radius 14cm is fully filled with sand. This sand is poured on a level ground. The heap of sand forms a cone shape of height 7cm. calculate the area of ground occupied by the circular base of the heap of the sand.

40. Prove that " The areas of two similar triangles is equal to the square of the ratio of their corresponding sides".

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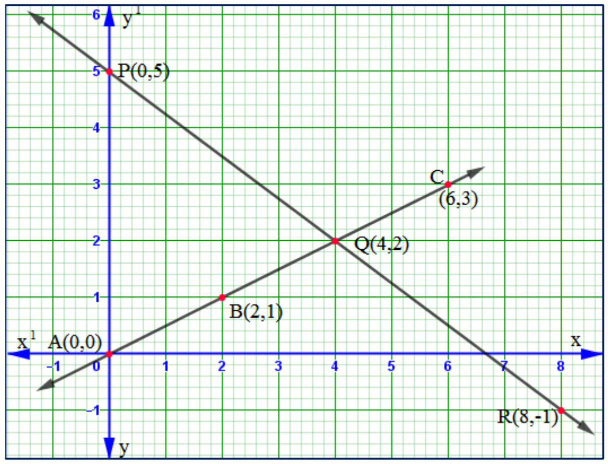
1. If n-th term of an arithmetic progression a_n=24-3n, then its 2nd term is
   (A) 18 (B) 15 (C) 0 (D) 2
   Ans: A) 18
   [$a_n = 24 - 3n \Rightarrow a_2 = 24 - 3x2 = 24 - 6 = 18$]
2. The lines represented by 2x+3y-9=0 and 4x+6y-18=0 are
   (A) Intersecting lines (B) Perpendicular lines to each other
   (C) Parallel lines (D) Coincident line.
   Ans: (D) Coincident line
   $\left[\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}\right]$
3. A straight line which passes through two points on a circle is
   (A) A chord (B) A secant (C) A tangent (D) the radius
   Ans: (B) A secant
4. If the area of a circle is 49 ∏ sq. units then its perimeter is
   (A) 7π units (B) 9π units (C) 14π units (D) 49π units
   Ans: (C) 14π units
5. " The product of two consecutive positive integers is 30. " this can be expressed algebraically as
   (A) x(x+2)=30 (A) x(x-2)=30 (A) x(x+3)=30 (A) x(x+1)=30
   Ans: (D) x(x + 1) = 30
6. If *a* and *b* are any two positive integers then HCF (a, b) × LCM (a, b) is equal to
   (A) a + b (B) a - b (C) a × b (D) a ÷ b
   Ans: (C) a x b
7. The value of $\cos 48^0 - \sin 42^0$ is
   (A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1
   Ans: A) 0 [$\cos48^0 - \sin(90 - 48^0) = \cos48^0 - \cos48^0 = 0$]
8. If P(A)= 0.05 then $P(\overline{A})$ is
   (A) 0.59 (B) 0.95 (C) 1 (D) 1.05
   Ans: (B) 0.95 [$1- P(A) = P(\bar{A}) \Rightarrow 1 - 0.05 = 0.95$]

II. Answer the following : **6 × 1= 6**

9. The given graph represents a pair of linear equations in two variables. Write how may solutions these pair of equations have.



Ans: Unique Solution

10. 17 = 6 x 2 + 5 is compared with Euclid's Division lemma a = bq + r, then which number is representing the remainder?

Ans : r = 5

11. Find the zeros of the polynomial $p(x) = x^2 - 3$

Ans: $x^2 = 3$

$\Rightarrow x = \pm\sqrt{3}$

12. Write the degree of the polynomial $p(x) = 2x^2 - x^3 + 5$

Ans: 3

13. Find the value of the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$

Ans: $\Delta = b^2 - 4ac$

$= (-4)^2 - 4(2)(3)$

$= 16 - 24 = \mathbf{-8}$

14. Write the formula to calculate the curved surface area of the frustum of a cone.

Ans $\boldsymbol{\pi(r_1 + r_2)l}$

III. Answer the following : **16 × 2= 32**

15. Find the sum of first twenty terms of Arithmetic series 2 + 7 + 12 + ….. using suitable formula.

Ans: a = 2, d = 5, n = 20

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = \frac{20}{2}[2x2 + (20-1)5]$

$S_{20} = 10[4 + 19x5]$

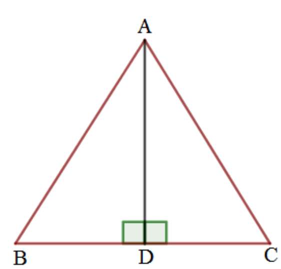
$S_{20} = 10[4 + 95]$

$S_{20} = 10[99]$

$\mathbf{S_{20} = 990}$

16. In Δ ABC, AD ⊥ BC and $AD^2 = BD \times CD$. Prove that $AB^2 + AC^2 = (BD + CD)^2$

Ans:In ΔABC AD⊥BC and $AD^2 = BDxCD$



$\Rightarrow AB^2 = AD^2 + BD^2$ - - -- --------(1)

