Physics

1. The Kirchhoff's first law $(\Sigma i = 0)$ and second law $(\Sigma iR = \Sigma E)$,
where the symbols have their usual meanings, are respectively based on :
(a) conservation of charge, conservation of momentum
(b) conservation of energy, conservation of charge
(c) conservation of momentum, conservation of charge
(d) conservation of charge, conservation of energy

2. Needles N_1, N_2 and N_3 are made of a ferromagnetic, a paramagnetic and a diamagnetic substance respectively. A magnet when brought close to them will :
(a) attract N_1 and N_2 strongly but repel N_3
(b) attract N_1 strongly, N_2 weakly and repel N_3 weakly
(c) attract N_1 strongly, but repel N_2 and N_3 weakly
(d) attract all three of them

3. A material 'B' has twice the specific resistance of 'A'. A circular wire made of 'B' has twice the diameter of a wire made of 'A'. Then for the two wires to have the same resistance, the ratio l_B / l_A of their respective lengths must be :
(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) 2

4. In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be a :
(a) helix (b) straight line
(c) ellipse (d) circle

5. An electric dipole is placed at an angle of 30° to a non-uniform electric field. The dipole will experience :
(a) a translational force only in the direction of the field
(b) a translational force only in a direction normal to the direction of the field
(c) a torque as well as a translational force
(d) a torque only

6. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity 'v' that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as :
(a) t^2 (b) t
(c) $t^{1/2}$ (d) t^3

7. A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is 4 ms^{-1}. The kinetic energy of the other mass is :
(a) 144 J (b) 288 J
(c) 192 J (d) 96 J

8. A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with speed v ms^{-1}. The velocity of sound in air is 300 ms^{-1}. If the person can hear frequencies upto a maximum of 10,000 Hz, the maximum value of v upto which he can hear the whistle is :
(a) $15\sqrt{2}$ ms^{-1} (b) $15/\sqrt{2}$ ms^{-1}
(c) 15 ms^{-1} (d) 30 ms^{-1}

9. A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string makes an angle of 45° with the initial vertical direction is :
(a) $Mg(\sqrt{2} + 1)$ (b) $Mg\sqrt{2}$
(c) $\frac{Mg}{\sqrt{2}}$ (d) $Mg(\sqrt{2} - 1)$

10. A particle of mass 100 g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is :

(a) – 0.5 J (b) –1.25 J

(c) 1.25 J (d) 0.5 J

11. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is :

(a) 0.01 s (b) 10 s

(c) 0.1 s (d) 100 s

12. Starting from the origin a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy ?

(a) $\frac{1}{6}$ s (b) $\frac{1}{4}$ s

(c) $\frac{1}{3}$ s (d) $\frac{1}{12}$ s

13. Assuming the sun to be a spherical body of radius R at a temperature of T K, evaluate the total radiant power, incident on earth, at a distance r from the sun :

(a) $4\pi r_0^2 R^2 \sigma T^4/r^2$ (b) $\pi r_0^2 R^2 \sigma T^4/r^2$

(c) $r_0^2 R^2 \sigma T^4/4\pi r^2$ (d) $R^2 \sigma T^4/r^2$

where r_0 is the radius of the earth and σ is Stefan's constant.

14. Which of the following units denotes the dimensions $[ML^2/Q^2]$, where Q denotes the electric charge ?

(a) Wb/m^2 (b) henry (H)

(c) H/m^2 (d) weber (Wb)

15. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m while applying the force and the ball goes upto 2 m height further, find the magnitude of the force. Consider $g = 10\ m/s^2$:

(a) 4 N (b) 16 N

(c) 20 N (d) 22 N

16. A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is :

(a) 105 Hz (b) 1.05 Hz

(c) 1050 Hz (d) 10.5 Hz

17. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved, so as to keep the centre of mass at the same position ?

(a) $\frac{m_2}{m_1} d$ (b) $\frac{m_1}{m_1 + m_2} d$

(c) $\frac{m_1}{m_2} d$ (d) d

18. A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching process is completed in 0.1s, the force of the blow exerted by the ball on the hand of the player is equal to :

(a) 150 N (b) 3 N

(c) 30 N (d) 300 N

19. In a common-base mode of a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of the base current amplification factor (β) will be :

(a) 49 (b) 50

(c) 51 (d) 48

20. A thermocouple is made from two metals, Antimony and Bismuth. If one junction of the couple is kept hot and the other is kept cold, then, an electric current will :

(a) flow from Antimony to Bismuth at the hot junction

(b) flow from Bismuth to Antimony at the cold junction

(c) not flow through the thermocouple

(d) flow from Antimony to Bismuth at the cold junction

21. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV and the stopping potential for a radiation incident on this surface is 5V. The incident radiation lies in :

(a) ultra-violet region

(b) infra -red region

(c) visible region

(d) X-ray region

22. An alpha nucleus of energy $\frac{1}{2} mv^2$ bombards a heavy nuclear target of charge Ze. Then the distance of closest approach for the alpha nucleus will be proportional to :

(a) v^2 (b) $1/m$

(c) $1/v^4$ (d) $1/Ze$

23. The time taken by a photoelectron to come out after the photon strikes is approximately :

(a) 10^{-4}s (b) 10^{-10} s

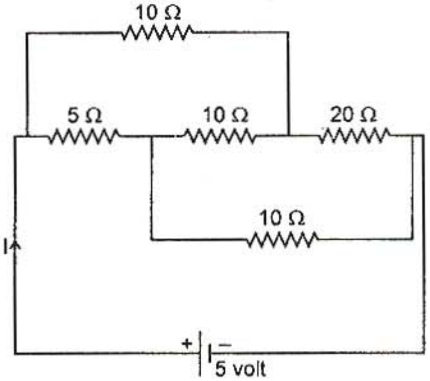
(c) 10^{-16} s (d) 10^{-1} s

24. When $_3Li^7$ nuclei are bombarded by protons, and the resultant nuclei are $_4Be^8$, the emitted particles will be :

(a) alpha particles (b) beta particles

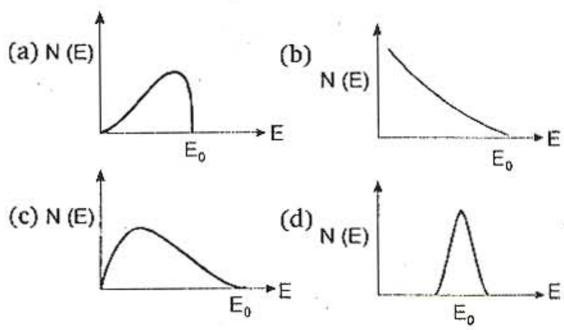
(c) gamma photons (d) neutrons

25. The current I drawn from the 5 volt source will be :



(a) 0.33 A (b) 0.5 A
(c) 0.67 A (d) 0.17 A

26. The energy spectrum of β-particles [number $N(E)$ as a function of β-energy E] emitted from a radioactive source is :



27. In a series resonant LCR circuit, the voltage across R is 100 volts and $R = 1\ k\Omega$ with $C = 2\mu F$. The resonant frequency ω is 200 rad/s. At resonance the voltage across L is :

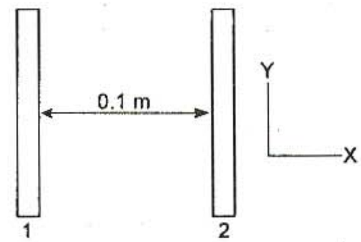
(a) 2.5×10^{-2} V (b) 40 V
(c) 250 V (d) 4×10^{-3} V

28. The resistance of a bulb filament is 100 Ω at a temperature of 100°C. If its temperature coefficient of resistance be 0.005 per °C, its resistance will become 200 Ω at a temperature of :

(a) 300°C (b) 400°C
(c) 500°C (d) 200°C

29. Two insulating plates are both uniformly charged in such a way that the potential difference between them is $V_2 - V_1 = 20$ V. (*i.e.*, plate 2 is at a higher potential). The plates are separated by $d = 0.1$ m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate 1. What is its speed when it hits plate 2 ?

$(e = 1.6 \times 10^{-19}\text{C},\ m_0 = 9.11 \times 10^{-31}\text{kg})$



(a) 2.65×10^6 m/s (b) 7.02×10^{12} m/s
(c) 1.87×10^6 m/s (d) 32×10^{-19} m/s

30. In an AC generator, a coil with N turns, all of the same area A and total resistance R, rotates with frequency ω in a magnetic field B. The maximum value of emf generated in the coil is :

(a) $N.A.B.R.\ \omega$ (b) $N.A.B$
(c) $N.A.B.R$ (d) $N.A.B.\ \omega$

31. A solid which is not transparent to visible light and whose conductivity increases with temperature is formed by :

(a) ionic binding
(b) covalent binding
(c) Van der Waal's binding
(d) metallic binding

32. The refractive index of glass is 1.520 for red light and 1.525 for blue light. Let D_1 and D_2 be angles of minimum deviation for red and blue light respectively in a prism of this glass. then, :

(a) $D_1 < D_2$
(b) $D_1 = D_2$
(c) D_1 can be less than or greater than D_2 depending upon the angle of prism
(d) $D_1 > D_2$

33. If the ratio of the concentration of electrons to that of holes in a semiconductor is $\frac{7}{5}$ and the ratio of currents is $\frac{7}{4}$, then what is the ratio of their drift velocities ?

(a) $\frac{5}{8}$ (b) $\frac{4}{5}$
(c) $\frac{5}{4}$ (d) $\frac{4}{7}$

34. In a Wheatstone's bridge, three resistances P, Q and R are connected in the three arms and the fourth arm is formed by two resistances S_1 and S_2 connected in parallel. The condition for the bridge to be balanced will be :

(a) $\frac{P}{Q} = \frac{2R}{S_1 + S_2}$ (b) $\frac{P}{Q} = \frac{R(S_1 + S_2)}{S_1 S_2}$

(c) $\frac{P}{Q} = \frac{R(S_1 + S_2)}{2S_1 S_2}$ (d) $\frac{P}{Q} = \frac{R}{S_1 + S_2}$

35. The flux linked with a coil at any instant '*t*' is given by :

$\phi = 10t^2 - 50t + 250$

The induced emf at $t = 3$ s is :

(a) −190 V (b) −10 V

(c) 10 V (d) 190 V

36. A long solenoid has 200 turns per cm and carries a current *i*. The magnetic field at its centre is 6.28×10^{-2} weber/m^2. Another long solenoid has 100 turns per cm and it carries a current *i*/3. The value of the magnetic field at its centre is :

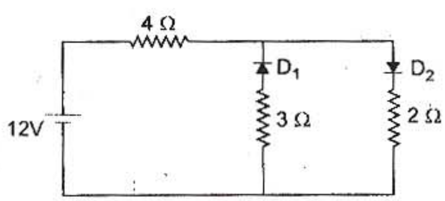
(a) 1.05×10^{-2} weber/m^2

(b) 1.05×10^{-5} weber/m^2

(c) 1.05×10^{-3} weber/m^2

(d) 1.05×10^{-4} weber/m^2

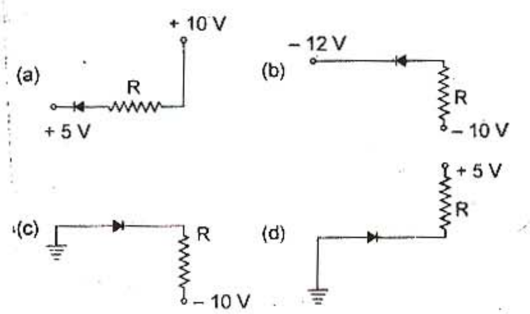
37. The circuit has two oppositely connected ideal diodes in parallel. What is the current flowing in the circuit ?



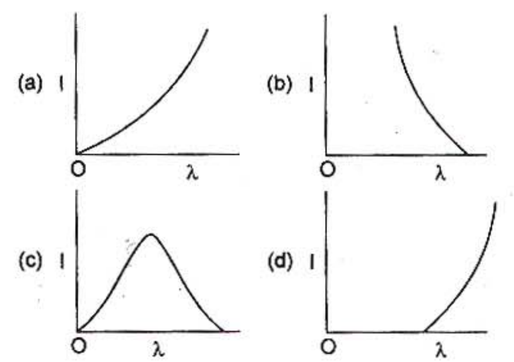
(a) 1.71 A (b) 2.00 A

(c) 2.31 A (d) 1.33 A

In the following, which one of the diodes is reverse biased ?



The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current *I* of the photocell varies as follows :



40. If the binding energy per nucleon in $^7_3$Li and $^4_2$He nuclei are 5.60 MeV and 7.06 MeV respectively, then in the reaction :

$p + {}^7_3\text{Li} \rightarrow 2\,{}^4_2\text{He}$

energy of proton must be :

(a) 28.24 MeV (b) 17.28 MeV

(c) 1.46 MeV (d) 39.2 MeV

41. An electric bulb is rated 220 volt − 100 watt. The power consumed by it when operated on 110 volt will be :

(a) 75 watt (b) 40 watt

(c) 25 watt (d) 50 watt

42. The 'rad' is the correct unit used to report the measurement of :

(a) the ability of a beam of gamma ray photons to produce ions in a target

(b) the energy delivered by radiation to a target

(c) the biological effect of radiation

(d) the rate of decay of a radioactive source

43. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω . The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time :

(a) at the mean position of the platform

(b) for an amplitude of g/ω^2

(c) for an amplitude of g^2/ω^2

(d) at the highest position of the platform

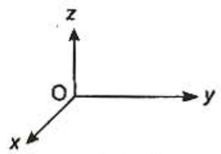
44. Four point masses, each of value *m*, are placed at the corners of a square *ABCD* of side *l*. The moment of inertia of this system about an axis passing through *A* and parallel to *BD* is :

(a) $2\,ml^2$ (b) $\sqrt{3}\,ml^2$

(c) $3ml^2$ (d) ml^2

45. A force of $-F\hat{\mathbf{k}}$ acts on *O*, the origin of the co-ordinate system. The torque about the point (1, −1) is :

(a) $F(\hat{\mathbf{i}} - \hat{\mathbf{j}})$
(b) $-F(\hat{\mathbf{i}} + \hat{\mathbf{j}})$
(c) $F(\hat{\mathbf{i}} + \hat{\mathbf{j}})$
(d) $-F(\hat{\mathbf{i}} - \hat{\mathbf{j}})$



46. The potential energy of a 1 kg particle free to move along the x-axis is given by $V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$ J.

