Mathematics

1. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals :

(a) $\frac{1}{2}(1-\sqrt{5})$ (b) $\frac{1}{2}\sqrt{5}$

(c) $\sqrt{5}$ (d) $\frac{1}{2}(\sqrt{5}-1)$

2. If $\sin^{-1}\left(\frac{x}{5}\right)+\text{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$, then a value of x is :

(a) 1 (b) 3

(c) 4 (d) 5

3. In the binomial expansion of $(a-b)^n$, $n \geq 5$, the sum of 5th and 6th terms is zero, then $\frac{a}{b}$ equals :

(a) $\frac{5}{n-4}$ (b) $\frac{6}{n-5}$

(c) $\frac{n-5}{6}$ (d) $\frac{n-4}{5}$

4. The set $S := \{1, 2, 3, \ldots, 12\}$ is to be partitioned into three sets A, B, C of equal size. Thus, $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is :

(a) $12!/3!\,(4!)^3$ (b) $12!/3!(3!)^4$

(c) $12!/(4!)^3$ (d) $12!/(3!)^4$

5. The largest interval lying in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ for which the function

$$\left[f(x)=4^{-x^2}+\cos^{-1}\left(\frac{x}{2}-1\right)+\log(\cos x)\right]$$

is defined, is :

(a) $[0, \pi]$ (b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(c) $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$ (d) $\left[0, \frac{\pi}{2}\right)$

6. A body weighing 13 kg is suspended by two strings 5 m and 12 m long, their other ends being fastened to the extremities of a rod 13 m long. If the rod be so held that the body hangs immediately below the middle point. The tensions in the strings are :

(a) 12 kg and 13 kg (b) 5 kg and 5 kg

(c) 5 kg and 12 kg (d) 5 kg and 13 kg

7. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is :

(a) 1/729 (b) 8/9

(c) 8/729 (d) 8/243

8. Consider a family of circles which are passing through the point $(-1, 1)$ and are tangent to x-axis. If (h, k) are the coordinates of the centre of the circles, then the set of values of k is given by the interval :

(a) $0 < k < 1/2$ (b) $k \geq 1/2$

(c) $-1/2 \leq k \leq 1/2$ (d) $k \leq 1/2$

9. Let L be the line of intersection of the planes $2x+3y+z=1$ and $x+3y+2z=2$. If L makes an angle α with the positive x-axis, then $\cos\alpha$ equals :

(a) $1/\sqrt{3}$ (b) $1/2$

(c) 1 (d) $1/\sqrt{2}$

10. The differential equation of all circles passing through the origin and having their centres on the x-axis is :

(a) $x^2 = y^2 + xy\frac{dy}{dx}$

(b) $x^2 = y^2 + 3xy\frac{dy}{dx}$

(c) $y^2 = x^2 + 2xy\frac{dy}{dx}$

(d) $y^2 = x^2 - 2xy\frac{dy}{dx}$

11. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of $(p + q)$ is :

(a) 2 (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

12. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that $AB (= a)$ subtends an angle of 60° at the foot of the tower and the angle of elevation of the top of the tower from A or B is 30°. The height of the tower is :

(a) $2a/\sqrt{3}$ (b) $2a\sqrt{3}$

(c) $a/\sqrt{3}$ (d) $a\sqrt{3}$

13. The sum of the series $^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots - \ldots + {}^{20}C_{10}$ is :

(a) $-{}^{20}C_{10}$ (b) $\frac{1}{2}\,{}^{20}C_{10}$

(c) 0 (d) $^{20}C_{10}$

14. The normal to a curve at $P(x, y)$ meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a :

(a) ellipse (b) parabola

(c) circle (d) hyperbola

15. If $|z + 4| \le 3$, then the maximum value of $|z + 1|$ is :

(a) 4 (b) 10

(c) 6 (d) 0

16. The resultant of two forces P N and 3 N is a force of 7 N. If the direction of the 3 N force were reversed, the resultant would be $\sqrt{19}$ N. The value of P is :

(a) 5 N (b) 6 N

(c) 3 N (d) 4 N

17. Two aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is :

(a) 0.06 (b) 0.14

(c) 0.2 (d) 0.7

18. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \ne 0$, $y \ne 0$, then D is :

(a) divisible by neither x nor y

(b) divisible by both x and y

(c) divisible by x but not y

(d) divisible by y but not x

19. For the Hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant when α varies ?

(a) Eccentricity

(b) Directrix

(c) Abscissae of vertices

(d) Abscissae of foci

20. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of each of x-axis and y-axis, then the angle that the line makes with the positive direction of the z-axis is :

(a) $\pi/6$ (b) $\pi/3$

(c) $\pi/4$ (d) $\pi/2$

21. A value of C for which the conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval $[1, 3]$ is :

(a) $2\log_3 e$ (b) $\frac{1}{2}\log_e 3$

(c) $\log_3 e$ (d) $\log_e 3$

22. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in :

(a) $(\pi/4, \pi/2)$ (b) $(-\pi/2, \pi/4)$

(c) $(0, \pi/2)$ (d) $(-\pi/2, \pi/2)$

23. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$

If $|A^2| = 25$, then $|\alpha|$ equals :

(a) 5^2 (b) 1

(c) $\frac{1}{5}$ (d) 5

24. The sum of the series $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots$ upto infinity is :

(a) e^{-2} (b) e^{-1}

(c) $e^{-\frac{1}{2}}$ (d) $e^{+\frac{1}{2}}$

25. If $\hat{\mathbf{u}}$ and $\hat{\mathbf{v}}$ are unit vectors and θ is the acute angle between them, then $2\hat{\mathbf{u}} \times 3\hat{\mathbf{v}}$ is a unit vector for :

(a) exactly two values of θ

(b) more than two values of θ

(c) no value of θ

(d) exactly one value of θ

26. A particle just clears a wall of height b at a distance a and strikes the ground at a distance c from the point of projection. The angle of projection is :

(a) $\tan^{-1}\frac{b}{ac}$ (b) $45°$
(c) $\tan^{-1}\frac{bc}{a(c-a)}$ (d) $\tan^{-1}\frac{bc}{a}$

27. The average marks of boys in a class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is :
(a) 40 (b) 20
(c) 80 (d) 60

28. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is :
(a) $(-1, 1)$ (b) $(0, 2)$
(c) $(2, 4)$ (d) $(-2, 0)$

29. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the coordinates of the other end of the diameter are :
(a) $(4, 9, -3)$ (b) $(4, -3, 3)$
(c) $(4, 3, 5)$ (d) $(4, 3, -3)$

30. Let $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}, \vec{\mathbf{b}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\vec{\mathbf{c}} = x\hat{\mathbf{i}} + (x-2)\hat{\mathbf{j}} - \hat{\mathbf{k}}$. If the vector $\vec{\mathbf{c}}$ lies in the plane of $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$, then x equals :
(a) 0 (b) 1
(c) -4 (d) -2

31. Let $A(h, k)$, $B(1, 1)$ and $C(2, 1)$ be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which 'k' can take is given by :
(a) {1,3} (b) {0, 2}
(c) {−1, 3} (d) {−3, −2}

32. Let $P = (-1, 0)$, $Q = (0, 0)$ and $R = (3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR is :
(a) $\sqrt{3}x + y = 0$ (b) $x + \frac{\sqrt{3}}{2}y = 0$
(c) $\frac{\sqrt{3}}{2}x + y = 0$ (d) $x + \sqrt{3}y = 0$

33. If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines $xy = 0$, then m is :
(a) $-\frac{1}{2}$ (b) -2
(c) ± 1 (d) 2

34. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, where $f(x) = \int_1^x \frac{\log t}{1+t}\,dt$. Then $F(e)$ equals :
(a) $\frac{1}{2}$ (b) 0
(c) 1 (d) 2

35. Let $f : R \to R$ be a function defined by $f(x) = \min\{x+1, |x|+1\}$. Then which of the following is true ?
(a) $f(x) \geq 1$ for all $x \in R$
(b) $f(x)$ is not differentiable at $x = 1$
(c) $f(x)$ is differentiable everywhere
(d) $f(x)$ is not differentiable at $x = 0$

36. The function $f : R/\{0\} \to R$ given by

$$f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$$

can be made continuous at $x = 0$ by defining $f(0)$ as :
(a) 2 (b) -1
(c) 0 (d) 1

37. The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2 - 1}} = \pi/2$ is :
(a) 2 (b) π
(c) $\sqrt{3}/2$ (d) $2\sqrt{2}$

38. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$ equals :
(a) $\frac{1}{2}\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + c$
(b) $\frac{1}{2}\log\tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + c$
(c) $\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + c$
(d) $\log\tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + c$

39. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is :
(a) $\frac{2}{3}$ (b) 1
(c) $\frac{1}{6}$ (d) $\frac{1}{3}$

40. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is :
(a) $(-3, 3)$ (b) $(-3, \infty)$
(c) $(3, \infty)$ (d) $(-\infty, -3)$

Physics

41. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metre. The time at which the maximum speed first occurs is :

(a) 0.5 s (b) 0.75 s
(c) 0.125 s (d) 0.25 s

42. In an AC circuit the voltage applied is $E = E_0 \sin \omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by :

(a) $P = \frac{E_0 I_0}{\sqrt{2}}$ (b) $P = \text{zero}$
(c) $P = \frac{E_0 I_0}{2}$ (d) $P = \sqrt{2}\, E_0 I_0$

43. An electric charge $10^{-3} \mu C$ is placed at the origin (0, 0) of X-Y coordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. The potential difference between the points A and B will be :

(a) 9 V (b) zero
(c) 2 V (d) 4.5 V

44. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be :

(a) 1 (b) 2
(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

45. An ideal coil of 10 H is connected in series with a resistance of 5Ω and a battery of 5 V. 2 s after the connection is made, the current flowing (in ampere) in the circuit is :

(a) $(1 - e)$ (b) e
(c) e^{-1} (d) $(1 - e^{-1})$

46. A long straight wire of radius a carries a steady current i. The current is uniformly distributed across its cross-section. The ratio of the magnetic field at $\frac{a}{2}$ and $2a$ is :

(a) $\frac{1}{4}$ (b) 4
(c) 1 (d) $\frac{1}{2}$

47. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then :

(a) the magnetic field is zero only on the axis of the pipe
(b) the magnetic field is different at different points inside the pipe
(c) the magnetic field at any point inside the pipe is zero
(d) the magnetic field at all points inside the pipe is the same, but not zero

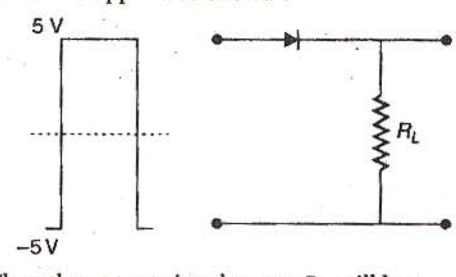
48. If M_o is the mass of an oxygen isotope ${}_8O^{17}$, M_p and M_n are the masses of a proton and a neutron, respectively, the nuclear binding energy of the isotope is :

(a) $(M_o - 8M_p)c^2$
(b) $(M_o - 8M_p - 9M_n)c^2$
(c) $M_o c^2$
(d) $(M_o - 17M_n)c^2$

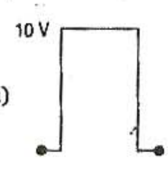
49. In gamma ray emission from a nucleus :

