ANSWERS KEY

CHEMISTRY

1. a	2. b	3. a	4. b	5. b	6. b	7. c	8. b	9. c	10. a	11. c	12. a	13. c
14. a	15. d	16. c	17. c	18. c	19. b	20. d	21. c	22. b	23. a	24. b	25. a	26. c
27. d	28. a	29. a	30. c									

PHYSICS

1. b	2. a	3. d	4. c	5. d	6. a	7. c	8. b	9. a	10. c	11. b	12. c	13. c
14. a	15. c	16. a	17. b	18. b	19. b	20. b	21. b	22. d	23. c	24. c	25. c	26. d
27. b	28. d	29. b	30. d									

MATHEMATICS

1. c	2. b	3. b	4. a	5. c	6. d	7. c	8. a	9. d	10. b	11. a	12. b	13. c
14. a	15. d	16. a	17. b	18. c	19. a	20. a	21. d	22. a	23. c	24. c	25. c	26. b
27. c	28. a	29. c	30. a									

HINTS & EXPLANATIONS

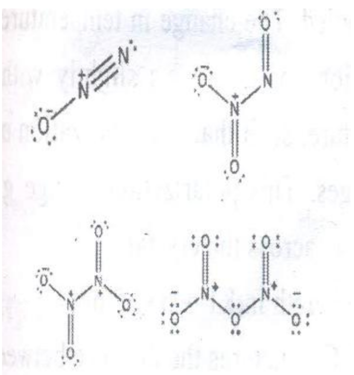
CHEMISTRY

Sol 1.

Ca imparts red colour to the flame.

Sol 2.

N_2O_5 does not have a N – N bond.



Sol 3.

In this process, the impure nickel is reached with excess carbon monoxide at 50- 60 °C to form nickel carbonyl. Ni (s) + 4 CO (g) → $Ni(CO)_4$ (g)

The mixture of excess carbon monoxide and nickel tetracarbonyl decomposes to give nickel:

$Ni(CO)_4$ (g) → Ni (s) + 4 CO (g)

Sol 4.

Orbitals corresponding to options are :

(a) 4s
(b) 3d
(c) 3d
(d) 3p;

Increasing order of energy is 3p, 4s, 3d = 3d or

$d < a < b = c$

Sol 5.

ΔT_r = i. K_f. m

For glycerine, ΔT_f = 1 x 0.030 x K_f. = 0.030 x K_f For KBr, ΔT_f = 2x 0.02 x K_f = 0.04 x K_f (KBr undergoes dissociation into K^+ and Br^- ions) for benzoic acid, ΔT_f = ½ x 0.030 x K_f = 0.015 x K_f (two molecules of benzoic acid undergo association) order : c < a < b

Sol 6.

108 g of A g requires 96500 C

1 g of Ag requires $\frac{69500}{108}$ = 893.52 C

Q = I x t or 893.52 = 30 x t

t = 893.52/30 = 29.78 s

Sol 7.

M.O. electronic configuration for

$$O_2 = (\sigma_{1s})^2(\sigma *_{1s})^2(\sigma_{2s})^2(\sigma *_{2s})^2(\sigma_{2pz})^2(\pi_{2px})^2(\pi_{2py})^2(\pi *_{2px})^1(\pi *_{2py})^1$$

	O_2	O_2^+	O_2^-
Number of bonding electrons:	10	10	10
Number of anti bonding electrons:	6	5	7
B.O. :	2	2.5	1.5

Sol 8.

The reaction of pentyl magnesium bromide with water gives pentane.

$CH_3CH_2CH_2CH_2CH_2MgBr + H_2\,O \rightarrow$

$CH_3CH_2CH_2CH_2CH_2Mg(Br)\,OH$

Sol 9.

5L of hydrocarbon requires 15L of oxygen for complete combustion or 1 mol of hydrocarbon requires 3 mol of oxygen, therefore the reaction can be represented as:

Hydrocarbon $+ 3O_2 \rightarrow CO_2 + H_2O$

Now let us consider complete combustion of hydrocarbons given in options.

$C_2H_6 + 3.5O_2 \rightarrow 2CO_2 + 3H_2O$

$C_2H_4 + 3O_2 \rightarrow 2CO_2 + 2H_2O$

$C_2H_2 + 2.5O_2 \rightarrow 2CO_2 + H_2O$

$C_3H_8 + 5O_2 \rightarrow 3CO_2 + H_2O$

Therefore the hydrocarbon is ethane

Sol 10.

