SET-2

Series BVM/2

Code No. **55/2/2**

Roll No. | | | | | | | |

Candidates must write the Code on the title page of the answer-book

PHYSICS (Theory) & SOLUTION

Time allowed : 3 hours **Maximum Marks : 70**

General Instructions :

(i) ***All*** *questions are* ***compulsory.*** *There are* ***27*** *questions in all.*

(ii) *This question paper has* ***four*** *sections : Section A, Section B, Section C, and Section D.*

(iii) *Section A contains five questions of one mark each, Section B contains seven questions of two marks each, Section C contains twelve question of three marks each, Section D contains three questions of five marks each.*

(iv) *There is no overall choice. However, an internal choice has been provided in two question of one marks, two question of two marks, four questions of three marks and three questions of five marks weightage. You have to attempt only one of the choices in such questions.*

(v) *You may use the following values of physical constants wherever necessary :*

$c = 3\times10^{8}$ m/s

$h = 6.63\times10^{-34}$ Js

$e = 1.6\times10^{-19}$ C

$\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}

$\varepsilon_0 = 8.854 \times10^{-12}$ C^2 N^{-1} m^{-2}

$\frac{1}{4\pi\varepsilon_0} = 9\times10^{9}$ $Nm^2 C^{-2}$

Mass of electron = 9.1×10^{-31} kg

Mass of neutron = 1.675×10^{-27} kg

Mass of proton = 1.673×10^{-27} kg

Avogadro's number = 6.023×10^{23} per gram mole

Boltzmann constant = 1.38×10^{-23} JK^{-1}

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SECTION A

1. Write the relation for the force acting on a charged particle q moving with velocity $\vec{v}$ in the presence of a magnetic filed $\vec{B}$ **.**

Ans. $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$

2. The magnetic susceptibility of magnesium at 300 K is 1.2×10^5. At what temperature will its magnetic susceptibility become 1.44×10^5 ?

Ans. $x \propto \frac{1}{T}$ given $T_1 = 300$ K $\qquad X_1 = 1.2 \times 10^5$

$$\frac{X_1}{X_2} = \frac{T_2}{T_1}$$

$$T_2 = \frac{X_1}{X_2} \times T_1 = \frac{1.2 \times 10^5}{1.44 \times 10^5} \times 300$$

$T_2 = 249.99 \text{ K} \approx 250 \text{ K}$

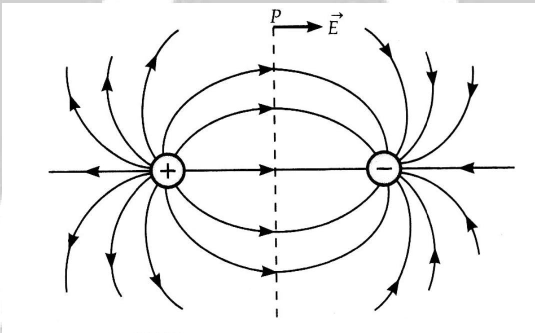
OR

The magnetic susceptibility x of a given material is – 0.5 Identify the magnetic material.

Ans. Diamagnetic material

3. Draw the pattern of electric filed lines due to an electric dipole.

Ans.



4. Which part of the electromagnetic spectrum is used in RADAR ? Give its frequency range.

Ans. Micro waves

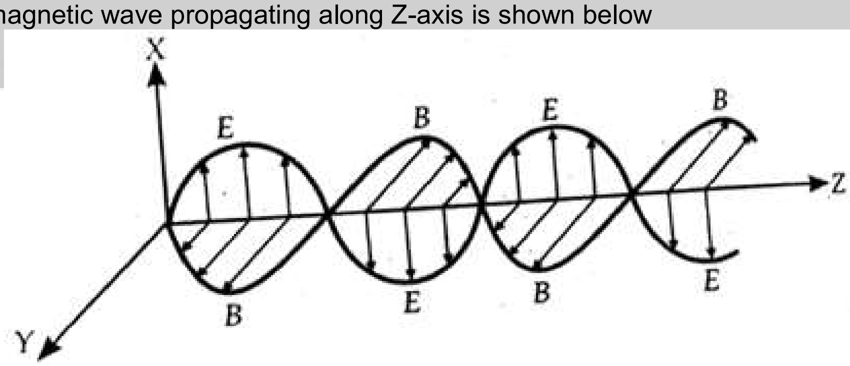
Frequency range = 10^9 to 10^{12} Hz

OR

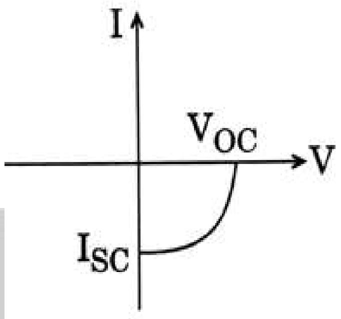
How are electromagnetic waves produced by accelerating charges?

Ans. A charge q oscillating at a frequency v produces oscillating electric field, which produces oscillating magnetic field, which in turn produces oscillating electric field, and so on. Hence an electromagnetic wave of frequency v originates from the charge q.

An electromagnetic wave propagating along Z-axis is shown below



5. Identify the semiconductor diode whose I – V characteristics are as shown.



Ans. Photo-diode

SECTION B

6. A photon and a proton have the same de – Broglie wavelength λ. Prove that the energy of the photon is $(2m\lambda c / h)$ times the kinetic energy of the proton.

Ans. For a particle of mass m,

$$K = \frac{p^2}{2m} \text{ and } \quad p = \frac{h}{\lambda}$$

$$\therefore \quad K = \frac{h^2}{2m\lambda^2}$$

Also, $E_{photon} = \frac{hc}{\lambda}$

$$\therefore \quad \frac{K}{E_{photon}} = \frac{h}{2mc\lambda}$$

or $E_{photon} = \left(\frac{2\lambda mc}{h}\right)K.$

7. How is the equation for Ampere's circuital law modified in the presence of displacement current? Explain.

Ans. Displacement current is that current which comes into existence, in addition to the conduction current, whenever the electric field and hence the electric flux changes with time.

To maintain the dimensional consistency, the displacement current is give the form :

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

Where ϕ_E = electric field × area = EA, is the electric flux across the loop.

$\therefore$ Total current across the closed loop

$$= I_c + I_d = I_c + \varepsilon_0 \frac{d\phi_E}{dt}$$

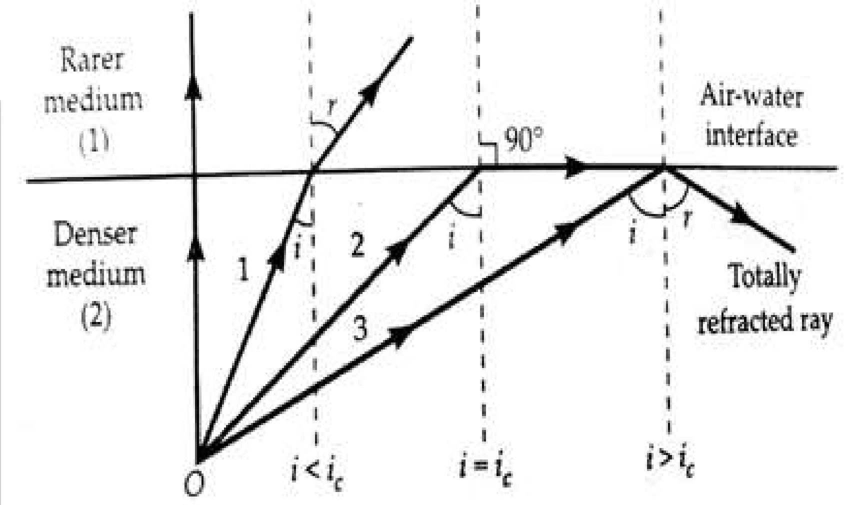
Hence the modified form of the Ampere's law is

$$\oint \vec{B}.\vec{dl} = \mu_0 \left[I_c + \varepsilon_0 \frac{d\phi_E}{dt} \right] \qquad \text{............(5)}$$

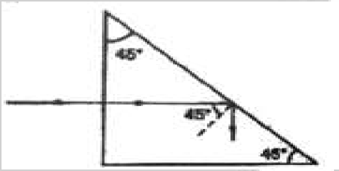
8. Under what conditions does the phenomenon of total internal reflection take place? Draw a ray diagram showing how a ray of light deviates by 90° after passing through a right – angled isosceles prism.

Ans. (i)Total internal Reflection

When a ray of light travelling from denser medium to rarer medium is incident at the interface of two medium at an angle greater than the critical angle for the two media, the ray is totally reflected back to denser medium. This phenomenon is called Total Internal Reflection.



(ii)



9. A photon emitted during the de – excitation of electron from a sate n to the first excited state in a hydrogen atom, irradiates a metallic cathode of work function 2eV, in a photo cell, with a stopping potential of 0.55 V. Obtain the value of the quantum number of the state n.