(a) both the neutron number and the proton number change
(b) there is no change in the proton number and the neutron number
(c) only the neutron number changes
(d) only the proton number changes

50. If in a p-n junction diode, a square input signal of 10 V is applied as shown :



Then the output signal across R_L will be :

(a)


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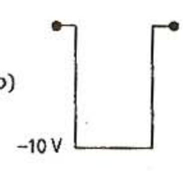
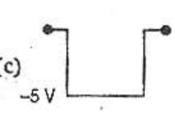


(c)


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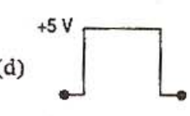


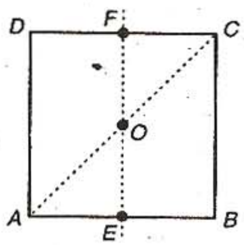
51. Photon of frequency ν has a momentum associated with it. If c is the velocity of light, the momentum is :

(a) ν/c (b) $h\nu c$
(c) $h\nu/c^2$ (d) $h\nu/c$

52. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is :

(a) $v_0 + 2g + 3f$ (b) $v_0 + g/2 + f/3$
(c) $v_0 + g + f$ (d) $v_0 + g/2 + f$

53. For the given uniform square lamina $ABCD$, whose centre is O :

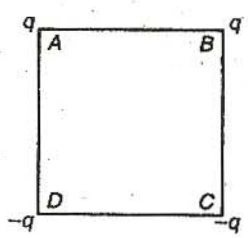


(a) $\sqrt{2} I_{AC} = I_{EF}$ (b) $I_{AD} = 3I_{EF}$
(c) $I_{AC} = I_{EF}$ (d) $I_{AC} = \sqrt{2} I_{EF}$

54. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as :
$a = A\cos(\omega t + \delta)$, then :

(a) $A = x_0,\ \delta = -\pi/4$
(b) $A = x_0\omega^2,\ \delta = \pi/4$
(c) $A = x_0\omega^2,\ \delta = -\pi/4$
(d) $A = x_0\omega^2,\ \delta = 3\pi/4$

55. Charges are placed on the vertices of a square as shown. Let $\vec{E}$ be the electric field and V the potential at the centre. If the charges on A and B are interchanged with those on D and C respectively, then :



(a) $\vec{E}$ remains unchanged, V changes
(b) both $\vec{E}$ and V change
(c) $\vec{E}$ and V remain unchanged
(d) $\vec{E}$ changes, V remains unchanged

56. The half-life period of a radioactive element X is same as the mean life time of another radioactive element Y. Initially they have the same number of atoms. Then :

(a) X will decay faster than Y
(b) Y will decay faster than X
(c) Y and X have same decay rate initially
(d) X and Y decay at same rate always

57. A Carnot engine, having an efficiency of $\eta = 1/10$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is :

(a) 99 J (b) 90 J
(c) 1 J (d) 100 J

58. Carbon, silicon and germanium have four valence electrons each. At room temperature which one of the following statements is most appropriate ?

(a) The number of free conduction electrons is significant in C but small in Si and Ge
(b) The number of free conduction electrons is negligibly small in all the three
(c) The number of free electrons for conduction is significant in all the three
(d) The number of free electrons for conduction is significant only in Si and Ge but small in C

59. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields $\vec{E}$ and $\vec{B}$ with a velocity $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$, and comes out without any change in magnitude or direction of $\vec{v}$. Then :

(a) $\vec{v} = \vec{E} \times \vec{B}/B^2$ (b) $\vec{v} = \vec{B} \times \vec{E}/B^2$
(c) $\vec{v} = \vec{E} \times \vec{B}/E^2$ (d) $\vec{v} = \vec{B} \times \vec{E}/E^2$

60. The potential at a point x (measured in μm) due to some charges situated on the x-axis is given by : $V(x) = 20/(x^2 - 4)$ volt
The electric field E at $x = 4\ \mu$m is given by :

(a) $\frac{5}{3}$ V/μm and in the –ve x direction
(b) $\frac{5}{3}$ V/μm and in the +ve x direction
(c) $\frac{10}{9}$ V/μm and in the –ve x direction
(d) $\frac{10}{9}$ V/μm and in the +ve x direction

61. Which of the following transitions in hydrogen atoms emit photons of highest frequency ?

(a) $n = 2$ to $n = 6$ (b) $n = 6$ to $n = 2$
(c) $n = 2$ to $n = 1$ (d) $n = 1$ to $n = 2$

62. A block of mass m is connected to another block of mass M by a spring (massless) of spring constant k. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then a constant force F starts acting on the block of mass M to pull it. Find the force on the block of mass m.

(a) $\frac{mF}{M}$ (b) $\frac{(M+m)F}{m}$

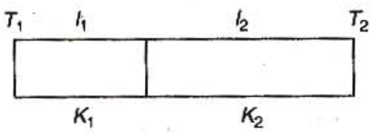
(c) $\frac{mF}{(m+M)}$ (d) $\frac{MF}{(m+M)}$

63. Two lenses of power −15D and +5D are in contact with each other. The focal length of the combination is :

(a) −20 cm (b) −10 cm

(c) +20 cm (d) +10 cm

64. One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of lengths l_1 and l_2 and thermal conductivities K_1 and K_2 respectively. The temperature at the interface of the two sections is



(a) $(K_2l_2T_1 + K_1l_1T_2)/(K_1l_1 + K_2l_2)$

(b) $(K_2l_1T_1 + K_1l_2T_2)/(K_2l_1 + K_1l_2)$

(c) $(K_1l_2T_1 + K_2l_1T_2)/(K_1l_2 + K_2l_1)$

(d) $(K_1l_1T_1 + K_2l_2T_2)/(K_1l_1 + K_2l_2)$

65. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of :

(a) 1000 (b) 10000

(c) 10 (d) 100

66. If C_P and C_V denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then :

(a) $C_P - C_V = R/28$ (b) $C_P - C_V = R/14$

(c) $C_P - C_V = R$ (d) $C_P - C_V = 28R$

67. A charged particle moves through a magnetic field perpendicular to its direction. Then :

(a) the momentum changes but the kinetic energy is constant

(b) both momentum and kinetic energy of the particle are not constant

(c) both momentum and kinetic energy of the particle are constant

(d) kinetic energy changes but the momentum is constant

68. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance d from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by :

(a) $\frac{\mu_0}{2\pi}\left(\frac{I_1 + I_2}{d}\right)^{1/2}$ (b) $\frac{\mu_0}{2\pi d}(I_1^2 + I_2^2)^{1/2}$

(c) $\frac{\mu_0}{2\pi d}(I_1 + I_2)$ (d) $\frac{\mu_0}{2\pi d}(I_1^2 + I_2^2)$

69. The resistance of a wire is 5 Ω at 50°C and 6 Ω at 100°C. The resistance of the wire at 0°C will be :

(a) 2Ω (b) 1Ω

(c) 4Ω (d) 3Ω

70. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volts. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is :

(a) $\frac{1}{2}(K-1)CV^2$ (b) $CV^2(K-1)/K$

(c) $(K-1)CV^2$ (d) zero

71. If g_E and g_M are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio $\frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}}$ to be :

(a) 1 (b) zero

(c) g_E/g_M (d) g_M/g_E

72. A circular disc of radius R is removed from a bigger circular disc of radius $2R$, such that the circumference of the discs coincide. The centre of mass of the new disc is $\frac{\alpha}{R}$ from the centre of the bigger disc. The value of α is :

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{6}$ (d) $\frac{1}{4}$

73. A round uniform body of radius R, mass M and moment of inertia I, rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is :

(a) $\frac{g\sin\theta}{1 + I/MR^2}$ (b) $\frac{g\sin\theta}{1 + MR^2/I}$

(c) $\frac{g\sin\theta}{1 - I/MR^2}$ (d) $\frac{g\sin\theta}{1 - MR^2/I}$

74. Angular momentum of the particle rotating with a central force is constant due to :

(a) constant force
(b) constant linear momentum
(c) zero torque
(d) constant torque

75. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes a uncompressed spring, and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10,000 N/m. The spring compresses by :

(a) 5.5 cm (b) 2.5 cm
(c) 11.0 cm (d) 8.5 cm

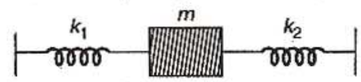
76. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is :

(a) K (b) zero
(c) $K/4$ (d) $K/2$

77. In a Young's double slit experiment the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of the light used) is I. If I_0 denotes the maximum intensity, I/I_0 is equal to :

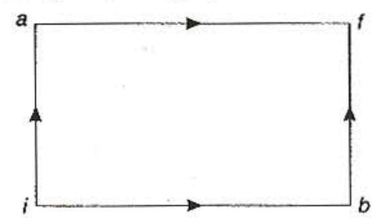
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$
(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

78. Two springs, of force constants k_1 and k_2, are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes :



(a) $f/2$ (b) $f/4$
(c) $4f$ (d) $2f$

79. When a system is taken from state i to state f along the path iaf, it is found that $Q = 50$ cal and $W = 20$ cal. Along the path ibf $Q = 36$ cal. W along the path ibf is :



(a) 6 cal (b) 16 cal
(c) 66 cal (d) 14 cal

80. A particle of mass m executes simple harmonic motion with amplitude a and frequency ν. The average kinetic energy during its motion from the position of equilibrium to the end is :

(a) $\pi^2 m a^2 \nu^2$ (b) $\frac{1}{4} m a^2 \nu^2$
(c) $4\pi^2 m a^2 \nu^2$ (d) $2\pi^2 m a^2 \nu^2$

Chemistry

81. The energies of activation for forward and reverse reactions for $A_2 + B_2 \rightleftharpoons 2AB$ are 180 kJ mol^{-1} and 200 kJ mol^{-1} respectively. The presence of a catalyst lowers the activation energy of both (forward and reverse) reactions by 100 kJ mol^{-1}. The enthalpy change of the reaction ($A_2 + B_2 \rightarrow 2AB$) in the presence of catalyst will be (in kJ mol^{-1}) :

(a) 300 (b) 120
(c) 280 (d) 20

82. The cell, $Zn|Zn^{2+}(1M)||Cu^{2+}(1M)|Cu$ ($E^\circ_{cell} = 1.10$ V), was allowed to be completely discharged at 298 K. The relative concentration of Zn^{2+} to Cu^{2+} $\left(\frac{[Zn^{2+}]}{[Cu^{2+}]}\right)$ is :

(a) antilog (24.08) (b) 37.3
(c) $10^{37.3}$ (d) 9.65×10^4

83. The pK_a of a weak acid (HA) is 4.5. The pOH of an aqueous buffered solution of HA in which 50% of the acid ionised is :

(a) 4.5 (b) 2.5
(c) 9.5 (d) 7.0

84. Consider the reaction,

$$2A + B \rightarrow \text{products}$$

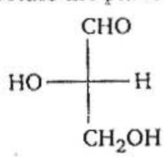
When concentration of B alone was doubled, the half-life did not change. When the concentration of A alone was doubled, the rate increased by two times. The unit of rate constant for this reaction is :

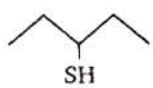
(a) $L\ mol^{-1}s^{-1}$ (b) no unit
(c) $mol\ L^{-1}s^{-1}$ (d) s^{-1}