$CH \equiv CH + NaNH_2 \rightarrow HC \equiv Can + NH_3$

$HC \equiv Cha + BrCH_2CH_3 \rightarrow$

$HC \equiv CCH_2CH_3 + NaBr$

Sod.acetylide ethyl bromide 1- Butyne

Sol 11.

$18\ M\ H_2SO_4 = 36\ N\ H_2SO_4$

$N_1V_1 = N_2V_2$

$36\ X\ 10 \quad = N_2\ X\ 10000$

$N_2 \frac{360}{10000} = 0.036\ N$

Sol 12.

Conversion of Cr^{3+} to $Cr_2O_7^{2-}$ involves loss of 3 electrons per Cr Atom, i.e.,

$Cr^{3+} \rightarrow Cr^{6+} + 3e^-$,This oxidation reactions takes place at anode.

Other reaction are reduction reactions which occur at cathode.

$O_2 + 4H^+ + 4e^- \rightarrow 2H_2O$

$F_2 + 2e^- \rightarrow 2F^-$ and $HAsO_2 + 3e^- + 3H^+ \rightarrow As + 2H_2O$

Sol 13.

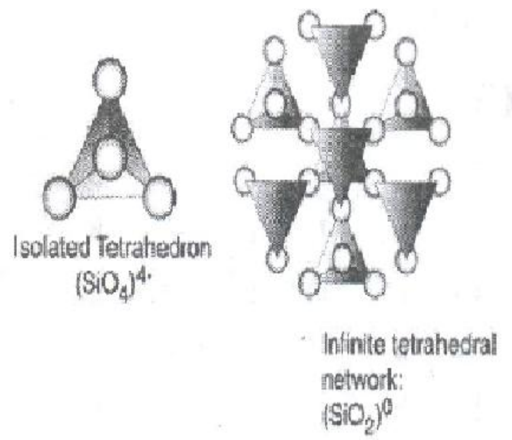
Zeise's salt, K[$PtCl_3$ ($\eta^2 - C_2H_4$)] is not a 18 e^- species. The 18-electron rule is a rule used primarily for predicting formulae for stable metal complex. The rule is based on the fact that the valence shells of transition metals consist of nine valence orbitals, which collectively can accommodate 18 electrons as either bonding or nonbonding electron pairs. This means that, the combination of these nine atomic orbitals with ligand orbitals creates nice molecular orbitals that are either metal-ligand bonding or non-bonding. When a metal complex has 18 valence electrons, it is said to have achieved the same electron configuration as the noble gas in the period. Zeise'ssald $[PtCl_3(C_2H_4)]^-$ violates the 18e rule and is an example of 16e complex.

Sol 14.

$Na_2Cr_2O_7 + 4H_2SO_4 \rightarrow Na_2SO_4 + Cr_2(SO_4)_3 + 4H_2O + 3O$

Sol 15.

The basic unit in all silicates is SiO_4^{4-}



Sol 16.

$$\text{Average} = \frac{\Delta[A]}{\Delta t} = \frac{(0.10-0.050)}{(20-10)}\frac{0.05}{10}$$

$= 0.005$ or 5×10^{-3}

Sol 17.

Pyroelectricity (from the Greek pyr, fire, and electricity) is the ability of certain materials to generate a temporary voltage when they are heated or cooled. The change in temperature modifies the positions of the atoms slightly within the crystal structure, such that the polarization of the material changes. This polarization change gives rise to a voltage across the crystal.

Sol 18.

Edge length in KF = 537. Pm

In FCC structures the distance between two ions will be half of the edge length of the unit cell, i.e., 268.75 pm.

Sol 19.

Greater is the valency of the oppositely charged ions of the electrolyte being added, the faster is the coagulation.

Sol 20.

In colloidal and surface chemistry, the critical micelle concentration (CMC) is defined s the concentration of surfactants above which micelles formation takes place by association of surfactant molecules.

Sol 21.

Number of moles of $CuCl_2 = \frac{13.44}{134.4} = 0.1$;

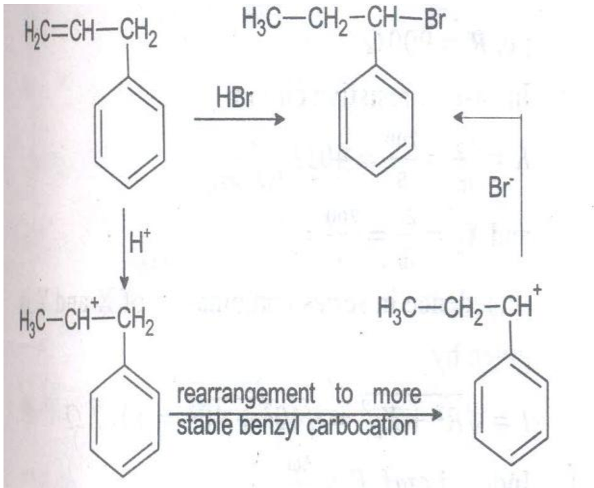
$CuCl_2 \rightarrow C^{2+} + 2Cl^-$;