Ans. $\phi = 2eV$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$E = \phi + K.E = \frac{hc}{\lambda} = hcR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \phi + K.E$$

K.E = eV_0

$n_1 = 2$, $n_2 = n$

$$= hcR\left(\frac{1}{(2)^2} - \frac{1}{n_2^2}\right) = 2 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19} \times 0.55$$

$$6.62 \times 10^{-34} \times 3 \times 10^8 \times 1.097 \times 10^7 \left[\frac{1}{4} - \frac{1}{n^2}\right] = 3.2 \times 1.6 \times 0.55 \times 10^{-38}$$

$$\frac{1}{4} - \frac{1}{n^2} = \frac{3.2 \times 1.6 \times 0.55 \times 10^{-38}}{6.62 \times 10^{-34} \times 3 \times 1.097 \times 10^{15}} = \frac{2.86 \times 10^{-38}}{21.786 \times 10^{-19}}$$

$$\frac{1}{4} - \frac{1}{n^2} = 0.129 \times 10^{-19}$$

$$\frac{1}{4} - \frac{1}{n^2} = 1.13 \times 10^{-20}$$

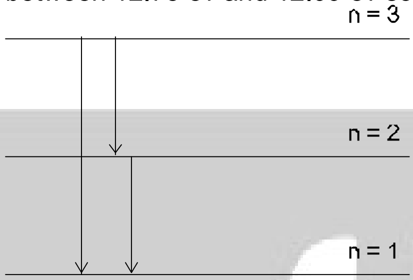
By solving n = 2

OR

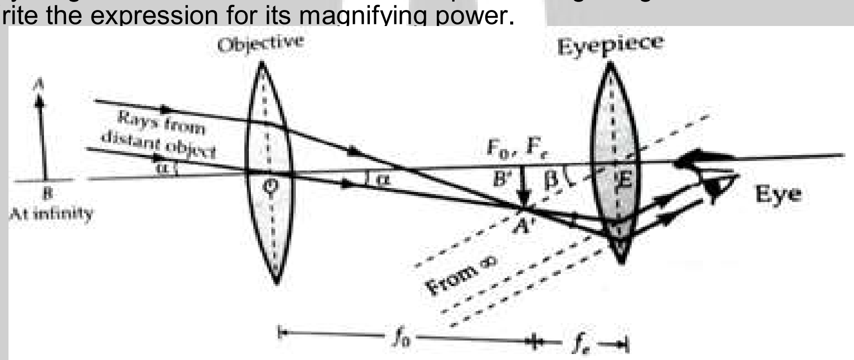
A hydrogen atom in the ground state is excited by an electron beam of 12.5 eV energy. Find out the maximum number of lines emitted by the atom from its excited state.

Ans. Given $E_n = 12.5ev$

This energy lies between 12.75 ev and 12.09 ev so, total number of transitions = 3



10. Draw the ray diagram of an astronomical telescope showing image formation in the normal adjustment position. Write the expression for its magnifying power.



$\therefore$ Magnifying power,

$$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} \quad \because \quad [\alpha, \beta \text{ are small angles}]$$

$$\frac{A'B'/B'E}{A'B'/OB'} = \frac{OB'}{B'E}$$

Applying new Cartesian sign convention,

$OB' = f_0 =$ Distance of $A'B'$ from the objective along the incident light

BE = $-f_e$ = Distance of $A'B'$ from the eyepiece against the incident light

$$m = -\frac{f_0}{f_e}$$

OR

Draw a labelled ray diagram to show image formation by a compound microscope and write the expression for its resolving power.

Ans.

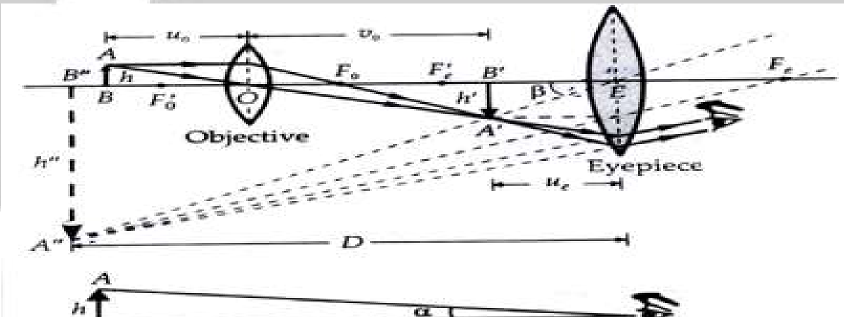
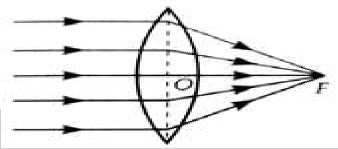


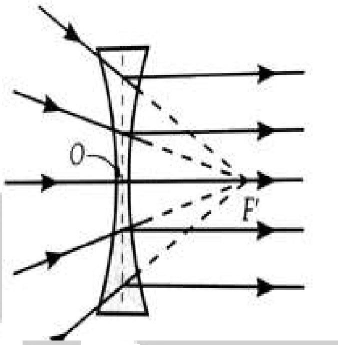
Fig. Compound Microscope, final image at D.

Resolving power of a microscope $= \frac{2\mu \sin\theta}{\lambda}$

11. A beam of light converges at a point P. Draw ray diagrams to show where the beam will converge if (i) a convex lens, and (ii) a concave lens is kept in the path of the beam.

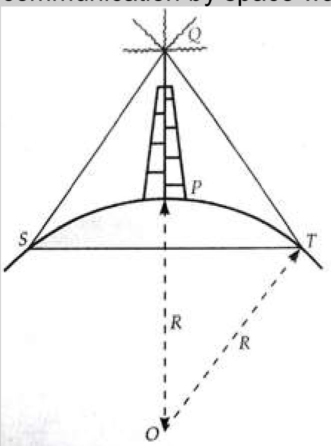
Ans. **(i) Convex lens** **(ii) Concave lens**





12. Write the relation between the height of a TV antenna and the maximum range up to which signals transmitted by the antenna can be received. How is this expression modified in the case of line of sight communication by space waves? In which range of frequencies, is this mode of communication used?

Ans.



PQ = h_T
PS = PT = d.
$d^2 = 2h_T R$

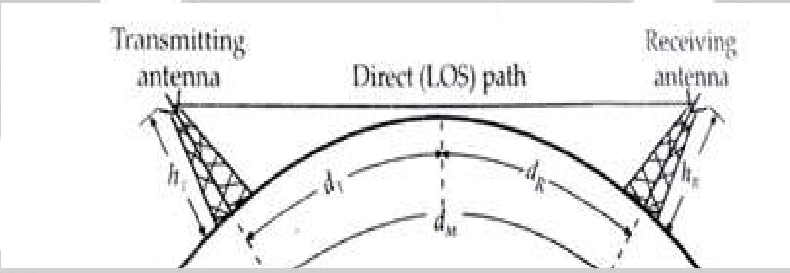


Figure shows the effect of antenna height on the radio horizon. The line of sight radio horizon for a single antenna at sea level is given by

$$d = \sqrt{2Rh}$$

Where d = distance to radio horizon
h = antenna height above sea level
R= Radius of the earth

The maximum line of sight distance d_M

$$d_M = d_T + d_R = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

Where d_T = radio horizon for transmitting antenna
d_R = radio horizon for receiving antenna
h_R = height of transmitting antenna
h_T = height of receiving antenna

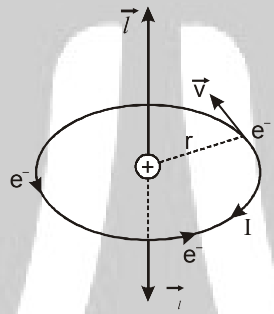
Range of frequency is 100 – 220 MHz

SECTION C

13. Prove that the magnetic moment of the electron revolving around a nucleus in an orbit of radius r with orbital speed v is equal to evr/2. Hence using Bohr's postulate of quantization of angular momentum, deduce the expression for the magnetic moment of hydrogen atom in the ground state.

Ans. According to Bohr model of hydrogen-like atoms, negatively charged electron revolves around the positively charge nucleus. This uniform circular motion of the electron is equivalent to a current loop which possesses a magnetic dipole moment of = IA.

Consider an electron revolving anticlockwise around a nucleus in an orbit of radius r with speed v and time period T.



Orbital magnetic moment of a revolving electron

Equivalent current,

$$I = \frac{\text{Charge}}{\text{Time}} = \frac{e}{T} = \frac{e}{2\pi r / v} = \frac{ev}{2\pi r}$$

Area of the current loop, A = πr^2

Therefore, the orbital magnetic moment (magnetic moment due to orbital motion) of the electron is :

$$m_l = \frac{evr}{2}$$

As the negatively charged electron is revolving anticlockwise, the associated current flows clockwise. According to right hand thumb rule, the direction of the magnetic dipole moment of the revolving electron will be perpendicular to the plane of its orbit and in the downward direction.

Also, the angular momentum of the electron due to its orbital motion is

$l = m_e vr$ .... (2)

The direction of $\vec{l}$ is normal to the plane of the electron orbit and in the upward direction, as shown in Figure.