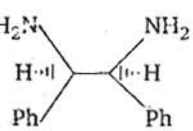
85. Identify the incorrect statement among the following :
(a) *d*-block elements show irregular and erratic chemical properties among themselves
(b) La and Lu have partially filled *d* orbitals and no other partially filled orbitals
(c) The chemistry of various lanthanoids is very similar
(d) 4*f* and 5*f* orbitals are equally shielded

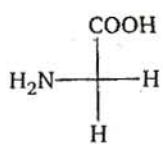
86. Which one of the following has a square planar geometry ?
(a) $[CoCl_4]^{2-}$ (b) $[FeCl_4]^{2-}$
(c) $[NiCl_4]^{2-}$ (d) $[PtCl_4]^{2-}$
(At. no. Co = 27, Ni = 28, Fe = 26, Pt = 78)

87. Which of the following molecules is expected to rotate the plane of plane-polarised light ?

(a) 

(b) 

(c) 

(d) 

88. The secondary structure of a protein refers to :
(a) α-helical backbone
(b) hydrophobic interactions
(c) sequence of α-amino acids
(d) fixed configuration of the polypeptide backbone

89. Which of the following reactions will yield, 2, 2-dibromopropane ?
(a) $CH_3-C\equiv CH + 2HBr \longrightarrow$
(b) $CH_3CH=CHBr + HBr \longrightarrow$
(c) $CH\equiv CH + 2HBr \longrightarrow$
(d) $CH_3-CH=CH_2 + HBr \longrightarrow$

90. In the chemical reaction,
$CH_3CH_2NH_2 + CHCl_3 + 3KOH \longrightarrow$
$(A) + (B) + 3H_2O$, the compounds (*A*) and (*B*) are respectively :
(a) C_2H_5CN and 3KCl
(b) $CH_3CH_2CONH_2$ and 3KCl
(c) C_2H_5NC and K_2CO_3
(d) C_2H_5NC and 3KCl

91. The reaction of toluene with Cl_2 in presence of $FeCl_3$ gives predominantly :
(a) benzoyl chloride
(b) benzyl chloride
(c) *o*-and *p*-chlorotoluene
(d) *m*-chlorotoluene

92. Presence of a nitro group in a benzene ring :
(a) activates the ring towards electrophilic substitution
(b) renders the ring basic
(c) deactivates the ring towards nucleophilic substitution
(d) deactivates the ring towards electrophilic substitution

93. In which of the following ionisation processes, the bond order has increased and the magnetic behaviour has changed ?
(a) $C_2 \longrightarrow C_2^+$ (b) $NO \longrightarrow NO^+$
(c) $O_2 \longrightarrow O_2^+$ (d) $N_2 \longrightarrow N_2^+$

94. The actinoids exhibit more number of oxidation states in general than the lanthanoids. This is because :
(a) the 5*f* orbitals are more buried than the 4*f* orbitals
(b) there is a similarity between 4*f* and 5*f* orbitals in their angular part of the wave function
(c) the actinoids are more reactive than the lanthanoids
(d) the 5*f* orbitals extend further from the nucleus than the 4*f* orbitals

95. Equal masses of methane and oxygen are mixed in an empty container at 25°C. The fraction of the total pressure exerted by oxygen is :
(a) $\frac{2}{3}$ (b) $\frac{1}{3}\times\frac{273}{298}$
(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

96. A 5.25% solution of a substance is isotonic with a 1.5% solution of urea (molar mass = 60 g mol^{-1}) in the same solvent. If the densities of both the solutions are assumed to be equal to $1.0\,g\,cm^{-3}$, molar mass of the substance will be :
(a) $90.0\ g\ mol^{-1}$ (b) $115.0\ g\ mol^{-1}$
(c) $105.0\ g\ mol^{-1}$ (d) $210.0\ g\ mol^{-1}$

97. Assuming that water vapour is an ideal gas, the internal energy change (ΔU) when 1 mole of water is vapourised at 1 bar pressure and 100°C, (Given : molar enthalpy of vaporisation of water at 1 bar and 373 K = 41 kJ mol^{-1} and $R = 8.3\,J\,mol^{-1}K^{-1}$) will be :

(a) $4.100\ kJ\ mol^{-1}$ (b) $3.7904\ kJ\ mol^{-1}$
(c) $37.904\ kJ\ mol^{-1}$ (d) $41.00\ kJ\ mol^{-1}$

98. In a saturated solution of the sparingly soluble strong electrolyte $AgIO_3$ (Molecular mass = 283) the equilibrium which sets in is

$$AgIO_3(s) \rightleftharpoons Ag^+(aq) + IO_3^-(aq)$$

if the solubility product constant K_{sp} of $AgIO_3$ at a given temperature is 1.0×10^{-8}, what is the mass of $AgIO_3$ contained in 100 mL of its saturated solution ?

(a) 28.3×10^{-2} g (b) 2.83×10^{-3} g
(c) 1.0×10^{-7} g (d) 1.0×10^{-4} g

99. A radioactive element gets spilled over the floor of a room. Its half-life period is 30 days. If the initial activity is ten times the permissible value, after how many days will it be safe to enter the room ?

(a) 1000 days (b) 300 days
(c) 10 days (d) 100 days

100. Which one of the following conformations of cyclohexane is chiral ?

(a) Twist boat (b) Rigid
(c) Chair (d) Boat

101. Which of the following is the correct order of decreasing S_N2 reactivity ?

(a) $RCH_2X > R_3CX > R_2CHX$
(b) $RCH_2X > R_2CHX > R_3CX$
(c) $R_3CX > R_2CHX > RCH_2X$
(d) $R_2CHX > R_3CX > RCH_2X$
(X = a halogen)

102. In the following sequence of reactions,

$$CH_3CH_2OH \xrightarrow{P + I_2} A \xrightarrow[\text{ether}]{Mg} B \xrightarrow{HCHO} C \xrightarrow{H_2O} D$$

the compound 'D' is :

(a) butanal (b) n-butyl alcohol
(c) n-propyl alcohol (d) propanal

103. Which of the following sets of quantum numbers represents the highest energy of an atom ?

(a) $n = 3, l = 1, m = 1, s = +1/2$
(b) $n = 3, l = 2, m = 1, s = +1/2$
(c) $n = 4, l = 0, m = 0, s = +1/2$
(d) $n = 3, l = 0, m = 0, s = +1/2$

104. Which of the following hydrogen bonds is the strongest ?

(a) O—H... N (b) F—H... F
(c) O—H...O (d) O—H... F

105. In the reaction,

$$2Al(s) + 6HCl(aq) \rightarrow 2Al^{3+}(aq) + 6Cl^-(aq) + 3H_2(g)$$

(a) 6 L $HCl(aq)$ is consumed for every 3L $H_2(g)$ produced
(b) 33.6 L H_2 (g) is produced regardless of temperature and pressure for every mole Al that reacts
(c) 67.2 L $H_2(g)$ at STP is produced for every mole Al that reacts
(d) 11.2 L $H_2(g)$ at STP is produced for every mole $HCl(aq)$ consumed

106. Regular use of which of the following fertilisers increases the acidity of soil ?

(a) Potassium nitrate
(b) Urea
(c) Superphosphate of lime
(d) Ammonium sulphate

107. Identify the correct statement regarding a spontaneous process :

(a) For a spontaneous process in an isolated system, the change in entropy is positive
(b) Endothermic processes are never spontaneous
(c) Exothermic processes are always spontaneous
(d) Lowering of energy in the reaction process is the only criterion for spontaneity

108. Which of the following nuclear reactions will generate an isotope ?

(a) Neutron particle emission
(b) Positron emission
(c) α-particle emission
(d) β-particle emission

109. The equivalent conductances of two strong electrolytes at infinite dilution in H_2O (where ions move freely through a solution) at 25°C are given below :

$$\Lambda^\circ_{CH_3COONa} = 91.0\ S\ cm^2/\text{equiv}$$
$$\Lambda^\circ_{HCl} = 426.2\ S\ cm^2/\text{equiv}$$

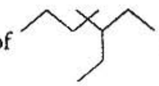
What additional information/quantity one needs to calculate Λ° of an aqueous solution of acetic acid ?

(a) Λ° of NaCl
(b) Λ° of CH_3COOK
(c) The limiting equivalent conductance of H^+ $(\lambda^\circ_{H^+})$
(d) Λ° of chloroacetic acid ($ClCH_2COOH$)

110. Which one of the following is the strongest base in aqueous solution ?

(a) Trimethylamine (b) Aniline
(c) Dimethylamine (d) Methylamine

111. The compound formed as a result of oxidation of ethyl benzene by $KMnO_4$ is :
(a) benzophenone (b) acetophenone
(c) benzoic acid (d) benzyl alcohol

112. The IUPAC name of  is :
(a) 1, 1-diethyl-2, 2-dimethylpentane
(b) 4, 4-dimethyl-5, 5-diethylpentane
(c) 5, 5-diethyl-4, 4-dimethylpentane
(d) 3-ethyl-4, 4-dimethylheptane

113. Which of the following species exhibits the diamagnetic behaviour ?
(a) O_2^{2-} (b) O_2^+
(c) O_2 (d) NO

114. The stability of dihalides of Si, Ge, Sn and Pb increases steadily in the sequence :
(a) $GeX_2 < SiX_2 < SnX_2 < PbX_2$
(b) $SiX_2 < GeX_2 < PbX_2 < SnX_2$
(c) $SiX_2 < GeX_2 < SnX_2 < PbX_2$
(d) $PbX_2 < SnX_2 < GeX_2 < SiX_2$

115. Identify the incorrect statement among the following :
(a) Ozone reacts with SO_2 to give SO_3
(b) Silicon reacts with NaOH(*aq*) in the presence of air to give Na_2SiO_3 and H_2O
(c) Cl_2 reacts with excess of NH_3 to give N_2 and HCl
(d) Br_2 reacts with hot and strong NaOH solution to give NaBr, $NaBrO_4$ and H_2O

116. The charge/size ratio of a cation determines its polarizing power. Which one of the following sequences represents the increasing order of the polarizing power of the cationic species, $K^+, Ca^{2+}, Mg^{2+}, Be^{2+}$?
(a) $Mg^{2+} < Be^{2+} < K^+ < Ca^{2+}$
(b) $Be^{2+} < K^+ < Ca^{2+} < Mg^{2+}$
(c) $K^+ < Ca^{2+} < Mg^{2+} < Be^{2+}$
(d) $Ca^{2+} < Mg^{2+} < Be^{2+} < K^+$

117. The density (in g mL^{-1}) of a 3.60 M sulphuric acid solution that is 29% H_2SO_4 (molar mass = 98 g mol^{-1}) by mass will be :
(a) 1.64 (b) 1.88
(c) 1.22 (d) 1.45

118. The first and second dissociation constants of an acid H_2A are 1.0×10^{-5} and 5.0×10^{-10} respectively. The overall dissociation constant of the acid will be :
(a) 5.0×10^{-5} (b) 5.0×10^{15}
(c) 5.0×10^{-15} (d) 0.2×10^{5}

119. A mixture of ethyl alcohol and propyl alcohol has a vapour pressure of 290 mm at 300 K. The vapour pressure of propyl alcohol is 200 mm. If the mole fraction of ethyl alcohol is 0.6, its vapour pressure (in mm) at the same temperature will be :
(a) 350 (b) 300
(c) 700 (d) 360

120. In conversion of limestone to lime,

$$CaCO_3(s) \longrightarrow CaO(s) + CO_2(g)$$

the values of $\Delta H°$ and $\Delta S°$ are +179.1 kJ mol^{-1} and 160.2 J/K respectively at 298 K and 1 bar. Assuming that $\Delta H°$ and $\Delta S°$ do not change with temperature, temperature above which conversion of limestone to lime will be spontaneous is :
(a) 1008 K (b) 1200 K
(c) 845 K (d) 1118 K

ANSWERS

MATHEMATICS

1.	(d)	**2.**	(b)	**3.**	(d)	**4.**	(c)	**5.**	(d)	**6.**	(c)	**7.**	(d)	**8.**	(b)
9.	(a)	**10.**	(c)	**11.**	(d)	**12.**	(c)	**13.**	(b)	**14.**	(c)	**15.**	(c)	**16.**	(a)
17.	(*)	**18.**	(b)	**19.**	(d)	**20.**	(d)	**21.**	(a)	**22.**	(b)	**23.**	(c)	**24.**	(b)
25.	(d)	**26.**	(c)	**27.**	(c)	**28.**	(d)	**29.**	(a)	**30.**	(d)	**31.**	(c)	**32.**	(a)
33.	(c)	**34.**	(a)	**35.**	(a)	**36.**	(d)	**37.**	(*)	**38.**	(a)	**39.**	(c)	**40.**	(a)

Note. None of the option is match (*).