$AC^2 = AD^2 + CD^2$ ---------------(2)

(1) + (2) = $AB^2 + AC^2 = BD^2 + CD^2 + 2AD^2$

$\Rightarrow AB^2 + AC^2 = BD^2 + CD^2 + 2BDxCD$

$\Rightarrow AB^2 + AC^2 = (BD + CD)^2$

17. In Δ ABC, DE ∥ BC, If AD = 5cm, BD = 7cm and AC = 18cm, find the length of AE.

In ΔABC, $DE \parallel BC$,

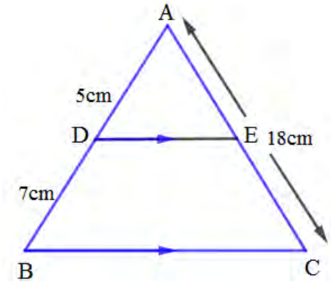
Therefore $\frac{AD}{AB} = \frac{AE}{AC}$

$\Rightarrow \frac{5}{12} = \frac{AE}{18}$

$\Rightarrow AE = \frac{5x18}{12}$

$\Rightarrow AE = \frac{5x3}{2}$

$\Rightarrow AE = \frac{15}{2} = \mathbf{7.5cm}$



OR

In the given figure if PQ ∥ RS, prove that Δ POQ ~ Δ SOR.

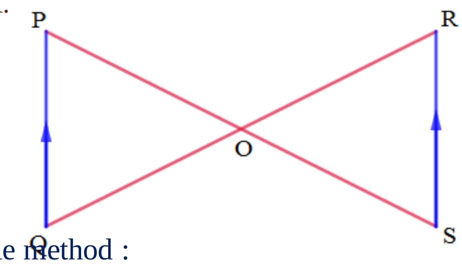
ΔPQR ಮತ್ತು ΔSOR ಗಳಲ್ಲಿ, PQ∥RS,ಆದ್ದರಿಂದ

$\angle OPQ = \angle OSR$ [Alternate angles]

$\angle OPQ = \angle OSR$ [Alternate angles]

$\angle POQ = \angle ROS$ [vertically opposite angles]

$\therefore \Delta PQR \sim \Delta SOR$ [Triangles are equiangular]



18. Solve the following pair of linear equations by any suitable method :

$x + y = 5$

$2x - 3y = 5$

Ans: $x + y = 5$ ------------(1)

$2x - 3y = 5$ ----------(2)

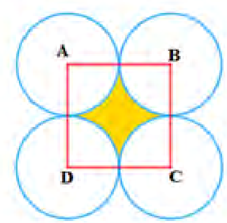
(1)x2 $\Rightarrow$ $2x + 2y = 10$ ---------(3)

(2) $\Rightarrow$ $2x - 3y = 5$

(2)-(3)$\Rightarrow$ $5y = 5$ $\Rightarrow \mathbf{y = 1}$

From(1), $x + 1 = 5 \Rightarrow x = 5 - 1 \Rightarrow \mathbf{x = 4}$

19. In the figure, ABCD is a square of side 14cm. A, B, C and D are the centers of four congruent circle such that each circle touches externally two of the remaining three circles. Find the area of the shaded region.



Ans: The side of the square = 14 cm

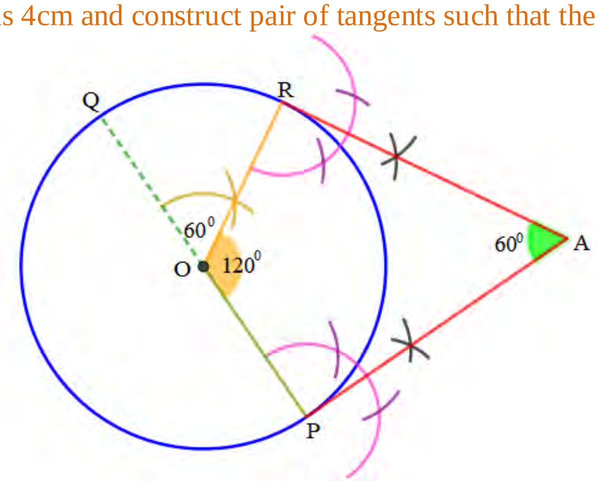
$\therefore$ The radius of the circle = $\frac{14}{2}$ = 7 cm

Area of the squre ABCD= $14^2 = 196\ cm^2$

Total area of the quadrants = $4\times\frac{\pi R^2}{4}\ cm^2 = \pi R^2 = \frac{22}{7}x7x7 = 154cm^2$

$\therefore$ **The area of the shaded region** = $196\ cm^2 - 154\ cm^2 = \mathbf{42cm^2}$

20. Draw a circle of radius 4cm and construct pair of tangents such that the angle between them is 60^0



21. Find the co-ordinates of point which divides the line segment joining the points A(4,-3) and B(8,5) in the ratio 3 : 1 internally.