Dividing equation (1) by (2), we get

$$\frac{m_l}{l} = \frac{evr/2}{m_e vr} = \frac{e}{2m_e}$$

The above ratio is a constant called gyromagnetic ratio. Its value is 8.8×10^{10} C kg^{-1}. So

$$\mu_l = \frac{e}{2m_e} l$$

Vector Form,

$$\vec{\mu}_l = -\frac{e}{2m_e}\vec{l}$$

The negative sign shows that the direction of $\vec{l}$ is opposite to that of $\vec{\mu}_l$. According to Bohr's quantisation condition, the angular momentum of an electron in any permissible orbit is integral multiple of $h/2\pi$, where h is Planck's constant, i.e.,

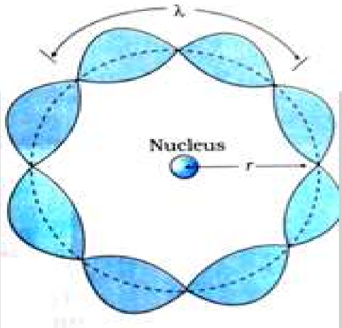
$$l = \frac{nh}{2\pi}, \text{ where } n = 1, 2, 3, \ldots\ldots\ldots\ldots$$

$$\therefore \mu_l = n\left(\frac{eh}{4\pi m_e}\right)$$

This equation gives orbital magnetic moment of an electron revolving in n^{th} orbit.

14. (a) How is the stability of hydrogen atom in Bohr model explained by de – Broglie's hypothesis?
(b) A hydrogen atom initially in the ground state absorbs a photon which excites it to n = 4 level. When it gets de- excited, find the maximum number of lines which are emitted by the atom. Identify the series to which these lines belong. Which of them has the shortest wavelength?

Ans. (a)



According to de – broglie, e^- can move in the orbit when circumference of orbit of e^- is equal to integral multiple of wavelength that is

$2\pi r = n\lambda$ ……….(1)

According to de – broglie, hypothesis the wavelength associate with e^- moving with velocity (V)

$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{h}{mv} \text{ ……….(2)}$$

By eq. 1

$$2\pi r = n\lambda$$

$$2\pi r = \frac{nh}{mv}$$

$$mvr = \frac{nh}{2\pi}$$

Hence de – broglie, proves bohr's second postulate that is orbit are quantized in the atom

(b) $n = 1, n = 4$

$$E_1 = \frac{-13.6}{(1)^2} = -13.6\text{eV}$$

$$E_4 = \frac{-13.6}{(4)^2} = -0.85 \text{ eV}$$

$$\Delta E = E_4 - E_1$$

$$= -0.85 - (13.6) = 12.75\text{eV}$$

$$= 12.75 \times 1.6 \times 10^{-19} \text{J}$$

$$\Delta E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{12.75 \times 1.6 \times 10^{-19}}$$

= 0.975×10^{-7}m

$= 975A^0$ Belong to Lyman series

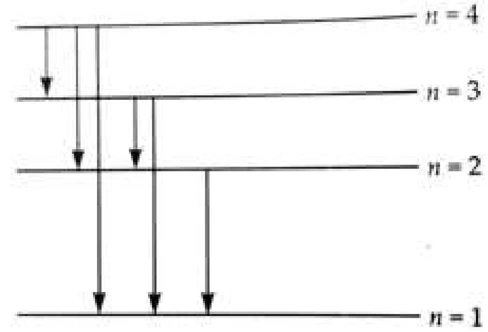
The possible transitions are

$n_i = 4$ to $n_f = 3, 2, 1$

$n_i = 3$ to $n_f = 2, 1$

$n_i = 2$ to $n_f = 1$

$\therefore$ Total number of transitions = 6, as shown in Fig.



15. A capacitor (C) and resistor (R) are connected in series with an ac source of voltage of frequency 50 Hz. The potential difference across C and R are respectively 120 V, 90 V, and the current in the circuit is 3 A. Calculate (i) the impedance of the circuit (ii) the value of the inductance, which when connected in series with C and R will make the power factor of the circuit unity.

Ans. $\varepsilon_{rms} = \sqrt{V_R^2 + V_C^2}$

$= \sqrt{90^2 + 120^2}$

= 150 Volt

(i) Impedance, $Z = \frac{\varepsilon_{rms}}{I_{rms}} = \frac{150}{3} = 50\Omega$.

(ii) $\because$ $V_c = IX_c$

$V_c = I\frac{1}{\omega c} \Rightarrow c = \frac{I}{\omega V_c}$ ....(1)

$L = \frac{1}{\omega^2 c}$ ....(2)

From eq. 1 & 2

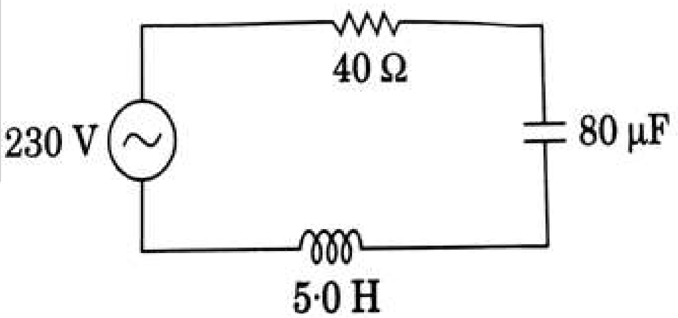
$L = \frac{V_c}{\omega I} = \frac{120}{2\pi f I}$

$L = \frac{120}{2 \times 3.14 \times 50 \times 3}$

$L = 0.127$ H

OR

The figure shows a series LCR circuit connected to a variable frequency 230 V source.



(a) Determine the source frequency which drives the circuit in resonance.

(b) Calculate the impedance of the circuit and amplitude of current at resonance.

(c) Show that potential drop across LC combination is zero at resonating frequency.

Ans. Here V_{rms} = 230 V, L = 5.0 H
= 80μF = 80 × 10–6 F, R = 40Ω

(a) The resonant angular frequency is

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(5\times 80\times 10^{-6})}} = 50\,\text{rad}\,s^{-1.}$$

(b) At $\omega = \omega_r, \omega L = \frac{1}{\omega C}$, therefore,

Impedance, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$ = R = 40Ω

Current amplitude

$$= \frac{V_0}{Z} = \frac{V_0}{R}(\text{at resonance})$$

$$= \frac{\sqrt{2}\ V_{rms}}{R} = \frac{1.414\times 230}{40} = 8.1\text{ A.}$$

(c) $I_{rms} = \frac{V_{rms}}{R} = \frac{230}{40} = \frac{23}{4}A$

∴ P.D. across L,

$$V_{rms}^{L} = I_{rms}\times\omega_r L = \frac{23}{4}\times 50\times 5 = 1437.5V$$

P.D. across C,

$$V_{rms}^{C} = I_{rms}\times\frac{1}{\omega_r C}$$

$$\frac{23}{4}\times\frac{1}{50\times 80\times 10^{-6}} = 1437.5V$$

P.D. across the LC – combination at the resonant frequency

$$= I_{rms}\left(\omega_r L - \frac{1}{\omega_r C}\right) = 0$$

P.D. across R,

$$V_{rms}^{R} = I_{rms}\times R = \frac{23}{4}\times 40 = 230\,V$$

= Applied rms voltage,

This is because the total potential drop across the LC- combination is zero

16. What is the reason to operate photodiodes in reverse bias ?
A p-n photodiode is fabricated from a semiconductor with a band gap of range of 2.5 to 2.8 eV Calculate the range of wavelengths of the radiation which can be detected by the photodiode.

Ans. (i) The fractional increase in majority carries is much less than the fractional increase in minority carriers. Consequently, the fractional change due to the photo – effects on the minority carrier dominated reverse bias current is more easily measurable than the fractional change in the majority carrier dominated forward bias current. Hence, photodiodes are preferable used in the reverse bias condition for measuring light intensity.

(ii) $\lambda_1 = \frac{hc}{E_1} = \frac{6.62\times 10^{-34}\times 3\times 10^{8}}{2.5\times 1.6\times 10^{-19}} = 4950A^0$

$$\lambda_2 = \frac{hc}{E_2} = \frac{6.62\times 10^{-34}\times 3\times 10^{8}}{2.8\times 1.6\times 10^{-19}} = 4410A^0$$

Range of wavelengths = $4410A^0 - 4950A^0$

17. Draw a labelled diagram of cyclotron. Explain its working principle. Show that cyclotron frequency is independent of the speed and radius of the orbit.

Ans. Cyclotron :

it is a device used to accelerate charged particles like protons, deuterons, α-particles, etc, to very high energies.

Principle :

A charged particle can be accelerated to high speeds (energies) by passing it through electric field many number of times and at the same time magnetic field makes the charged particle to move in a circular path.