➠ PHYSICS

41.	(a)	**42.**	(b)	**43.**	(b)	**44.**	(d)	**45.**	(d)	**46.**	(c)	**47.**	(c)	**48.**	(b)
49.	(b)	**50.**	(d)	**51.**	(d)	**52.**	(b)	**53.**	(c)	**54.**	(d)	**55.**	(d)	**56.**	(b)
57.	(b)	**58.**	(d)	**59.**	(a)	**60.**	(d)	**61.**	(c)	**62.**	(c)	**63.**	(b)	**64.**	(c)
65.	(d)	**66.**	(a)	**67.**	(a)	**68.**	(b)	**69.**	(c)	**70.**	(d)	**71.**	(a)	**72.**	(a)
73.	(a)	**74.**	(c)	**75.**	(a)	**76.**	(c)	**77.**	(d)	**78.**	(d)	**79.**	(a)	**80.**	(a)

➠ CHEMISTRY

81.	(d)	**82.**	(c)	**83.**	(c)	**84.**	(a)	**85.**	(d)	**86.**	(d)	**87.**	(a)	**88.**	(a)
89.	(a)	**90.**	(d)	**91.**	(c)	**92.**	(d)	**93.**	(b)	**94.**	(d)	**95.**	(c)	**96.**	(d)
97.	(c)	**98.**	(b)	**99.**	(d)	**100.**	(c)	**101.**	(b)	**102.**	(c)	**103.**	(b)	**104.**	(b)
105.	(d)	**106.**	(d)	**107.**	(a)	**108.**	(a)	**109.**	(a)	**110.**	(c)	**111.**	(c)	**112.**	(d)
113.	(a)	**114.**	(c)	**115.**	(d)	**116.**	(c)	**117.**	(c)	**118.**	(c)	**119.**	(a)	**120.**	(d)

HINTS & SOLUTIONS

Mathematics

1. Since, each term is equal to the sum of two preceeding terms.

$\therefore \quad ar^{n-1} = ar^n + ar^{n+1}$

$\Rightarrow \quad 1 = r + r^2 \quad \Rightarrow \quad r^2 + r - 1 = 0$

$\Rightarrow \quad r = \dfrac{\sqrt{5}-1}{2} \quad \left(\because \ r \neq \dfrac{-\sqrt{5}-1}{2} \right)$

2. $\because \sin^{-1}\left(\dfrac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\dfrac{5}{4}\right) = \dfrac{\pi}{2}$

$\Rightarrow \quad \sin^{-1}\left(\dfrac{x}{5}\right) + \sin^{-1}\left(\dfrac{4}{5}\right) = \dfrac{\pi}{2}$

$\Rightarrow \quad \sin^{-1}\left(\dfrac{x}{5}\right) = \dfrac{\pi}{2} - \sin^{-1}\left(\dfrac{4}{5}\right)$

$\Rightarrow \quad \sin^{-1}\left(\dfrac{x}{5}\right) = \cos^{-1}\left(\dfrac{4}{5}\right)$

$\Rightarrow \quad \sin^{-1}\left(\dfrac{x}{5}\right) = \sin^{-1}\left(\dfrac{3}{5}\right)$

$\Rightarrow \quad x = 3$

3. Since, in binomial expansion of $(a-b)^n$, $n \geq 5$, the sum of fifth and sixth term is equal to zero.

$\therefore \quad {}^nC_4 \, a^{n-4}(-b)^4 + {}^nC_5 \, a^{n-5}(-b)^5 = 0$

$\Rightarrow \dfrac{n!}{(n-4)!\,4!} a^{n-4} \cdot b^4 - \dfrac{n!}{(n-5)!\,5!} a^{n-5} b^5 = 0$

$\Rightarrow \dfrac{n!}{(n-5)!\,4!} a^{n-5} \cdot b^4 \left\{ \dfrac{a}{n-4} - \dfrac{b}{5} \right\} = 0$

$\Rightarrow \dfrac{a}{b} = \dfrac{n-4}{5}.$

4. Required number of ways $= {}^{12}C_4 \times {}^8C_4 \times {}^4C_4$

$= \dfrac{12!}{8! \times 4!} \times \dfrac{8!}{4! \times 4!} \times 1$

$= \dfrac{12!}{(4!)^3}.$

5. 4^{-x^2} is defined for $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$.

$\cos^{-1}\left(\dfrac{x}{2} - 1\right)$ is defined if $-1 \leq \dfrac{x}{2} - 1 \leq 1.$

$\Rightarrow \quad 0 \leq \dfrac{x}{2} \leq 2 \quad \Rightarrow \quad 0 \leq x \leq 4$

and $\log(\cos x)$ is defined, if $\cos x > 0$

$\Rightarrow \quad -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

Hence $f(x) = 4^{-x^2} + \cos^{-1}\left(\dfrac{x}{2} - 1\right) + \log(\cos x)$

is defined if $x \in \left[0, \dfrac{\pi}{2}\right)$.

6. $\because OC = CA = CB$

$\Rightarrow \angle AOC = \angle OAC$ and $\angle COB = \angle OBC$

$\therefore \quad \sin\theta = \sin A = \dfrac{5}{13}$

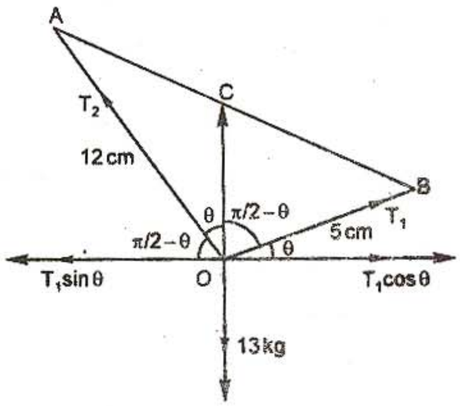
and $\quad \cos\theta = \dfrac{12}{13}$

Now by Lami's theorem

$$\frac{T_1}{\sin(180° - \theta)} = \frac{T_2}{\sin(90° + \theta)}$$

$$\Rightarrow \quad \frac{T_1}{\sin\theta} = \frac{T_2}{\cos\theta} \quad \Rightarrow \quad T_1 \cos\theta = T_2 \sin\theta$$

$$\Rightarrow \quad T_1\left(\frac{12}{13}\right) = T_2\left(\frac{5}{13}\right) \quad \Rightarrow \quad T_1 = \left(\frac{5}{12}\right)T_2.$$



Also $\quad T_1 \sin\theta + T_2 \cos\theta = 13$

$$\Rightarrow \quad T_2\left(\frac{5}{12}\cdot\frac{5}{13} + \frac{12}{13}\right) = 13$$

$$\Rightarrow \quad T_2\left(\frac{169}{12\cdot 13}\right) = 13$$

$$\Rightarrow \quad T_2 = 12 \text{ kg} \quad \text{and} \quad T_1 = 5 \text{ kg}.$$

7. Probability of getting score 9 in a single throw

$$= \frac{4}{36} = \frac{1}{9}.$$

Probability of getting score 9 exactly twice

$$= {}^3C_2 \times \left(\frac{1}{9}\right)^2 \times \frac{8}{9} = \frac{8}{243}.$$

8. Equation of circle which touches x-axis and coordinates of centre are (h, k), is

$$(x-h)^2 + (y-k)^2 = k^2$$

$\because$ It is passing through $(-1, 1)$, then

$$(-1-h)^2 + (1-k)^2 = k^2$$

$$\Rightarrow \quad h^2 + 2h - 2k + 2 = 0$$

$$\because \quad D \geq 0$$

$$\Rightarrow \quad 2k - 1 \geq 0 \quad \Rightarrow \quad k \geq \frac{1}{2}.$$

9. If direction cosines of L be l, m, n, then

$$2l + 3m + n = 0 \qquad \ldots\text{(i)}$$

and $\quad l + 3m + 2n = 0 \qquad \ldots$(ii)

On solving eqs. (i) and (ii), we get

$$\frac{l}{3} = \frac{m}{-3} = \frac{n}{3}$$

$$\therefore \quad l : m : n = \frac{1}{\sqrt{3}} : \left(-\frac{1}{\sqrt{3}}\right) : \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad l = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \cos\alpha = \frac{1}{\sqrt{3}}.$$

10. General equation of all such circles which pass through the origin and whose centre lie on x-axis, is

$$x^2 + y^2 + 2gx = 0. \qquad \ldots\text{(i)}$$

On differentiating w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} + 2g = 0$$

$$2g = -\left(2x + 2y\frac{dy}{dx}\right)$$

On putting the value of $2g$ in eq. (i), we get

$$x^2 + y^2 + \left(-2x - 2y\frac{dy}{dx}\right)x = 0$$

$$\Rightarrow \quad x^2 + y^2 - 2x^2 - 2xy\frac{dy}{dx} = 0$$

$$\Rightarrow \quad y^2 = x^2 + 2xy\frac{dy}{dx}$$

Which is required equation.

11. Using AM $\geq$ GM

$$\frac{p^2 + q^2}{2} \geq pq$$

$$\Rightarrow \quad pq \leq \frac{1}{2} \qquad (\because p^2 + q^2 = 1)$$

Now $\quad (p+q)^2 = p^2 + q^2 + 2pq$

$$\Rightarrow \quad (p+q)^2 = 1 + 2pq$$

$$\Rightarrow \quad (p+q)^2 \leq 1 + 1$$

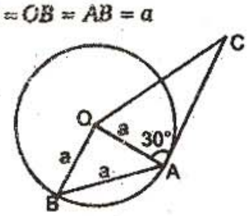
$$\Rightarrow \quad p + q \leq \sqrt{2}.$$

12. Let h be the height of a tower.

$\because \quad \angle AOB = 60°$

$\therefore \Delta OAB$ is equilateral.