Ans: $(x_1, y_1) = (4, -3)$, $(x_2, y_2) = (8,5)$, $m_1 : m_2 = 3 : 1$

x_1	y_1	x_2	y_2
4	-3	8	5

$$x = \frac{m_1x_2 + m_2\ x_1}{m_1+m_2} = \frac{3(8)+1(4)}{3+1} = \frac{24+4}{4} = \frac{28}{4} = 7$$

$$y = \frac{m_1y_2 + m_2\ y_1}{m_1+m_2} = \frac{3(5)+1(-3)}{3+1} = \frac{15-3}{4} = \frac{12}{4} = 3$$

∴The co-ordinates of point (7,3)

22. Prove that $3 + \sqrt{5}$ is an irrational number.

Ans:Let $3 + \sqrt{5}$ be a rational number

$\Rightarrow 3 + \sqrt{5} = \frac{p}{q}$ [p,q∈ Z, q ≠ 0 ಮತ್ತು (p,q)=1]

$\Rightarrow \sqrt{5} = \frac{p}{q} - 3$

$\Rightarrow \sqrt{5} = \frac{p-3q}{q}$

Here, $\frac{p-3p}{q}$ is a rational number but $\sqrt{5}$ is an irrational number

Therefore our assumption is wrong

∴ $3 + \sqrt{5}$ is an irrational number

23. The sum and product of zeroes of quadratic polynomial $p(x) = ax^2 + bx + c$ are -3 and 2 respectively. Show that b + c = 5a

Ans: Sum of the Zeroes $= \frac{-b}{a} \Rightarrow -3 = \frac{-b}{a} \Rightarrow -b = -3a \Rightarrow b = 3a$ -----(1)

The product of the zeroes $= \frac{c}{a} \Rightarrow 2 = \frac{c}{a} \Rightarrow c = 2a$ -----------------(2)

From (1) and (2) $b + c = 3a + 2a \Rightarrow b + c = 5a$

24. Find the quotient and remainder when $p(x) = 3x^3 + x^2 + 2x + 5$ is divided by $g(x) = x^2 + 2x + 1$

$1 + 2x + x^2$	$3x^3 + x^2 + 2x + 5$	$3x - 5$
	$3x^3 + 6x^2 + 3x$	
	$-5x^2 - x + 5$ $-5x^2 - 10x - 5$	
	$9x + 10$	

quotient = $3x - 5$; remainder = $9x + 10$

25. Solve $2x^2 - 5x + 3 = 0$ by using formula.

Ans: $2x^2 - 5x + 3 = 0$

$a = 2,\ b = -5,\ c = 3$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(3)}}{2(2)} = \frac{5 \pm \sqrt{25-24}}{4} = \frac{5 \pm \sqrt{1}}{4} = \frac{5 \pm 1}{4}$$

$$x = \frac{5+1}{4},\quad x = \frac{5-1}{4}$$

$$x = \frac{6}{4},\ x = \frac{4}{4} \Rightarrow \boldsymbol{x = \frac{3}{2},\ x = 1}$$

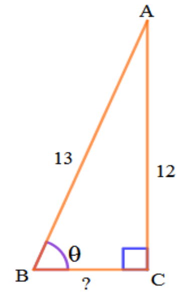
26. The length of a rectangular field is 3 times its breadth. If the area of the field is 147 sq.m, find its length and breadth.

Ans: Let breadth = x. There fore Length = 3x

Area of the field = x(3x)

$\Rightarrow 3x^2 = 147 \Rightarrow x^2 = 49 \Rightarrow x = 7$ ∴ **Breadth = 7m and Length = 3x7 = 21 m**

27. If $\sin\theta = \frac{12}{13}$, find the value of $\cos\theta$ and $\tan\theta$



Ans: $\sin\theta = \frac{12}{13}$ ⇒ In Right angle triangle ABC, AB = 13 and AC = 12

Therefore $BC^2 = 13^2 - 12^2 = 169 - 144 = 25 \Rightarrow BC = 5$

$\cos\theta = \frac{BC}{AB} = \frac{5}{13}$ ಮತ್ತು $\tan\theta = \frac{AC}{BC} = \frac{12}{5}$

OR

If $\sqrt{3}\tan\theta = 1$ and θ is acute, find the value of $\sin 3\theta + \cos 2\theta$

Ans: $\sqrt{3}\tan\theta = 1 \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^0$

$\therefore \sin 3(30^0) + \cos 2(30^0) \Rightarrow \sin 90^0 + \cos 60^0$

$\Rightarrow 1 + \frac{1}{2} = \mathbf{\frac{3}{2}}$

28. Prove that $\left(\frac{1+\cos\theta}{1-\cos\theta}\right) = (\operatorname{cosec}\theta + \cot\theta)^2$

Ans: L.H.S. $= \frac{1+\cos\theta}{1-\cos\theta}$

$= \frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$

$= \frac{(1+\cos\theta)^2}{1-\cos^2\theta} = \frac{(1+\cos\theta)^2}{\sin^2\theta} = \frac{1+\cos^2\theta+2\cos\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} + \frac{2\cos\theta}{\sin^2\theta}$

$= \operatorname{cosec}^2\theta + \cot^2\theta + 2 \,.\, \operatorname{cosec}\theta \,.\, \cot\theta$

$\mathbf{= (\operatorname{cosec}\theta + \cot\theta)^2}$

29. A cubical die numbered from 1 to 6 is rolled twice. Find the probability of getting the sum of numbers on its face is 10.