The total mechanical energy of the particle is 2 J. Then, the maximum speed (in m/s) is :

(a) $3/\sqrt{2}$ (b) $\sqrt{2}$
(c) $1/\sqrt{2}$ (d) 2

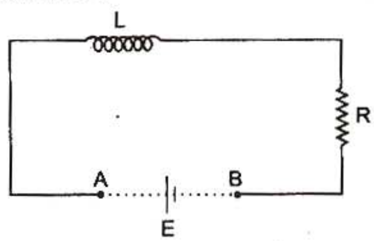
47. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity $\omega' =$:

(a) $\frac{\omega(m + 2M)}{m}$ (b) $\frac{\omega(m - 2M)}{(m + 2M)}$
(c) $\frac{\omega m}{(m + M)}$ (d) $\frac{\omega m}{(m + 2M)}$

48. A wire elongates by l mm when a load W is hanged from it. If the wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be (in mm) :

(a) l (b) $2l$
(c) zero (d) $l/2$

49. An inductor ($L = 100\,\text{mH}$), a resistor ($R = 100\,\Omega$) and a battery ($E = 100\,\text{V}$) are initially connected in series as shown in the figure. After a long time the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is :



(a) $1/e$ A (b) e A
(c) 0.1 A (d) 1 A

50. Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire then in equilibrium condition, the ratio of the magnitude of the electric fields at the surfaces of spheres A and B is :

(a) 4 : 1 (b) 1 : 2
(c) 2 : 1 (d) 1 : 4

51. Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0, while box B contains one mole of helium at temperature $(7/3)\,T_0$. The boxes are then put into thermal contact with each other, and heat flows between them until the gases reach a common final temperature (Ignore the heat capacity of boxes). Then, the final temperature of the gases, T_f, in terms of T_0 is :

(a) $T_f = \frac{3}{7}T_0$ (b) $T_f = \frac{7}{3}T_0$
(c) $T_f = \frac{3}{2}T_0$ (d) $T_f = \frac{5}{2}T_0$

52. The work of 146 kJ is performed in order to compress one kilo mole of a gas adiabatically and in this process the temperature of the gas increases by 7°C. The gas is :
($R = 8.3\,\text{J mol}^{-1}\,\text{K}^{-1}$)

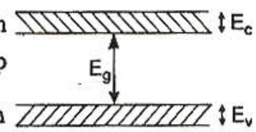
(a) diatomic
(b) triatomic
(c) a mixture of monoatomic and diatomic
(d) monoatomic

53. If the lattice constant of this semiconductor is decreased, then which of the following is correct ?

Conduction band width

Band gap

Valence band width



(a) All E_c, E_g, E_v increase
(b) E_c and E_v increase, but E_g decreases
(c) E_c and E_v decrease, but E_g increases
(d) All E_c, E_g, E_v decrease

54. The rms value of the electric field of the light coming from the sun is 720 N/C. The average total energy density of the electromagnetic wave is :

(a) 4.58×10^{-6} J/m^3 (b) 6.37×10^{-9} J/m^3
(c) 81.35×10^{-12} J/m^3 (d) 3.3×10^{-3} J/m^3

55. If the terminal speed of a sphere of gold (density $= 19.5$ kg/m^3) is 0.2 m/s in a viscous liquid (density $= 1.5$ kg/m^3), find the terminal speed of a sphere of silver (density $= 10.5$ kg/m^3) of the same size in the same liquid.

(a) 0.4 m/s (b) 0.133 m/s
(c) 0.1 m/s (d) 0.2 m/s

Chemistry

56. HBr reacts with $CH_2{=}CH{-}OCH_3$ under anhydrous conditions at room temperature to give :

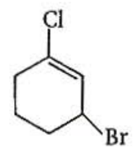
(a) CH_3CHO and CH_3Br

(b) $BrCH_2CHO$ and CH_3OH

(c) $BrCH_2-CH_2-OCH_3$

(d) $H_3C-CHBr-OCH_3$

57. The IUPAC name of the compound shown below is :



(a) 2-bromo-6-chlorocyclohex-1-ene

(b) 6-bromo-2-chlorocyclohexene

(c) 3-bromo-1-chlorocyclohexene

(d) 1-bromo-3-chlorocyclohexene

58. The increasing order of the rate of HCN addition to compounds A-D is :

(A) HCHO (B) CH_3COCH_3

(C) $PhCOCH_3$ (D) PhCOPh

(a) $A < B < C < D$ (b) $D < B < C < A$

(c) $D < C < B < A$ (d) $C < D < B < A$

59. How many moles of magnesium phosphate, $Mg_3(PO_4)_2$ will contain 0.25 mole of oxygen atoms ?

(a) 0.02 (b) 3.125×10^{-2}

(c) 1.25×10^{-2} (d) 2.5×10^{-2}

60. According to Bohr's theory, the angular momentum of an electron in 5th orbit is :

(a) $25 \frac{h}{\pi}$ (b) $1.0 \frac{h}{\pi}$

(c) $10 \frac{h}{\pi}$ (d) $2.5 \frac{h}{\pi}$

61. Which of the following molecules/ions does not contain unpaired electrons ?

(a) O_2^{2-} (b) B_2

(c) N_2^+ (d) O_2

62. Total volume of atoms present in a face-centred cubic unit cell of a metal is (r is atomic radius) :

(a) $\frac{20}{3} \pi r^3$ (b) $\frac{24}{3} \pi r^3$

(c) $\frac{12}{3} \pi r^3$ (d) $\frac{16}{3} \pi r^3$

63. A reaction was found to be second order with respect to the concentration of carbon monoxide. If the concentration of carbon monoxide is doubled, with everything else kept the same, the rate of reaction will :

(a) remain unchanged

(b) triple

(c) increases by a factor of 4

(d) double

64. Which of the following chemical reactions depicts the oxidizing behaviour of H_2SO_4 ?

(a) $2HI + H_2SO_4 \rightarrow I_2 + SO_2 + 2H_2O$

(b) $Ca(OH)_2 + H_2SO_4 \rightarrow CaSO_4 + 2H_2O$

(c) $NaCl + H_2SO_4 \rightarrow NaHSO_4 + HCl$

(d) $2PCl_5 + H_2SO_4 \rightarrow 2POCl_3 + 2HCl + SO_2Cl_2$

65. The IUPAC name for the complex $[Co(NO_2)(NH_3)_5]Cl_2$ is :

(a) nitrito-N-pentaminecobalt (III) chloride

(b) nitrito-N-pentamminecobalt (II) chloride

(c) pentammine nitrito-N-cobalt (II) chloride

(d) pentaamine nitrito-N-cobalt (III) chloride

66. The term anomers of glucose refers to :

(a) isomers of glucose that differ in configurations at carbons one and four (C–1 and C–4)

(b) a mixture of (D)–glucose and (L)–glucose

(c) enantiomers of glucose

(d) isomers of glucose that differ in configuration at carbon one (C–1)

67. In the transformation of ${}^{238}_{92}U$ to ${}^{234}_{92}U$, if one emission is an α-particle, what should be the other emission (s) ?

(a) Two β^- (b) Two β^- and one β^+

(c) One β^- and one γ

(d) One β^+ and one β^-

68. Phenyl magnesium bromide reacts with methanol to give :

(a) a mixture of anisol and Mg(OH)Br

(b) a mixture of benzene and Mg(OMe)Br

(c) a mixture of toluene and Mg(OH)Br

(d) a mixture of phenol and Mg(Me)Br

69. $CH_3Br + Nu^- \rightarrow CH_3-Nu + Br^-$

The decreasing order of the rate of the above reaction with nucleophiles (Nu^-) A to D is :

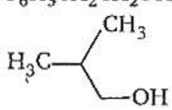
$[Nu^- = (A)PhO^-, (B)AcO^-, (C)HO^-, (D)CH_3O^-]$

(a) $D > C > A < B$ (b) $D > C > B > A$
(c) $A > B > C > D$ (d) $B > D > C > A$

70. The pyrimidine bases present in DNA are :

(a) cytosine and adenine
(b) cytosine and guanine
(c) cytosine and thymine
(d) cytosine and uracil

71. Among the following the one that gives positive iodoform test upon reaction with I_2 and NaOH is :

(a) $CH_3CH_2CH(OH)CH_2CH_3$
(b) $C_6H_5CH_2CH_2OH$
(c) 
(d) $PhCHOHCH_3$

72. The increasing order of stability of the following free radicals is :

(a) $(CH_3)_2\dot{C}H < (CH_3)_3\dot{C} < (C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C}$

(b) $(C_6H_5)_3\dot{C} < (C_6H_5)_2\dot{C}H < (CH_3)_3\dot{C} < (CH_3)_2\dot{C}H$

(c) $(C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C} < (CH_3)_3\dot{C} < (CH_3)_2\dot{C}H$

(d) $(CH_3)_2\dot{C}H < (CH_3)_3\dot{C} < (C_6H_5)_3\dot{C} < (C_6H_5)_2\dot{C}H$

73. Uncertainty in the position of an electron (mass $= 9.1 \times 10^{-31}$ kg) moving with a velocity $300\ ms^{-1}$, accurate upon 0.001% will be :

(a) 19.2×10^{-2} m (b) 5.76×10^{-2} m
(c) 1.92×10^{-2} m (d) 3.84×10^{-2} m

($h = 6.63 \times 10^{-34}$ Js)

74. Phosphorus pentachloride dissociates as follows, in a closed reaction vessel,

$$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$$

If total pressure at equilibrium of the reaction mixture is P and degree of dissociation of PCl_5 is x, the partial pressure of PCl_3 will be :

(a) $\left(\frac{x}{x+1}\right)P$ (b) $\left(\frac{2x}{1-x}\right)P$
(c) $\left(\frac{x}{x-1}\right)P$ (d) $\left(\frac{x}{1-x}\right)P$

75. The standard enthalpy of formation (ΔH_f°) at 298 K for methane, $CH_4(g)$, is -74.8 kJ mol^{-1}. The additional information required to determine the average energy for C—H bond formation would be :

(a) the dissociation energy of H_2 and enthalpy of sublimation of carbon
(b) latent heat of vaporization of methane
(c) the first four ionization energies of carbon and electron gain enthalpy of hydrogen
(d) the dissociation energy of hydrogen molecule, H_2

76. Among the following mixtures, dipole-dipole as the major interaction, is present in :

(a) benzene and ethanol
(b) acetonitrile and acetone
(c) KCl and water
(d) benzene and carbon tetrachloride

77. Fluorobenzene (C_6H_5F) can be synthesized in the laboratory :

(a) by heating phenol with HF and KF
(b) from aniline by diazotisation followed by heating the diazonium salt with HBF_4
(c) by direct fluorination of benzene with F_2 gas
(d) by reacting bromobenzene with NaF solution

78. A metal, M forms chlorides in its +2 and +4 oxidation states. Which of the following statements about these chlorides is correct ?

(a) MCl_2 is more volatile than MCl_4
(b) MCl_2 is more soluble in anhydrous ethanol than MCl_4
(c) MCl_2 is more ionic than MCl_4
(d) MCl_2 is more easily hydrolysed than MCl_4

79. Which of the following statement is true ?

(a) H_3PO_3 is a stronger acid than H_2SO_3
(b) In aqueous medium HF is a stronger acid than HCl
(c) $HClO_4$ is a weaker acid than $HClO_3$
(d) HNO_3 is a stronger acid than HNO_2

80. The molar conductivities Λ°_{NaOAc} and Λ°_{HCl} at infinite dilution in water at 25°C are 91.0 and 426.2 S cm^2/ mol respectively. To calculate Λ°_{HOAc}, the additional value required is :

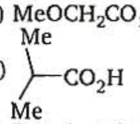
(a) $\Lambda^\circ_{H_2O}$ (b) Λ°_{KCl}
(c) Λ°_{NaOH} (d) Λ°_{NaCl}

81. Which one of the following sets of ions represents a collection of isoelectronic species ?

(a) $K^+, Cl^-, Ca^{2+}, Sc^{3+}$
(b) $Ba^{2+}, Sr^{2+}, K^+, S^{2-}$
(c) $N^{3-}, O^{2-}, F^-, S^{2-}$
(d) $Li^+, Na^+, Mg^{2+}, Ca^{2+}$

82. The correct order of increasing acid strength of the compounds :

(A) CH_3CO_2H (B) $MeOCH_2CO_2H$

(C) CF_3CO_2H (D) 

is :

(a) $B < D < A < C$ (b) $D < A < C < B$

(c) $D < A < B < C$ (d) $A < D < C < B$

83. In which of the following molecules/ions all the bonds are not equal ?

(a) SF_4 (b) SiF_4

(c) XeF_4 (d) BF_4^-

84. What products are expected from the disproportionation reaction of hypochlorous acid ?

(a) $HClO_3$ and Cl_2O (b) $HClO_2$ and $HClO_4$

(c) HCl and Cl_2O (d) HCl and $HClO_3$

85. Nickel $(Z = 28)$ combines with a uninegative monodentate ligand X^- to form a paramagnetic complex $[NiX_4]^{2-}$. The number of unpaired electron(s) in the nickel and geometry of this complex ion are respectively :

(a) one, tetrahedral (b) two, tetrahedral

(c) one, square planar (d) two, square planar

86. In $Fe(CO)_5$, the Fe—C bond possesses :

(a) π-character only

(b) both σ and π characters

(c) ionic character

(d) σ-character only

87. The increasing order of the first ionization enthalpies of the elements B, P, S and F (lowest first) is :

(a) F < S < P < B (b) P < S < B < F

(c) B < P < S < F (d) B < S < P < F

88. An ideal gas is allowed to expand both reversibly and irreversibly in an isolated system. If T_i is the initial temperature and T_f is the final temperature, which of the following statements is correct ?

(a) $(T_f)_{irrev} > (T_f)_{rev}$

(b) $T_f > T_i$ for reversible process but $T_f = T_i$ for irreversible process

(c) $(T_f)_{rev} = (T_f)_{irrev}$

(d) $T_f = T_i$ for both reversible and irreversible processes

89. In Langmuir's model of adsorption of a gas on a solid surface :

(a) the rate of dissociation of adsorbed molecules from the surface does not depend on the surface covered

(b) the adsorption at a single site on the surface may involve multiple molecules at the same time

(c) the mass of gas striking a given area of surface is proportional to the pressure of the gas

(d) the mass of gas striking a given area of surface is independent of the pressure of the gas

90. Rate of a reaction can be expressed by Arrhenius equation as :

$$k = Ae^{-E/RT}$$

In this equation, E represents :

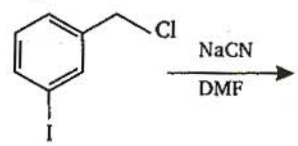
(a) the energy above which all the colliding molecules will react

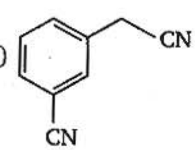
(b) the energy below which colliding molecules will not react

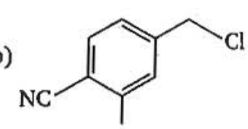
(c) the total energy of the reacting molecules at a temperature, T

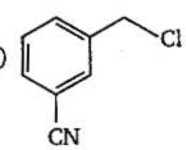
(d) the fraction of molecules with energy greater than the activation energy of the reaction

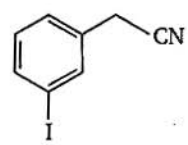
91. The structure of the major product formed in the following reaction is :



(a) 

(b) 

(c) 

(d) 

92. Reaction of *trans*-2-phenyl-1-bromocyclopentane on reaction with alcoholic KOH produces :

(a) 4-phenylcyclopentene

(b) 2-phenylcyclopentene

(c) 1-phenylcyclopentene

(d) 3-phenylcyclopentene

93. Increasing order of stability among the three main conformations (*i.e.*, Eclipse, Anti, Gauche) of 2-fluoroethanol is :

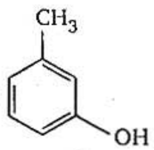
(a) Eclipse, Gauche, Anti

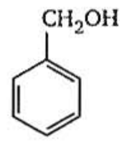
(b) Gauche, Eclipse, Anti

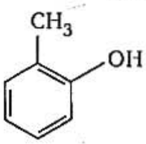
(c) Eclipse, Anti, Gauche

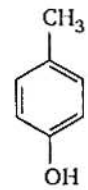
(d) Anti, Gauche, Eclipse

94. The structure of the compound that gives a tribromo derivative on treatment with bromine water is :

(a) 

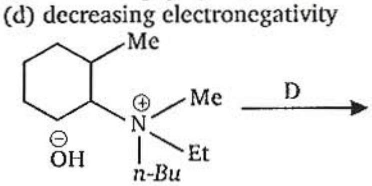
(b) 

(c) 

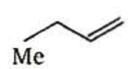
(d) 

95. The decreasing values of bond angles from NH_3 (107°) to SbH_3 (91°) down group-15 of the periodic table is due to :

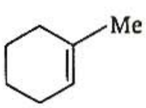
(a) increasing bp-bp repulsion
(b) increasing *p*-orbital character in sp^3
(c) decreasing lp-bp repulsion
(d) decreasing electronegativity

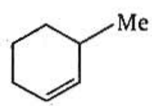
96. 

The alkene formed as a major product in the above elimination reaction is :

(a) 

(b) $CH_2{=}CH_2$

(c) 

(d) 

97. The 'spin-only' magnetic moment [in units of Bohr magneton, (μ_β)] of Ni^{2+} in aqueous solution would be (Atomic number of Ni = 28) :

(a) 2.84 (b) 4.90
(c) 0 (d) 1.73

98. The equilibrium constant for the reaction

$$SO_3(g) \rightleftharpoons SO_2(g) + \frac{1}{2}O_2(g)$$

is $K_c = 4.9 \times 10^{-2}$. The value of K_c for the reaction

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$$

will be :

(a) 416 (b) 2.40×10^{-3}
(c) 9.8×10^{-2} (d) 4.9×10^{-2}

99. Following statements regarding the periodic trends of chemical reactivity of the alkali metals and the halogens are given. Which of these statements gives the correct picture ?