$\therefore \quad OA = OB = AB = a$



Now in ΔOAC, $\tan 30° = \frac{h}{a} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{a}$

$$\Rightarrow \quad h = \frac{a}{\sqrt{3}}.$$

13. We know that

$$(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1 x + \ldots + {}^{20}C_{10} x^{10} + \ldots + {}^{20}C_{20} x^{20}$$

On putting $x = -1$ in above expansion, we get

$$0 = {}^{20}C_0 - {}^{20}C_1 + \ldots - {}^{20}C_9 + {}^{20}C_{10} - {}^{20}C_{11} + \ldots + {}^{20}C_{20}$$

$\Rightarrow 0 = {}^{20}C_0 - {}^{20}C_1 + \dots - {}^{20}C_9 + {}^{20}C_{10} - {}^{20}C_9 + \dots + {}^{20}C_0$

$\Rightarrow 0 = 2({}^{20}C_0 - {}^{20}C_1 + \dots - {}^{20}C_9) + {}^{20}C_{10}$

$\Rightarrow {}^{20}C_{10} = 2({}^{20}C_0 - {}^{20}C_1 + \dots + {}^{20}C_{10})$

$\Rightarrow \quad {}^{20}C_0 - {}^{20}C_1 + \dots + {}^{20}C_{10} = \frac{1}{2}\,{}^{20}C_{10}.$

14. Equation of normal is $Y - y = -\frac{dx}{dy}(X - x)$

$\Rightarrow \quad G = \left(x + y\frac{dy}{dx},\ 0\right)$

According to question

$$\left|x + y\frac{dy}{dx}\right| = |2x|$$

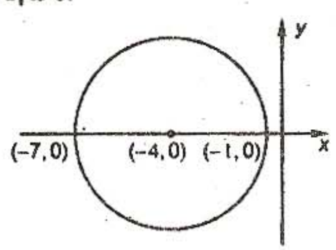
$\Rightarrow \quad y\frac{dy}{dx} = x \quad$ or $\quad y\frac{dy}{dx} = -3x$

$\Rightarrow \quad y\,dy = x\,dx \quad$ or $\quad y\,dy = -3x\,dx$

$\Rightarrow \quad \frac{y^2}{2} = \frac{x^2}{2} + c \quad$ or $\quad \frac{y^2}{2} = -\frac{3x^2}{2} + c$

$\Rightarrow \quad x^2 - y^2 = -2c \quad$ or $\quad 3x^2 + y^2 = 2c.$

15. From the Argand diagram maximum value of $|z+1|$ is 6.

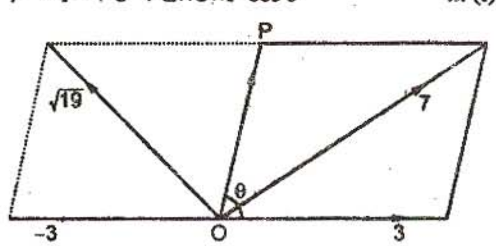


Aliter :

$|z+1| = |z+4-3| \le |z+4| + |-3| \le 6$

Thus maximum value of $|z+1|$ is 6.

16. $7^2 = P^2 + 3^2 + 2\times3\times P\cos\theta \quad \dots$ (i)



and $(\sqrt{19})^2 = P^2 + (-3)^2 + 2\times(-3)\times P\cos\theta \quad \dots$(ii)

On adding eqs. (i) and (ii), we get

$68 = 2P^2 + 18 \Rightarrow P = 5.$

17. The desired probability

$= (0.7)(0.2) + (0.7)(0.8)(0.7)(0.2) + (0.7)(0.8)(0.7)(0.8)(0.7)(0.2) + \dots$

$= 0.14[1 + (0.56) + (0.56)^2 + \dots]$

$= 0.14\left[\frac{1}{1-0.56}\right] = \frac{0.14}{0.44} = \frac{7}{22} = 0.32$

From above it is clear that no option is correct.

18. $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

On applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix} = xy.$$

Hence, D is divisible by both x and y.

19. The given equation of hyperbola is $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$. Here, $a^2 = \cos^2\alpha$ and $b^2 = \sin^2\alpha$

Coordinates of foci are $(\pm ae, 0)$.

$\because \quad e = \sqrt{1 + \frac{b^2}{a^2}}$

$\Rightarrow \quad e = \sqrt{1 + \frac{\sin^2\alpha}{\cos^2\alpha}} = \sqrt{1 + \tan^2\alpha}$

$\Rightarrow \quad e = \sec\alpha.$

Hence, abscissae of foci remain constant when α varies.

20. Since, a line makes an angle of $\frac{\pi}{4}$ with positive direction of each of x-axis and y-axis, therefore

$$\alpha = \frac{\pi}{4}, \beta = \frac{\pi}{4}$$

we know that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \quad \cos^2\frac{\pi}{4} + \cos^2\frac{\pi}{4} + \cos^2\gamma = 1$

$\Rightarrow \quad \frac{1}{2} + \frac{1}{2} + \cos^2\gamma = 1$

$\Rightarrow \quad \cos^2\gamma = 0 \Rightarrow \gamma = 90°$

21. Using mean value theorem

$\therefore \quad f'(c) = \frac{f(3) - f(1)}{3-1}$

$\Rightarrow \quad \frac{1}{c} = \frac{\log 3 - \log 1}{2}$

$\Rightarrow \quad c = \frac{2}{\log_e 3} = 2\log_3 e.$

22. $\because\ f(x) = \tan^{-1}(\sin x + \cos x)$

$$\therefore\ f'(x) = \frac{1}{1+(\sin x + \cos x)^2}(\cos x - \sin x)$$

$$= \frac{\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)}{1+(\sin x+\cos x)^2}$$

$f(x)$ is increasing if $-\frac{\pi}{2} < x + \frac{\pi}{4} < \frac{\pi}{2}$

$\Rightarrow \quad -\frac{3\pi}{4} < x < \frac{\pi}{4}$

Hence, $f(x)$ is increasing when $x \in \left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$.

23. $\because A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$

$$\therefore\ A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 25\alpha+5\alpha^2 & 10\alpha+25\alpha^2 \\ 0 & \alpha^2 & 5\alpha^2+25\alpha \\ 0 & 0 & 25 \end{bmatrix}$$

$$|A^2| = \begin{vmatrix} 25 & 25\alpha+5\alpha^2 & 10\alpha+25\alpha^2 \\ 0 & \alpha^2 & 5\alpha^2+25\alpha \\ 0 & 0 & 25 \end{vmatrix}$$

$$= 25\begin{vmatrix} 25 & 25\alpha+5\alpha^2 \\ 0 & \alpha^2 \end{vmatrix} = 625\alpha^2$$

But $\quad |A^2| = 25$

$\therefore \quad 625\alpha^2 = 25 \Rightarrow |\alpha| = \frac{1}{5}$.

24. $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots$

$$= 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots = e^{-1}.$$

25. Let $(2\hat{\mathbf{u}} \times 3\hat{\mathbf{v}})$ is a unit vector.

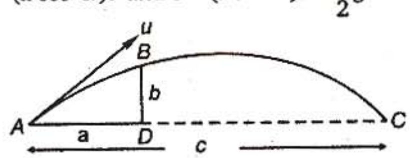
$$|2\hat{\mathbf{u}} \times 3\hat{\mathbf{v}}| = 1$$

$\Rightarrow \quad 6|\hat{\mathbf{u}}||\hat{\mathbf{v}}||\sin\theta| = 1$

$\Rightarrow \quad \sin\theta = \frac{1}{6}$

Hence, there is exactly one value of θ for which $(2\hat{\mathbf{u}} \times 3\hat{\mathbf{v}})$ is a unit vector.

26. $a = (u\cos\alpha)t$ and $b = (u\sin\alpha)t - \frac{1}{2}gt^2$



$$b = a\tan\alpha - \frac{1}{2}g\frac{a^2}{u^2\cos^2\alpha}$$

Also, $\quad c = \frac{u^2\sin 2\alpha}{g}$

$$b = a\tan\alpha - \frac{a^2 g}{2}\left(\frac{\sin 2\alpha}{cg}\right)\sec^2\alpha$$

$\Rightarrow \quad b = a\tan\alpha - \frac{a^2}{2c}2\tan\alpha$

$\Rightarrow \quad \left(a - \frac{a^2}{c}\right)\tan\alpha = b$

$\Rightarrow \quad \tan\alpha = \frac{bc}{a(c-a)}$.

$$\alpha = \tan^{-1}\frac{bc}{a(c-a)}$$

27. Let the number of boys and girls are x and y.

$\therefore \quad 52x + 42y = 50(x+y)$

$\Rightarrow \quad 52x + 42y = 50x + 50y$

$\Rightarrow \quad 2x = 8y \Rightarrow x = 4y$

$\therefore$ Total number of students in the class

$= x + y = 4y + y = 5y$

$\therefore$ Required % of boys $= \frac{4y}{5y} \times 100\% = 80\%$.

28. $\because$ Required point must be on the directrix of the parabola.

Hence the required point is $(-2, 0)$.

29. Equation of sphere is

$$x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$$

whose coordinates of centre are $(3, 6, 1)$

Let the coordinates of the other end of diameter are

(α, β, γ), then $\frac{\alpha+2}{2} = 3$, $\frac{\beta+3}{2} = 6$, $\frac{\gamma+5}{2} = 1$

Hence $\alpha = 4$, $\beta = 9$ and $\gamma = -3$.

Thus the coordinates of other point are $(4, 9, -3)$.

30. $\because \vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\vec{\mathbf{b}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

and $\quad \vec{\mathbf{c}} = x\hat{\mathbf{i}} + (x-2)\hat{\mathbf{j}} - \hat{\mathbf{k}}$ are coplanar

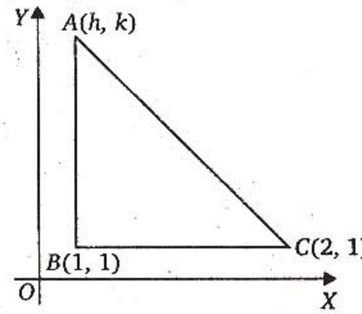
$$\therefore \quad \begin{vmatrix} x & x-2 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 0$$

$\Rightarrow \quad 3x + 2 - x + 2 = 0$

$\Rightarrow \quad 2x = -4$

$\Rightarrow \quad x = -2.$

31. $\because A(h, k), B(1, 1)$ and $C(2, 1)$ are the vertices of a right angled triangle ABC.



Now, $AB = \sqrt{(1-h)^2 + (1-k)^2}$

or $BC = \sqrt{(2-1)^2 + (1-1)^2} = 1$

or $CA = \sqrt{(h-2)^2 + (k-1)^2}$

Now, Pythagorus theorem

$$AC^2 = AB^2 + BC^2$$

$$4 + h^2 - 4h + k^2 + 1 - 2k = h^2 + 1 - 2h + k^2 + 1 - 2k + 1$$

$$5 - 4h = 3 - 2h$$

$\Rightarrow \quad h = 1 \quad \ldots(i)$

Now, given that area of the triangle is 1.

Then, area $(\Delta ABC) = \frac{1}{2} \times AB \times BC$

$$1 = \frac{1}{2} \times \sqrt{(1-h)^2 + (1-k)^2} \times 1$$

$2 = \sqrt{(1-h)^2 + (1-k)^2} \quad \ldots(ii)$

Putting $h = 1$ from Eq. (i), we get

$$2 = \sqrt{(k-1)^2}$$

Squaring both the sides, we get

$$4 = k^2 + 1 - 2k$$

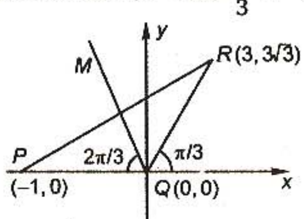
or $k^2 - 2k - 3 = 0$

or $(k-3)(k+1) = 0$

So, $k = -1, 3$

Thus the set of values of k is $\{-1, 3\}$.