Ans: S – { die numbered from 1 to 6 is rolled twice }

S = { (a,b)/a,b = 1,2,3,4,5,6}

n(S) = 36

A = { getting the sum of numbers on its face is 10.}

A = { (4,6), (5,5), (6,4)}

n(A) = 3

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{36}$ [or $\frac{1}{12}$]

30. The radii of two circular ends of a frustum of a cone shaped dustbin are 15cm and 8cm. if fits depth is 63cm, find the volume of the dustbin.

Ans: Volume of the dust bin $= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

$\pi = \frac{22}{7}$; h = 63cm; r_1 = 15 = 2cm; r_2 = 8cm

$= \frac{1}{3} \times \frac{22}{7} \times 63(15^2 + 8^2 + 15 \times 8)$

$= 22 \times 3(225 + 64 + 120)$

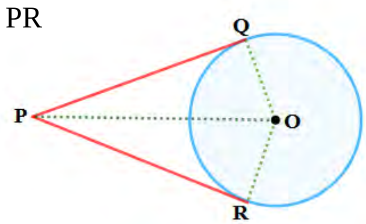
$= 66 \times 409$

$= \mathbf{26994 cm^3}$

IV. Answer the following : $6 \times 3 = 18$

31. Prove that " The lengths of tangents drawn from an external point to a circle are equal " .

Data: O is the center, P is an external point PQ and PR are the tangents drawn from the point P



Join OP, OQ, OR

To Prove: PQ = PR

Proof: In right angle triangle OQP and ORP,

OQ = OR (Radius of the same circle)

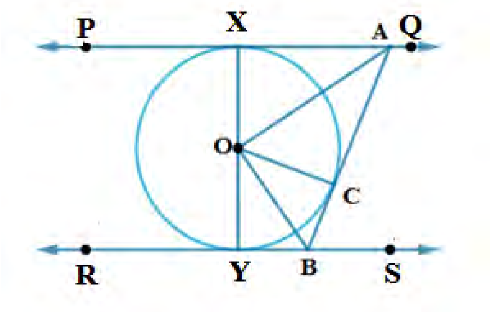
OP = OP (Common side)

∴ ΔOQ P ≅Δ ORP (R.H.S)

⇒ PQ = PR (CPST)

OR

In the given figure PQ and RS are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting PQ at A and RS at B. prove that ∟AOB = 90^0



The tangent AB touches the circle at point C. Join OC

Proof: In Quadrilateral XOCA ,

∠OXA = ∠OCA = 90^0 [OX⊥PQ; OC⊥AB]

AX = AC (∵The tangents drawn from the point A)

Here, the opposite sides and adjacent angles are equal

∴ XOCA is a square

∴ ∠XOC = 90^0 [The diagonals bisects the angles]

⇒ ∠AOC = 45^0

ll^{ly} ∠BOC = 45^0

⇒ ∠AOC + ∠BOC = 90°

⇒ ∠AOB = 90°

Alternate Method:

The tangent AB touches the circle at point C. Join OC

In ΔAXO and ΔACO ,

OX = OC (∵ radius of the same circle)

AX = AC (∵The tangents drawn from the point A)

OA = OA (∵Common side)

∴ ΔAXO ≅ ΔACO (SSS Axiom)

⇒ ∠XOA = ∠CAO (1)

ll^{ly} ∠BOY = ∠BOC (2)

XOY is a diameter ∴ ∠XOY = 180^0

⇒∠XOA + ∠COA + ∠BOY + ∠BOC = 180 °

from (1) and (2)

2∠AOC + 2∠BOC = 180°

⇒ ∠AOC + ∠BOC = 90°

⇒ ∠AOB = 90°

32. Calculate the median of the following frequency distribution table.

Class interval	1- 4	4-7	7-10	10-13	13-16	16-19
Frequency(f_i)	6	30	40	6	4	4

$\sum f_i = 100$

Ans:

C.I.	Freequency(f_i)	(cf_i)
1-4	6	6
4-7	30	36
7-10	40	76
10-13	16	92
13-16	4	96
16-19	4	100

$n = \sum f_i = 100$

Now $n = 100$, $\therefore \frac{n}{2} = 50$ this is in a class interval $7 - 10$

l (Lower limit) $= 7$; cf $= 36$; $f = 40$; h $= 3$

Median $= l + \left[\frac{\frac{n}{2} - cf}{f}\right] \text{x h}$

$= 7 + \left[\frac{50 - 36}{40}\right] \text{x } 3$

$= 7 + \left[\frac{14}{40}\right] \text{x } 3$

$= 7 + 1.05$

Median = 8.05

OR

Calculate the mode for the following frequency distribution table.