(a) The reactivity decreases in the alkali metals but increases in the halogens with increase in atomic number down the group
(b) In both the alkali metals and the halogens the chemical reactivity decreases with increase in atomic number down the group
(c) Chemical reactivity increases with increase in atomic number down the group in both the alkali metals and halogens
(d) In alkali metals the reactivity increases but in the halogens it decreases with increase in atomic number down the group

100. Given the data at 25°C,

$$Ag + I^- \rightarrow AgI + e^-; \quad E° = 0.152\ V$$
$$Ag \rightarrow Ag^+ + e^-; \quad E° = -0.800\ V$$

What is the value of log K_{sp} for AgI ?

$\left(2.303\frac{RT}{F} = 0.059\ V\right)$

(a) − 8.12 (b) + 8.612
(c) − 37.83 (d) − 16.13

101. The following mechanism has been proposed for the reaction of NO with Br_2 to form NOBr :

$$NO(g) + Br_2(g) \rightleftharpoons NOBr_2(g)$$
$$NOBr_2(g) + NO(g) \rightarrow 2NOBr(g)$$

If the second step is the rate determining step, the order of the reaction with respect to NO (*g*) is :

(a) 1 (b) 0
(c) 3 (d) 2

102. Lanthanoid contraction is caused due to :

(a) the appreciable shielding on outer electrons by 4*f* electrons from the nuclear charge
(b) the appreciable shielding on outer electrons by 5*d* electrons from the nuclear charge
(c) the same effective nuclear charge from Ce to Lu
(d) the imperfect shielding on outer electrons by 4f electrons from the nuclear charge

103. Resistance of a conductivity cell filled with a solution of an electrolyte of concentration 0.1 M is 100 Ω. The conductivity of this solution is 1.29 S m^{-1}. Resistance of the same cell when filled with 0.2 M of the same solution is 520 Ω. The molar conductivity of 0.02 M solution of the electrolyte will be :

(a) 124×10^{-4} S m^2 mol^{-1}

(b) 1240×10^{-4} S m^2 mol^{-1}
(c) 1.24×10^{-4} S m^2 mol^{-1}
(d) 12.4×10^{-4} S m^2 mol^{-1}

104. The ionic mobility of alkali metal ions in aqueous solution is maximum for :

(a) K^+ (b) Rb^+
(c) Li^+ (d) Na^+

105. Density of a 2.05 M solution of acetic acid in water is 1.02 g/mL. The molality of the solution is :

(a) 1.14 mol kg^{-1} (b) 3.28 mol kg^{-1}
(c) 2.28 mol kg^{-1} (d) 0.44 mol kg^{-1}

106. The enthalpy changes for the following processes are listed below :

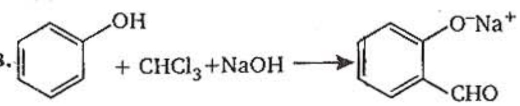
$Cl_2\ (g) = 2Cl\ (g)$, 242.3 kJ mol^{-1}
$I_2\ (g) = 2I\ (g)$, 151.0 kJ mol^{-1}
$ICl\ (g) = I\ (g) + Cl\ (g)$, 211.3 kJ mol^{-1}
$I_2\ (s) = I_2\ (g)$, 62.76 kJ mol^{-1}

Given that the standard states for iodine and chlorine are $I_2(s)$ and $Cl_2\ (g)$, the standard enthalpy of formation of ICl (g) is :

(a) – 14.6 kJ mol^{-1} (b) – 16.8 kJ mol^{-1}
(c) + 16.8 kJ mol^{-1} (d) + 244.8 kJ mol^{-1}

107. How many EDTA (ethylenediaminetetra acetic acid) molecules are required to make an octahedral complex with a Ca^{2+} ion ?

(a) six (b) three
(c) one (d) two

108. 

The electrophile involved in the above reaction is :

(a) dichloromethyl cation ($\overset{\oplus}{C}HCl_2$)
(b) dichlorocarbene ($: CCl_2$)
(c) trichloromethyl anion ($\overset{-}{C}Cl_3$)
(d) formyl cation ($\overset{\oplus}{C}HO$)

109. 18 g of glucose ($C_6H_{12}O_6$) is added to 178.2 g of water. The vapour pressure of water for this aqueous solution at 100°C is :

(a) 759·00 torr (b) 7·60 torr
(c) 76·00 torr (d) 752·40 torr

110. $(\Delta H - \Delta U)$ for the formation of carbon monoxide (CO) from its elements at 298 K is :
$(R = 8 \cdot 314\ JK^{-1}\ mol^{-1})$

(a) – 1238.78 J mol^{-1} (b) 1238.78 J mol^{-1}
(c) – 2477.57 J mol^{-1} (d) 2477.57 J mol^{-1}

Part-C

Mathematics

111. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30°$, and $\tan 15°$ respectively, then the value of $2 + q - p$ is :

(a) 3 (b) 0
(c) 1 (d) 2

112. The value of the integral $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}}\,dx$ is :

(a) $\frac{3}{2}$ (b) 2
(c) 1 (d) $\frac{1}{2}$

113. Let W denote the words in the English dictionary. Define the relation R by :
$R = \{(x, y) \in W \times W |$ the words x and y have at least one letter in common$\}$. Then R is :

(a) reflexive, symmetric and not transitive
(b) reflexive, symmetric and transitive
(c) reflexive, not symmetric and transitive
(d) not reflexive, symmetric and transitive

114. The number of values of x in the interval $[0, 3\pi]$ satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is :

(a) 6 (b) 1
(c) 2 (d) 4

115. If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true ?

(a) $AB = BA$
(b) either of A or B is a zero matrix
(c) either of A or B is an identity matrix
(d) $A = B$

116. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$ is :

(a) 1 (b) –1
(c) $-i$ (d) i

any three vectors such that $\vec{a} \cdot \vec{b} \neq 0, \vec{b} \cdot \vec{c} \neq 0$, then $\vec{a}$ and $\vec{c}$ are :

(a) inclined at an angle of $\frac{\pi}{6}$ between them
(b) perpendicular
(c) parallel
(d) inclined at an angle of $\frac{\pi}{3}$ between them

118. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4 lie in the interval :

(a) $m > 3$ (b) $-1 < m < 3$
(c) $1 < m < 4$ (d) $-2 < m < 0$

119. ABC is triangle, right angled at A. The resultant of the forces acting along $\overrightarrow{\mathbf{AB}}, \overrightarrow{\mathbf{AC}}$ with magnitudes $\frac{1}{AB}$ and $\frac{1}{AC}$ respectively is the force along $\overrightarrow{\mathbf{AD}}$, where D is the foot of the perpendicular from A onto BC. The magnitude of the resultant is :

(a) $\frac{(AB)(AC)}{AB + AC}$ (b) $\frac{1}{AB} + \frac{1}{AC}$
(c) $\frac{1}{AD}$ (d) $\frac{AB^2 + AC^2}{(AB)^2 (AC)^2}$

120. Suppose a population A has 100 observations 101, 102, ... , 200 and another population B has 100 observations 151, 152, ..., 250. If V_A and V_B represent the variances of the two population respectively, then $\frac{V_A}{V_B}$ is :

(a) $\frac{9}{4}$ (b) $\frac{4}{9}$
(c) $\frac{2}{3}$ (d) 1

121. $\int_{-3\pi/2}^{-\pi/2} [(x + \pi)^3 + \cos^2 (x + 3\pi)]\, dx$ is equal to :

(a) $\left(\frac{\pi^4}{32}\right) + \left(\frac{\pi}{2}\right)$ (b) $\frac{\pi}{2}$
(c) $\left(\frac{\pi}{4}\right) - 1$ (d) $\frac{\pi^4}{32}$

122. In an ellipse, the distances between its foci is 6 and minor axis is 8. Then its eccentricity is :

(a) $\frac{1}{2}$ (b) $\frac{4}{5}$
(c) $\frac{1}{\sqrt{5}}$ (d) $\frac{3}{5}$

123. The locus of the vertices of the family of parabolas $y = \frac{a^3 x^2}{3} + \frac{a^2 x}{2} - 2a$ is :

(a) $xy = \frac{3}{4}$ (b) $xy = \frac{35}{16}$
(c) $xy = \frac{64}{105}$ (d) $xy = \frac{105}{64}$

124. A straight line through the point A (3, 4) is such that its intercept between the axes is bisected at A. Its equation is :

(a) $3x - 4y + 7 = 0$ (b) $4x + 3y = 24$
(c) $3x + 4y = 25$ (d) $x + y = 7$

125. The value of a, for which the points A, B, C with position vectors $2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ and $a\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ respectively are the vertices of a right angled triangle with $C = \frac{\pi}{2}$ are :

(a) -2 and -1 (b) -2 and 1
(c) 2 and -1 (d) 2 and 1

126. $\int_0^{\pi} x f(\sin x)\, dx$ is equal to :

(a) $\pi \int_0^{\pi} f(\sin x)\, dx$ (b) $\frac{\pi}{2} \int_0^{\pi/2} f(\sin x)\, dx$
(c) $\pi \int_0^{\pi/2} f(\cos x)\, dx$ (d) $\pi \int_0^{\pi} f(\cos x)\, dx$

127. The two lines $x = ay + b, z = cy + d$ and $x = a'y + b', z = c'y + d'$ are perpendicular to each other, if :

(a) $aa' + cc' = 1$ (b) $\frac{a}{a'} + \frac{c}{c'} = -1$
(c) $\frac{a}{a'} + \frac{c}{c'} = 1$ (d) $aa' + cc' = -1$

128. At an election, a voter may vote for any number of candidates not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote, is :

(a) 6210 (b) 385
(c) 1110 (d) 5040

129. If the expansion in powers of x of the function $\frac{1}{(1 - ax)(1 - bx)}$ is $a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$, then a_n is :

(a) $\frac{a^n - b^n}{b - a}$ (b) $\frac{a^{n+1} - b^{n+1}}{b - a}$
(c) $\frac{b^{n+1} - a^{n+1}}{b - a}$ (d) $\frac{b^n - a^n}{b - a}$

130. For natural numbers m, n if

$(1-y)^m (1+y)^n = 1 + a_1 y + a_2 y^2 + \dots$ and $a_1 = a_2 = 10$, then (m, n) is :

(a) (35, 20) (b) (45, 35)
(c) (35, 45) (d) (20, 45)

131. A particle has two velocities of equal magnitude inclined to each other at an angle θ. If one of them is halved, the angle between the other and the original resultant velocity is bisected by the new resultant. Then θ is :

(a) 120° (b) 45°
(c) 60° (d) 90°

132. At a telephone enquiry system the number of phone calls regarding relevant enquiry follow Poisson distribution with an average of 5 phone calls during 10 minute time intervals. The probability that there is at the most one phone call during a 10 minute time period, is :

(a) $\frac{5}{6}$ (b) $\frac{6}{55}$
(c) $\frac{6}{e^5}$ (d) $\frac{6}{5^e}$

133. A body falling from rest under gravity passes a certain point P. It was at a distance of 400 m from P, 4 seconds prior to passing through P. If $g = 10\ m/s^2$, then the height above the point P from where the body began to fall is :

(a) 900 m (b) 320 m
(c) 680 m (d) 720 m

134. The set of points, where $f(x) = \frac{x}{1+|x|}$ is differentiable, is :

(a) $(-\infty, -1) \cup (-1, \infty)$
(b) $(-\infty, \infty)$
(c) $(0, \infty)$
(d) $(-\infty, 0) \cup (0, \infty)$

135. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $a, b, \in N$. Then :

(a) there exist more than one but finite number of B's such that $AB = BA$
(b) there exists exactly one B such that $AB = BA$
(c) there exists infinitely many B's such that $AB = BA$
(d) there cannot exist any B such that $AB = BA$

136. Let $a_1, a_2, a_3, \dots$ cannot be terms of an AP. If $\frac{a_1 + a_2 + \dots + a_p}{a_1 + a_2 + \dots + a_q} = \frac{p^2}{q^2}, p \neq q$, then $\frac{a_6}{a_{21}}$ equals:

(a) $\frac{7}{2}$ (b) $\frac{2}{7}$
(c) $\frac{11}{41}$ (d) $\frac{41}{11}$

137. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at :

(a) $x = -2$ (b) $x = 0$
(c) $x = 1$ (d) $x = 2$

138. Angle between the tangents to the curve $y = x^2 - 5x + 6$ at the points (2, 0) and (3, 0) is :

(a) $\pi/2$ (b) $\pi/6$
(c) $\pi/4$ (d) $\pi/3$

139. If x is real, the maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is :

(a) 41 (b) 1
(c) 17/7 (d) 1/4

140. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is :

(a) $\sqrt{\frac{x^3}{8}}$ (b) $\frac{1}{2}x^2$
(c) πx^2 (d) $\frac{3}{2}x^2$

141. If (a, a^2) falls inside the angle made by the lines $y = \frac{x}{2}$, $x > 0$ and $y = 3x$, $x > 0$, then a belongs to :

(a) $(3, \infty)$ (b) $\left(\frac{1}{2}, 3\right)$
(c) $\left(-3, -\frac{1}{2}\right)$ (d) $\left(0, \frac{1}{2}\right)$

142. If $x^m y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is :

(a) $\frac{x+y}{xy}$ (b) xy
(c) $\frac{x}{y}$ (d) $\frac{y}{x}$

143. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, the equation of the circle is :

(a) $x^2 + y^2 + 2x - 2y - 62 = 0$
(b) $x^2 + y^2 - 2x + 2y - 62 = 0$
(c) $x^2 + y^2 - 2x + 2y - 47 = 0$
(d) $x^2 + y^2 + 2x - 2y - 47 = 0$

144. The image of the point (–1, 3, 4) in the plane $x - 2y = 0$ is :

(a) (15, 11, 4) (b) $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$
(c) (8, 4, 4) (d) $\left(-\frac{17}{3}, -\frac{19}{3}, 4\right)$

145. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constant, is of :

(a) first order and second degree
(b) first order and first degree
(c) second order and first degree
(d) second order and second degree

146. The value of $\int_1^a [x] f'(x)\,dx$, $a > 1$, where $[x]$ denotes the greatest integer not exceeding x, is :

(a) $[a]\, f(a) - \{f(1) + f(2) + \ldots\ldots + f([a])\}$
(b) $[a]\, f([a]) - \{f(1) + f(2) + \ldots\ldots + f(a)\}$
(c) $a\, f([a]) - \{f(1) + f(2) + \ldots + f(a)\}$
(d) $a f(a) - \{f(1) + f(2) + \ldots\ldots + f([a])\}$

147. Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its centre, is :

(a) $x^2 + y^2 = 1$ (b) $x^2 + y^2 = \frac{27}{4}$
(c) $x^2 + y^2 = \frac{9}{4}$ (d) $x^2 + y^2 = \frac{3}{2}$

148. If $a_1, a_2, \ldots, a_n$ are in HP, then the expression $a_1a_2 + a_2a_3 + \ldots\ldots + a_{n-1}\, a_n$ is equal to :

(a) $(n-1)(a_1 - a_n)$ (b) na_1a_n
(c) $(n-1)\, a_1a_n$ (d) $n\,(a_1 - a_n)$

149. If $z^2 + z + 1 = 0$, where z is complex number, then the value of

$$\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \ldots + \left(z^6 + \frac{1}{z^6}\right)^2 \text{ is :}$$

(a) 54 (b) 6
(c) 12 (d) 18

150. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is :

(a) $\frac{(4 - \sqrt{7})}{3}$
(b) $-\frac{(4 + \sqrt{7})}{3}$
(c) $\frac{(1 + \sqrt{7})}{4}$
(d) $\frac{(1 - \sqrt{7})}{4}$