Theory:

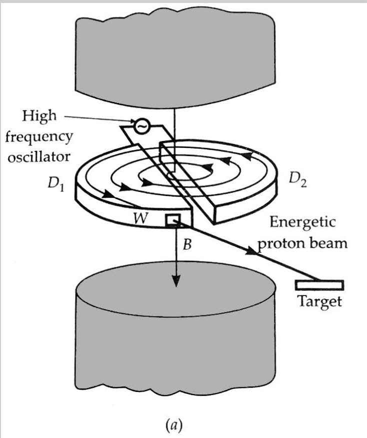
Let a particle of charge q and mass m enters a region of magnetic field $\vec{B}$ with a velocity $\vec{v}$, normal to the field $\vec{B}$. The particle follows a circular path, the necessary centripetal force is provided by the magnetic field. Therefore,

Magnetic force on charge q = Centripetal force on charge q

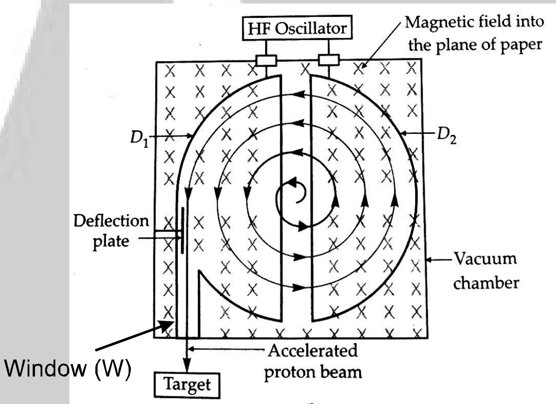
or $qv B \sin 90° = \frac{mv^2}{r}$ or $\boxed{r = \frac{mv}{qB}}$

Period of revolution of the charged particle is given by

$$\boxed{T = \frac{2\pi r}{v} = \frac{2\pi}{v}, \frac{mv}{qB} = \frac{2\pi m}{qB}}$$



(a)



(b)

Hence frequency of revolution of the particle will be

$$\boxed{f_c = \frac{1}{T} = \frac{qB}{2\pi m}}$$

Clearly, this frequency is independent of both the velocity of the particle and the radius of the orbit and is called cyclotron frequency or magnetic resonance frequency. This is the key fact which is used in the operation of a cyclotron.

Working :

• Suppose a positive ion, say a proton, enters the gap between the two dees and finds dee D_1 to be negative. It gets accelerated towards dee D_1.

• As it enters the dee D_1, it does not experience any electric field due to shielding effect of the metallic dee. The perpendicular magnetic field throws it into a circular path with constant speed.

• At the instant the proton comes out of dee D_1, it finds dee D_1 positive and dee D_2 negative. It now gets accelerated towards dee D_2.

• It moves faster through D_2 describing a larger semicircle than before.

• Thus if the frequency of the applied voltage is kept exactly the same as the frequency of revolution of the proton, then every time the proton reaches the gap between the two dees, the electric field is reversed and proton receives a push and finally it acquires very high energy.

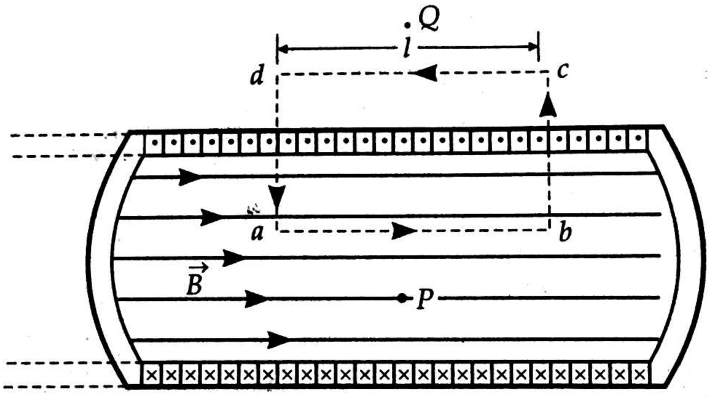
• This condition in which frequency of applied voltage is equal to the frequency of revolution of charged particle is called the cyclotron's resonance condition.

• The accelerated proton is ejected through a window by a deflecting voltage and hits the target.

OR

(a) Derive, with the help of a diagram, the expression for the magnetic field inside a very long solenoid having n turns per unit length carrying a current I.

Ans. Calculation of magnetic field inside a long straight solenoid :



The magnetic field of a very long solenoid

The magnetic field inside a closely wound long solenoid is uniform everywhere and zero outside it. Figure shows the sectional view of a long solenoid. At various turns of the solenoid, current comes out of the plane of paper at points marked (integration) and enters the plane of paper at points marked $\otimes$.

To determine the magnetic field $\vec{B}$ at any inside point consider a rectangular closed path (Amperean loop) abcd as the Amperean loop. According to Ampere's circuital law,

$\oint \vec{B}\vec{dl} = \mu_0 \times$ Total current through the loop abcd .... (i)

Now $\oint \vec{B}.\vec{dl} = \int_b^c \vec{B}.\vec{dl} + \int_d^a \vec{B}.\vec{dl}$

But $\int_b^c \vec{B}.\vec{dl} + \int_d^c \vec{B}.\vec{dl} \cos 90^\circ = 0$

$\int_c^d \vec{B}.\vec{dl} = 0$ as B = 0 for points outside the solenoid.

$\therefore \oint \vec{B}.\vec{dl} = \int_a^b \vec{B}.\vec{dl}$

$\int_a^b B dl \cos 0^\circ = B\int_a^b dl = Bl$

where,

l = length of the side ab of the rectangular loop abcd.

Let number of turns per unit length of the solenoid = n

Then number of turns in length l of the solenoid = nl

Thus the current I of the solenoid threads the loop abcd, nl times.

$\therefore$ Total current threading the loop abcd = n l I .... (iii)

Put (ii) and (iii) in (i)

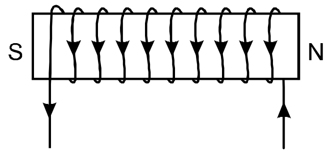
The magnetic field at the end of the solenoid is just one half of that at its middle.

So,

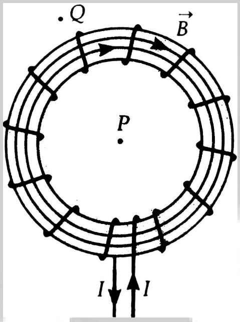
$$B_{end} = \frac{1}{2}\mu_0 n I$$

(b) How is a toroid different from a solenoid ?

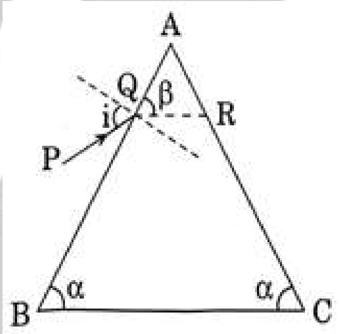
Ans. A solenoid means an insulated copper wire wound closely in the form of a helix such that the length of solenoid is very large as compared to its diameter.



A solenoid bent into the form of a closed ring is called a toroidal solenoid or A solenoid infinite length is called a toroid.



18. A ray of light incident on the face AB of an isosceles triangular prism makes an angle of incidence (i) and deviates by angle β as shown in the figure. Show that in the position of minimum deviation, $\angle\beta = \angle\alpha$. Also find out the condition when the refracted ray QR suffers total internal reflection.



Ans. here $\alpha = 60°$ (for isosceles triangle)

Now $r_1 = 90° - \beta$ from diagram

and $r_2 = \beta - 30$ from diagram

for minimum deviation

$r_1 = r_2$

$90° - \beta = \beta - 30$

$2\beta = 120$

$\beta = 60° = \alpha$ H.P

For TIR

$r_2 = I_c$

$r_2 = 30°$

Now from condition of TIR

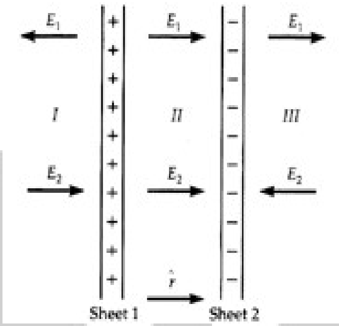
$\frac{1}{\sin i_c} \le \mu$

$\frac{1}{\sin 30°} \le \mu$

$\mu >, 2$

19. Two large charged plane sheets of charge densities σ and $-2\sigma\ C/m^2$ are arranged vertically with a separation of d between them. Deduce expression for the electric field at points (i) to the left of the first sheet, (ii) to the right of the second sheet, and (iii) between the two sheets.

Ans.



(a) E.F. left of the plate 1(region I)

$$\vec{E}_I = \vec{E}_I + \vec{E}_2 = -\frac{\sigma}{2\varepsilon_0}\hat{r} + \frac{2\sigma}{2\varepsilon_0}\hat{r}$$

(b) II (between the plates)

$$\vec{E}_{II} = -\frac{\sigma}{2\varepsilon_0}\hat{r} + \frac{2\sigma}{2\varepsilon_0}\hat{r}$$

(c) Region III (Right of plate 2)

$$\vec{E}_{III} = -\frac{\sigma}{2\varepsilon_0}\hat{r} - \frac{2\sigma}{2\varepsilon_0}\hat{r}$$

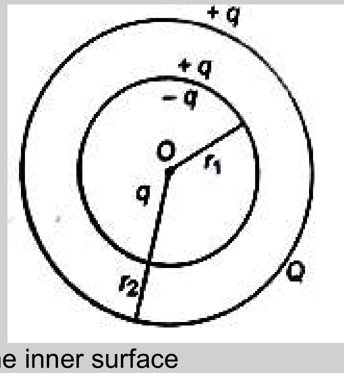
OR

A spherical conducting shell of inner radius r_1 and outer r_2 has a charge Q.

