ANSWERS XIII

CHEMISTRY

1.a	2.a	3.c	4.a	5.c	6.b	7.c	8.a	9.b	10.a	11.c	12.b	13.d
14.c	15.b	16.c	17.c	18.a	19.c	20.a	21.b	22.d	23.b	24.c	25.b	26.b
27.d	28.d	29.b	30.b									

PHYSICS

1.d	2.b	3.b	4.c	5.d	6.b	7.c	8.b	9.b	10.a	11.d	12.a	13.d
14.a	15.b	16.c	17.c	18.a	19.a	20.a	21.c	22.c	23.b	24.a	25.c	26.c
27.d	28.c	29.d	30.d									

MATHEMATICS

1.a	2.c	3.c	4.c	5.b	6.a	7.c	8.d	9.a	10.b	11.a	12.b	13.c
14.d	15.a	16.b	17.a	18.b	19.a	20.d	21.a	22.c	23.c	24.a	25.c	26.c
27.b	28.b	29.d	30.a									

HINTS AND EXPLANATIONS XIII
CHEMISTRY

Sol.1

$NaHCO_3$ reacts with NaOH to form Na_2CO_3.

$NaHCO_3 + NaOH \rightleftharpoons Na_2CO_3 + H_2O$

Sol.2

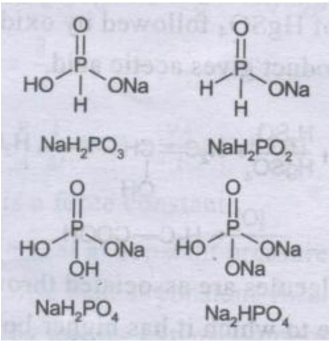
From the values given in the table, it can be seen that when the conc of Cl_2 is doubled as in entry 2, the rate of reaction becomes double, so the rate depends upon $[Cl_2]$. Similarily when conc of NO is doubled (compare entries 2 and 3), the rate of reaction becomes 4 times indicating that rate depends upon $[NO]^2$. Therefore rate expression for the reaction is rate=$k[NO]^2[Cl_2]$

Sol.3

The reaction that occurs at the anode when the electrolysis of $CuCl_2$ is done using platinum electrode is :
$2\ H_2O \rightarrow O_2 + 4\ H^+ + 4\ e$

Sol.4

NaH_2PO_2 does not contain any –O-H so it not an acid salt.



Sol.5

$KF + HF \rightarrow KHF_2; \quad K^+HF_2^-$

Sol.6

Solubility of iodine in water=0.35 g/L = 0.35/254=0.001378 mol

$K = \frac{[I_2]_{CCl4}}{[I_2]_{water}}$ or $600 = \frac{[I_2]_{CCl4}}{0.001378}$

Solubility of iodine in CCl_4 = 600 x 0.001378 = 0.8268 mol or o.8268 x 254 = 210 gL^{-1}.

Sol.7

Mn^{4+} = [Ar]$3d^3$;no. of unpaired electrons = 3

$\mu_g = \sqrt{n(n+2)} = \sqrt{3(3+2)} = \sqrt{15} = 3.89 \approx 4.0$

Sol.8

(a)

Sol.9

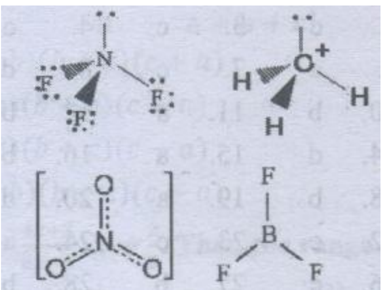
Van der walls's constant 's' has the dimension of atm $L^2 mol^{-2}$

Sol.10

Osmotic pressure (π) is given as: $\pi V = nRT$

Sol.11

Isostrucural pairs are



Sol.12

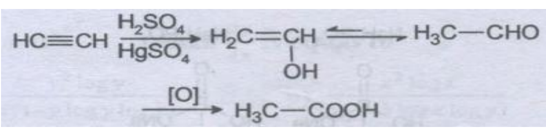
The value of equilibrium constant remains the same.

Sol.13

Ca^{2+} plays an important role in muscle contraction. The release of Ca^{2+} after receiving the nerve impulse, liberates the myosin's binding site on actin filaments. This enables a contraction, and a return of the calcium in the sarcoplasmic reticulum allows the muscle to relax.