ANSWERS

1.	(d)	**2.**	(b)	**3.**	(d)	**4.**	(b)	**5.**	(c)	**6.**	(a)	**7.**	(b)	**8.**	(c)
9.	(d)	**10.**	(b)	**11.**	(a)	**12.**	(a)	**13.**	(b)	**14.**	(b)	**15.**	(d)	**16.**	(a)
17.	(c)	**18.**	(c)	**19.**	(a)	**20.**	(d)	**21.**	(a)	**22.**	(b)	**23.**	(b)	**24.**	(c)
25.	(b)	**26.**	(c)	**27.**	(c)	**28.**	(b)	**29.**	(a)	**30.**	(d)	**31.**	(b)	**32.**	(a)
33.	(c)	**34.**	(b)	**35.**	(b)	**36.**	(a)	**37.**	(b)	**38.**	(d)	**39.**	(*)	**40.**	(b)
41.	(c)	**42.**	(c)	**43.**	(b)	**44.**	(c)	**45.**	(c)	**46.**	(a)	**47.**	(d)	**48.**	(a)
49.	(a)	**50.**	(a)	**51.**	(c)	**52.**	(a)	**53.**	(c)	**54.**	(a)	**55.**	(c)	**56.**	(d)
57.	(c)	**58.**	(c)	**59.**	(b)	**60.**	(d)	**61.**	(a)	**62.**	(d)	**63.**	(c)	**64.**	(a)
65.	(d)	**66.**	(d)	**67.**	(a)	**68.**	(b)	**69.**	(a)	**70.**	(c)	**71.**	(d)	**72.**	(a)
73.	(c)	**74.**	(a)	**75.**	(a)	**76.**	(b)	**77.**	(b)	**78.**	(c)	**79.**	(d)	**80.**	(d)
81.	(a)	**82.**	(c)	**83.**	(a)	**84.**	(d)	**85.**	(b)	**86.**	(b)	**87.**	(d)	**88.**	(a)
89.	(c)	**90.**	(b)	**91.**	(d)	**92.**	(d)	**93.**	(c)	**94.**	(a)	**95.**	(d)	**96.**	(b)
97.	(a)	**98.**	(a)	**99.**	(d)	**100.**	(d)	**101.**	(d)	**102.**	(d)	**103.**	(d)	**104.**	(b)
105.	(c)	**106.**	(c)	**107.**	(c)	**108.**	(b)	**109.**	(d)	**110.**	(b)	**111.**	(a)	**112.**	(a)
113.	(a)	**114.**	(d)	**115.**	(a)	**116.**	(c)	**117.**	(c)	**118.**	(b)	**119.**	(c)	**120.**	(d)
121.	(b)	**122.**	(d)	**123.**	(d)	**124.**	(b)	**125.**	(d)	**126.**	(c)	**127.**	(d)	**128.**	(b)
129.	(c)	**130.**	(c)	**131.**	(a)	**132.**	(c)	**133.**	(d)	**134.**	(b)	**135.**	(c)	**136.**	(c)
137.	(d)	**138.**	(a)	**139.**	(a)	**140.**	(b)	**141.**	(b)	**142.**	(d)	**143.**	(c)	**144.**	(*)
145.	(c)	**146.**	(a)	**147.**	(c)	**148.**	(c)	**149.**	(c)	**150.**	(b)				

* No option is matching

HINTS & SOLUTIONS

Part-A

Physics

1. Kirchhoff's Ist law or KCL states that the algebaric sum of current meeting at any junction is equal to zero. In other words we can say that " the sum of all the currents directed towards a junction in a circuit is equal to the sum of all the currents directed away from that junction." Thus, no charge has been accumulated at any junction *i.e.*, charge is conserved, and hence, we can say that KCL ($\Sigma i = 0$) is based on conservation of charge.

Kirchhoff's IInd law or KVL states that algebraic sum of changes in potential around any closed resistor loop must be zero. In other words "around any closed loop, voltage drops are equal to voltage rises". No energy is gained or lost in circulating a charge around a loop, thus, we can say that KVL is based on conservation of energy.

2. Ferromagnetic substances have strong tendency to get magnetised (induced magnetic moment) in the same direction as that of applied magnetic field, so magnet attracts N_1 strongly.

Paramagnetic substances get weakly magnetised (magnetic moment induced is small) in the same direction as that of applied magnetic field, so magnet attracts N_2 weakly.

Diamagnetic substances also get weakly magnetised when placed in an external magnetic field but in opposite direction and hence, N_3 is weakly repelled by magnet.

3. Let (ρ_A, l_A, r_A, A_A) and (ρ_B, l_B, r_B, A_B) are specific resistances, lengths, radii and areas of wires A and B respectively.

Resistance of $A = R_A = \frac{\rho_A l_A}{A_A} = \frac{\rho_A l_A}{\pi r_A^2}$

Resistance of $B = R_B = \frac{\rho_B l_B}{A_B} = \frac{\rho_B l_B}{\pi r_B^2}$

From given information

$$\rho_B = 2\rho_A$$

$$r_B = 2r_A$$

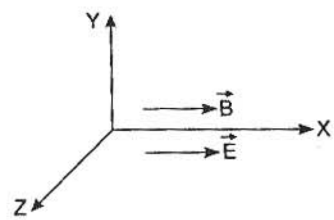
and $$R_A = R_B$$

$\therefore$ $$\frac{\rho_A l_A}{\pi r_A^2} = \frac{\rho_B l_B}{\pi r_B^2}$$

$\Rightarrow$ $$\frac{\rho_A l_A}{\pi r_A^2} = \frac{2\rho_A \times l_B}{\pi (2r_A)^2}$$

$\Rightarrow$ $$\frac{l_B}{l_A} = \frac{2}{1} = 2:1$$

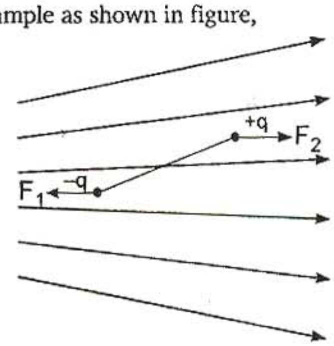
4. Let $\vec{E}$ and $\vec{B}$ be along X-axis. When a charged particle is released from rest, it will experience an electric force along the direction of electric field or opposite to the direction of electric field depending on the nature of charge. Due to this force, it acquires some velocity along X-axis. Due to this motion of charge, magnetic force can not have non-zero value because angle between $\vec{v}$ and $\vec{B}$ would be either 0° or 180°.



So, only electric force is acting on particle and hence, it will move along a straight line.

5. In a non-uniform electric field, the dipole may experience both non-zero torque as well as translational force.

For example as shown in figure,



$F_1 \neq F_2$ as E is non-uniform.

Torque would also be non-zero.

6. $v = \alpha\sqrt{x}$

$$\frac{dx}{dt} = \alpha\sqrt{x} \qquad \left(\because v = \frac{dx}{dt}\right)$$

$$\frac{dx}{\sqrt{x}} = \alpha\, dt$$

Perform integration

$$\int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\, dt$$

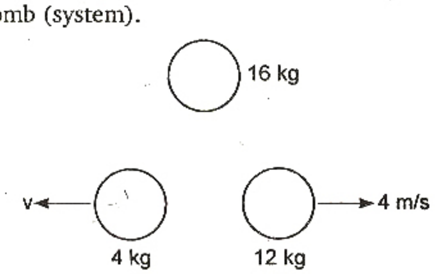
[$\because$ at $t = 0$, $x = 0$ and let at any time t, particle is at x]

$$\Rightarrow \qquad \left.\frac{x^{1/2}}{1/2}\right|_0^x = \alpha t$$

$$\Rightarrow \qquad x^{1/2} = \frac{\alpha}{2} t$$

$$\Rightarrow \qquad x = \frac{\alpha^2}{4} \times t^2 \Rightarrow x \propto t^2$$

7. Here momentum of the system is remaining conserved as no external force is acting on the bomb (system).



Initial momentum (before explosion) = Final momentum (after explosion)

Let velocity of 4 kg mass is v m/s. From momentum conservation we can say that its direction is opposite to velocity of 12 kg mass.

From $\qquad \vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f$

$\Rightarrow \quad 0 = 12 \times 4 - 4 \times v \Rightarrow v = 12$ m/s

KE of 4 kg mass $= \dfrac{4 \times (12)^2}{2} = 288$ J

8. Velocity of sound in air = 300 m/s

If a source of sound is moving towards a stationary listener, the frequency heard by the listener would be different from the actual frequency of the source, this apparent frequency is given by

$f_{app.} = \left(\dfrac{v_{\text{sound in air}}}{v_{\text{sound in air}} \pm v_{\text{source}}}\right)$, where symbols have their usual meanings.

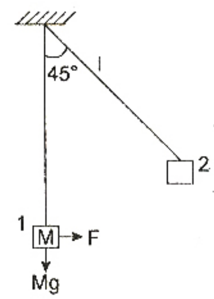
In the denominator +ve sign would be taken when source is receding away from the listener, while –ve sign would be taken when source is approaching the listener.

Let v be the maximum value of source velocity for which the person is able to hear the sound, then

$$10{,}000 = f_{app.} = \left(\frac{300}{300 - v}\right) \times 9500$$

$$\Rightarrow \qquad v = 15 \text{ m/s}$$

9. Here, the constant horizontal force required to take the body from position 1 to position 2 can be calculated by using work-energy theorem. Let us assume that body is taken slowly so that its speed doesn't change, then



$\Delta K = 0$

$= W_F + W_{Mg} + W_{\text{tension}}$

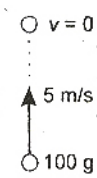
[symbols have their usual meanings]

$$W_F = F \times l \sin 45°,$$

$$W_{Mg} = Mg(l - l\cos 45°),\ W_{\text{tension}} = 0$$

$$\therefore \qquad F = Mg(\sqrt{2} - 1)$$

10. The height (h) traversed by particle while going up is



$$h = \frac{u^2}{2g} = \frac{25}{2 \times 9.8}$$

Work done by gravity force $= m\vec{g}\cdot\vec{h}$

$$= 0.1 \times g \times \frac{25}{2 \times 9.8} \cos 180°$$

[Angle between force and displacement is 180°]

$$\therefore \qquad W = -0.1 \times \frac{25}{2} = -1.25 \text{ J}$$

11. The maximum velocity of a particle performing SHM is given by $v = A\omega$, where A is the amplitude and ω is the angular frequency of oscillation.

$$\therefore \qquad 4.4 = (7 \times 10^{-3}) \times 2\pi / T$$

$$\Rightarrow \qquad T = \frac{7 \times 10^{-3}}{4.4} \times \frac{2 \times 22}{7} = 0.01 \text{ s}$$

12. KE of a body undergoing SHM is given by

$\text{KE} = \dfrac{1}{2} m\omega^2 A^2 \cos^2 \omega t$ and $\text{KE}_{\max} = \dfrac{m\omega^2 A^2}{2}$

[symbols represent standard quantities]

From given information

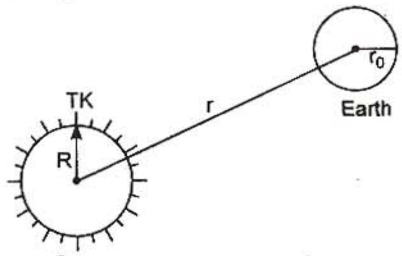
$$\text{KE} = (\text{KE}_{\max}) \times \frac{75}{100}$$

$$\Rightarrow \quad \frac{m\omega^2 A^2}{2} \cos^2 \omega t = \frac{m\omega^2 A^2}{2} \times \frac{3}{4}$$

$$\Rightarrow \cos \omega t = \pm \frac{\sqrt{3}}{2}$$

$$\Rightarrow \omega t = \frac{\pi}{6} \Rightarrow \frac{2\pi}{T} \times t = \frac{\pi}{6} \Rightarrow t = \frac{T}{12} = \frac{1}{6} \text{ s}$$

13. From Stefan's law, the rate at which energy is radiated by sun at its surface is



$$P = \sigma \times 4\pi R^2 \times T^4$$

[Sun is a perfectly black body as it emits radiations of all wavelengths and so for it $e = 1$.]

The intensity of this power at earth's surface [under the assumption $r >> r_0$] is

$$I = \frac{P}{4\pi r^2} = \frac{\sigma \times 4\pi R^2 T^4}{4\pi r^2} = \frac{\sigma R^2 T^4}{r^2}$$

The area of earth which receives this energy is only one half of total surface area of earth, whose projection would be πr_0^2.

$\therefore$ Total radiant power as received by earth

$$= \pi r_0^2 \times I$$
$$= \frac{\pi r_0^2 \times \sigma R^2 T^4}{r^2}$$
$$= \frac{\pi r_0^2 R^2 \sigma T^4}{r^2}$$

14. Magnetic energy $= \frac{1}{2}Li^2 = \frac{LQ^2}{2t^2}$

[$L \rightarrow$ inductance, $i \rightarrow$ current]

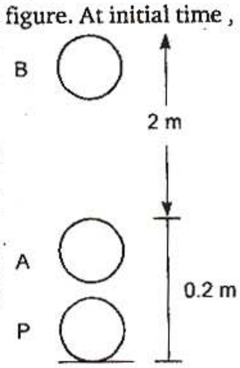
Energy has the dimensions $[ML^2T^{-2}]$.

Equate the dimensions, we have

$$[ML^2T^{-2}] = (\text{henry}) \times \frac{[Q^2]}{[T^2]}$$

$$\Rightarrow \quad [\text{henry}] = \frac{[ML^2]}{[Q^2]}$$

15. The situation is shown in figure. At initial time, the ball is at P, then under the action of a force (exerted by hand) from P to A and then from A to B let acceleration of ball during PA is a m/s^2 [assumed to be constant] in upward direction and velocity of ball at A is v m/s.



Then for PA, $v^2 = 0^2 + 2a \times 0.2$

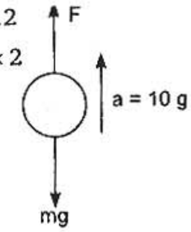
For AB, $\quad 0 = v^2 - 2 \times g \times 2$

$\Rightarrow \quad v^2 = 2g \times 2$

From above equations,

$$a = 10\,g = 100 \text{ m/s}^2$$

Then for PA, FBD of ball is



$F - mg = ma$[F is the force exerted by hand on ball]

$$\Rightarrow \quad F = m(g + a) = 0.2(11g)$$
$$= 22\,\text{N}$$

Alternate solution :

Using work-energy theorem

$$W_{mg} + W_F = 0$$

$\Rightarrow -mg \times 2.2 + F \times 0.2 = 0 \Rightarrow F = 22\,\text{N}$

16. For string fixed at both the ends, resonant frequency are given by $f = \frac{nv}{2L}$, where symbols have their usual meanings. It is given that 315 Hz and 420 Hz are two consecutive resonant frequencies, let these are nth and $(n+1)th$ harmonics.

$$315 = \frac{nv}{2L} \quad \ldots\text{(i)}$$

$$420 = \frac{(n+1)v}{2L} \quad \ldots\text{(ii)}$$

$\Rightarrow$ Eq. (i) $\div$ Eq. (ii) $\Rightarrow \frac{315}{420} = \frac{n}{n+1} \Rightarrow n = 3$

From Eq. (i), lowest resonant frequency

$$f_0 = \frac{v}{2L} = \frac{315}{3} = 105\,\text{Hz}$$

17. To keep the COM at the same position, velocity of COM is zero, so

$$\frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} = 0$$

[where $\vec{v}_1$ and $\vec{v}_2$ are velocities of particles 1 and 2 respectively.]

$$\Rightarrow \quad m_1 \frac{d\vec{r}_1}{dt} + m_2 \frac{d\vec{r}_2}{dt} = 0$$

$$[\because \vec{v}_1 = \frac{d\vec{r}_1}{dt} \;\&\; \vec{v}_2 = \frac{d\vec{r}_2}{dt}]$$

$\Rightarrow m_1 d\vec{r}_1 + m_2 d\vec{r}_2 = 0$ [$d\vec{r}_1$ and $d\vec{r}_2$ represent the change in displacement of particles]

Let 2nd particle has been displaced by distance x.

$$\Rightarrow \quad m_1(d) + m_2(x) = 0 \Rightarrow x = -\frac{m_1 d}{m_2}$$

–ve sign shows that both the particles have to move in opposite directions.