(a) A charge q is placed at the centre of the shell. Find out the surface charge density on the inner and outer surface of the shell.

(b) Is the electric filed inside a cavity (with no charge) zero; independent of the fact whether the shell is spherical or not? Explain.

Ans. (a) The charge + Q resides on the outer surface of the shell. As the charge q is placed at the centre of the shell, there is charge –q induced on the inner surface and a charge + q is induced on the outer surface of the shell. Thus, the total charge on the inner surface of the shell is – q and on the outer surface of the shell is (Q+ q).



The surface charge density on the inner surface

$$\sigma_1 = -\frac{q}{4\pi r_1^2}$$

The surface charge density on the outer surface

$$\sigma_2 = -\frac{Q+q}{4\pi r_2^2}$$

(c) Yes, the electric field inside a cavity is zero, if the shape of the shell is not spherical. If we take a closed – loop which is inside the cavity along a field line and the rest outside it, the net work done by the field in carrying a test charge over the closed – loop will not be zero. So, the electric field inside a cavity with no charge is always zero.

20. A signal of low frequency f_m is to be transmitted using a carrier wave of frequency f_c. Derive the expression for the amplitude modulated wave and deduce expressions for the lower and upper sidebands produced. Hence, obtain the expression for modulation index.

Ans. Let modulating signal, $m(t) = A_m \sin\omega_m t$

And carrier signal, $c(t) = A_c \sin\omega_c t$.

Where, m(t) = instantaneous voltage of modulating wave

$A_m =$ amplitude of modulating wave

$\omega_m = 2\pi f_m$ = angular frequency of modulating wave

$\omega_c t =$ instantaneous voltage of carrier wave

$A_c =$ amplitude of carrier wave

$\omega_c = 2\pi f_c$ = angular frequency of carrier wave.

The amplitude of the carrier wave varies at the frequency ω_m of the modulating wave. So amplitude of the modulated wave is given by

$A = A_c + m(t) = A_c + A_m \sin\omega_m t$

$= A_c + \mu A_c \sin\omega_m t = A_c(1 + \mu \sin\omega_m t)$

Here $\mu = \frac{A_m}{A_c}$ is the modulation index.

The sum of the carrier frequency and the modulating frequency $(f_c + f_m)$ is called upper side band frequency (USB). Thus

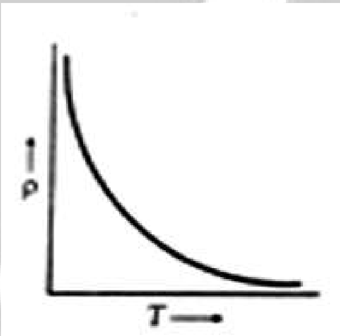
$USB = f_c + f_m$

The difference of the carrier frequency and the modulating frequency $(f_c - f_m)$ is called lower side band frequency (LSB). Thus

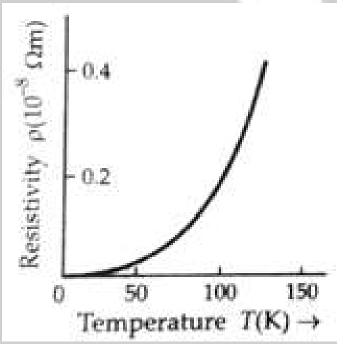
$LSB = f_c - f_m$

21. Show, on a plot, variation of resistivity of (i) a conductor, and (ii) a typical semiconductor as a function of temperature.
Using the expression for the resistivity in terms of number density and relaxation time between the collisions, explain how resistivity in the case of a conductor increases while it decreases in a semiconductor, with the rise of temperature.

Ans. Temperature dependence of resistivity. The resistivity of any material depends on the number density n of free electrons and the mean collision time τ.



For conductors



For semiconductors

$\rho = \frac{m}{ne^2\tau}$

1. **Conductors :** For conductors, the number density n of free electrons is almost independent of temperature. As temperature increases, the thermal speed of free electrons increases and also the amplitude of vibration of the metal ions increases. Consequently, the free electrons collide more frequently with the metal ions. The mean collision time τ decreases. Hence the resistivity of a metal $(\rho \propto 1/\tau)$ increases and the conductivity decreases with the increases in temperature.
2. **Semiconductors :** In case of insulators and semiconductors, the relaxation time τ does not change with temperature but the number density of free electrons increases exponentially with the increase in temperature. Consequently, the conductivity increases or resistivity decreases exponentially with the increase in temperature.

22. A 100 μF parallel plate capacitor having plate separation of 4 mm is charged by 200 V dc. The source is now disconnected. When the distance between the plates is doubled and a dielectric slab of thickness 4 mm. and dielectric constant 5 is introduced between the plates, how will (i) it capacitance, (ii) the electric field between the plates, and (iii) energy density of the capacitor get affected ? Justify your answer in each case.

Ans. Given, $c = 100\mu F$, d = 4 mm , t = 4mm, v = 200 V,

(i) $d' = 2d,\ K = \varepsilon_r = 5$

$$C = \frac{\varepsilon_0 A}{d} \Rightarrow 100 \times 10^{-6} = \frac{\varepsilon_0 A}{d} \Rightarrow \quad A = \frac{100 \times 10^{-6} \times d}{\varepsilon_0}$$

$$= \frac{\varepsilon_0 A}{2d - t + \frac{t}{K}}$$

$$\Rightarrow C' = 83.33\mu F$$

(ii) Charge on capacitor plates, when 200 V p.d is applied, becomes

$$q = C_0 V_0$$

$$= 100 \times 10^{-6} \times 200$$

$$q = 2 \times 10^{-2} c$$

Even after the battery is removed, the charge of $2 \times 10^{-2} c$ on the capacitor plates remains the same.

So, $C_0 V_0 = CV$

$$V = \frac{C_0 V_0}{C}$$

$$V = \frac{100 \times 10^{-6}}{83.33 \times 10^{-6}} \times 200$$

V = 240 V

$$E_0 = \frac{V_0}{d}$$

$$= \frac{200}{4 \times 10^{-3}}$$

$$E_0 = 50 \times 10^3 \ \frac{V}{m}$$

$$E' = \frac{V}{2d}$$

$$E' = \frac{240}{2 \times 4 \times 10^{-3}}$$

$$E' = 30 \times 10^3 \ \frac{V}{m}$$

(iii) $\overline{U} = \frac{1}{2} \varepsilon_0 E_0^2$

$$\overline{U} = \frac{1}{2} \times 8.854 \times 10^{-12} \times \left(50 \times 10^3\right)^2$$

$$\overline{U} = 11067 \times 10^{-6} J/m^3$$

$$\left(\overline{U}\right)' = \frac{1}{2} \varepsilon_0 \left(E'\right)^2$$

$$\overline{U} = \frac{1}{2} 8.854 \times 10^{-12} \left(30 \times 10^3\right)^2$$

$$\overline{U} = 3984.3 \times 10^{-6} \ \frac{J}{m^3}$$

23. Why is it difficult to detect the presence of an anti-neutrino during β – decay? Define the term decay constant of a radioactive nucleus and derive the expression for its mean life in terms of the decay constant.

Ans. (i) Beta minus (β^-) decay : It is the process of spontaneous emission of an electron from a nucleus.

• In this a neutron (n) changes into a proton (p), releasing an electrons (e^-) (Called β^- particle) and a chargeless massless particle called antineutrino ().

$$n \rightarrow p + e^- + \bar{\nu}$$

• In β^- decay, mass number remains same, atomic number increases by one.

$${}_ZX^A \longrightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \bar{\nu}$$

Anti neutrinos interact very weakly with matter. So they have a very high penetrating power. Hence the detection of antineutrinos is very difficult.

Definition equation $t = \frac{1}{\lambda}$, then $N = N_0e^{-\lambda t}$

$$N = N_0e^{-1} = \frac{N_0}{2.718} = 0.368N_0$$

or $N = \frac{N_0}{e} = 36.8\%$ of N_0

The radioactive decay constant may be defined as the reciprocal of the time interval during which the number of active nuclei in a given radioactive sample reduces to 36.8% (or 1/e times) of its initial value.

Relation between mean life and decay constant (Expression of mean life) :

Suppose a radioactive sample contains N_0 nuclei at time t = 0. After time t, this number reduces to N Furthermore, suppose dN nuclei disintegrate in time t to t + dt. As dt is small, so the life of each of the dN nuclei can be approximately taken equal to t.