Complete ionization of $CaCl_2$ gives 3 ions; value of i = 3

Δ Tb = i. Kb. m = 3x 0.52 x 0.1 = 0.156 = 0.16

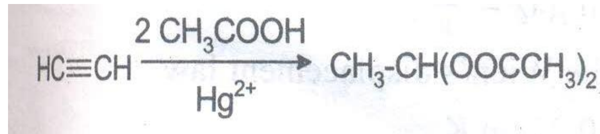
Sol 22.

the reaction involves the formation of a more stable benzyl carbocation by the rearrangement of initially formed secondary carbocation.



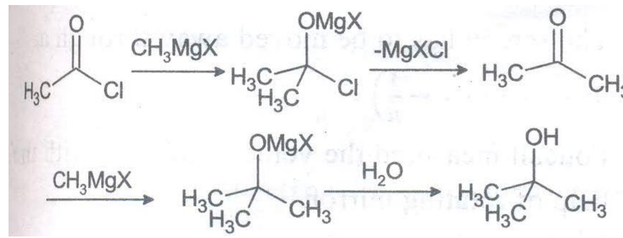
Sol 23.

The reaction involves addition of 2 moles of acetic acid to acetylene.



Sol 24.

The reaction between acetyl chloride and CH_3MgX results in the formation of a tertiary alcohol.



(a) $CH_3\,CH_2\,CH_2CI + Mg \rightarrow CH_3\,CH_2\,CH_2Mg\,CI$
(b) $CH_3\,CH_2\,CH_2Mg\,CI + H_2O \rightarrow$

$CH_3\,CH_2\,CH_3 + Mg\,(OH)\,CI$

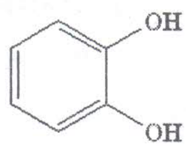
(c) $CH_3\,CH = CH_2 + B_2\,H_6 \rightarrow (CH_3\,CH_2\,CH_2)_3\,B$
$(CH_3\,CH_2\,CH_2)_3\,B + CH_3\,COOH \rightarrow$
$3CH_3CH_2CH_3$
(d) $CH_3\,CH\,(OH)\,CH_3 + 2\,[H] \rightarrow$
$CH_3\,CH_2\,CH_3 + H_2O$

Sol 25.

NBS is a specific reagent for allylicbromination

Sol 26.

Catechol is



Sol 27.

Positive iodoform test is given by ph $CHOHCH_3$

$PhCHOHCH_3 + I_2 \rightarrow PhCOCH_2 + HI$

$C_6H_5COCH_3 + 3I_2 + 3NaOH \rightarrow C_6H_5COONa + CHI_3 + 3NaI + H_2O$

Sol 28.

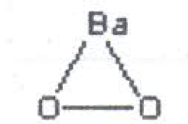
In solution, when stored at temperatures ranging from 30 to 80°C, aspartame is progressively degraded into diketopiperazineIt is therefore not usable in foods heated at higher temperature.

Sol 29.

When H_2S gas is passed in a metal sulphate solution in the presence of NH_4OH, 4th group cations are precipitated as corresponding sulphides. Zn gives white ppt in 4th group analysis.

Sol 30.

BaO_2 is a peroxide.



<u>**PHYSICS**</u>

Sol 1.

In a VernierCallipers

Length measured = Reading before zero + number of coinciding Vernierdivistion x Vernier Constant

= 9 x 10^{-3} + 0 x 1 x 10^{-4}

= 0.009 m = 9mm

Sol 2.

Maximum height of a projectile is given by

$$H_{max} = \frac{u^2}{2g} for\theta\ 90°$$

$\Rightarrow H_{max}$ is independent of mass

Sol 3.

Only the total momentum of an isolated system remains unchanged.

Both statements are not correct.

Sol 4.

Given $p^1 = \frac{3}{2}p \Rightarrow v' = \frac{3}{2}v$

As K.E. ∝ V^2

$\Rightarrow \frac{K.E.'}{K.E.} = \frac{9}{4}$

And increase in K.E. = $\frac{E.E.^1 - K.E.}{K.E.}$ X 100

$$= \frac{5}{4} X\ 100\ = 125\%$$

Sol 5.

Kinetic energy of rotation

$$K.E._{rot} = \frac{1}{2} I\omega^2$$

$\Rightarrow I = \frac{2\, K.E._{rot}}{\omega^2}$

$= \frac{2x\ 360}{30\ x\ 30} = 0.8\ kgm^2$

Sol 6.