So, $\frac{m_1 d}{m_2}$ is the distance moved by 2nd particle to keep COM at the same position.

18. This is the question based on impulse-momentum theorem.

$|F \cdot \Delta t| = |$ change in momentum$|$

$\Rightarrow \quad F \times 0.1 = |p_f - p_i|$

As the ball will stop after catching;

$$P_i = mv_i = 0.15 \times 20 = 3,\ p_f = 0$$

$\Rightarrow \quad F \times 0.1 = 3 \Rightarrow F = 30\text{ N}$

19. $\beta = \frac{I_C}{I_B}$ and $I_E = I_C + I_B$

$\therefore \quad \beta = \frac{I_C}{I_E - I_C} = \frac{5.488}{5.60 - 5.488} = 49$

20. When a circuit is made up of any two metals in thermoelectric series, the current flows across the cold junction from the later occuring metal in the series to the one occuring earlier. In thermoelectric series Bismuth comes earlier than Antimony. So, at cold junction current flows from Antimony to Bismuth and at hot junction it flows from Bismuth to Antimony.

21. $h\nu_0 = 6.2\text{ eV}$, $eV_0 = 5\text{ eV}$

From Einstein's photoelectric equation

$$h\nu = h\nu_0 + eV_0 = 6.2 + 5 = 11.2\text{ eV}$$

$\Rightarrow \quad \frac{hc}{\lambda} = 11.2\text{ eV}$

$\therefore \quad \lambda = \frac{hc}{11.2(\text{eV})} = 1108.9\ \text{Å}$

which belongs to ultra-violet region.

22. Since, here nuclear target is heavy it can be assumed safely that it will remain stationary and will not move due to the Coulombic interaction force.

At distance of closest approach relative velocity of two particles is v. Here target is considered as stationary, so α-particle comes to rest instantaneously at distance of closest approach. Let required distance is r, then from work-energy theorem

$$0 - \frac{mv^2}{2} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze \times 2e}{r}$$

$\Rightarrow \quad r \propto \frac{1}{m}$

$$\propto \frac{1}{v^2}$$

$$\propto Ze^2$$

23. The photoelectric effect is an instantaneous phenomenon (experimentally proved). It takes approx. time of the order of 10^{-10} s.

24. The nuclear reaction can be represented as

$${}_3\text{Li}^7 + {}_1\text{H}^1 \longrightarrow {}_4\text{Be}^8 + {}_Z\text{X}^A$$

Applying conservation of atomic number (charge)

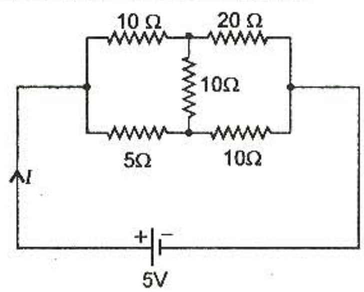
$$3 + 1 = 4 + Z \Rightarrow Z = 0$$

Applying conservation of atomic mass

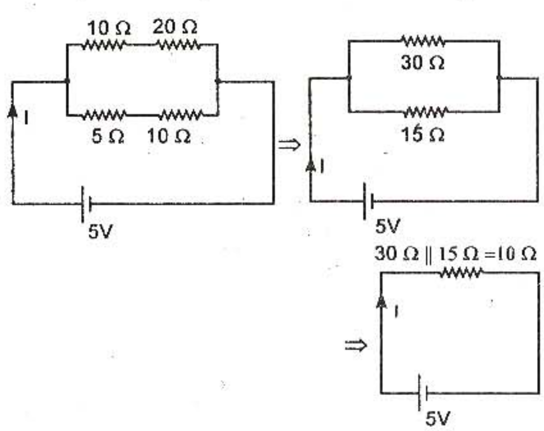
$$7 + 1 = 8 + A \Rightarrow A = 0$$

Thus, the emitted particles are γ-photons (${}_0\text{X}^0$).

25. The given circuit can be redrawn as

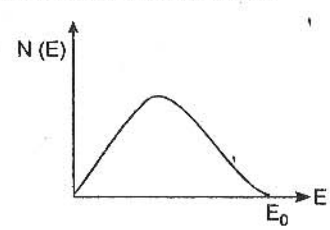


which is a balanced Wheatstone's bridge and hence, no current flows in the centre resistor, so equivalent circuit would be as shown below.



So, $\quad I = \frac{V}{R} = 5/10 = 0.5\text{ A}$

26. Energy spectrum of emitted β-particles from a radioactive source is drawn as



27. At resonance, $\omega L = \frac{1}{\omega C}$

Current flowing through the circuit,

$$I = \frac{V_R}{R} = \frac{100}{1000} = 0.1\text{A}$$

So, voltage across L is given by

$$V_L = I X_L = I\omega L$$

but $\quad \omega L = \frac{1}{\omega C}$

$\therefore \quad V_L = \frac{I}{\omega C} = \frac{0.1}{200 \times 2 \times 10^{-6}} = 250\text{ V}$

28. Let resistance of bulb filament is R_0 at 0°C then from expression

$$R_\theta = R_0[1 + \alpha\Delta\theta]$$

we have, $100 = R_0 [1 + 0.005 \times 100]$

and $\quad 200 = R_0[1 + 0.005 \times x]$

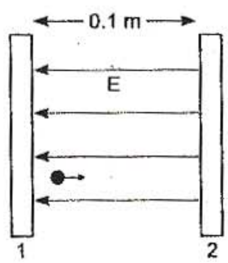
where x is temperature in °C at which resistance become 200 Ω.

Dividing the above two equations

$$\frac{200}{100} = \frac{1 + 0.005x}{1 + 0.005 \times 100} \Rightarrow x = 400°\text{C}$$

29. Since $V_2 > V_1$, so electric field will point from plate 2 to plate 1.

The electron will experience an electric force, opposite to the direction of electric field, and hence move towards the plate 2.



Use work-energy theorem to find speed of electron when it strikes the plate 2.

$$\frac{m_e v^2}{2} - 0 = e(V_2 - V_1)$$

where v is the required speed.

$$\therefore \quad \frac{9.11 \times 10^{-31}}{2} v^2 = 1.6 \times 10^{-19} \times 20$$

$$\Rightarrow v = \sqrt{\frac{1.6 \times 10^{-19} \times 40}{9.11 \times 10^{-31}}} = 2.65 \times 10^6 \text{ m/s}$$

30. The emf generated would be maximum when flux (cutting) would be maximum *i.e.*, angle between area vector of coil and magnetic field is 0°. The emf generated is given by [as a function of time]

$$e = NBA\omega \cos \omega t$$

$$\Rightarrow \quad e_{max} = NAB\omega$$

32. $D = (\mu - 1) A$

For blue light μ is greater than that for red light, so $D_2 > D_1$.

33. $I = n_e A v_d$

$$\therefore \quad \frac{I_e}{I_h} = \frac{n_e \times (v_d)_e}{n_h \times (v_d)_h}$$

Here, $\quad \frac{n_e}{n_h} = \frac{7}{5}, \frac{I_e}{I_h} = \frac{7}{4}$

$$\therefore \quad \frac{7}{4} = \frac{7}{5} \times \frac{(v_d)_e}{(v_d)_h}$$

$$\Rightarrow \quad \frac{(v_d)_e}{(v_d)_h} = \frac{5}{7} \times \frac{7}{4} = \frac{5}{4}$$

34. For balanced Wheatstone's bridge

$$\frac{P}{Q} = \frac{R}{S} \text{ here } S = S_1 \parallel S_2 = \frac{S_1 S_2}{S_1 + S_2}$$

$$\Rightarrow \quad \frac{P}{Q} = \frac{R(S_1 + S_2)}{S_1 S_2}$$

35. $\phi = 10t^2 - 50t + 250$

From Faraday's law of electromagnetic induction $e = -\frac{d\phi}{dt}$

$\therefore \quad e = -[10 \times 2t - 50]$

$\therefore \quad e|_{t=3s} = -[10 \times 6 - 50] = -10\text{ V}$

36. Magnetic field due to a long solenoid is given by

$$B = \mu_0 n i$$

From given data,

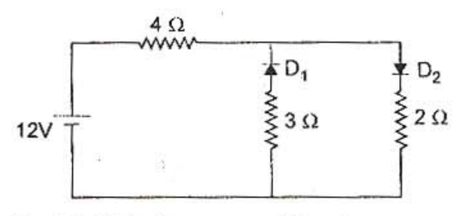
$6.28 \times 10^{-2} = \mu_0 \times 200 \times 10^2 \times i \quad \ldots\text{(i)}$

and $\quad B = \mu_0 \times 100 \times 10^2 \times (i/3) \ldots\text{(ii)}$

Solving Eqs.(i) and (ii), we get

$$B \approx 1.05 \times 10^{-2} \text{ Wb/m}^2$$

37. In the given circuit diode D_1 is reverse biased while D_2 is forward biased, so the circuit can be redrawn as :



[∵ For ideal diodes, reverse biased means open and forward biased means short]

Apply KVL to get current flowing through the circuit

$$-12 + 4i + 2i = 0 \Rightarrow i = \frac{12}{6} = 2\text{A}$$

38. For reverse biasing of an ideal diode, the potential of n-side should be higher than potential of p-side. Only option (d) is satisfying the criterion for reverse biasing.

39. According to the photoelectric effect in a photocell, if a light of wavelength λ is incident on a cathode, then electrons are emitted, which constitute the photoelectric current.

Photocell is based on the principle of photoelectric effect. As the wavelength of light changes, there is no change in number of electrons emitted and hence, no change in current (plate current of photocell). Thus, the two wavelength of incident light and plate current are independent to each other.

Plate current depends on intensity of light used.

Note : Here, no option is matching.

40. Energy of proton $+7 \times 5.60 = 2 \times [4 \times 7.06]$

$\therefore$ Energy of proton $= 17.28$ MeV

41. Resistance of electric bulb $R = \frac{V^2}{P}$ where subscripts denote for rated parameters.

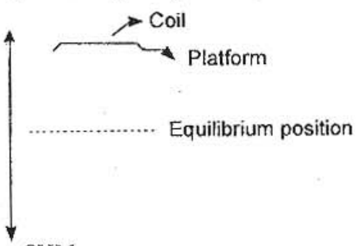
$$R = \frac{(220)^2}{100}$$

Power consumed at 110 volt, $P_{consumed} = \frac{V^2}{R}$

$$\therefore \quad P_{consumed} = \frac{(110)^2}{(220)^2/100} = 25\,W$$

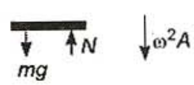
42. 'rad' is used to measure biological effect of radiation.

43. As the amplitude is increased, the max. acceleration of the platform (along with coin as long as they doesn't get separated) increases.



Performing SHM

If we draw the FBD for coin at one of the extreme positions as shown



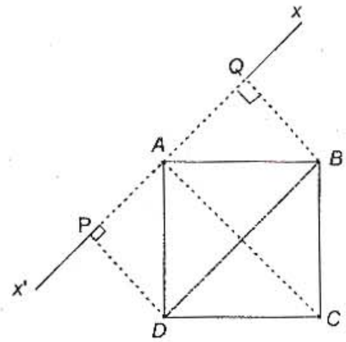
then from Newton's law, $mg - N = m\omega^2 A$

For loosing contact with the platform, $N = 0$

So, $\quad A = g/\omega^2$

44. The situation is shown in figure.

$$I_{xx'} = m \times DP^2 + m \times BQ^2 + m \times CA^2$$



$$= m \times 2 \times \left(\frac{\sqrt{2}l}{2}\right)^2 + m \times (\sqrt{2}l)^2$$

$$= 3ml^2$$

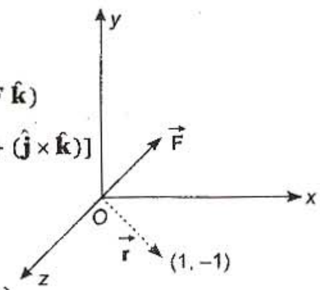
45. $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau} = (\hat{i} - \hat{j}) \times (-F\,\hat{k})$

$= F[(-\hat{i} \times \hat{k}) + (\hat{j} \times \hat{k})]$

$= F[\hat{j} + \hat{i}]$

$= F\,[\hat{i} + \hat{j}]$



46. $V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$

For minimum value of V, $\frac{dV}{dx} = 0$

$\Rightarrow \quad \frac{4x^3}{4} - \frac{2x}{2} = 0 \Rightarrow x = 0,\ x = \pm 1$

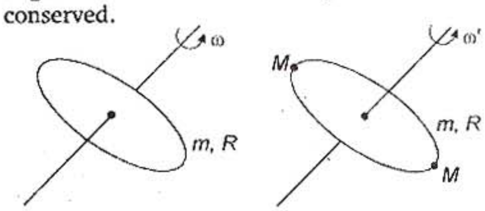
So, $\quad V_{min.}\ (x = \pm 1) = \frac{1}{4} - \frac{1}{2} = \frac{-1}{4}$ J

$\therefore K_{max.} + V_{min.} =$ Total mechanical energy

$K_{max} = (1/4) + 2 \Rightarrow K_{max.} = 9/4$

$\Rightarrow \quad \frac{mv^2}{2} = \frac{9}{4} \Rightarrow v = \frac{3}{\sqrt{2}}$ m/s

47. As no external torque is acting about the axis, angular momentum of system remains conserved.

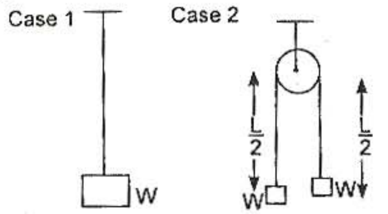


$\therefore \quad I_1\,\omega = I_2\omega'$

$\Rightarrow \quad mR^2\omega = (mR^2 + 2MR^2)\omega'$

$\Rightarrow \quad \omega' = \left(\frac{m}{m + 2M}\right)\omega$

48. Let us consider the length of wire as L and cross-sectional area A, the material of wire has Young's modulus as Y.



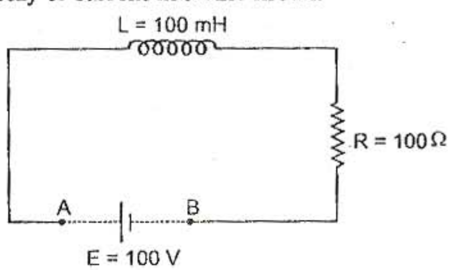
Then for 1st case $Y = \frac{W/A}{l/L}$

For 2nd case, $Y = \frac{W/A}{2l'/L}$

$\therefore \quad l' = l/2$

So, total elongation of both sides $= 2l' = l$

49. This is a combined example of growth and decay of current in an LR circuit.

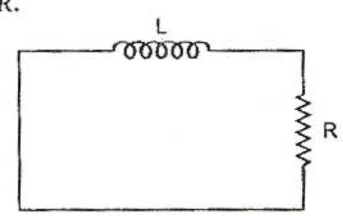


The current through circuit just before shorting the battery,

$$I_0 = E/R = 1\text{A}$$

[as inductor would be shorted in steady state]

After this decay of current starts in the circuit according to the equation $I = I_0 e^{-t/\tau}$ where $\tau = L/R$.



$$I = 1 \times e^{-(1 \times 10^{-3})/(100 \times 10^{-3}/100)}$$

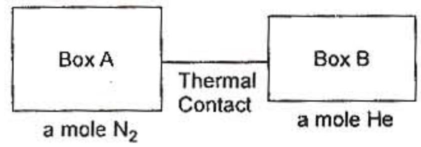
$$= (1/e)\,\text{A}$$

50. When the two conducting spheres are connected by a conducting wire, charge will flow from one sphere (having higher potential) to other (having lower potential) till both acquire the same potential.