$\therefore$ Total life of dN nuclei = t dN

Sum of lives of N_0 nuclei = $\int_0^{N_0} t\,dN$

Mean life = $\frac{\text{Sum of lives } N_0 \text{ nuclei}}{N_0}$

or $\tau = \frac{1}{N_0}\int_0^{N_0} t\,dN$

As N = $N_0e^{-\lambda t}$

$\therefore$ dN = $-\lambda N_0e^{-\lambda t}$ dt

For limits :

When N = N_0 ; t = 0 and when N = 0 ; t = ∞

Changing the limits of integration in terms of time, we get

$$\tau = \frac{-1}{N_0}\int_{\infty}^{0} t\lambda N_0e^{-\lambda t}dt$$

$$\tau = -\lambda\int_{\infty}^{0} t e^{\lambda t}dt$$

$$= -\lambda\left[\left\{\frac{te^{-\lambda t}}{-\lambda}\right\}_{\infty}^{0} - \int_{\infty}^{0}\frac{e^{-\lambda t}}{-\lambda}dt\right]$$

$$= 0 - \frac{\lambda}{\lambda}\int_{\infty}^{0} e^{-\lambda t}dt = -\int_{\infty}^{0} e^{-\lambda t}dt = -\left[\frac{e^{-\lambda t}}{-\lambda}\right]_{\infty}^{0}$$

$$= \frac{1}{\lambda}\left[e^{-0} - e^{-\infty}\right]$$

$$\boxed{\tau = \frac{1}{\lambda}}$$

OR

(a) State two distinguishing features of nuclear force.

(b) Draw a plot showing the variation of potential energy of a pair of nucleons as a function of their separation. Mark the regions on the graph where the force is (i) attractive, and (ii) repulsive.

Ans. (a)

1. Strongest interaction :

Nuclear force is the strongest interaction known in nature that holds the nucleons together despite the strong electrostatic repulsion between the protons. The relative strength of gravitational, electrostatic and nuclear forces is

$$F_g : F_E : F_n = 1 : 10^{36} : 10^{38}$$

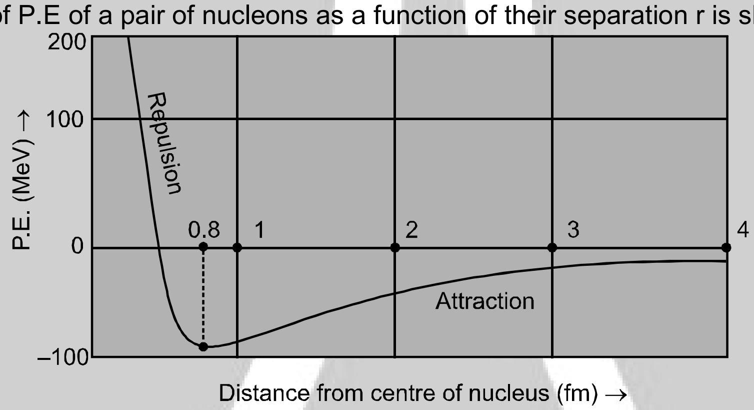
2. Short-range force :

Unlike gravitational and electrostatic forces, nuclear force is a short-range force. It operates only up to a very short distance of about 2-3 fm from a nucleon.

Ans. (b)

Variation with distance :

The graph of P.E of a pair of nucleons as a function of their separation r is shown.



24. Prove that in a common – emitter amplifier, the output and input differ in phase by 180°.

In a transistor, the change of base current by 30 μA produces change of 0.02 V in the base –emitter voltage and a change of 4 mA in the collector current. Calculate the current amplification factor and the load resistance used, if the voltage gain of the amplifier is 400.

Ans. (a) Phase relationship between input and output signals-

When an a.c. signal is fed to the input circuit, its positive half cycle increases the forward bias of the circuit which, in turn, increases the emitter current and hence the collector current. The increase in collector current increases the potential drop across R_L, which makes the output voltage V_0 less positive or more negative. So as the input signal goes through its positive half cycle, the amplified output signal goes through a negative half cycle. Similarly, as the input signal goes through its negative half cycle, the amplified output signal goes through its positive half cycle. Hence in a common emitter amplifier, the input and output voltages are 180° out of phase or in opposite phases.

(b) $\Delta I_B = 30\mu\ A$

$\Delta I_{BE} = 0.02\ V$

$\Delta I_C = 4mA$ $\qquad A_v = 400$

Current amplification factor

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{4mA}{30\mu A} = \frac{4\times10^{-3}}{30\times10^{-6}}$$

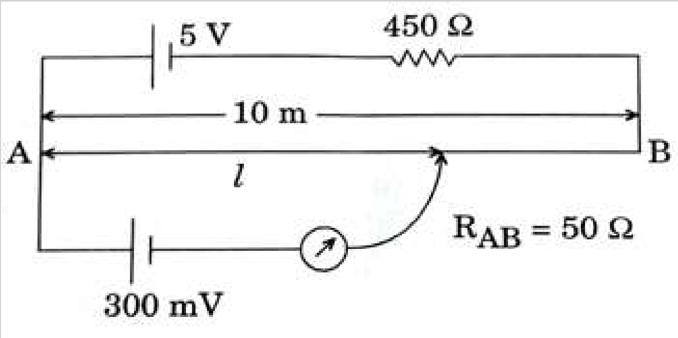
$$A_v = \beta\frac{R_L}{R_i}$$

$$R_L = \frac{A_v \times R_i}{\beta} \quad = 400\times\frac{\frac{V_{BE}}{I_B}}{1.33\times10^2}$$

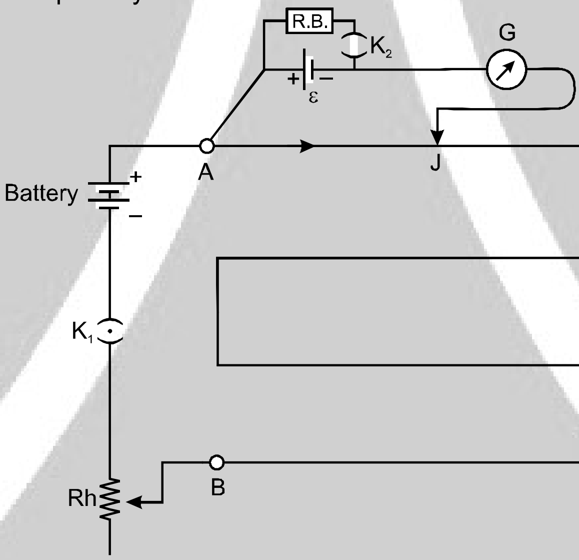
$$R_L = 0.200\times10^4\,\Omega$$

SECTION D

25. (a) Describe briefly, with the help of a circuit diagram, the method of measuring the internal resistance of a cell.
(b) Give reason why a potentiometer is preferred over a voltmeter for the measurement of emf of a cell.
(b) In the potentiometer circuit given below, calculate the balancing length l. Give reason, whether the circuit will work, if the driver cell of emf 5 V is replaced with a cell of 2 V, keeping all other factors constant.



Ans.
(a) Internal resistance of primary cell :



Internal resistance of primary cell

Close the key K_1. A constant current flows through the potentiometer wire. With key K_2 kept open, move the jockey along AB till it balances the emf e of the cell. Let l_1 be the balancing length of the wire. If k is the potential gradient, then emf of the cell will be :
$e = kl_1$
With the help of resistance box R.B., introduce a resistance R and close key K_2. Find the balance point for the terminal potential difference V of the cell. If l_2 is the balancing length, then
$V = kl_2$
Divide the above to equation
$$\frac{\varepsilon}{V} = \frac{l_1}{l_2}$$
Let r be the internal resistance of the cell. If current I flows through cell when it is shunted with resistance R, then from Ohm's law we get
$\varepsilon = I(R + r)$ and $V = IR$

$\therefore \quad \frac{\varepsilon}{V} = \frac{R+r}{R} = \frac{l_1}{l_2}$; $1 + \frac{r}{R} = \frac{l_1}{l_2}$ or $\boxed{\frac{r}{R} = \frac{l_1 - l_2}{l_2}}$

$\therefore$ Internal resistance, $r = R\left[\frac{l_1 - l_2}{l_2}\right]$

(b) Superiority of a potentiometer to a voltmeter :
Potentiometer is a null method device. At null point, it does not draw any current from the cell and thus there is no potential drop due to the internal resistance of the cell. It measures the p.d. in an open circuit which is equal to the actual emf of the cell.
On the other hand, a voltmeter draws a small current from the cell for its operation. So it measures the terminal p.d. in a closed circuit which is less than the emf of a cell. That is why a potentiometer is preferred over a voltmeter for measuring the emf of a cell.

(c) $I_{AB} = \frac{5}{450+50} = \frac{5}{500} = \frac{1}{100}$ Amp.

$V_{AB} = I_{AB}R_{AB} = \frac{1}{100} \times 50 = 0.5V$

Potential Gradient $K = \frac{V_{AB}}{L} = \frac{0.5}{10}$

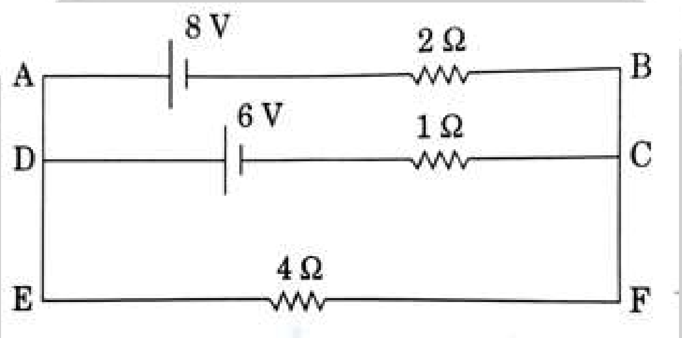
Balancing length, $L = \frac{\text{Potential Difference}}{\text{Potential gradient}}$

$= \frac{300 \times 10^{-3} v}{0.05 \ v/m} \qquad = \frac{0.3}{0.05} = 6 \ m$

If the driver cell of emf 5 V is replaced with a cell of 2 V keeping all other factors constant then potential drop along AB is 0.2 Volt.
The balance point cannot be obtained on the potentiometer if the fall of potential along the potentiometer wire due to the auxiliary battery is less than the emf of the cell to be measured.