Sol.14

Acetylene on heating with dilute H_2SO_4 followed by oxidation of the formed product gives acetic acid.

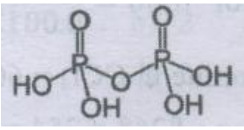


Sol.15

Water molecules are associated through hydrogen bonds due to which it has higher boiling point than H_2S (having no H-bonding).

Sol.16

Structure of pyrophosphoric acid is given below. Oxidation number of P in this compound is +5 and basicity is 4(due to 4 OH groups)



Sol.17

HS^- can accept as well as donate a proton

$HS^- + H^+ \rightleftharpoons H_2S, HS^- - H^+ \rightleftharpoons S^{2-}$

Sol.18

Since the rate of reaction doubles with doubling the initial concentration of salt, it is a first order reaction.

Sol.19

$\Delta T_f = i.K_f.m$; for NaCl, $i = 2$

$\Delta T_f = 2 \text{ x } 1.86 \text{ x } 0.1 = 0.372$ Therefore, freezing point of solution = -0.372°

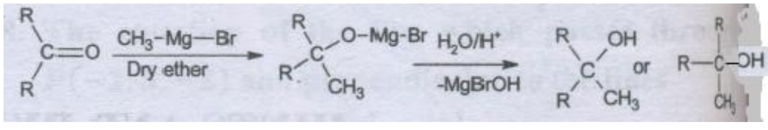
Sol.20

$\mu_{rms} = \sqrt{\frac{3RT}{M}} \mu_{so_2} = \sqrt{\frac{3RT}{64}} \mu_{o_2} = \sqrt{\frac{3Rx\,300}{32}}$ Root mean square speed of SO_2 will become the same as that of O_2 at 300K, when the temperature is 600K or 327°C.

$\mu_{so_2} = \sqrt{\frac{3R\text{ x }600}{64}} = \sqrt{\frac{3R\text{ x }300}{32}} \mu'_{o_2} = \sqrt{\frac{3Rx\,300}{32}}$

Sol.21

A ketone on reaction with CH_3MgI forms a tertiary alcohol; formaldehlyde gives a primary alcohol; acetaldehyde gives secondary alcohol and acetic acid does not form an alcohol.



Sol.22

If the electronegative difference between two atoms is more than 1.7, the bond is predominantly ionic.

Sol.23

Rolled gold is composed of a solid layer of gold bonded with heat and pressure to a base metal such as brass.

Sol.24

$CaCO_3 \rightleftharpoons CaO + CO_2$; One mol of $CaCO_3$ (100 g) gives 22.4 L of $CO_2(g)$ at STP, 50 g will give 11.2 L of $CO_2(g)$

Sol.25

$Zn + CuSO_4 \rightarrow ZnSO_4 + Cu + heat$

X 3.175 g 20J heat evolved for production of 1 mol (63.5 g) of Cu $= \frac{20 \; x \; 63{,}5}{3.175} = 400J$

Sol.26

In the reaction, $3Cl_2 + 60H \rightarrow 5Cl^- + ClO_3^- + 3H_2O$ chloride us oxidize as well as reduced.

$^{0}Cl_2 \rightarrow {}^{-1}Cl^-$ (reduction) $^{0}Cl_2 \rightarrow {}^{-5}ClO_3$- (oxidation)

Sol.27

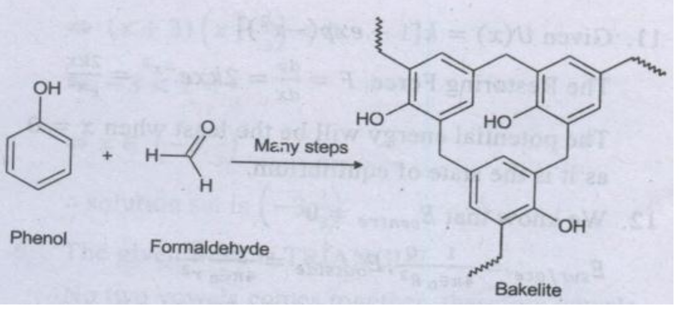
$BiCl_5$

Sol.28

Mg burns in air or oxygen with a dazzling light to from MgO and Mg_3N_2. $2Mg + O_2 \rightarrow 2MgO$ $3Mg + N_2 \rightarrow Mg_3N_2$

Sol.29

Bakelite is a polymer of phenol and formaldehyde.