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

So, $\quad \frac{E_1}{E_2} = \left(\frac{r_2}{r_1}\right)^2 = 4:1$

51. Here, change in internal energy of the system is zero. *i.e.*, increase in internal energy of one is equal to decrease in internal energy of other.



$$\Delta U_A = 1 \times \frac{5R}{2}(T_f - T_0)$$

$$\Delta U_B = 1 \times \frac{3R}{2}\left(T_f - \frac{7}{3}T_0\right)$$

Now $\quad \Delta U_A + \Delta U_B = 0$

$$\frac{5R}{2}(T_f - T_0) + \frac{3R}{2}\left(T_f - \frac{7T_0}{3}\right) = 0$$

$$5T_f - 5T_0 + 3T_f - 7T_0 = 0$$

$$\Rightarrow \quad 8T_f = 12T_0 \Rightarrow T_f = \frac{12}{8}T_0 = \frac{3}{2}T_0$$

52. For adiabatic process, $dQ = 0$

So, $\quad dU = -\Delta W$

$\Rightarrow \quad nC_V dT = +146 \times 10^3\text{J}$

$\Rightarrow \quad \frac{nfR}{2} \times 7 = 146 \times 10^3$

[$f \rightarrow$ Degree of freedom]

$\Rightarrow \quad \frac{10^3 \times f \times 8.3 \times 7}{2} = 146 \times 10^3$

$f = 5.02 \approx 5.$

So, it is a diatomic gas.

53. If lattice constant of semiconductor is decreased, then E_c and E_v decrease but E_g increases.

54. Total average energy $= \varepsilon_0 E_{rms}^2$

$$= 8.85 \times 10^{-12} \times (720)^2$$

$$= 4.58 \times 10^{-6}\ \text{J/m}^3$$

55. Terminal speed of spherical body in a viscous liquid is given by

$$v_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

where ρ = density of substance of body
σ = density of liquid

From given data

$$\frac{v_T(Ag)}{v_T(\text{Gold})} = \frac{\rho_{Ag} - \sigma_l}{\rho_{\text{Gold}} - \sigma_l}$$

$$\Rightarrow v_T(Ag) = \frac{10.5 - 1.5}{19.5 - 1.5} \times 0.2 = \frac{9}{18} \times 0.2$$

$$= 0.1\ \text{m/s}$$

Part-B

Chemistry

56. $CH_2{=}CH{-}O{-}CH_3 \xrightarrow{HBr} \underset{\displaystyle Br}{CH_3{-}\overset{|}{C}H{-}O{-}CH_3}$

First protonation occurs, two possible intermediates are

$\overset{+}{C}H_2 \rightarrow\!\!-\underset{\displaystyle H}{\overset{}{CH}} \rightarrow\!\!-OCH_3$ (I)

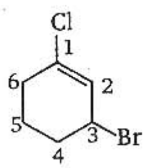
(–*I* effect destabilizes carbocation)

and $CH_3{-}\overset{+}{C}H{-}\ddot{O}CH_3$ II

(+*M* effect stabilizes carbocation)

II, is more favourable Hence, $Br^{\ominus}$ attacks, and product is

$CH_3{-}\underset{\displaystyle Br}{CH}{-}O{-}CH_3$

57. 

Unsaturation (double bond) is given priority over halogen, then lowest set of locants. So the correct IUPAC name is 3-bromo-1-chlorocyclohexene

58. Addition of HCN is nucleophilic addition. Greater the electron deficiency of carbonyl group higher the rate of reaction.

Hence,

$Ph{-}\overset{O}{\overset{||}{C}}{-}Ph < Ph{-}\overset{O}{\overset{||}{C}}{-}CH_3 < CH_3{-}\overset{O}{\overset{||}{C}}{-}CH_3 < H\overset{O}{\overset{||}{C}}H$

59. $Mg_3(PO_4)_2$; mole

8 moles of O-atom are contained by 1 mole $Mg_3(PO_4)_2$.

Hence, 0.25 moles of O-atom $= \frac{1}{8} \times 0.25$

$= 3.125 \times 10^{-2}$

60. Angular momentum of an electron

$= mvr = \frac{nh}{2\pi}$ (*n* is orbit number)

in 5th orbit $= \frac{5h}{2\pi}$

$= \frac{2.5\,h}{\pi}$

61. O_2^{2-} (Total number of electrons = 18)

$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2$

$= \pi 2p_y^2 \;\; \pi^* 2p_x^2 = \pi^* 2p_y^2$

62. Total number of atoms in face centred cubic unit cell = 4

At corner $= 8 \times \frac{1}{8} = 1$

On faces $= 6 \times \frac{1}{2} = 3$

Total = 4

Total volume $= 4 \times \frac{4\pi r^3}{3}$

$= \frac{16}{3}\pi r^3$

63.

$r \propto [CO]^2$

$r' \propto [2CO]^2$

$r' \propto 4\,[CO]^2$

Hence $\frac{r'}{r} = \frac{4\,[CO]^2}{[CO]^2}$

$\mathbf{r' = 4\,r}$

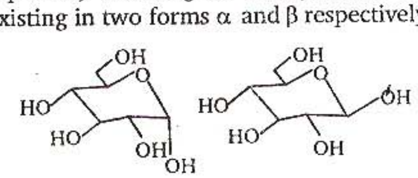
64. $2\overset{-1}{H}I + H_2\overset{+6}{S}O_4 \rightarrow \overset{0}{I_2} + \overset{+4}{S}O_2 + 2H_2O$

$\underset{\text{Oxidising agent}}{H_2SO_4}$ — Reduced to→ SO_2

65. $[Co(NO_2)(NH_3)_5]Cl_2$

Pentaamine nitrito-N-cobalt (III)chloride

66. Anomers of glucose are cyclic diastereomers (epimers) differing in configuration at C–1, existing in two forms α and β respectively.



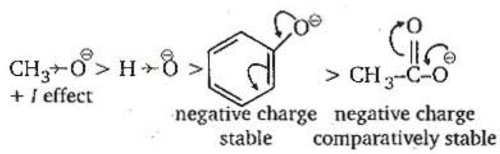
α-form β-form

67. $_{92}U^{238} \rightarrow _{92}U^{234} + _2He^4 + 2\,_{-1}e^0$

68. PhMgBr can be protonated by any of the protic solvent *e.g.*, CH_3OH.

$$PhMgBr + CH_3OH \rightarrow \underset{\text{Benzene}}{Ph-H} + Mg\begin{matrix} Br \\ OCH_3 \end{matrix}$$

69. Nucleophilicity order is :



70. In DNA, cytosine and thymine are pyrimidine bases.

71. For positive iodoform test, alcohol molecule must have $CH_3-\underset{OH}{CH}-$ group

$$Ph-\underset{OH}{CH}-CH_3 \xrightarrow{I_2 + NaOH} CHI_3 + Ph-COO^-$$

72. Free radicals stability :

$$\underset{\text{Highly stable by delocalisation}}{C_6H_5-\underset{C_6H_5}{\dot{C}}-C_6H_5} > C_6H_5-\underset{C_6H_5}{\dot{C}H}$$

$$> \underset{\substack{\text{9-Hyperconjugative}\\ \text{Hydrogens and } +I \text{ effect}}}{CH_3-\underset{CH_3}{\dot{C}}-CH_3} > H_3C\underset{CH_3}{\dot{C}}-H$$

73. $\Delta x \cdot \Delta v \geq \dfrac{h}{4\pi m}$

$$\Delta x = \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 300 \times 0.001 \times 10^{-2}}$$

$= 0.01933$

$= 1.93 \times 10^{-2}$

74. $\underset{(1-x)}{PCl_5(g)} \rightleftharpoons \underset{x}{PCl_3(g)} + \underset{x}{Cl_2(g)}$

Total number of moles at equilibrium

$= (1 - x) + x + x$

$= 1 + x$

$$P_{PCl_3} = \left[\frac{x}{1+x}\right] \times P$$

75. Carbon is found in solid state. The state of substance affects the enthalpy change.

$C(s) \longrightarrow C(g)$ Sublimation

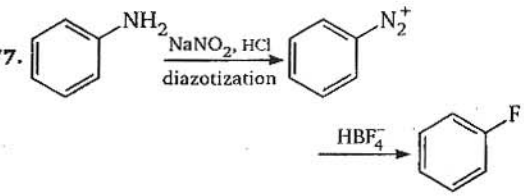
and $H_2(g) \longrightarrow 2H(g)$ dissociation

are required for C—H bond

76.

Molecules	**Interaction**
Benzene and ethanol	dispersion force
Acetonitrile and acetone	dipole-dipole
KCl and water	ion-dipole
Benzene and carbon tetrachloride	dispersion (London) force

77.



78. MCl_2 Oxidation state of $M = +2$

MCl_4 Oxidation state of $M = +4$

Higher the oxidation state, smaller the size.

Greater the polarizing power, greater the covalent characteristics

Hence MCl_4 is more covalent and MCl_2 is more ionic.

79. $H \rightarrow O \rightarrow \rightarrow \overset{+}{N}\begin{matrix} O \\ O^- \end{matrix}$ $\quad H \rightarrow O \rightarrow N = O$

Polarity along O—H in HNO_3 is more in comparison to —O—H in HNO_2.

80. According to Kohlrausch's law-

$$[NaOAc = CH_3\overset{O}{\overset{\|}{C}}-O^-Na^+]$$

$$\lambda^\circ_{CH_3COOH} = \lambda^\circ_{CH_3COO^-} + \lambda^\circ_{H^+} \quad ...(I)$$

$$\lambda^\circ_{HCl} = \lambda^\circ_{H^+} + \lambda^\circ_{Cl^-} \quad ...(II)$$

$$\lambda^\circ_{CH_3COONa} = \lambda^\circ_{CH_3COO^-} + \lambda^\circ_{Na^+} \quad ...(III)$$

thus on adding (II) and (III) if $\lambda^\circ_{Na^+}$ and $\lambda^\circ_{Cl^-}$ are subtracted we can obtained the value of λ°_{HOAC}. Thus, additional value required is λ°_{NaCl}.

81. isoelectronic means having same no. of electons.K^+, Cl^-, Ca^{2+}, Sc^{3+} (all are having 18 electrons).

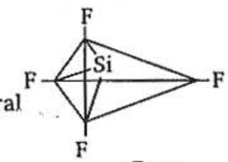
82. $\begin{matrix} Me \\ Me \end{matrix}\!\!>CH-\overset{O}{\overset{\|}{C}}-O-H < CH_3-\overset{O}{\overset{\|}{C}}-O-H$

$< Me-O-CH_2-\overset{O}{\overset{\|}{C}}-OH < F_3C-\overset{O}{\overset{\|}{C}}-O-H$

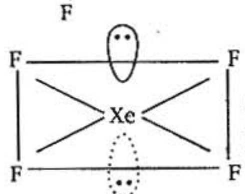
–*I* effect increases acidity.
+*I* effect decreases acidity.
–CF_3 exerting more –*I* effect than MeO –
Me_2CH– exerting more +*I* effect than —CH_3

83. SF_4 has trigonal bipyramidal geometry, lone pair of electrons repels the axial bond pair and decreases the bond angle to 173°.

SiF_4 Regular tetrahedral

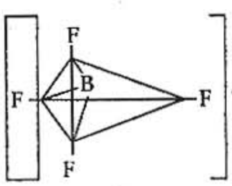


XeF_4



Square planar

BF_4^-

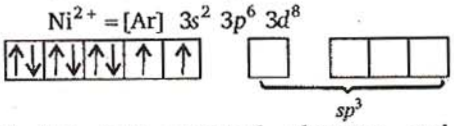


Tetrahedral

84. $3HOCl \longrightarrow 2HCl + HClO_3$

85. $_{28}Ni = [Ar]\ 3s^2\ 3p^6 4s^2\ 3d^8$

$Ni^{2+} = [Ar]\ 3s^2\ 3p^6\ 3d^8$



Nickel has two unpaired electrons and geometry is tetrahedral due to sp^3 hybridisation.

86. Metal carbonyl organometallic compounds possess both σ and π characters.

87. Examine the positions in periodic table.

B C N O F
P S

phosphorus is having stable half filled configuration.

Hence order is B < S < P < F.

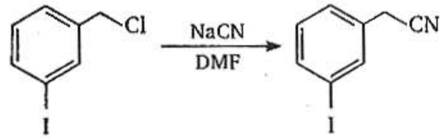
88. $T_{f\,(\text{irreversible})} > T_{f(\text{reversible})}$
It is an adiabatic expansion and *W* (rev) is maximum.

89. The adsorption of a gas is directly proportional to the pressure of the gas.

90. $k = Ae^{-E/RT}$

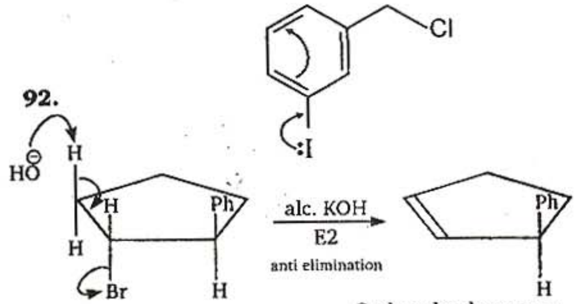
E is activation energy, it is that energy, which molecule must have to give the product.

91.



Chloride is on 1° aliphatic carbon which is substituted easier in comparison to iodide which is arylic and more stable due to delocalisation hence diffic ult to substitute.

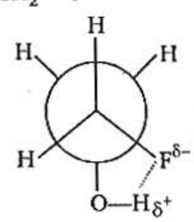
92.



3-phenylcyclopentene

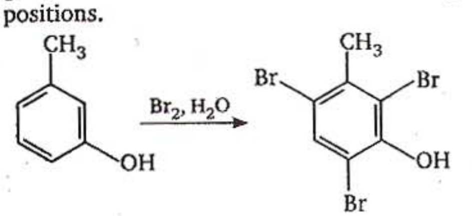
Anti-elimination, means —H and the —Br both departing group must be present at dihedral angle of 180° (anti).

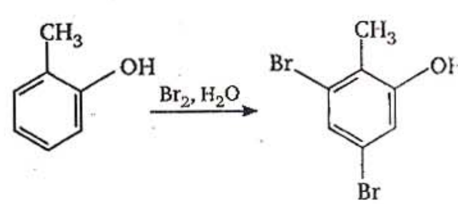
93. HO —CH_2 —CH_2 —F



Gauche conformation is comparatively more stable due to hydrogen linkage in between F and H (at O-atom), hence order is Eclipse, Anti (staggered), Gauche.

94. *m*-cresol due to phenoxide ion in H_2O solvent, gives tribromoderivative at all *ortho* and *para* positions.

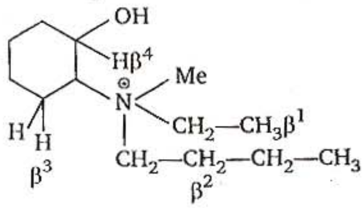




Dibromo derivative

95. $NH_3 > PH_3 > AsH_3 > SbH_3$

As the electronegativity of central atom increases, bonded electron polarise towards central atom more, so, repulsion increases and bond angle increases.

96. 

There are four β-hydrogens, in this quaternary ammonium salt.

On heating quaternary ammonium salt gives Hofmann elimination (abstraction of most acidic hydrogen which is $\beta^{1'}$)

Hence major product is $CH_2{=}CH_2$. (Least substituted alkene)

97. $Ni^{2+} = [Ar]\ 3d^8$

↑↓	↑↓	↑↓	↑	↑

Number of unpaired electrons = 2

Hence, magnetic moment $= \sqrt{n\,(n+2)}$

$= \sqrt{8} = 2.84$

98. Equilibrium constant for the reaction :

$$SO_2(g) + \frac{1}{2}O_2 \rightleftharpoons SO_3\ (g)$$

$$K_c = \frac{1}{4.9 \times 10^{-2}}$$

and for $2SO_2 + O_2 \rightleftharpoons 2SO_3\,(g)$

$$K_c = \left(\frac{1}{4.9 \times 10^{-2}}\right)^2$$

$$= \frac{10^4}{(4.9)^2} = 416.49$$

99. Alkali metals reactivity increases down the group as electropositivity increases, but for halogens F_2 is more reactive as moving down molecular stability increases.