OR

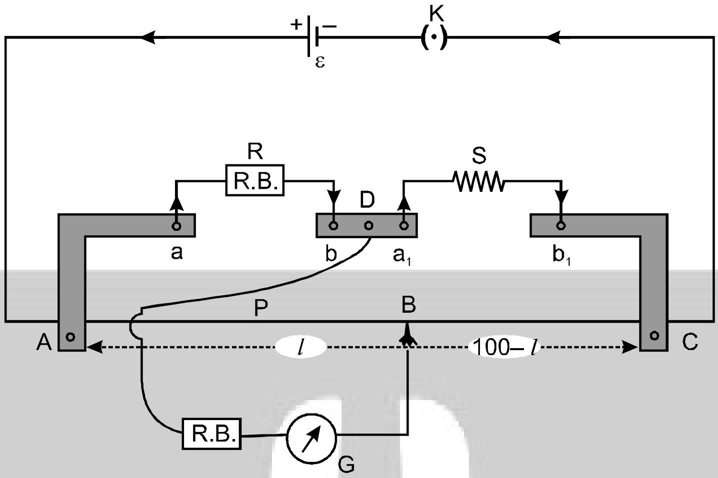
(a) State the working principle of a meter bridge used to measure an unknown resistance.
(b) Give reason
 (i) why the connections between the resistors in a meter bridge are made of thick copper strips,
 (ii) why is it generally preferred to obtain the balance length near the mid – point of the bridge wire.
(c) Calculate the potential difference across the $4\ \Omega$ resistor in the given electrical circuit, using Kirchhoff's rules.



OR

Ans. (a) Meter bridge (or slide wire bridge) :
It is the simplest practical application of the Wheatstone bridge that is used to measure an unknown resistance.
Principle :
Its working is based on the principle of Wheatstone bridge.
When the bridge is balanced,

$\boxed{\frac{P}{Q} = \frac{R}{S}}$



Measurement of unknown resistance by a metre bridge

Working :

After selecting a suitable resistance R from the resistance box, the jockey is moved along the wire AC till there is no deflection in the galvanometer. This is the balanced condition of the Wheatstone bridge. If P and Q are resistances of the parts AB and BC of the wire, then for the balanced condition of the bridge, we have

$$\frac{P}{Q} = \frac{R}{S}$$

Let total length of wire AC = 100 cm and AB = l cm, then BC = (100 – l) cm. Since the bridge wire is of uniform cross-section, therefore,

resistance of wire µ length of wire or $\boxed{R \propto l}$

or $\frac{P}{Q} = \frac{\text{resistance of AB}}{\text{resistance of BC}} = \frac{\sigma l}{\sigma(100 - l)} = \frac{l}{100 - l}$

where s is the resistance per unit length of the wire

Hence

$$\frac{R}{S} = \frac{l}{100 - l} \Rightarrow \qquad S = \frac{R(100 - l)}{l}$$

Knowing l and R, unknown resistance S can be determined.

(b)

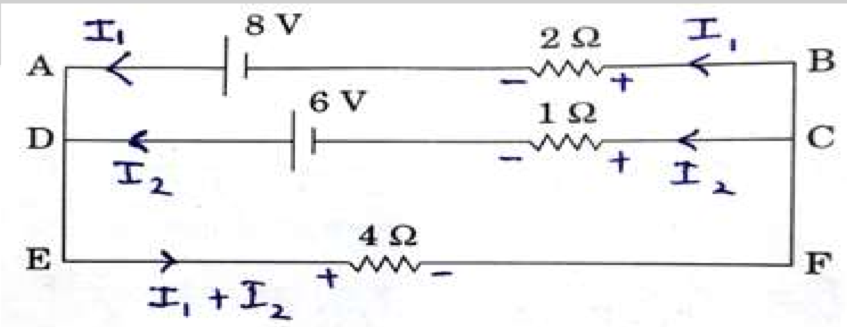
(i) The connections are made of thick copper wires to minimise the resistance of connecting wire Because the connection resistance have not been accounted in the formula.

$$R \propto \frac{1}{A}$$

Resistance is inversely proportional to cross sectional area so thick wire has low resistance

(ii) The error in the measured value of unknown resistance S using bridge wire will be minimum, when the null point is obtained at the middle of bridge wire. In this situation, the end error of the bridge will be ineffective.

(c)



For loop ADCBA ;

$-2I_1 + 8 - 6 + I_2 = 0$

$I_2 - 2I_1 = -2$ ....(1)

For loop DEFCD

$-4(I_1 + I_2) - I_2 + 6 = 0$

$-4I_1 - 4I_2 - I_2 + 6 = 0$

$-4I_1 - 5I_2 = -6$

$4I_1 + 5I_2 = 6$ ....(2)

By solving eq. (1) and (2)

$2I_2 - 4I_1 = -4$

$5I_2 + 4I_1 = 6$

$7I_2 = 2$

$I_2 = \frac{2}{7}$

By eq. (1)

$\frac{2}{7} - 2I_1 = -2$

$2I_1 = -2 - \frac{2}{7}$

$-2I_1 = \frac{-14-2}{7}$

$I_1 = \frac{16}{14} = \frac{8}{7}$

Current across 4 – Ω Resistor

So,

$I_1 + I_2$

$= \frac{2}{7} + \frac{8}{7}$

$= \frac{10}{7}$

So,

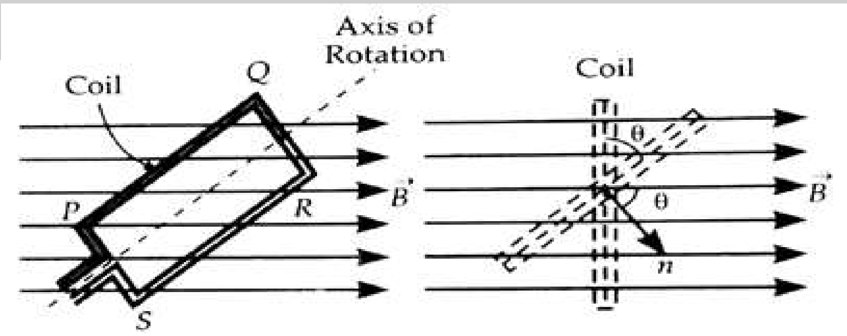
V = IR

$= \frac{10}{7} \times 4 = \frac{40}{7}$ V

26. (a) Derive an expression for the induced emf developed when a coil of N turns, and area of cross-section A, is rotated at a constant angular speed ω in a uniform magnetic field B.

(b) A wheel with 100 metallic spokes each 0.5 m long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of the Earth's magnetic field. If the resultant magnetic field at that place is 4×10^{-4} T and the angle of dip at the place is 30°, find the emf induced between the axle and the rim of the wheel.

Ans. (a)



Let any time t the angle B/W $\vec{B}$ and $\vec{A}$ is θ then

$$\omega = \frac{\theta}{t}$$

$$\Rightarrow \theta = \omega t \quad ....(1)$$

The linked magnetic flux with the coil at time t is

$$\phi_n = NBA\cos\theta$$

From eq. (1)

$$\phi_n = NBA\cos\omega t \quad ....(2)$$

According to Faraday's law the induced emf is.

$$\varepsilon = \frac{-d}{dt}(\phi_n)$$

$$\varepsilon = \frac{-d}{dt}(NBA\cos\omega t)$$

$$\varepsilon = -NBA\frac{d}{dt}(\cos\omega t)$$

$$\varepsilon = NBA\sin\omega t(\omega)$$

$$\varepsilon = NBA\omega\sin\omega t \quad ....(3)$$

(b)

spokes = 100

length (l) = 0.5 m

$$\omega = 120\frac{rev}{min}$$

$$B_e = 4\times10^{-4}T$$

$$\varepsilon = \frac{1}{2}Bl^2\omega$$

$$\because\ B_H = |B_e|\cos\theta$$

$$B_V = |B_e|\sin\theta$$

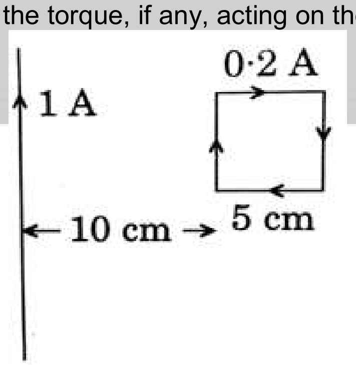
$$\varepsilon = \frac{1}{2}|B_e|\cos\theta.\ell^2\omega \qquad = \frac{1}{2}\times4\times10^{-4}\times\cos30\times(0.5)^2\times2\pi v$$

$$= \frac{1}{2} = 4\times10^{-4}\times\frac{\sqrt{3}}{2}\times(0.5)^2\times2\pi\times\left(\frac{120}{60}\right) \qquad = 5.441\text{ Volt}$$

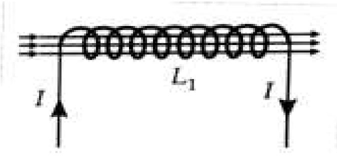
OR

(a) Derive the expression for the magnetic energy stored in an inductor when a current I develops in it. Hence, obtain the expression for the magnetic energy density.