32. Slope of the line $QM = \tan \frac{2\pi}{3} = -\sqrt{3}$



Hence equation is line QM is $y = -\sqrt{3}x$

or $\sqrt{3}x + y = 0$.

33. Equation of bisectors of lines $xy = 0$ are $y = \pm x$

put $y = \pm x$ in $my^2 + (1 - m^2)xy - mx^2 = 0$, we get

$$mx^2 + (1 - m^2)x^2 - mx^2 = 0$$

$\Rightarrow \quad (1 - m^2)x^2 = 0 \Rightarrow m = \pm 1.$

34. $\because \quad f(x) = \int_1^x \frac{\log t}{1+t}\, dt$

and $\quad F(e) = f(e) + f\left(\frac{1}{e}\right)$

$$\Rightarrow \quad F(e) = \int_1^e \frac{\log t}{1+t}\, dt + \int_1^{1/e} \frac{\log t}{1+t}\, dt$$

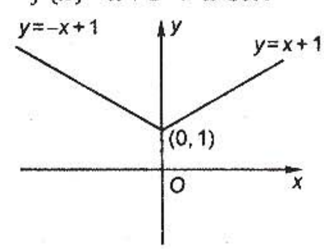
$$= \int_1^e \frac{\log t}{1+t}\, dt + \int_1^e \frac{\log t}{t(1+t)}\, dt$$

$$= \int_1^e \frac{\log t}{t}\, dt = \left[\frac{(\log t)^2}{2}\right]_1^e$$

$$= \frac{1}{2}[(\log e)^2 - (\log 1)^2] = \frac{1}{2}$$

35. $f(x) = \min\{x + 1, |x| + 1\}$

$f(x) = x + 1 \quad \forall\ x \in R.$



From figure it is clear that $f(x) \geq 1$, when $x \in R$.

36. $$\lim_{x \to 0} \left\{\frac{1}{x} - \frac{2}{e^{2x} - 1}\right\}$$

$$= \lim_{x \to 0} \frac{e^{2x} - 1 - 2x}{x(e^{2x} - 1)}$$

$$= \lim_{x \to 0} \frac{2e^{2x} - 2}{(e^{2x} - 1) + 2xe^{2x}}$$

$$= \lim_{x \to 0} \frac{4e^{2x}}{4e^{2x} + 4xe^{2x}} = 1$$

$\therefore f(x)$ is continuous at $x = 0$, then

$$\lim_{x \to 0} f(x) = f(0)$$

$\Rightarrow \quad 1 = f(0).$

37. $$\int_{\sqrt{2}}^{x} \frac{dt}{t\sqrt{t^2 - 1}} = \frac{\pi}{2}$$

$\Rightarrow \quad [\sec^{-1} t]_{\sqrt{2}}^{x} = \frac{\pi}{2}$

$\Rightarrow \quad \sec^{-1} x - \frac{\pi}{4} = \frac{\pi}{2}$

$\Rightarrow \quad \sec^{-1} x = \frac{3\pi}{4}$

$\Rightarrow \quad x = -\sqrt{2}.$

38. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$

$$= \frac{1}{2}\int \sec\left(x - \frac{\pi}{3}\right) dx$$

$$= \frac{1}{2}\log\tan\left(\frac{x}{2} - \frac{\pi}{6} + \frac{\pi}{4}\right) + c$$

$$= \frac{1}{2}\log\tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + c.$$

39. Required area $= A = \int_0^1 (\sqrt{x} - x)\,dx$

$$= \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}.$$

40. Let α and β be the roots of equation $x^2 + ax + 1 = 0$, then

$$\alpha + \beta = -a \quad \text{and} \quad \alpha\beta = 1$$

Now $\quad |\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$\Rightarrow \quad |\alpha - \beta| = \sqrt{a^2 - 4}$

According to question,

$\sqrt{a^2 - 4} < \sqrt{5}$

$\Rightarrow \quad a^2 - 4 < 5$

$\Rightarrow \quad a^2 - 9 < 0$

$\Rightarrow \quad a \in (-3, 3).$

Physics

41. $\because \quad x = (2 \times 10^{-2})\cos \pi t$

Here, $\quad a = 2 \times 10^{-2}\text{ m} = 2\text{ cm}$

At $t = 0$, $x = 2$ cm, *i.e.*, the object is at positive extreme, so to acquire maximum speed (*i.e.*, to reach mean position) it takes $\frac{1}{4}$ th of time period.

$\therefore \quad$ Required time $= \frac{T}{4}$

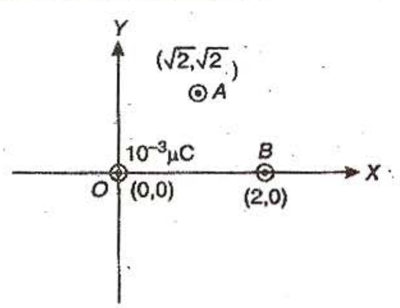
where $\quad \omega = \frac{2\pi}{T} = \pi$

$\Rightarrow \quad T = 2\text{s}$

So, $\quad$ required time $= \frac{T}{4} = \frac{2}{4} = 0.5$ s

42. For given circuit current is lagging the voltage by $\pi/2$, so circuit is purely inductive and there is no power consumption in the circuit. The work done by battery is stored as magnetic energy in the inductor.

43. Potential at A due to charge at O



$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{(10^{-3})}{OA}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{(10^{-3})}{\sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}}$$

Potential at B due to charge at O

$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(10^{-3})}{OB}$$

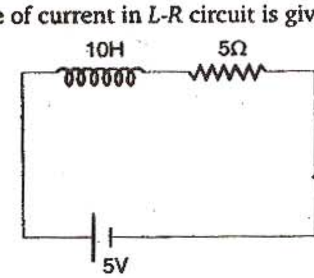
$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{(10^{-3})}{2}$$

So, $\quad V_A - V_B = 0$

44. Ratio of energy stored in the capacitor and the work done by the battery

$$= \frac{\frac{1}{2}qV}{qV} = \frac{1}{2}$$

45. Rise of current in L-R circuit is given by



$$I = I_0(1 - e^{-t/\tau})$$

where $\quad I_0 = \frac{E}{R} = \frac{5}{5} = 1\text{ A}$

Now, $\tau = \frac{L}{R} = \frac{10}{5} = 2s$

After 2s, *i.e.*, at $t = 2s$

Rise of current $I = (1 - e^{-1})$ A

46. Current density $J = \frac{i}{\pi a^2}$

From Ampere's circuital law

$$\oint B \cdot dl = \mu_0 \cdot i_{\text{enclosed}}$$

For $r < a$

$$B \times 2\pi r = \mu_0 \times J \times \pi r^2$$

$\Rightarrow$ $$B = \frac{\mu_0 i}{\pi a^2} \times \frac{r}{2}$$

At $r = a/2$

$$B_1 = \frac{\mu_0 i}{4\pi a}$$

For $r > a$

$$B \times 2\pi r = \mu_0 i \Rightarrow B = \frac{\mu_0 i}{2\pi r}$$

At $r = 2a$, $B_2 = \frac{\mu_0 i}{4\pi a}$

So, $\frac{B_1}{B_2} = 1$

47. Using Ampere's circuital law the magnetic field at any point inside the pipe is zero.

48. Binding energy

$$\text{BE} = (M_{\text{nucleus}} - M_{\text{nucleons}})c^2$$
$$= (M_o - 8M_p - 9M_n)c^2$$

49. In gamma ray emission the energy is released from nucleus, so that nucleus get stabilised.

50. For $V_i < 0$

the diode is reverse biased and hence offer infinite resistance, so circuit would be like as shown in Fig. (2) and $V_o = 0$.

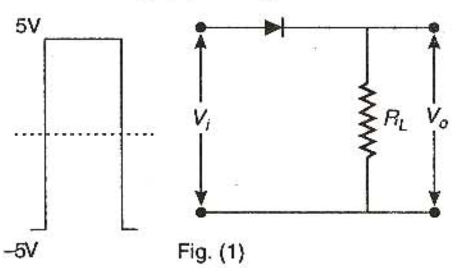


Fig. (1)

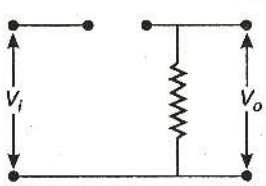


Fig. (2)

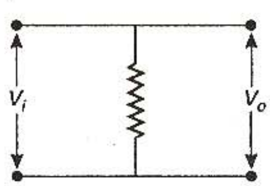


Fig. (3)

For $V_i > 0$, the diode is forward biased and circuit would be as shown in Fig. (3) and $V_o = V_i$.

Hence, the opitcal (d) is correct.

51. The momentum of the photon

$$p = \frac{h}{\lambda} = \frac{h\nu}{c}$$

52. $$v = v_0 + gt + ft^2$$

or $$\frac{dx}{dt} = v_0 + gt + ft^2$$

$\Rightarrow$ $$dx = (v_0 + gt + ft^2)dt$$

So, $$\int_0^x dx = \int_0^1 (v_0 + gt + ft^2)\,dt$$

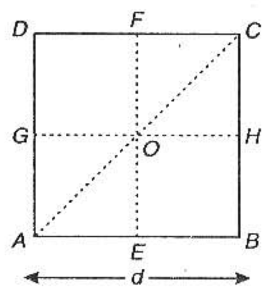
$\Rightarrow$ $$x = v_0 + g/2 + f/3$$

53. Let the each side of square lamina is d.

So, $I_{EF} = I_{GH}$ (due to symmetry)

and $I_{AC} = I_{BD}$ (due to symmetry)

Now, according to theorem of perpendicular axis,



$$I_{AC} + I_{BD} = I_0$$

$\Rightarrow$ $$2I_{AC} = I_0 \quad \ldots\text{(i)}$$

and $$I_{EF} + I_{GH} = I_0$$

$\Rightarrow$ $$2I_{EF} = I_0 \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get $I_{AC} = I_{EF}$

$\therefore$ $$I_{AD} = I_{EF} + \frac{md^2}{4}$$

$$= \frac{md^2}{12} + \frac{md^2}{4} \quad \left(\text{as } I_{EF} = \frac{md^2}{12}\right)$$

So, $$I_{AD} = \frac{md^2}{3} = 4I_{EF}$$

54. $x = x_0 \cos(\omega t - \pi/4)$

Acceleration, $a = \frac{d^2x}{dt^2}$

$$= -\omega^2 x_0 \cos(\omega t - \pi/4)$$
$$= \omega^2 x_0 \cos(\omega t + 3\pi/4)$$

So, $A = \omega^2 x_0$

and $\delta = 3\pi/4$

55. Direction of $\vec{\mathbf{E}}$ reverses while magnitude remains same and V remains unchanged.

56.
$$T_{1/2}(X) = \tau(Y)$$
$$\Rightarrow \quad \frac{0.693}{\lambda_X} = \frac{1}{\lambda_Y}$$
$$\Rightarrow \quad \lambda_Y = \frac{\lambda_X}{0.693}$$
$$\Rightarrow \quad \lambda_Y > \lambda_X$$
So, Y will decay faster than X.