The speed of a planet orbiting the sum always increases in going from aphelion to perihelion

Sol 7.

Rise in capillary tube $= \frac{h}{\sin\ 30°} = \frac{h}{1/2} = 2h$

Sol 8.

Here (m x 540 + m x 1 x 95) x 10^3

= 10 x 10^{-3} x 80 x 10^3 + 94 10^{-3} x 1 x 5 x 10^3

= 2g

Sol 9.

Using the relation

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$1930 = \sqrt{\frac{3\ X\ 8.314\ 10^{11} X\ 300}{m}}$

$\Rightarrow$ M = 2 $\Rightarrow$ H_2 gas

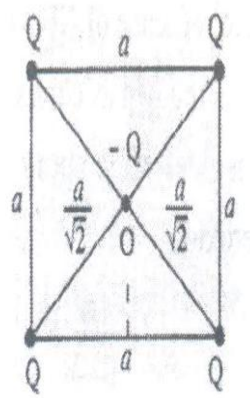
Sol 10.

$\frac{V_H}{V_0} = \sqrt{\frac{32}{2}} = 4$

Sol 11.

The source of sound can be identified by the overtones present in the sound.

Sol 12.



Potential at O is given by V = 4. $\frac{1}{4\pi\epsilon_0}\frac{Q}{r}$

= 4 x$\frac{1}{4\pi\epsilon_0}\frac{Q}{a/\sqrt{2}}$

= $\frac{Q\sqrt{2}}{\pi\epsilon_0 a}$

and work done = V x Q = $\frac{Q\sqrt{2}}{\pi\epsilon_0 a}Q = \frac{\sqrt{2}Q^2}{\pi\epsilon_0 a}$

Sol 13.

The resistance will be greater when the lamp is switched on

Sol 14.

Given $E_{mag} = MB$ And $E_{th} = \frac{3}{2}KT_1 \Rightarrow \frac{E_{th}}{E_{mag}} = \frac{\frac{3}{2}KT_1}{MB} = \frac{3KT_1}{2MB}$

Sol 15.

Using R $\frac{V}{I_g}$ - 100

100 + R = $\frac{V}{I_g}$

Also 1000 + R = $\frac{2v}{I_g}$ - 100 ⇒ 1100 + R $\frac{2v}{I_g}$

∴ $\frac{1100 + R}{100 + R} = \frac{2v}{I_g} x \frac{I_g}{V}$ = 2⇒ 1100 + R = 200 + 2R

i.e. R = 900 Ω

Sol 16.

In case of resistive circuit

R = $\frac{E_0}{I_0} = \frac{200}{5} = 40\Omega$

And $X_L = \frac{E_0}{I_0} = \frac{200}{5} = 40\Omega$

Impedence in series combination of X and Y is given by

$$I = \sqrt{R^2 + X_L^2} = \sqrt{40^2 + 40^2} = 40\sqrt{2}\Omega$$

Sol 17.

Induced $emf. E = \frac{\Delta\varphi}{\Delta t x R}$

current, $I = \frac{Q}{\Delta t} = \frac{E}{R} = \frac{\Delta\varphi}{\Delta t x R}$

comparing $Q = \frac{\Delta\varphi}{R}$

Sol 18.

Using the Wien's displacement law

$\lambda_m T = 0.29$ cm K

$$\lambda_m = \frac{0.29}{T} = \frac{0.29}{2.7} = 0.11\ cm$$

This wavelength belongs to microwave.

Sol 19.

The screen has to be moved away through a distance t $\left(1 - \frac{1}{▯}\right) = \frac{t}{▯}\ (▯ - 1)$

Sol 20.

Foucalt measured the velocity of light with the help of rotating mirror.

Sol 21.

Deviation $\delta = (\mu - 1)$ A

For glass ▯ = 1.5

$\Rightarrow \delta_1 = (1.5 - 1)$ A $= 0.5$A

When prism is dipped into water, the new refractive index

$\mu' = \frac{3/2}{4/3} = \frac{9}{8}$

$\therefore \delta_2 = \left(\frac{9}{8} - 1\right) A = \frac{1}{8} A$

Dividing $\frac{\delta_2}{\delta_1} = \frac{1}{8\ x\ 0.5} = \frac{1}{4}$

Sol 22.

Stopping potential does not depend upon the distance of source from photocell

$$Saturation current \ \propto \frac{1}{square of distance of source}$$

$\therefore I_1 = 18 \ \propto \frac{1}{(0.2)^2} and I_2 \ \propto \frac{1}{(0.6)^2} \Rightarrow \frac{I_2}{18} = \frac{(0.2)^2}{(0.6)^2} x\ 18 = 2mA$

Sol 23.