(b) A square loop of sides 5 cm carrying a current of 0.2A in the clockwise direction is placed at a distance of 10 cm from an infinitely long wire carrying a current of 1 A as shown. Calculate (i) the resultant magnetic force, and (ii) the torque, if any, acting on the loop.



Ans. (a)



The magnitude of induced emf

$$\varepsilon = \frac{LdI}{dt}$$

If dq extra charge supply by the battery against induced emf then small work done. By the battery is

$$dw = \varepsilon dq \qquad (\therefore w = qv)$$

$$dw = \frac{LdI}{dt} \times dq$$

$$dw = L\left(\frac{dq}{dt}\right)dI$$

$$dw = LIdI \qquad \text{..... (i)}$$

For total w.d. integrate eq (i) for limit 0 to I_0

$$w = \int_0^{I_0} LIdI$$

$$w = L\int_0^{I_0} IdI$$

$$w = L\left[\frac{I^2}{2}\right]_0^{I_0}$$

$$w = \frac{LI^2{}_0}{2}$$

This w.d. is stored as magnetic potential energy then

$$U_m = \frac{LI^2{}_0}{2}$$

Magnetic energy density stored in the inductor

$$U_m = \frac{1}{2}LI^2 \qquad \text{............(1)}$$

The M.E produced in the inductor

$$B = \frac{\mu_0 NI}{L}$$

$$I = \frac{BL}{\mu_0 N} \qquad \text{............(2)}$$

The self-inductance of inductor

$$I = \frac{\mu_0 N^2 A}{e} \qquad \text{.............(3)}$$

Substitute eq (2) & (3) in (1)

$$U_m = \frac{1}{2}\left(\frac{\mu_0 N^2 A}{L}\right)\left(\frac{B^2L^2}{\mu_0{}^2N^2}\right)$$

$$U_m = \frac{B^2}{2\mu_0}(AL)$$

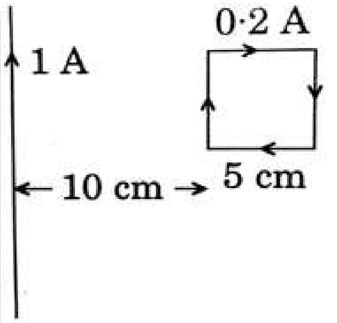
$$\frac{U_m}{AL} = \frac{B^2}{2\mu_0} \qquad \text{............(4)}$$

The energy stored per unit volume called energy density.

$$\bar{U}_m = \frac{U_m}{AL}$$

From eq (4) $\bar{U}_m = \frac{B^2}{2\mu_0}$

(a) (i)



$$F_{AB} = \frac{\mu_0 I_1 I_2}{2\pi r}\ell$$

$$= \frac{2\times10^{-7}\times1\times0.2\times5\times10^{-2}}{10\times10^{-2}} = 2\times10^{-8}\text{Attractive}$$

$$F_{CD} = \frac{2\times10^{-7}\times1\times0.2\times5\times10^{-2}}{15\times10^{-2}} = 1.3\times10^{-8}\text{ Repulsive}$$

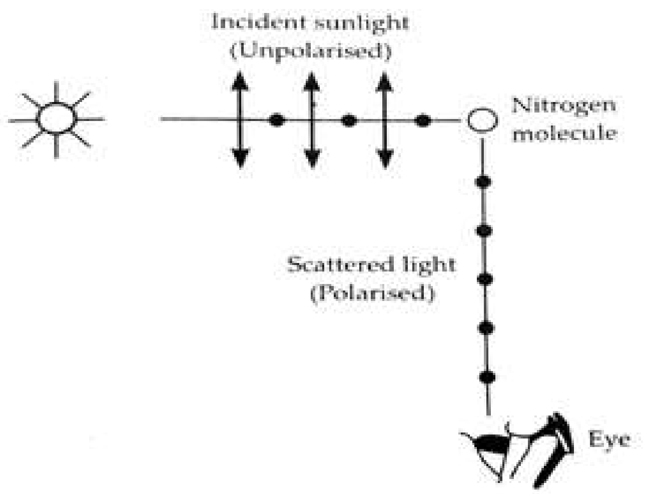
$$F_{net} = F_{AB} - F_{CD} \qquad = 7\times10^{-9}\text{N}$$

(ii) F_1 & F_2 are collinear, hence they do not produce torque on the loop

27. With the help of a diagram, how plane polarized light can be produced by scattering of light from the Sun.

Two polaroids P_1 and P_2 are placed with their pass axes perpendicular to each other. Unpolarised light of intensity I is incident on P_1. A third polaroid P_3 is kept between P_1 and P_2 such that its pass axis makes an angle of 45° with that of P_1. Calculate the intensity of light transmitted through P_1, P_2 and P_3.

Ans. (a) The unpolarised light incident on a molecule. The dots show vibrations perpendicular to the plane of paper and double arrows show vibration in the plane of paper. The electron in the molecule begin to vibrate in both of these directions. The electrons vibrating parallel to the double arrows cannot send energy towards an observer looking at 90° to the direction of the sun because their acceleration has no transverse component. The light scattered by the molecules in this direction has only dots. It is polarized perpendicular to the plane of paper. This explains the polarization of light scattered from the sky.



(b) Intensity of light through $P_1 = \frac{I_0}{2}$

Intensity of light through P_3

$= \frac{I_0}{2}\cos^2 45 = \frac{I_0}{2}.\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I_0}{4}$

Intensity of light through P_2

$= \frac{I_0}{8}.\cos^2(90° - 45°) = \frac{I_0}{8}.\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I_0}{16}$

OR

(a) Why cannot the phenomenon of interference be observed by illuminating two pin holes with two sodium lamps ?

(b) Two monochromatic waves having displacements y_1 = a cos ωt and y_2 = a cos (ωt + φ) from two coherent sources interfere to produce an interference pattern. Derive the expression for the resultant intensity and obtain the conditions for constructive and destructive interference.

(c) Two wavelengths of sodium light of 590 nm and 596 nm are used in turn to study the diffraction taking place at a single slit of aperture 2 × 10^{-6}m. If the distance between the slit and the screen in 1.5 m, calculate the separation between the positions of the second maxima of diffraction pattern obtained in the two cases.

Ans. (a) two pin holes with two sodium lamps cannot produce coherent source of light, so the phenomenon of interference cannot be observed.

(b) By the principle of superposition, the resultant displacement at the observation point will be

$y_1 = a\cos\omega t$, $y_2 = a\cos(\omega t + \phi)$

The resultant displacement is given by

$y = y_1 + y_2 = a\cos\omega t + a\cos(\omega t + \phi)$

$= a\cos\omega t(1 + \cos\phi) - a\sin\omega t\sin\phi$ = $a\cos\omega t + a\cos\omega t\cos\phi - a\sin\omega t\sin\phi$

Put $R\cos\theta = a(1 + \cos\phi)$ ....(i) $R\sin\theta = a\sin\phi$ ....(ii)

By squaring and adding Eqs. (i) and (ii)

$R^2 = a^2\left(1 + \cos^2\phi + 2\cos\phi\right) + a^2\sin^2\phi$

$= 2a^2(1 + \cos\phi) = 4a^2\cos^2\frac{\phi}{2}$

$\therefore \quad I = R^2 = 4a^2\cos^2\frac{\phi}{2} = 4I_0\cos^2\frac{\phi}{2}$

For constructive interference :

$\cos\frac{\phi}{2} \pm 1$ or $\frac{\phi}{2} = n\pi$ or $\phi = 2n\pi$

For destructive interference :

$\cos\frac{\phi}{2} = 0$ or $\frac{\phi}{2} = (2n+1)\frac{\pi}{2}$ or $\phi = (2n+1)\pi$

(c)Here, $\lambda_1 = 590$ nm $= 596 \times 10^{-9}$m, $\lambda_2 = 596$ nm $= 596 \times 10^{-9}$m,
d = 2 × 10^{-4} m, D = 1.5 m

Distance of first secondary maximum from the centre of the screen is $x = \frac{3}{2}\frac{D\lambda}{d}$

For the two wavelengths, we have

$x_1 = \frac{3}{2}\frac{D\lambda_1}{d}$ and $x_2 = \frac{3}{2}\frac{D\lambda_2}{d}$

Spacing between the first two maximum of sodium lines

$= x_2 - x_1 = \frac{3D}{2d}(\lambda_2 - \lambda_1) = \frac{3 \times 1.5}{2 \times 2 \times 10^{-6}}\left(596 \times 10^{-9} - 590 \times 10^{-9}\right) = \frac{3 \times 1.5 \times 6 \times 10^{-3}}{4}$

$= 6.75 \times 10^{-3}\text{m} = 6.75\,\text{mm}$