Class interval	10-25	25-40	40-55	55-70	70-85	85-100
Frequency(f_i)	2	3	7	6	6	6

$\sum f_i = 30$

Ans: Maximum number of students are in the class interval 40 - 55

Therefore 40 – 55 is the modal calss interval

$\therefore$ $l = 40$; h = 15; $f_1 = 7$; $f_0 = 3$; $f_2 = 6$

Mode $= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \text{x } h$

$= 40 + \left[\frac{7 - 3}{2(7) - 3 - 6}\right] \text{x } 15$

$= 40 + \left[\frac{4}{14 - 9}\right] \text{x } 15$

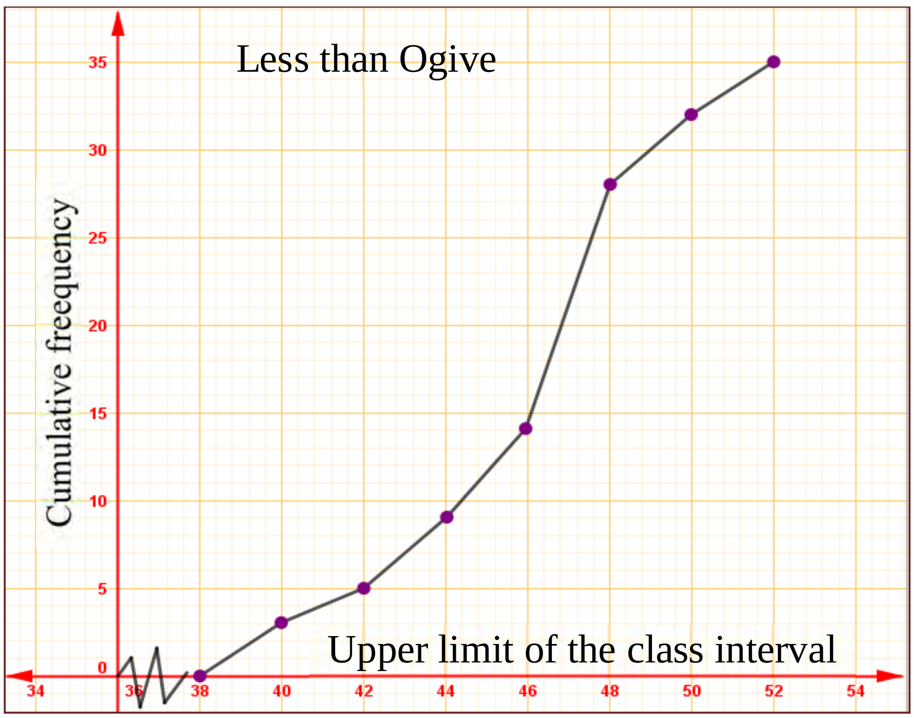
$= 40 + \frac{4}{5} \text{ x } 15$

$= 40 + 12$

$\therefore$ **The mode of the above data = 52**

33. During the medical check-up 35 students of a class, their weights were recorded as follows. Draw a less than type of ogive for the given data.

Weight (in kg)	Number of studens
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35



34. The seventh term of an Arithmetic progression is four times its second term and twelfth term is 2 more than three times of its fourth term. Find the progression.

Ans: $a_7 = 4a_2 \Rightarrow$ a + 6d = 4(a + 2d) ⇒ a + 6d = 4a + 8d

⇒ 3a + 2d = 0 -------------- (1)

$a_{12} = 3a_4 + 2 \Rightarrow$ a + 11d = 3(a + 3d) + 2

⇒ a + 11d = 3a + 9d + 2 ⇒ 2a - 2d = -2

⇒ a – d = -1 ⇒ a = d - 1 --------------(2)

(1) ⇒ 3(d - 1) = 2d ⇒ 3d – 2d = 3 ⇒ **d = 3**

(2) ⇒ a = 3 – 1 ⇒ **a = 2**

The progression.: **2, 5, 8, 11 ------**

OR

A line segment is divided into four parts forming an Arithmetic progression. The sum of the lengths of 3rd and 4th parts is three times the sum of the lengths of first two parts. If the length of fourth part is 14cm, find the total length of the line segment.