Using $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T} \Rightarrow \left(1 - \frac{3}{4}\right) = \left(\frac{1}{2}\right)^{3/4^T}$

$or \frac{1}{4} = \left(\frac{1}{2}\right)^{\frac{3}{4T}}$

Or $2 = \frac{3}{4T} \Rightarrow \mathrm{T} = \frac{3}{8}$ S

Sol 24.

Using $hv = Rchz^2 \left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = Rchz^2 [\frac{n^2(n-1)^2}{(n-1)^2 n^2}]$

$= Rchz^2 [\frac{2n-1}{n^2(n-1)^2}]$

or$\mathrm{hv} = \frac{2Rcz^2}{n^3}$

$\Rightarrow \mathrm{v} \propto \frac{1}{n^3}$

Sol 25.

Band gap $\mathrm{E_g} = \mathrm{hv} = \frac{hc}{\lambda}$

Given $\lambda = 2480$ nm $= 2480 \times 10^{-9}$m

$= 248 \times 10^{-8}$ m

$\Rightarrow \mathrm{E_g} = \frac{6.6\ x 10^{-34} x\ 34\ x 10^8}{248\ x 10^{-8}}$

$= 7.984 \times 10^{-20} \times 10^{-20}$ J

$= \frac{7.984\ \ x 10^{-20}}{1.6\ \ x 10^{-19}}$ eV $= 0.499$ eV

$= 0.5$ eV

Sol 26.

$f_c \propto \sqrt{N_{max}}$

Where

N_{max} = maximum electron density of ionosphere

Here $\frac{f_{c1}}{f_{c1}} = \left(\frac{N_{\max\,1}}{N_{\max\,2}}\right)^{1/2}$

$\Rightarrow \frac{N_{\max 1}}{N_{\max 2}} = \left(\frac{f_{c1}}{f_{C2}}\right)^2 = \left(\frac{10}{8}\right)^2 = \frac{25}{16}$

Sol 27.

Using the formula

$f' = f\left(\frac{v+v_0}{v}\right)$where v = speed of sound

$\Rightarrow 5.5 = 5\left(\frac{v+va}{v}\right)$ where v_A = speed of train A and $6.0 = 5\left(\frac{v+v_B}{v}\right)$ where V_B = speed of train B

Diving I $= \frac{V_B}{V_A} = 2$

Sol 28.

Using I = $neAv_d$

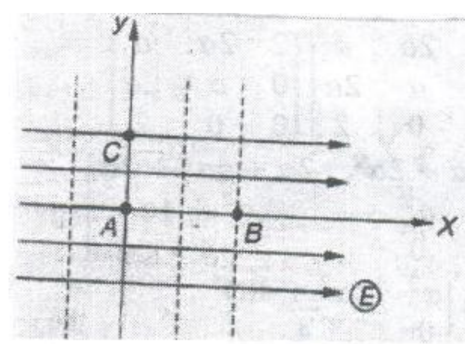
Drift speed $v_d = \frac{1}{neA}$

$\Rightarrow v_d \propto \frac{1}{8}$

⇒ Fornon uniform cross section or different values of A, drift speed will e different at different section. Only current or rate of flow of change will be same.

Sol 29.

Potential decreases in the direction of electric field. The dotted lines represent the equipotential lines



$\therefore V_A = V_C$ and $V_A > V_B$

Sol 30.

X – rays suffers diffraction when the width of slit is of the order of wavelength of x-rays. Here wavelength of x-rays (1 – 100 A°) is very much less than slit width (0.6 mm). Therefore no diffraction will occur.

MATHEMATICS

Sol 1.

$f(\theta) = \sin^4 \theta + \cos^4\theta$

$f(\theta) = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2 \theta cos^2\theta$

$= 1 - \frac{1}{2}(2\sin\theta cos\theta)^2$

$= 1 - \frac{1}{2}(sin2\,\theta)^2$

$= 1 - \frac{1}{2}\left(\frac{1-\cos\,4\theta}{2}\right)$

$= = \frac{3}{4} + \frac{1}{4}cos4\,\theta$

Period of Cos4θ is $\frac{2\pi}{4}$

$\therefore f(\theta)$ is periodic with period $\frac{\pi}{2}$

Sol 2.

xRy ⇒ yRx ∴ R is symmetric

Sol 3.