57. For Carnot engine using as refrigerator
$$W = Q_2\left(\frac{T_1}{T_2} - 1\right)$$
It is given $\eta = \frac{1}{10}$
$$\Rightarrow \quad \eta = 1 - \frac{T_2}{T_1}$$
$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{9}{10}$$
So, $Q_2 = 90\,\text{J}$ (as $W = 10$ J)

58. The number of free electrons for conduction is significant only in Si and Ge but small in C, as C is an impurity.

59. As $\vec{\mathbf{v}}$ of charged particle is remaining constant, it means force acting on charged particle is zero.

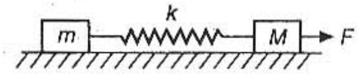
So, $$q(\vec{\mathbf{v}} \times \vec{\mathbf{B}}) = q\vec{\mathbf{E}}$$
$$\Rightarrow \quad \vec{\mathbf{v}} \times \vec{\mathbf{B}} = \vec{\mathbf{E}}$$
$$\Rightarrow \quad \vec{\mathbf{v}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{B^2}$$

60.
$$\vec{\mathbf{E}} = -\frac{\partial V}{\partial x}\hat{\mathbf{i}} - \frac{\partial V}{\partial y}\hat{\mathbf{j}} - \frac{\partial V}{\partial z}\hat{\mathbf{k}}$$
$$\Rightarrow \quad E_x = -\frac{\partial V}{\partial x} = -\frac{d}{dx}\left[\frac{20}{x^2 - 4}\right]$$
$$= \frac{40\,x}{(x^2 - 4)^2}$$
$$\Rightarrow \quad E_x \text{ at } x = 4\ \mu\text{m} = \frac{10}{9}\ \text{V}/\mu\text{m}$$
and is along positive x-direction.

61. Emission spectrum would be when electron makes a jump from higher energy level to lower energy level.

Frequency of emitted photon is proportional to change in energy of two energy levels, *i.e.*,
$$\nu = RcZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

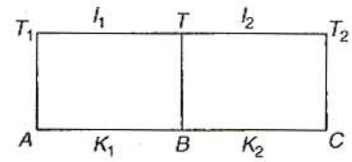
62. Acceleration of system, $a = \frac{F}{m + M}$



So, force acting on mass $m = ma$
$$= \frac{mF}{m + M}$$

63. Power of a lens is reciprocal of its focal length. Power of combined lens is
$$P = P_1 + P_2$$
$$= -15 + 5 = -10\,\text{D}$$
$$\therefore \quad f = \frac{1}{P} = \frac{100}{-10}\ \text{cm}$$
$$f = -10\ \text{cm}$$

64. Let temperature at the interface is T.
For part AB,



$$\frac{Q_1}{t} \propto \frac{(T_1 - T)K_1}{l_1}$$
For part BC,
$$\frac{Q_2}{t} \propto \frac{(T - T_2)K_2}{l_2}$$
At equilibrium, $\frac{Q_1}{t} = \frac{Q_2}{t}$
$$\therefore \quad \frac{(T_1 - T)K_1}{l_1} = \frac{(T - T_2)K_2}{l_2}$$
$$\Rightarrow \quad T = \frac{T_1K_1l_2 + T_2K_2l_1}{K_1l_2 + K_2l_1}$$

65. Let intensity of sound be I and I'.
Loudness of sound initially
$$\beta_1 = 10\log\left(\frac{I}{I_0}\right)$$
Later, $$\beta_2 = 10\log\left(\frac{I'}{I_0}\right)$$
Given, $\beta_2 - \beta_1 = 20$
$$\therefore \quad 20 = 10\log\left(\frac{I'}{I}\right)$$
$$\therefore \quad I' = 100\,I$$

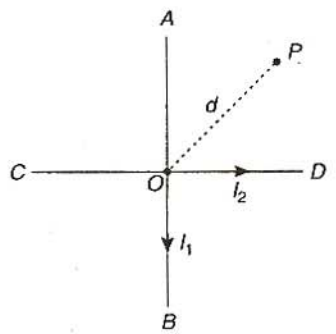
66. According to Mayer's relation,
$$C_P - C_V = \frac{R}{m} = \frac{R}{28}$$

67. In case of motion of a charged particle perpendicular to the motion, *i.e.*, displacement, the work done

$$W = \int \vec{\mathbf{F}} \cdot \vec{\mathbf{ds}} = \int F ds \cos\theta = 0 \qquad (\text{as } \theta = 90°)$$

and by work-energy theorem, $W = \Delta KE$, the kinetic energy and hence speed v remains constant. But $\vec{\mathbf{v}}$ changes, so, momentum changes.

68. The magnetic field induction at a point P, at a distance d from O in a direction perpendicular to the plane $ABCD$ due to currents through AOB and COD are perpendicular to each other, is



Hence, $B = \sqrt{B_1^2 + B_2^2}$

$$= \left[\left(\frac{\mu_0}{4\pi}\frac{2I_1}{d}\right)^2 + \left(\frac{\mu_0}{4\pi}\frac{2I_2}{d}\right)^2\right]^{1/2}$$

$$= \frac{\mu_0}{2\pi d}\sqrt{(I_1^2 + I_2^2)}$$

69. From $R_t = R_0(1 + \alpha t)$

$$5 = R_0(1 + 50\alpha) \qquad \ldots(i)$$

and $6 = R_0(1 + 100\alpha) \qquad \ldots(ii)$

$\therefore \quad \frac{5}{6} = \frac{1 + 50\alpha}{1 + 100\alpha} \Rightarrow \alpha = \frac{1}{200}$

Putting value of α in Eq (i), we get

$$5 = R_0(1 + 50 \times 1/200)$$

$\therefore \quad R_0 = 4\,\Omega$

70. On introduction and removal and again on introduction, the capacity and potential remain same. So, net work done by the system in this process

$$W = U_f - U_i$$
$$= \frac{1}{2}C V^2 - \frac{1}{2}C V^2$$
$$= 0$$

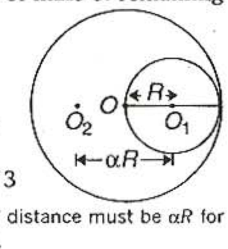
71. According to Millikan's oil drop experiment, electronic charge is given by,

$$q = \frac{6\pi\eta r(v_1 + v_2)}{E}$$

which is independent of g.

So, $\frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}} = 1$

72. In this question distance of centre of mass of new disc from the centre of mass of remaining disc is αR.



Mass of remaining disc

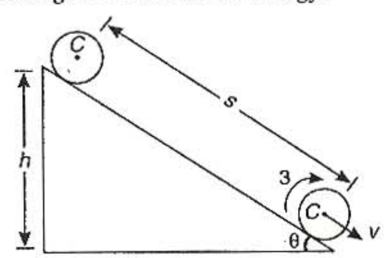
$= M - M/4 = 3M/4$

$\therefore \quad -\frac{3M}{4}\alpha R + \frac{M}{4}R = 0$

$\Rightarrow \quad \alpha = 1/3$

Note : In Q.No. 72, the given distance must be αR for real approach to the solution.

73. Assuming that no energy is used up against friction, the loss in potential energy is equal to the total gain in the kinetic energy.



Thus, $Mgh = \frac{1}{2}I(v^2/R^2) + \frac{1}{2}Mv^2$

or $\frac{1}{2}v^2(M + I/R^2) = Mgh$

or $v^2 = \frac{2Mgh}{M + I/R^2} = \frac{2gh}{1 + I/MR^2}$

If s be the distance covered along the plane, then

$$h = s\sin\theta$$
$$v^2 = \frac{2gs\sin\theta}{1 + I/MR^2}$$

Now, $v^2 = 2as$

$\therefore \quad 2as = \frac{2gs\sin\theta}{1 + I/MR^2}$

or $a = \frac{g\sin\theta}{1 + I/MR^2}$

74. According to the principle of conservation of angular momentum, in the absence of external torque, the total angular momentum of the system is constant.

75. $a = \frac{F_k}{m} = \frac{15}{2} = 7.5\ \text{m/s}^2$

Now, $ma = \frac{1}{2}kx^2$

or $2 \times 7.5 = \frac{1}{2} \times 10000 \times x^2$

or $x^2 = 3 \times 10^{-3}$

or $x = 0.055$ m

$\Rightarrow$ $x = 5.5$ cm

76. Kinetic energy at highest point

$$(KE)_H = \frac{1}{2}mv^2\cos^2\theta$$
$$= K\cos^2\theta$$
$$= K(\cos 60°)^2$$
$$= \frac{K}{4}$$

77. Phase difference $= \frac{2\pi}{\lambda} \times$ path difference

i.e., $\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{6}$

$$= \frac{\pi}{3}$$

As, $I = I_{max}\cos^2(\phi/2)$

or $\frac{I}{I_{max}} = \cos^2(\phi/2)$

or $\frac{I}{I_0} = \cos^2(\pi/6)$

$$= \frac{3}{4}$$

78. $f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}$

and $f' = \frac{1}{2\pi} \cdot 2\sqrt{\frac{k_1 + k_2}{m}} = 2f$

79. From first law of thermodynamics,

$$Q = \Delta U + W$$

For path *iaf*,

$$50 = \Delta U + 20$$

$\therefore$ $\Delta U = U_f - U_i = 30$ cal

For path *ibf*,

$$Q = \Delta U + W$$

or $W = Q - \Delta U$

$$= 36 - 30 = 6 \text{ cal}$$

80. Average kinetic energy of particle

$$= \frac{1}{4}ma^2\omega^2$$
$$= \frac{1}{4}ma^2(2\pi\nu)^2$$
$$= \pi^2\nu^2ma^2$$

Chemistry

81. $A_2 + B_2 \rightleftharpoons 2AB$

E_a(forward) = 180 kJ mol^{-1}

E_a (backward) = 200 kJ mol^{-1}

In the presence of catalyst :

E_a (forward) = 180 − 100 = 80 kJ mol^{-1}

E_a (backward) = 200 − 100 = 100 kJ mol^{-1}

$$\Delta H = E_a \text{ (forward)} - E_a \text{ (backward)}$$
$$= 80 - 100$$
$$= -20 \text{ kJ mol}^{-1}$$

82. Cell is completely discharged, it means equilibrium gets established, $E_{cell} = 0$

$$Zn/Zn^{2+} (1M) || Cu^{2+} (1M) | Cu$$

cell reaction : $Zn + Cu^{2+} \rightleftharpoons Zn^{2+} + Cu$

$$K_{eq} = \frac{[Zn^{2+}]}{[Cu^{2+}]}$$

We know,

$$E_{cell} = E°_{cell} - \frac{0.0591}{n}\log K_{eq}$$

$$E°_{cell} = \frac{0.0591}{n}\log K_{eq}$$

or $1.10 = \frac{0.0591}{2}\log K_{eq}$

$$K_{eq} = \frac{[Zn^{2+}]}{[Cu^{2+}]} = \text{antilog}\frac{2.20}{0.0591}$$
$$= \text{antilog } 37.3$$

83. Aqueous buffered solution of HA

50% HA is ionised

$\Rightarrow$ $[HA] = [A^-]$

Buffer solution of weak acid H$A \rightarrow$ acidic buffer

$$pH = pK_a + \log\frac{[A^-]}{[HA]}$$

or $pH = pK_a = 4.5$

$$pOH = pK_w - pH$$
$$pOH = 14 - 4.5 = 9.5$$

84. $2A + B \longrightarrow$ products

$[B]$ is doubled, half-life didn't change

Half-life is independent of change in conc. of reactant ⇒ First order

First order w.r.t. to B

$[A]$ is doubled, rate increased by two times

⇒ First order w.r.t A

Hence, net order of reaction = 1 + 1 = 2

Unit for the rate constant = $\text{conc.}^{(1-n)}t^{-1}$

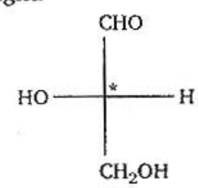
$= (\text{mol}\cdot L^{-1})^{-1}\cdot s^{-1}$

$= L\cdot mol^{-1}s^{-1}$

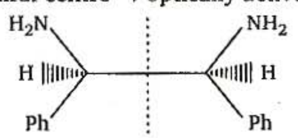
85. $4f$ and $5f$ belongs to different shell, experience different amount of shielding.