Ans: $a_3 + a_4 = 3(a + a_2)$

$\Rightarrow$ a + 2d + a + 3d = 3(a + a + d)

$\Rightarrow$ 2a + 5d = 3(2a + d)

$\Rightarrow$ 2a + 5d = 6a + 3d

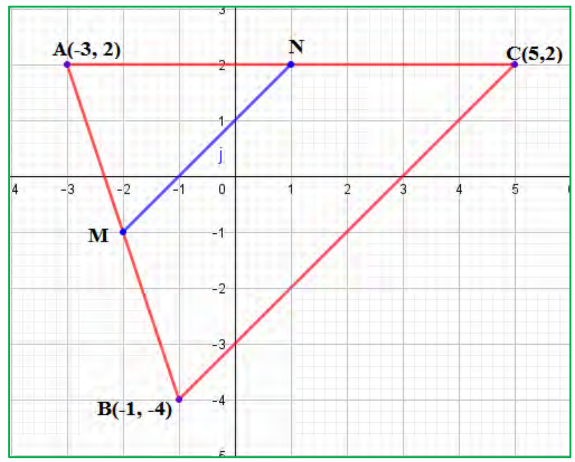
$\Rightarrow$ 4a = 2d $\Rightarrow$ 2a = d --------------------(1)

a_4 = 14 $\Rightarrow$ a + 3d = 14 $\Rightarrow$ a + 3(2a) = 14 $\Rightarrow$ a + 6a = 14 $\Rightarrow$ 7a = 14 $\Rightarrow$ **a = 2**

$\Rightarrow$ d = 2x2 $\Rightarrow$ **d = 4**

Therefore the length of the line segments: **2cm, 6cm, 10cm, 14cm**

35. The vertices of a ΔABC are A(-3,2), B(-1,-4) and C(5,2). If M and N are the mid – points of AB and AC respectively, show that 2MN = BC.



$M\left[\frac{-3+-1}{2}, \frac{2+-4}{2}\right]$; $N\left[\frac{-3+5}{2}, \frac{2+2}{2}\right]$

$M\left[\frac{-4}{2}, \frac{-2}{2}\right]$; $N\left[\frac{2}{2}, \frac{4}{2}\right]$

$M[-2, -1]$; $N[1, 2]$

MN = $\sqrt{(1+2)^2 + (2+1)^2}$

= $\sqrt{3^2 + 3^2}$ = $\sqrt{9+9}$ = $\sqrt{18}$ = $3\sqrt{2}$

BC = $\sqrt{(5+1)^2 + (2+4)^2}$

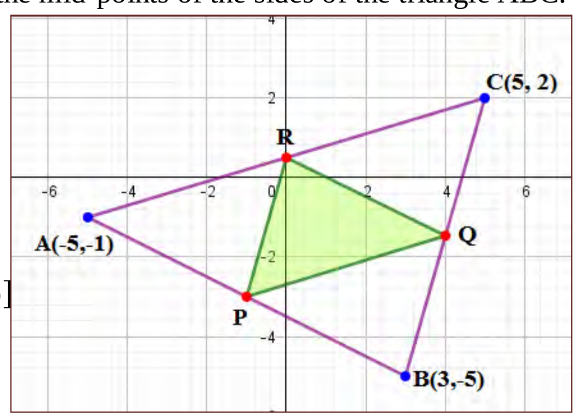
= $\sqrt{6^2 + 6^2}$ = $\sqrt{36+36}$

= $\sqrt{72}$ = $6\sqrt{2}$ = 2x $3\sqrt{2}$ = 2MN

∴ **BC = 2MN**

OR

The vertices of a ΔABC are A(-5,-1), B(3,-5) and C(5,2). Show that the area of the ΔABC is four times the area of the triangle formed by joining the mid-points of the sides of the triangle ABC.



Ans: $P\left(\frac{3-5}{2}, \frac{-5-1}{2}\right)$, $Q\left(\frac{5+3}{2}, \frac{2-5}{2}\right)$, $R\left(\frac{5-5}{2}, \frac{2-1}{2}\right)$

$P\left(\frac{-2}{2}, \frac{-6}{2}\right)$, $Q\left(\frac{8}{2}, \frac{-3}{2}\right)$, $R\left(\frac{0}{2}, \frac{1}{2}\right)$

$P(-1, -3)$, $Q\left(4, \frac{-3}{2}\right)$, $R\left(0, \frac{1}{2}\right)$

$\Rightarrow$ $P(-1, -3)$, $Q(4, -1.5)$, $R(0, 0.5)$

Area of ΔABC

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

x_1	y_1	x_2	y_2	x_3	y_3
−5	−1	3	−5	5	2

Area of ΔABC, $= \frac{1}{2}[-5(-5-2) + 3(2+1) + 5(-1+5)]$

$= \frac{1}{2}[-5(-7) + 3(3) + 5(4)] = \frac{1}{2}[35 + 9 + 20] = = \frac{1}{2}[64)] =$ **32 sq.units ------(1)**