Let n and (n + 1) be the two consecutive roots of

$x^2 + bx + c = 0$

∴ n + (n + 1) = - b and n (n + 1) = c

∴ $b^2 - 4c = (-b)^2 - 4c$

$= (2n + 1)^2 - 4c\,(n + 1)$

$= 4n^2 + 4n + 1 - 4n^2 - 4n$

= 1

Sol 4.

Let z = $\frac{i}{1+i}$

$\bar{z} = \frac{-i}{1+i} = \frac{i}{1-1}$

$= \frac{i}{1-i} x \frac{i+1}{i+1} = \frac{i^2+i}{i^2+1} = \frac{-1+i}{-1-1} = \frac{1-i}{2}$

Sol 5.

$$A = \begin{bmatrix} 2 & 2\alpha & \alpha \\ 0 & \alpha & 2\alpha \\ 0 & 0 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 2\alpha & \alpha \\ 0 & \alpha & 2\alpha \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 2\alpha & \alpha \\ 0 & \alpha & 2\alpha \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4\alpha + 2\alpha^2 & 2\alpha + 4\,\alpha^2 + \; 2\alpha \\ 0 & \alpha^2 & 2\alpha^2 + \; 4\alpha \\ 0 & 0 & 4 \end{bmatrix}$$

$|A^2| = 4 \begin{vmatrix} \alpha^2 & 2\alpha^2 + \; 4\alpha \\ 0 & 0 \end{vmatrix}$

$= 16\ \alpha^2 = 16$ Given $|A^2| = 16$

$\Rightarrow \alpha^2 = 1$ $\Rightarrow |\alpha| = 1$

Sol 6.

Given $A^2 + A - I = 0$

$\Rightarrow A^{-1} A^2 + A^{-1} A - A^{-1} I = 0$

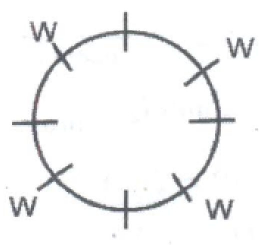
$\Rightarrow (A^{-1} A) A + I - A^{-1} I = 0$

$\Rightarrow IA + I - A^{-1} = 0$

$\Rightarrow A + I = A^{-1}$

Sol 7.

First fix the position of one women, the number of ways to sit 3 women is 3! and the number of ways to sit 3 men is ${}^4P_3$



$\therefore$ Total number of ways

$= 3! \;\; X\ {}^4P_3$

$= 3!\ X\ 4!$

Sol 8.

Given that

P (A) = 0.20, P(B) = 0.30, P (A ∩ B) = 0.10

P (A ∪ B) = P (A) + P (B) – P (A ∪ B) = 0.20 + 0.30 – 0.10

P (A ∪ B) = 0.40

P ($\bar{A}$ ∩ $\bar{B}$) = P ($\overline{A \cup B}$) = 1 – P (A ∪ B) = 1 – 0.40

= 0.60

Sol 9.

Let P (n) = 2^{3n} – 1

Step I : P (1) = $2^{3 \times 1}$ – 1 = 8 – 1 = 7 which is divisible by 7

So P (1) is true.

Step II : Let P (m) be true. The n

2^{3m} – 1 is divisible by 7 ⇒ 2^{3m-1} = 7 λ for some λϵN

Step III : P(m + 1) = $2^{3(m+1)}$ – 1

= 2^{3m} 2^3- 1

= (7 λ + 1) 2^3 – 1

= 56λ + 8 – 1

= 7 (8λ + 1), which is divisible by 7

∴ P (m + 1) is true Hence P (n) divisible by 7.

Sol 10.

$T_{r+1} = n_{c_r} a^r isthe\ (r+1)$th term is the expansion of (x + a)^n.

∴ In the expansion of $\left(x^2 + \frac{1}{x}\right)^8$

$T_4 = T_{3+1} = {}^8c_3 (x^2)^{8-3} \left(\frac{1}{x}\right)^5$

= ${}^8c_3\ x^{10} \frac{1}{x^5}$

= $\frac{8 \times 7 \times 6}{3 \times 2}$ x^5 = 56 x^5

Sol 11.