100. $AgI(s) + e^- \rightleftharpoons Ag(s) + I^-;\ E° = -0.152$

$Ag\,(s) \to Ag^+ + e^- \qquad E° = -0.8$

$AgI(s) \to Ag^+ + I^- \qquad E° = -0.952$

$$E°_{cell} = \frac{0.059}{n}\log K_{sp}$$

$$-0.952 = \frac{0.059}{1}\log K_{sp}$$

$$\log K_{sp} = \frac{-0.952}{0.059} = -16.135$$

101. Rate $= k\ [NOBr_2][NO]$ ...(1)

But $NOBr_2$ is in equilibrium.

$$K_{eq} = \frac{[NOBr_2]}{[NO]\ [Br_2]}$$

$$[NOBr_2] = K_{eq}[NO]\ [Br_2] \qquad ...(2)$$

Putting the $[NOBr_2]$ in (1)

Rate $= k \cdot K_{eq}\ [NO]\ [Br_2]\ [NO]$

Hence Rate $= k \cdot K_{eq}\ [NO]^2[Br_2]$

$$\text{Rate} = k'[NO]^2[Br_2]$$

where $k' = k \cdot K_{eq}$.

102. Lanthanoid contraction is due to ineffective shielding produced by larger *f*-subshell.

103.

$$R = 100\,\Omega$$

$$\kappa = \frac{1}{R}\left(\frac{l}{a}\right)$$

$\frac{l}{a}$ (cell constant) $= 1.29 \times 100\ m^{-1}$

Given $R = 520\ \Omega$; $C = 0.2\ M$

μ (molar conductivity) = ?

$$\mu = k \times V$$

(κ can be calculated as $\kappa = \frac{1}{R}\left(\frac{l}{a}\right)$ now cell constant is known)

Hence $\mu = \frac{1}{520} \times 129 \times \frac{1000}{0.2} \times 10^{-6}\ m^3$

$= 12.4 \times 10^{-4}\ Sm^2\ mol^{-1}$

104. Li^+ is having largest hydrated ionic size while Rb^+ is having smallest.

Smaller the size, greater the mobility.

105. molality $(m) = \frac{M}{1000\,d - MM_1} \times 100$

M = Molarity

M_1 = Molecular mass

d = density

$$= \frac{2.05}{(1000 \times 1.02) - (2.05 \times 60)} \times 1000$$

$$= 2.28\ mol\ kg^{-1}$$

106. $\frac{1}{2}I_2(s) + \frac{1}{2}Cl_2(g) \to ICl(g)$

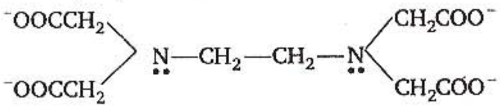
$$\Delta H = \left[\frac{1}{2}\Delta H_{s\to g} + \frac{1}{2}\Delta H_{diss.}\,(Cl_2) + \frac{1}{2}\Delta H_{diss.}\,(I_2)\right] - \Delta H_{ICl}$$

$$= \left(\frac{1}{2} \times 62.76 + \frac{1}{2} \times 242.3 + \frac{1}{2} \times 151.0\right) - 211.3$$

$= 228.03 - 211.3$

$\Delta H = 16.73$

107. EDTA (Ethylenediaminetetra acetic acid)



It is hexadentate (6 electron pairs) that's why for octahedral complex only one EDTA is required.

108. $CHCl_3 + OH^{\ominus} \rightarrow {}^{\ominus}CCl_3 + H_2O$

$${}^{\ominus}CCl_3 \rightarrow :CCl_2 + Cl^-$$

109. $\frac{p^\circ - P_s}{P_s} = \frac{n}{N}$

$$\frac{760 - P_s}{P_s} = \frac{18/180}{178.2/18}$$

$$= \frac{0.1}{9.9}$$

$$760 - P_s = \frac{1}{99}P_s$$

$$P_s = 752.4$$

110. $\Delta H - \Delta U = \Delta n_g RT$

$$= +\frac{1}{2} \times 8.314 \times 298$$

$$= +\ 1238.78 \text{ J mol}^{-1}$$

$$\left[\begin{array}{l} C(s) + \frac{1}{2}O\ 2(g) \rightarrow CO\ (g) \\ \Delta n_g = 1 - \frac{1}{2} \\ = \frac{1}{2} \end{array} \right]$$

Part-C

Mathematics

111. Since tan 30° and tan 15° are roots of equation $x^2 + px + q = 0$.

$\therefore$ $\tan 30° + \tan 15° = -p$

and $\tan 30° \tan 15° = q$

Therefore

$$2 + q - p = 2 + \tan 30° \tan 15° + (\tan 30° + \tan 15°)$$

$$= 2 + \tan 30° \tan 15° + 1 - \tan 30° \tan 15°$$

$$\left(\because \tan 45° = \frac{\tan 30° + \tan 15°}{1 - \tan 30° \tan 15°} \right)$$

$$\Rightarrow \quad 2 + q - p = 3$$

112. Let $I = \int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}}\,dx$ ... (i)

$$= \int_3^6 \frac{\sqrt{9-x}}{\sqrt{9-9+x} + \sqrt{9-x}}\,dx$$

$$\Rightarrow I = \int_3^6 \frac{\sqrt{9-x}}{\sqrt{x} + \sqrt{9-x}}\,dx \quad \text{... (ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_3^6 \frac{\sqrt{x} + \sqrt{9-x}}{\sqrt{x} + \sqrt{9-x}}\,dx$$

$$= \int_3^6 1\,dx = [x]_3^6$$

$$= 6 - 3 = 3$$

$$\Rightarrow \quad I = \frac{3}{2}$$

113. Let $W = \{CAT, TOY, YOU, \ldots\ldots\}$

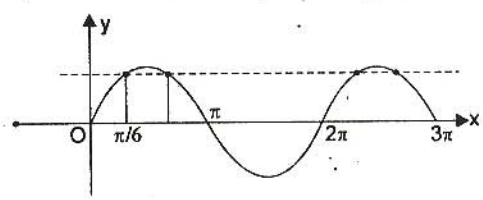
Clearly R is reflexive and symmetric but not transitive.

Since ${}_{CAT}R_{TOY}$, ${}_{TOY}R_{YOU} \not\Rightarrow {}_{CAT}R_{YOU}$

114. Since $2\sin^2 x + 5\sin x - 3 = 0$

$$\Rightarrow \quad (2\sin x - 1)(\sin x + 3) = 0$$

$$\Rightarrow \quad \sin x = 1/2 \qquad (\because \sin x \neq -3)$$



From figure it is clear that the number of solutions of given equation in $[0, 3\pi]$ is 4.

115. $\because A^2 - B^2 = (A - B)(A + B)$

$$\Rightarrow \quad A^2 - B^2 = A^2 - B^2 + AB - BA$$

$$\Rightarrow \quad AB = BA$$

116. $\sum_{k=1}^{10} \left(\sin \frac{2k\pi}{11} + i \cos \frac{2k\pi}{11} \right)$

$$= i \sum_{k=1}^{10} \left(\cos \frac{2k\pi}{11} - i \sin \frac{2k\pi}{11} \right)$$

$$= i \sum_{k=1}^{10} \left(e^{-\frac{2k\pi}{11}} \right)$$

$$= i \left\{ \sum_{k=0}^{10} \left(e^{-\frac{2k\pi}{11}} \right) - 1 \right\}$$

117. $\because (\vec{a}\times\vec{b})\times\vec{c} = \vec{a}\times(\vec{b}\times\vec{c})$

$\Rightarrow (\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a} = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

$\Rightarrow \quad (\vec{b}\cdot\vec{c})\vec{a} = (\vec{a}\cdot\vec{b})\vec{c}$

$\Rightarrow$ $\vec{a}$ is parallel to $\vec{c}$.

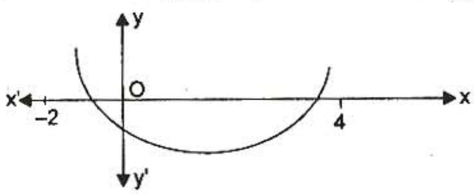
118. Since both roots of equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4.

$\therefore \quad D \geq 0$

$\Rightarrow \quad 4m^2 - 4m^2 + 4 \geq 0 \Rightarrow m \in R \quad$ ...(i)

and $\quad -2 < \frac{-b}{2a} < 4 \Rightarrow -2 < \left(\frac{2m}{2\cdot 1}\right) < 4$

$\Rightarrow \quad -2 < m < 4 \quad$ ...(ii)



Also $f(4) > 0 \Rightarrow 16 - 8m + m^2 - 1 > 0$

$\Rightarrow \quad m^2 - 8m + 15 > 0$

$\Rightarrow \quad (m-3)(m-5) > 0$

$\Rightarrow \; -\infty < m < 3$ and $5 < m < \infty \quad$ ...(iii)

Also $f(-2) > 0 \Rightarrow 4 + 4m + m^2 - 1 > 0$

$\Rightarrow \quad m^2 + 4m + 3 > 0$

$\Rightarrow \quad (m+3)(m+1) > 0$

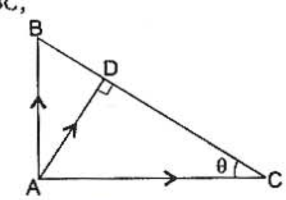
$\Rightarrow \; -\infty < m < -3$ and $-1 < m < \infty \quad$ ...(iv)

$\therefore$ From (i), (ii), (iii) and (iv)

m lies between -1 and 3.

119. Let $|\overrightarrow{BC}| = l$

$\therefore$ In ΔABC,



$l = \sqrt{AB^2 + AC^2}$

and $\quad \tan\theta = \frac{AB}{AC}$

$\Rightarrow \quad \sin\theta = \frac{AB}{l}$ and $\cos\theta = \frac{AC}{l}$

$\therefore$ Resultant vector

$= \frac{1}{AB}\hat{i} + \frac{1}{AC}\hat{j} = \left(\frac{1}{l\sin\theta}\hat{i} + \frac{1}{l\cos\theta}\hat{j}\right)$

$= k\,\overrightarrow{AD}$

Now, $\quad AD = AC\sin\theta = l\sin\theta\cos\theta$

$= \frac{AB\cdot AC}{l}$

Magnitude of resultant vector

$= \sqrt{\frac{1}{l^2}\left(\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}\right)}$

$= \frac{l}{(AB)(AC)} = \frac{1}{AD}$

120. Since variance is independent of change in origin. Hence variance of observations 101, 102, ..., 200 is same as variance of 151, 152, ..., 250

$\therefore \quad V_A = V_B$

$\Rightarrow \quad \frac{V_A}{V_B} = 1$

121. Let $I = \int_{-3\pi/2}^{-\pi/2} [(x+\pi)^3 + \cos^2(x+3\pi)]\, dx \quad$ ...(i)

and $\quad I = \int_{-3\pi/2}^{-\pi/2} \left[\left(-\frac{\pi}{2} - \frac{3\pi}{2} - x + \pi\right)^3 + \cos^2\left(-\frac{\pi}{2} - \frac{3\pi}{2} - x + 3\pi\right)\right] dx$

$\Rightarrow \quad I = \int_{-3\pi/2}^{-\pi/2} [-(x+\pi)^3 + \cos^2(\pi - x)]\, dx \quad$ ...(ii)

On adding Eqs. (i) and (ii), we get

$2I = \int_{-3\pi/2}^{-\pi/2} 2\cos^2 x\, dx$

$= \int_{-3\pi/2}^{-\pi/2} (1 + \cos 2x)\, dx$

$= \left[x + \frac{\sin 2x}{2}\right]_{-3\pi/2}^{-\pi/2}$

$= -\frac{\pi}{2} + \frac{3\pi}{2} = \pi$

$\Rightarrow \quad I = \frac{\pi}{2}$

122. Given that $2ae = 6$ and $2b = 8$

$\Rightarrow \quad ae = 3$ and $b = 4$

$\Rightarrow \quad \frac{ae}{b} = \frac{3}{4}$

$\Rightarrow \quad e^2 = \frac{9}{16}\left(\frac{b^2}{a^2}\right)$

We know that $\frac{b^2}{a^2} = 1 - e^2$

$$\Rightarrow \quad e^2 = \frac{9}{16}(1 - e^2)$$

$$\Rightarrow \quad \left(\frac{16+9}{9}\right) e^2 = 1 \Rightarrow e = \frac{3}{5}$$

123. The given equation of parabola is

$$y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$$

$$\Rightarrow \quad y + 2a = \frac{a^3}{3}\left[x^2 + \frac{3}{2a}x\right]$$

$$\Rightarrow \quad y + 2a = \frac{a^3}{3}\left[x^2 + \frac{3}{2a}x + \frac{9}{16a^2} - \frac{9}{16a^2}\right]$$

$$\Rightarrow \quad y + 2a = \frac{a^3}{3}\left[x + \frac{3}{4a}\right]^2 - \frac{9}{16a^2} \times \frac{a^3}{3}$$

$$\Rightarrow \quad y + 2a + \frac{3a}{16} = \frac{a^3}{3}\left(x + \frac{3}{4a}\right)^2$$

$$\Rightarrow \quad \left(y + \frac{35a}{16}\right) = \frac{a^3}{3}\left(x + \frac{3}{4a}\right)^2$$

Thus the vertices of parabola is $\left(-\frac{3}{4a}, -\frac{35a}{16}\right)$.

Let $\quad h = -\frac{3}{4a}$

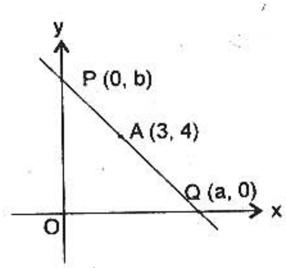
and $\quad k = -\frac{35a}{16}$

$\Rightarrow \quad hk = \frac{105}{64}$

Thus the locus of vertices of a parabola is

$$xy = \frac{105}{64}$$

124. Since A is midpoint of line PQ



$\therefore \quad 3 = \frac{a+0}{2} \Rightarrow a = 6$

and $\quad 4 = \frac{0+b}{2} \Rightarrow b = 8$

Thus the equation of line is

$$\frac{x}{6} + \frac{y}{8} = 1$$

$$\Rightarrow \quad 4x + 3y = 24$$

125. Since position vectors of A, B, C are $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $a\hat{i} - 3\hat{j} - \hat{k}$ respectively.

$$\therefore \quad \overrightarrow{AC} = (a\hat{i} - 3\hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$$

$$= (a-2)\hat{i} - 2\hat{j}$$

and $\quad \overrightarrow{BC} = (a\hat{i} - 3\hat{j} + \hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k})$

$$= (a-1)\hat{i} + 6\hat{k}$$

Since the ΔABC is right angled at C, then

$$\overrightarrow{AC} \cdot \overrightarrow{BC} = 0$$

$$\Rightarrow \{(a-2)\hat{i} - 2\hat{j}\} \cdot \{(a-1)\hat{i} + 6\hat{k}\} = 0$$

$$\Rightarrow \quad (a-2)(a-1) = 0$$

$$\Rightarrow \quad a = 1 \text{ and } 2$$

126. Let $I = \int_0^\pi x\, f(\sin x)\, dx$ ...(i)

$$\Rightarrow \quad I = \int_0^\pi (\pi - x)\, f(\sin(\pi - x))\, dx$$

$$\Rightarrow \quad I = \int_0^\pi (\pi - x)\, f(\sin x)\, dx \quad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^\pi \pi\, f(\sin x)\, dx$$

$$\Rightarrow \quad I = \frac{\pi}{2}\int_0^\pi f(\sin x)\, dx$$

$$\Rightarrow \quad I = \pi \int_0^{\pi/2} f(\sin x)\, dx$$

Let $\quad \frac{\pi}{2} - x = t \Rightarrow x = \frac{\pi}{2} - t$

$$\therefore \quad I = \frac{\pi}{2} \cdot \int_{-\pi/2}^{\pi/2} f(\cos t)\, dt$$

$$= \frac{\pi}{2}\int_{-\pi/2}^{\pi/2} f(\cos x)\, dx$$

$$= \pi \int_0^{\pi/2} f(\cos x)\, dx$$

($\because f(\cos x)$ is an even function)