86. Cl^- is a weak ligand but Cl^- cause the pairing of electron with large Pt^{2+} and consequently give dsp^2 hybridisation and square planar geometry.

87. The molecule, which is optically active, has chiral centre, is expected to rotate the plane of polarised light.



One chiral centre ⇒ optically active

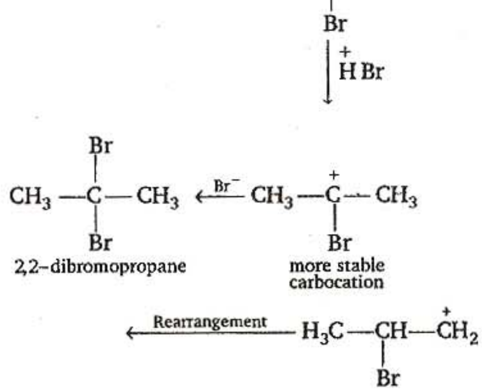


Two chiral centres, but plane of symmetry within molecule ⇒ optically inactive

88. Primary structure involves sequence of α-amino acids polypeptide chain.

Secondary structure involves α-helical and β-pleated sheet like structures.

89. $CH_3—C\equiv CH + HBr \rightarrow CH_3—C(Br)=CH_2$



$CH_3—CH=CHBr \xrightarrow{HBr} CH_3—CH_2—CHBr_2$

$CH\equiv CH \xrightarrow{2HBr} CH_3CHBr_2$

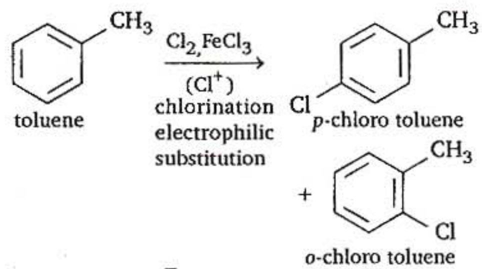
$CH_3—CH=CH_2 \xrightarrow{HBr} CH_3—CHBr—CH_3$

90. $CH_3CH_2NH_2 + CHCl_3 + KOH \rightarrow A + B + 3H_2O$

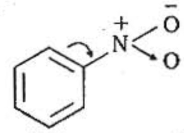
$CHCl_3 + KOH \longrightarrow :CCl_2 + KCl + H_2O$

$CH_3CH_2NH_2 + :CCl_2 \rightarrow CH_3CH_2NC + 2HCl$
carbylamine

91.



92.



$—NO_2$ group withdraw electron from the ring, shows $-M$ effect makes ring electron deficient, thus, deactivates ring for electrophilic substitution.

93. $NO \longrightarrow NO^+$

$(\overset{+}{NO})$ Total $e^- = 14$

$\sigma 1s^2\sigma^* 1s^2\sigma 2s^2\sigma^* 2s^2\pi 2p_x^{1+1} = 2p_y^{1+1}\sigma 2p_z^2$

Diamagnetic

Bond order $= \frac{10-4}{2} = 3$

(NO) Total $e^- = 15$

$\sigma 1s^2\sigma^* 1s^2\sigma 2s^2\sigma^* 2s^2, \sigma 2p_z^2\ \pi 2p_x^{1+1}\ \pi 2p_y^{1+1}$

$\pi^* 2p_x^1 = \pi 2p_y$

Paramagnetic

Bond order $= \frac{10-5}{2} = 2.5$

Electron is taken away from non-bonding molecular orbital, that's why bond order increases.

94. The actinoid ($5f$-elements) exhibits more number of oxidation states in general than the lanthanoid because $5f$ orbitals extend farther from the nucleus than the $4f$ orbitals.

95. Suppose the equal mass of methane and oxygen = $w = 1$ g

Mole fraction of oxygen $= \dfrac{w/32}{w/32 + w/16}$

$$= \frac{\frac{1}{32}}{3/32} = \frac{1}{3}$$

Let the total pressure = P

Pressure exerted by oxygen (partial pressure)

$$= X_{O_2} \times P_{total} = P \times \frac{1}{3}$$

96. Solution is isotonic.

$$\Rightarrow \quad C_1RT = C_2RT$$
$$C_1 = C_2$$

Density of both the solutions are assumed to be equal to 1.0 g cm^{-3} $\Rightarrow$ molality = molarity 5.25%

in 100 g 5.25 g of substance in 1000 g 52.5 g of substance

Hence, $\dfrac{52.5}{M} = \dfrac{15}{60}$,

M = molecular mass of the substance

$$M = \frac{52.5 \times 60}{15} = 210$$

97. $H_2O\,(l) \longrightarrow H_2O\,(g)$

$$\Delta n = 1 - 0 = 1$$
$$\Delta E = \Delta H - \Delta nRT$$
$$= 41 - 1 \times 8.3 \times 373 \times 10^{-3}\ (R = 8.3 \times 10^{-3})$$
$$= 37.9 \text{ kJ mol}^{-1}$$

98. $AgIO_3(s) \rightleftharpoons Ag^+(aq) + IO_3^-(aq)$

Let solubility of $AgIO_3$ be S

$$K_{sp} = [Ag^+][IO_3^-]$$
$$1.0 \times 10^{-8} = S^2 \text{ or } S = 1 \times 10^{-4} \text{ mol/L}$$

In 1000 mL mol of $AgIO_3$ dissolved

$= 1 \times 10^{-4}$ mol

In 100 mL of mole of $AgIO_3$ dissolved

$= 1 \times 10^{-5}$ mol

Mass of $AgIO_3$ in 100 mL $= 1 \times 10^{-5} \times 283$

$= 2.83 \times 10^{-3}$

99. Activity $\propto N$

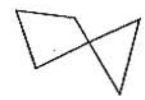
$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

or $\quad \dfrac{1}{10} = \left(\dfrac{1}{2}\right)^n$

or $\quad 10 = 2^n$

Taking log on both sides

$$\log 10 = n \log 2$$
$$n = \frac{1}{0.301} = 3.32$$
$$\text{time} = n \times \text{half-life}$$
$$= 3.32 \times 30 = 99.6 \text{ days}$$

100. 

Chair form is unsymmetrical and absence of any element of symmetry.

101. S_N2 reactions are greatly controlled by steric factor.

$$\underset{1°}{R—CH_2—X} \quad \underset{2°}{R_2CH—X} \quad \underset{3°}{R_3C—X}$$

S_N2 reactivity decreases as bulkyness of alkyl group increases.

102. $CH_3CH_2OH \xrightarrow{P + I_2} \underset{'A'}{CH_3CH_2—I} \xrightarrow{\text{Mg, ether}} \underset{'B'}{CH_3CH_2MgI} \xrightarrow{H—\overset{O}{\overset{\|}{C}}—H} \underset{'C'}{CH_3—CH_2—\overset{OMgI}{\overset{|}{CH_2}}} \xrightarrow{H_2O}$

$\underset{'D'}{CH_3CH_2\overset{OH}{\overset{|}{CH_2}}} + Mg(OH)I$

n-propyl alcohol

103. $n + l = 5$ maximum.

104. Hydrogen bond is strongest in HF due to higher electronegativity of F.

105. $2Al(s) + \underset{1\,mol}{6HCl(aq)} \rightarrow 2Al^{3+}(aq) + 6Cl^-(aq) + 3H_2\,(g)$

106. $(NH_4)_2SO_4 \longrightarrow \underset{acidic}{2\overset{+}{N}H_4} + SO_4^{2-}$

107. In an isolated system where either mass and energy are not exchanged with surrounding for the spontaneous process, the change in entropy is positive.

108. ${}^{A}_{Z}X \longrightarrow {}^{A-1}_{Z}X + {}_0n^1$

Isotopes are species having same number of proton, but different number of neutron.

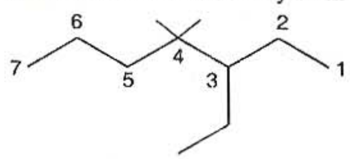
109. We know from Kohlrausch's law

$$\lambda°_{CH_3COOH} = \lambda°_{CH_3COONa} + \lambda°_{HCl} - \lambda°_{NaCl}$$

110. In aqueous solution, basicity order :

$\underset{2°}{\text{Dimethylamine}} > \underset{1°}{\text{methylamine}} > \underset{3°}{\text{trimethylamine}} > \text{aniline}$

111. Any aliphatic carbon with hydrogen attached to it, in combination with benzene ring, will be oxidised to benzoic acid by $KMnO_4/H^+$.

112. 

3-ethyl-diethylheptane

113. The correct option is O_2^{2-}. This species has 18 electrons, which are filled in such a way that all molecular orbitals are fully filled, so diamagnetic.

$$\sigma 1s^2 \sigma^* 1s^2, \sigma 2s^2 \sigma^* 2s^2\ \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2, \pi^* 2p_x^2 = \pi^* 2p_y^2$$

114. Due to inert pair effect, the stability of +2 oxidation state increases as we move down this group.

$$\therefore \quad SiX_2 < GeX_2 < SnX_2 < PbX_2$$

115. Br_2 reacts with hot and strong NaOH to give NaBr, $NaBrO_3$ and H_2O.

116. Higher the charge/size ratio, more is the polarising power.

$$\therefore \quad K^+ < Ca^{2+} < Mg^{2+} < Be^{2+}$$

117. $\text{Molarity} = \dfrac{10 \times \text{density} \times \text{wt. of solute}}{\text{mol. wt. of the solute}}$

$$\text{density} = \frac{3.60 \times 98}{10 \times 29} = 1.21$$

118.

$$K = k_1 \times k_2 = 1.0 \times 10^{-5} \times 5.0 \times 10^{-10} = 5 \times 10^{-15}$$

119. According to Raoult's law :

$$P = P^\circ_A X_A + P^\circ_B X_B$$

$$290 = 200 \times 0.4 + P^\circ_B \times 0.6$$

$$P^\circ_B = 350$$

120. $\Delta S = \dfrac{\Delta H}{T}$

$$\Delta S = 160.2 \text{ J/K}$$

$$\Delta H = 179.1 \times 10^3 \text{ J/ mol}$$

$$T = \frac{179.1 \times 10^3 \text{ J/ mol}}{160.2 \text{ J/K}} = 1117.97 \text{ K} \approx 1118 \text{ K.}$$