$T_{r+1} = {}^{10}c_r(x^3)^r \left(-\frac{1}{x^2}\right)^{10-r}$

$= {}^{10}c_r(-1)^{10-r} \frac{x^{3r}}{x^{20-2r}}$

$T_{r+1} = {}^{10}c_r (-1)^{10-r}\ x^{-20+5r}$

The term is independent of x if

$-20 + 5r = 0$

$5r = 20 \Rightarrow r = 4$

We get

$T_5 = {}^{10}c_4 (-1)^{10-4} = {}^{10}c_4 (-1)^6$

$= \frac{10 \ x\ 9\ x\ 8\ x\ 7}{4\ x\ 3\ x\ 2\ x\ 1} = 210$

Sol 12.

$$Theknowe \ = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ \ldots\ldots (i)$$

$$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} - \frac{1}{4!} \ + \ldots\ldots\ldots\ldots\ldots (ii)$$

Adding (i) and (ii), we get

$e + e - 1 = 2 + \frac{2}{2!} + \frac{2}{4!} + \ldots\ldots\ldots\ldots$

$\Rightarrow \frac{e^2+1}{e} - 2 = \frac{2}{2!} + \frac{2}{4!} + \ \ldots\ldots\ldots$

$$\Rightarrow \frac{e^2 + 1 - 2e}{e} = 2\left(\frac{1}{2!} + \frac{1}{4!} + \ldots.\right)$$

$$\Rightarrow \frac{(e-1)^2}{2e} = \frac{1}{2!} + \frac{1}{4!} + \ldots\ldots$$

Sol 13.

$$Givethat \frac{x^m}{y^n} = (x-y)^{(m-n)}$$

m log x – n log y = (m - n) log (x – y)

$$\frac{m}{n} - \frac{n}{y}\frac{dy}{dx} = \frac{(m-n)}{(x-y)}\left(1 - \frac{dy}{dx}\right)$$

$$\frac{dy}{dx}\left[\frac{m-n}{x-y}\right] = -\frac{m}{x} + \frac{m-n}{x-y}$$

$$\frac{dy}{dx}\left[\frac{my-ny-nx+ny}{y(x-y)}\right] = \frac{-mx+my+mx-nx}{x(x-y)}$$

$$\frac{dy}{dx}\frac{x}{y} = \frac{my-nx}{my-nx} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

Sol 14.

Using mean value through for $f(x) = x^2 + 3x$in interval [2, 4]

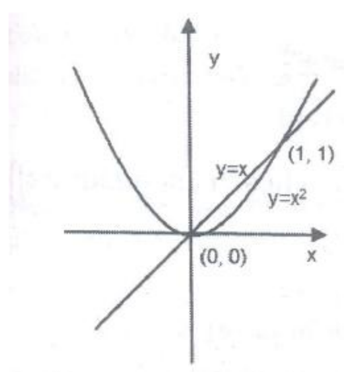
$$f'(c) = \frac{f(4) - f(2)}{4-2}$$

$2c + 3 = \frac{(16+12)-(4+6)}{2}$

$2c + 3 = 9 \Rightarrow 2c = 6$

$\Rightarrow c = 3$

Sol 15.



The equations of given curve and give as

$y = x^2$ and $y = x$

The point of intersection

Are (0, 0) and (1, 1)

$Area = \int_{x^2}^{x} dydx = \int_0^1 [y]_{x^2}^{x}\, dx$

$= \int_0^1 (x - x^2)dx$

$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ sq. units

Sol 16.

$$I = \int \frac{dx}{x\,(x^n - 1)} = \frac{\int x^{n-1}}{x^n\,(x^n - 1)}\,dx$$

$$put x^n - 1 = t \Rightarrow nx^{n-1}dx = dt$$

$$I = \frac{1}{n}\int \frac{dt}{(t+1)t} = \frac{1}{n}\int \left[\frac{1}{t} - \frac{1}{t+1}\right] dt$$

$$= \frac{1}{n}[logt - \log(t+1)] + c$$

$$= \frac{1}{n}\log\left[\frac{t}{t+1}\right] + c$$

$$= \frac{1}{n}\log\left[\frac{x^n - 1}{x^n}\right] + c$$

Sol 17.

$$I = \int_0^{\pi/2} \frac{sinx}{\sin x + \cos x}\,dx \;\ldots\ldots\ldots\; (i)$$

$$= \int_0^{\frac{\pi}{2}} \frac{sinx}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}$$

$$I = \int_0^{\pi/2} \frac{cosx}{cosx + sinx}\,dx \ldots\ldots\ldots\ldots (ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{sinx + \cos x}{sinx + \cos x}\,dx$$

$$2I = \int_0^{\pi/2} 1\,dx$$

$$2I = [x]_0^{\frac{\pi}{2}} 1 \Rightarrow 2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

Sol 18.

The differential equation is $\frac{d^2y}{dx^2} = e^{2x}$ (i) Integrating (i) both sides, we get

$$\frac{dy}{dx} = \frac{e^{2x}}{2} + c \text{ (ii)}$$

Integrating (ii) on both sides, we get

$$y = \frac{e^{2x}}{2} + cx + d$$

Sol 19.