Area of ΔPQR = $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

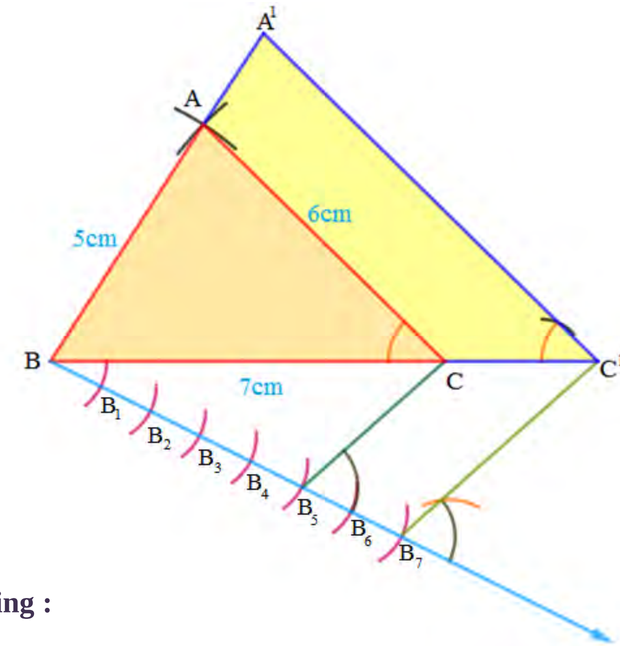
x_1	y_1	x_2	y_2	x_3	y_3
−1	−3	4	−1.5	0	0.5

ΔPQR ನ ವಿಸ್ತೀರ್ಣ $= \frac{1}{2}[-1(-1.5-0.5)+\ 4(0.5+3)+0(-3+1.5)]$

$= \frac{1}{2}[-1(-2)+\ 4(3.5)+0] = \frac{1}{2}[2+\ 14] = \frac{1}{2}[16] =$ **8 sq.units ------(2)**

From (1) and (2), Area of ΔABC = 4Area of ΔPQR

36. Construct a triangle with sides 5cm, 6cm, and 7cm and then construct another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.



V. Answer the following : 4 × 4= 16

37. Find the solution of the following pairs of linear equation by the graphical method :

2x + y = 6

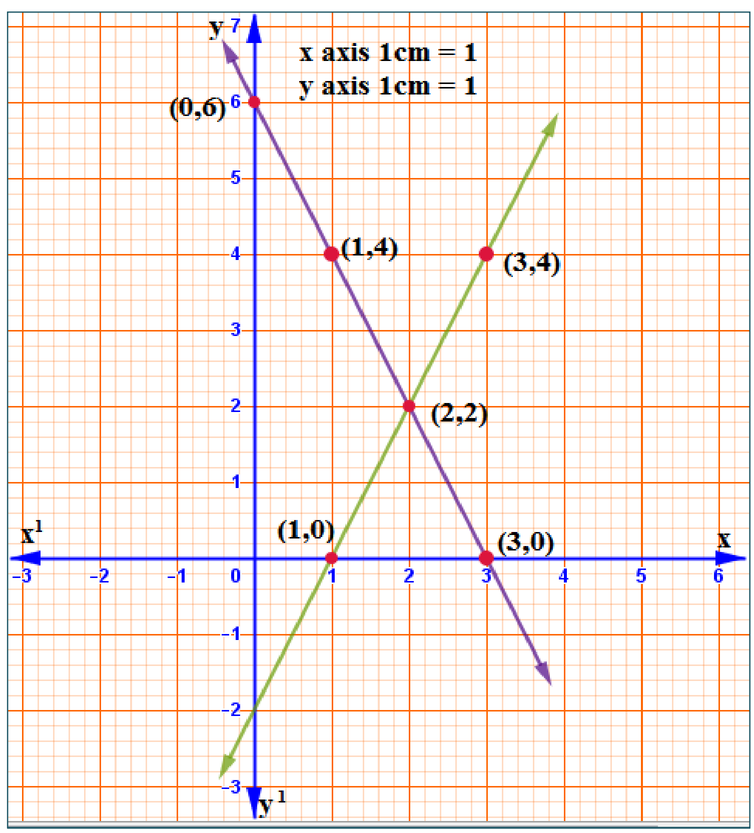
2x – y = 2

Ans: 2x + y = 6

x	0	1	2
y	6	4	2

2x - y = 2

x	1	2	3
y	0	2	4



ಪರಿಹಾರ: x = 2; y = 2

38. The angles of elevation of the top of a tower from two points at a distance of 4m and 9m from the base of the tower and in the same straight line with it are complimentary. Find the height of the tower.