127. The equations of given lines are

$$x = ay + b,\ z = cy + d$$

and $\quad x = a'y + b',\ z = c'y + d'$

These equations can be rewritten as

$$\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c}$$

and $\quad \frac{x-b'}{a'} = \frac{y-0}{1} = \frac{z-d'}{c'}$

These lines will perpendicular, if

$$aa' + cc' + 1 = 0$$

128. Total number of ways

$$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$$
$$= 10 + 45 + 120 + 210$$
$$= 385$$

129. $\because (1-ax)^{-1}(1-bx)^{-1}$

$$= (1 + ax + a^2x^2 + \dots)(1 + bx + b^2x^2 + \dots)$$

Hence $a_n =$ coefficient of x^n in $(1-ax)^{-1}(1-bx)^{-1}$

$$= a^0b^n + ab^{n-1} + \dots + a^nb^0$$
$$= a^0b^n\left(\frac{\left(\frac{a}{b}\right)^{n+1} - 1}{\frac{a}{b} - 1}\right)$$
$$= \frac{b^n(a^{n+1} - b^{n+1})}{a-b} \cdot \frac{b}{b^{n+1}} = \frac{a^{n+1} - b^{n+1}}{a-b}$$

130. $\because (1-y)^m(1+y)^n = ({}^mC_0 - {}^mC_1y + {}^mC_2y^2 - \dots)({}^nC_0 + {}^nC_1y + {}^nC_2y^2 + \dots)$

$\therefore a_1 =$ coefficient of y in $(1-y)^m(1+y)^n$

$$= {}^nC_1 - {}^mC_1 = 10$$
$$\Rightarrow \quad n - m = 10$$
$$\Rightarrow \quad n = m + 10 \quad \dots\text{(i)}$$

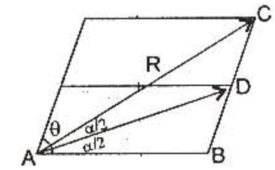
and $a_2 =$ coefficient of y^2 in $(1-y)^m(1+y)^n$

$$= {}^nC_2 - {}^mC_1 \cdot {}^nC_1 + {}^mC_2$$
$$\therefore {}^nC_2 - {}^mC_1 \cdot {}^nC_1 + {}^mC_2 = 10$$
$$\Rightarrow \frac{n(n-1)}{2} + \frac{m(m-1)}{2} - mn = 10$$
$$\Rightarrow \frac{(10+m)(9+m)}{2} + \frac{m(m-1)}{2} - m(10+m) = 10$$
(using Eq. (i))
$$\Rightarrow 45 - 5m + \frac{9m}{2} - \frac{m^2}{2} + \frac{m^2}{2} - \frac{m}{2} = 10$$
$$\Rightarrow \quad 45 - m = 10 \Rightarrow m = 35$$
$$\therefore \quad n = 45 \quad \text{(using Eq. (i))}$$
$$\therefore \quad (m, n) = (35, 45)$$

131. Let the magnitude of two velocities is u.

We know that

$$R = \sqrt{u^2 + u^2 + 2u^2\cos\theta}$$
$$R = \sqrt{2}u\sqrt{1 + \cos\theta}$$
$$R = 2u\cos\frac{\theta}{2}$$



We know that by angle bisector theorem

$$\frac{AB}{AC} = \frac{BD}{DC}$$
$$\Rightarrow \quad \frac{u}{2u\cos\frac{\theta}{2}} = 1$$
$$\Rightarrow \quad \cos\frac{\theta}{2} = \frac{1}{2} = \cos\frac{\pi}{3}$$
$$\Rightarrow \quad \theta = 120^\circ$$

132. Required probability $= P(X=0) + P(X=1)$

$$= \frac{e^{-5}}{0!}5^0 + \frac{e^{-5}}{1!} \cdot 5^1$$
$$= e^{-5} + 5e^{-5}$$
$$= \frac{6}{e^5}$$

133. Let the body is at a height h_1 at a time t and is at height h at a time $(t-4)$ from above.

$$\therefore \quad h_1 - h = 400$$
$$\frac{1}{2}gt^2 - \frac{1}{2}g(t-4)^2 = 400$$
$$\Rightarrow \quad t^2 - (t-4)^2 = 80$$
$$\Rightarrow \quad (2t-4)4 = 80$$
$$\Rightarrow \quad t = 12 \text{ seconds}$$
$$\therefore \quad h = \frac{1}{2}g(t-4)^2$$
$$= 320 \text{ m}$$

Hence total distance of point P from the point from where body began to fall

$$= 320 + 400$$
$$= 720 \text{ m}$$

134. $\because \quad f(x) = \frac{x}{1+|x|}$

Let $\quad f(x) = \frac{g(x)}{h(x)} = \frac{x}{1+|x|}$

It is clear that $g(x) = x$ and $h(x) = 1 + |x|$ are differentiable on $(-\infty, \infty)$ and $(-\infty, 0) \cup (0, \infty)$ respectively.

Thus $f(x)$ is differentiable on $(-\infty, 0) \cup (0, \infty)$.

For $x = 0$

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{\frac{x}{1+|x|}}{x}$$
$$= \lim_{x \to 0} \frac{1}{1+|x|} = 1$$

Thus $f(x)$ is differentiable on $(-\infty, \infty)$.

135. $\because A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

Now $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$= \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$

and $BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$

If $AB = BA \Rightarrow a = b$

Hence $AB = BA$ is possible for infinitely many B's.

136. $\because \frac{a_1 + a_2 + \ldots + a_p}{a_1 + a_2 + \ldots + a_q} = \frac{p^2}{q^2}$

$\Rightarrow \frac{\frac{p}{2}[2a_1 + (p-1)d]}{\frac{q}{2}[2a_1 + (q-1)d]} = \frac{p^2}{q^2}$

$\Rightarrow \frac{(2a_1 - d) + pd}{(2a_1 - d) + qd} = \frac{p}{q}$

$\Rightarrow (2a_1 - d)(p - q) = 0$

$\Rightarrow a_1 = \frac{d}{2}$

Now $\frac{a_6}{a_{21}} = \frac{a_1 + 5d}{a_1 + 20d} = \frac{\frac{d}{2} + 5d}{\frac{d}{2} + 20d}$

$= \frac{11d}{41d} = \frac{11}{41}$

137. $\because f(x) = \frac{x}{2} + \frac{2}{x}$

$\therefore f'(x) = \frac{1}{2} - \frac{2}{x^2}$

Put $f'(x) = 0$ for maxima or minima

$\frac{1}{2} - \frac{2}{x^2} = 0$

$\Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

Now $f''(x) = \frac{4}{x^3} \Rightarrow f''(2) = \frac{4}{8} = \frac{1}{2} > 0$

and $f''(-2) = -\frac{4}{8} = -\frac{1}{2} < 0$

$\therefore f(x)$ is minimum at $x = 2$.

138. $\because y = x^2 - 5x + 6$

$\therefore \frac{dy}{dx} = 2x - 5$

Now $m_1 = \left(\frac{dy}{dx}\right)_{(2,0)} = 4 - 5 = -1$

and $m_2 = \left(\frac{dy}{dx}\right)_{(3,0)} = 6 - 5 = 1$

Since $m_1 m_2 = -1$

$\therefore$ Angle between the tangents is $\frac{\pi}{2}$.

139. $\because \frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$

$= 1 + \frac{10}{3\left(x^2 + 3x + \frac{7}{3}\right)}$

$= 1 + \frac{10}{3\left[\left(x + \frac{3}{2}\right)^2 + \frac{1}{12}\right]}$

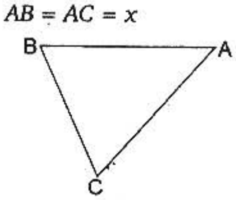
$\Rightarrow$ Maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ occurs at $x = -\frac{3}{2}$.

$\therefore$ Maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$

$= 1 + \frac{10}{3\left(\frac{1}{12}\right)}$

$= 1 + 40 = 41$

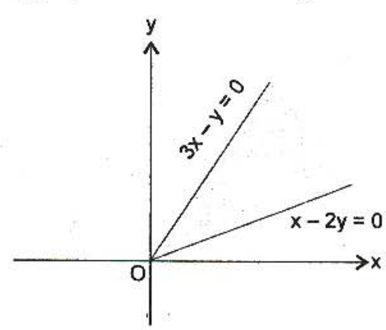
140. Given that $AB = AC = x$



We know that area of isosceles triangle is maximum, if it is right angled triangle.

$\therefore$ Maximum area of triangle $= \frac{1}{2}x^2$

141. The graph of equations $x - 2y = 0$ and $3x - y = 0$ is as shown in the figure. Since given point (a, a^2) lies in the shaded region.



Then $a - 2a^2 < 0$ and $3a - a^2 > 0$

$\Rightarrow \quad a \in (-\infty, 0) \cup \left(\frac{1}{2}, \infty\right)$

and $\quad a \in (0, 3) \Rightarrow a \in \left(\frac{1}{2}, 3\right)$

142. $\because x^m \cdot y^m = (x + y)^{m+n}$

Taking log on both sides, we get

$m \log x + n \log y = (m + n) \log (x + y)$

On differentiating with respect to x, we get

$$\frac{m}{x} + \frac{n}{y}\frac{dy}{dx} = \frac{(m+n)}{(x+y)}\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{m+n}{x+y} - \frac{n}{y}\right) = \frac{m}{x} - \frac{m+n}{x+y}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{my + ny - nx - ny}{y(x+y)}\right) = \frac{mx + my - mx - nx}{x(x+y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

143. The given equations of diameters are

$$3x - 4y - 7 = 0 \quad \text{...(i)}$$

and

$$2x - 3y - 5 = 0 \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$$x = 1 \text{ and } y = -1$$

$\therefore$ Centre of circle is $(1, -1)$.

Let r be the radius of circle, then

$$\pi r^2 = 49\pi$$

$$\Rightarrow \quad r = 7 \text{ units}$$

$\therefore$ Equation of required circle is

$$(x-1)^2 + (y+1)^2 = 49$$

$$\Rightarrow x^2 + y^2 - 2x + 2y + 1 + 1 = 49$$

$$\Rightarrow x^2 + y^2 - 2x + 2y - 47 = 0$$

144. We know that the image (x, y, z) of a point (x_1, y_1, z_1) in a plane $ax + by + cz + d = 0$ is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \frac{-2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$$

Thus image of point $(-1, 3, 4)$ in a plane $x - 2y = 0$ is given by

$$\frac{x+1}{1} = \frac{y-3}{-2} = \frac{z-4}{0} = \frac{-2(1 \times (-1) + 3 \times (-2) + 0)}{1 + 4}$$

$$\Rightarrow \frac{x+1}{1} = \frac{y-3}{-2} = \frac{z-4}{0} = \frac{-2(-7)}{5}$$

$$\Rightarrow x = \frac{14}{5} - 1 = \frac{9}{5},\ y = -\frac{28}{5} + 3 = -\frac{13}{5}$$

and $\quad z = 4$

Thus image of point $(-1, 3, 4)$ is $\left(\frac{9}{5}, -\frac{13}{5}, 4\right)$.

Hence no option is correct.

145. The given equation is $Ax^2 + By^2 = 1$

On differentiating with respect to x, we get

$$2Ax + 2By\frac{dy}{dx} = 0 \quad \text{...(i)}$$

On again differentiating

$$2A + 2B\left\{\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2}\right\} = 0 \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - \frac{y}{x}\cdot\frac{dy}{dx} = 0$$

This is the required differential equation whose order is 2 and degree is 1.

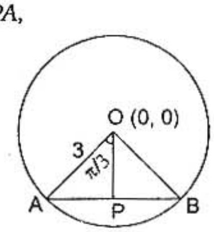
146. $\because \int_1^a [x] f'(x)\,dx = \int_1^2 f'(x)\,dx + \int_2^3 2f'(x) + \ldots + \int_{[a]}^a [a] f'(x)\,dx$

$= [a] f(x) - \{f(1) + f(2) + \ldots + f([a])\}$

147. Let the co-ordinates of a point P be (h, k) which is mid point of the chord AB.

$$OP = \sqrt{(h-0)^2 + (k-0)^2} = \sqrt{h^2 + k^2}$$

Now in ΔOPA,



$$\cos\frac{\pi}{3} = \frac{OP}{OA}$$

$$\Rightarrow \frac{1}{2} = \frac{\sqrt{h^2 + k^2}}{3}$$

$$\Rightarrow h^2 + k^2 = \left(\frac{3}{2}\right)^2$$

$$\Rightarrow h^2 + k^2 = \frac{9}{4}$$

Thus the required locus is

$$x^2 + y^2 = \frac{9}{4}$$

148. $\because a_1, a_2, a_3, \ldots, a_n$ are in HP

$\therefore \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots, \frac{1}{a_n}$ are in AP

Let d be the common difference of AP.

$$\therefore \quad \frac{1}{a_2} - \frac{1}{a_1} = d \Rightarrow a_1 - a_2 = a_1 a_2 d$$

Similarly $\quad a_2 - a_3 = a_2 a_3 d$

....

....

$$a_{n-1} - a_n = a_{n-1} a_n d$$

On adding all of these, we get

$$a_1 - a_n = d\{a_1 a_2 + a_2 a_3 + \ldots\ldots + a_{n-1} a_n\} \quad \ldots(i)$$

Also $\quad \frac{1}{a_n} = \frac{1}{a_1} + (n-1)d$

$$\Rightarrow \quad d = \frac{a_1 - a_n}{a_1 a_n (n-1)}$$

On putting this value of d in equation (i), we get

$$a_1 - a_n = \frac{a_1 - a_n}{a_1 a_n (n-1)} \{a_1 a_2 + a_2 a_3 + \ldots\ldots + a_{n-1} a_n\}$$

$$\Rightarrow \quad a_1 a_2 + a_2 a_3 + \ldots + a_{n-1} a_n = a_1 a_n (n-1)$$

149. Given equation is

$$z^2 + z + 1 = 0$$

$$\Rightarrow \quad z = \omega, \omega^2$$

Now $\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \left(z^4 + \frac{1}{z^4}\right)^2 + \left(z^5 + \frac{1}{z^5}\right)^2 + \left(z^6 + \frac{1}{z^6}\right)^2$

$$= (-1)^2 + (-1)^2 + (1+1)^2 + (-1)^2 + (-1)^2 + (1+1)^2$$

$$= 1 + 1 + 4 + 1 + 1 + 4 = 12$$

150. $\because \quad \cos x + \sin x = \frac{1}{2}, \; 0 < x < \pi \quad \ldots(i)$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \quad \cos\left(x - \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \quad \frac{\pi}{4} < x - \frac{\pi}{4} < \frac{\pi}{2}$$

$$\Rightarrow \quad \frac{\pi}{2} < x < \frac{3\pi}{4}$$

$\therefore$ $\tan x$ lies in second quadrant.

Now on squaring Eq. (i)

$$(\sin x + \cos x)^2 = \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \quad \sin^2 x + \cos^2 x + 2 \sin x \cos x = \frac{1}{4}$$

$$\Rightarrow \quad 1 + \sin 2x = \frac{1}{4}$$

$$\Rightarrow \quad \sin 2x = -\frac{3}{4}$$

$$\Rightarrow \quad \frac{2 \tan x}{1 + \tan^2 x} = -\frac{3}{4}$$

$$\Rightarrow \quad 8 \tan x = -3 - 3 \tan^2 x$$

$$\Rightarrow \quad 3 \tan^2 x + 8 \tan x + 3 = 0$$

$$\Rightarrow \quad \tan x = \frac{-4 \pm \sqrt{7}}{3}$$

But $\quad \tan x < -1$

$$\therefore \quad \tan x = \frac{-4 - \sqrt{7}}{3} = -\left(\frac{4 + \sqrt{7}}{3}\right)$$