Given equation is

$\left(1+\frac{dy}{dx}\right)^{\frac{4}{5}}=\frac{d^4y}{dx^4}$

$\left(1+\frac{dy}{dx}\right)^{4}=\left(\frac{d^4y}{dx^4}\right)^{5}$

Sol 20.

Given that $e=\frac{1}{3}, ae=3 \ \Rightarrow a=9$

Now $b^2=a^2(1-e^2)=81\left(1-\frac{1}{9}\right)= \ 72$

∴ Equation of ellipse is $\frac{x^2}{81}+\frac{y^2}{72}=1$

Sol 21.

Given equations of the circle is

$x^2+\ y^2+\ 4x-4y+2=0$ Centre of circle is (- 2.2)

Radius of circle = $\sqrt{6}$ Let equation of tanget be x + y = c

The perpendicular distance from (-2. 2) to the circle is equal to the radius of circle.

∴ $\left|\frac{2+2-c}{\sqrt{1+1}}\right|=\sqrt{6}\Rightarrow\frac{c}{\sqrt{2}}=\sqrt{6}\ \Rightarrow c=\sqrt{12}$ Hence equation of tangent is x + y = $\sqrt{12}$

Sol 22.

The given of the circle is

$x^2+y^2-4x-4y+4=0$

whose centre is C(2, 2) and radius = 2

$S_1=5^2+6^2-20-24+4$

$=25+36-20-24+4=21$

∴ P lies outside the circle

PC = $\sqrt{(5-2)^2+\ (6-2\,)^2}=\sqrt{9+16}\ =5$

The least distance between circle and the point P = 5 – 2 = 3

Sol 23.

A parallelpipied is formed by planes drawn through the points (1, 2, 3) and (5, 7, 9) parallel to the coordinate planes.

Let a, b, c be the length of edges, then

A = 5 – 1 = 4, be = 7 – 2 = 5, c = 9 – 3 = 6

So length of the diagonal of a parallelepiped

$= \sqrt{a^2 + b^2 + c^2}$

$= \sqrt{16 + 25 + 36} = \sqrt{77}$

Sol 24.

Give equation of the sphere is

$X^2 + y^2 + z^2 + 4x - 2y + 6z + 5 = 0$

and plane is $x + 2y + 3z - 4 = 0$

centre of the sphere is (-2, 1, -3) and

radius $= \sqrt{4 + 1 + 9 - 5} = 3$

Length of perpendicular from centre of sphere to the plane $= \frac{|-2+2-9-4\,|}{\sqrt{1+4+9}} = \frac{13}{\sqrt{14}}$

Sol 25.

Since vectors $\vec{a}, \vec{c} and \vec{b}$ from a right handed system

$\therefore \vec{c} = \vec{b} x \vec{a} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & -y & z \\ y & 0 & 0 \end{vmatrix}$

$= z\hat{\jmath} + y\hat{k}$

Sol 26.

By triangle law

$\hat{c} = \hat{a} + \hat{b}$

$\vec{c} x \vec{c} = (\vec{a} + \vec{b}) x \vec{c}$

$0 = \vec{a} x \vec{c} + \vec{b} x \vec{c}$

Sol 27.

A total of 8 is obtained in following cases

{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

∴ probability of getting a total of 8 $\frac{5}{36}$

Sol 28.

Probability of getting a score of 7 in a single throw

$= \frac{6}{36} = \frac{1}{6}$

Probability of getting score of 7 in throw of four times = ${}^4c_2\left(\frac{1}{6}\right)^2\left(1-\frac{1}{6}\right)^2 = 6\,x\,\frac{1}{36}\,x\,\frac{25}{36} = \frac{25}{216}$

Sol 29.

$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\text{x}\frac{1}{3}}\right)$

$= \tan^{-1}\left(\frac{\frac{1}{6}}{\frac{5}{6}}\right)$

$= \tan^{1}(1) = \frac{\pi}{4}$

Sol 30.

Given that α is root of

$25 \sin^2 \theta + 5 \sin \theta - 12 = 0$

∴ $25 \sin^2 \alpha + 5 \sin \alpha - 12 = 0$

⇒ $(5 \sin \alpha - 3)(5 \sin \alpha + 4) = 0$

⇒ $\sin \alpha = \frac{3}{5}, -\frac{4}{5}$

⇒ $\sin \alpha = \frac{3}{5}$ [because α lies in 1st quadrant]

Now $\cos \alpha = \pm\sqrt{1-\sin^2\alpha}$

$= \pm\sqrt{1-\frac{9}{25}} = \pm\frac{4}{5}$

⇒ $\cos \alpha = \frac{4}{5}$ [[because α lies in 1st quadrant]]