AB is the height of the tower

C and D are the points 4 m and 9 m from the base of the tower

$\tan x = \frac{AB}{BC}$

$\Rightarrow \tan x = \frac{AB}{4}$ ----------------- (1)

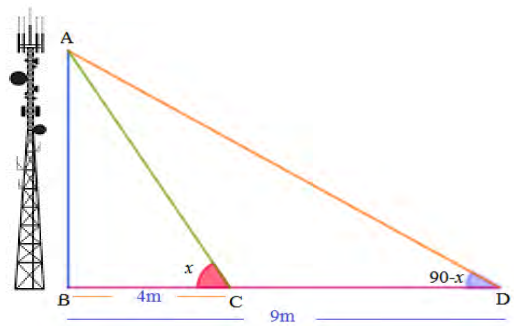
$\tan (90°-x) = \frac{AB}{BD}$

$\Rightarrow \cot x = \frac{AB}{9} \Rightarrow \tan x = \frac{9}{AB}$ -----(2)

From eqn (1) and (2)

$\frac{AB}{4} = \frac{9}{AB} \Rightarrow AB^2 = 36$

$\Rightarrow AB = 6m$



39. The bottom of a right cylindrical shaped vessel as shown in the figure. The radius of the circular base of the cylinder and radius of the circular base of the cone are each is equal to 7cm. if the height of the cylinder is 20cm and height of cone is 3cm, calculate the cost of milk to fill completely this vessel at the rate of Rs. 20 per liter.

Ans:The cost of the Milk = The quantity of the milk x Rs 20

The quantity of Milk =

[The Volume of the cylinder – The volume of the Cone]

$= \left[\pi r^2 H - \frac{1}{3}\pi r^2 h\right]$ x Rs20

$= \pi r^2 \left[H - \frac{1}{3}h\right]$

$= \frac{22}{7} x7x7 \left[20 - \frac{1}{3} x3\right]$

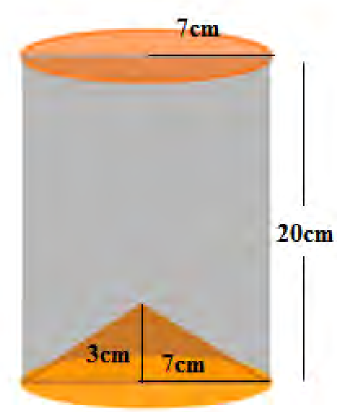
$= 22\ x7[20 - 1]$

$= 154[19]$

$= 2926 = 2.926$ ltr

The total cost $= 2.926$ x Rs 20

$= \boldsymbol{Rs\ 58.52}$



OR

A hemispherical vessel of radius 14cm is fully filled with sand. This sand is poured on a level ground. The heap of sand forms a cone shape of height 7cm. calculate the area of ground occupied by the circular base of the heap of the sand.

Ans: The volume of the hemisphere = The volume of the Cone

$\Rightarrow \frac{2}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$

$\Rightarrow 2R^3 = r^2 h$

$\Rightarrow 2(14)^3 = 7r^2$

$\Rightarrow 2x\ 2744 = 7r^2$

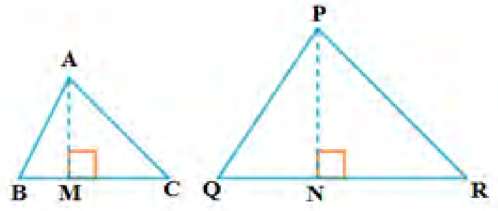
$\Rightarrow r^2 = 784 \Rightarrow r = 28cm$

The area of the circular Base $= \pi r^2$

$= \frac{22}{7}$ x 28 x28 $\Rightarrow$ 22 x 28 x 4

$= \mathbf{2464\ cm^2}$

40. Prove that " The areas of two similar triangles is equal to the square of the ratio of their corresponding sides".



Given: $\Delta ABC \sim \Delta PQR$

To Prove: $\frac{Area(ABC)}{Area(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{PR}\right)^2$

Construction: Draw $AM \perp BC$ and $PN \perp QR$

Proof: $\frac{Area(ABC)}{Area(PQR)} = \frac{\frac{1}{2} x\ BC\ x\ AM}{\frac{1}{2} x\ QR\ x\ PN} = \frac{BC\ x\ AM}{QR\ x\ PN}$ - - -(1) [Area of triangle=$\frac{1}{2}$xbasexheight]

In ΔABM and ΔPQN,

$\angle B = \angle Q$ [Corresponding angles of the similar triangle]

$\angle M = \angle N = 90^0$ [Construction]

$\therefore \Delta ABM \sim \Delta PQN$ [AA similarity criteria]

$\Rightarrow \frac{AM}{PN} = \frac{AB}{PQ}$ - - - - - - (2)

But, $\Delta ABC \sim \Delta PQR$ [Given]

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{PR}$ - - - - (3)

$\Rightarrow \frac{AM}{PN} = \frac{BC}{QR}$ [From (2) and (3)]

$\therefore \frac{Area(ABC)}{Area(PQR)} = \frac{BC}{QR} x \frac{BC}{QR}$ - -------- [From (1) and (3)]

$\Rightarrow \frac{Area(ABC)}{Area(PQR)} = \left(\frac{BC}{QR}\right)^2$

$\frac{Area(ABC)}{Area(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{PR}\right)^2$ [From (3)]

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