Sol.30

CuS (black), Na_2S (white), PbS (black), ZnS (dirty white)

PHYSICS

Sol.1

Least count $\frac{1}{100} = 0.01mm$

Thickness of one paper = 0 +25 × 0.01 = 0.25mm

Thickness of pile = 0.25 × 50 = 12.5mm

Sol.2

As $x = (u\cos\theta) \Rightarrow \frac{x}{t} = ucos\theta$

Given $x = 6t \therefore \frac{6t}{t} = u\cos\theta \;\Rightarrow u\cos\theta = 6$ (a)

$y = (u\sin\theta)t - \frac{1}{2}gt^2$

i.e.$y = (u\sin\theta)t - \frac{10}{2}t^2$

or$8t - 5t^2 = (u\sin\theta)t - 5t^2$

$u\sin\theta = 8$ (b)

From (a) and (b)

$u^2sin^2\theta + u^2cos^2\theta = 64 + 36 = 100$ Or $u^2 = 100 \Rightarrow u = 10m/s$

Sol.3

Momentum, p $= mv = 3.513 \times 5 = 17.565$

According to rules of significant figures

P = 17.6kgm/s

Sol.4

We know that Potential energy = $mgh \Rightarrow h = \frac{P.E.}{mg} = \frac{100}{5\times9.8} = 2.04m$

Sol.5

For bodies (1) and (2)

$mv = (m_1 + m_2)v_1 = (,m + m)v_1$

or$v_1 = {}^{v}/_{2}$

for bodies (2) and (3)

$2m\frac{v}{2} = 3mv_2 \Rightarrow v_2 = \frac{v}{3}$

Final speed therefore will be $v_{n-1} = \frac{v}{n}$

Sol.6

Given Gravitational force between star of mass M and planet of mass m = $certripetal\ force$

i.e.$\frac{GMm}{R^{5/2}} = \frac{mv^2}{R} \Rightarrow v^2 = \frac{GM}{R^{3/2}}$

As $\omega = \frac{2\pi}{T} \Rightarrow \frac{V}{R} = \frac{2\pi}{T} orV = \frac{2\pi R}{T}$

$\Rightarrow T = \frac{2\pi R}{V} otT^2 = \frac{4\pi^2 R^2}{v^2} = \frac{4\pi^2 R^2 R^{3/2}}{GM}$

Or $T^2 \propto R^{7/2}$

Sol.7

For solid $P\frac{1}{2}V_p \times 1 \times g = V_p\delta_p g$

For solid $Q\frac{2}{3}V_Q \times 1 \times g = V_Q\delta_Q g$

$\therefore \frac{\delta_P}{\delta_Q} = \frac{3}{4}$

Sol.8

As $Y = \frac{F}{A}.\frac{l}{\Delta l} \Rightarrow F = \frac{VA}{l}\Delta l = k\Delta l$

Here k is a force constant.

Sol.9

Specific heat at constant pressure is greater than the specific heat at constant volume because heat is used by gas for expansion purposes at constant pressure.

Sol.10

As $d\theta = dU + dW we get dQ = dU + O$

As $dQ < O \Rightarrow dU < 0$

Now because final internal energy is less than initial internal energy the temperature will decrease

Sol.11

Given $U(x) = k[1 - \exp(-x^2)]$

The restoring Force, $F = \frac{dv}{dx} = 2kxe^{-x^2} = \frac{2kx}{e^{x^2}}$

The potential energy will be the least when $x = 0$ as it is the state of equilibrium.

Sol.12

We know that $E_{center} = 0$

$E_{surface} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}, E_{outside} = \frac{1}{4\pi\epsilon_0}\frac{Qr}{r^2}$

$E_{outside} \propto \frac{1}{r^2} and E_{inside} = \frac{1}{4\pi\epsilon_0}\frac{Qr}{r^3}$

Sol.13

Using $H = I^2Rt \Rightarrow I^2 = \frac{H}{Rt} = \frac{15000}{5\times30} = 100$ i.e.$I = 10A$

Sol.14

When a magnetic needle is kept in a non uniform magnetic field it experiences both force and torque.

Sol.15

As $B = \frac{\mu_0 I}{2a}, I = \frac{2aB}{\mu_0} = \frac{(5\times10^5)(2)(5\times10^{-2})}{4\pi\times10^{-7}} = 0.4A$

Sol.16

We know that $E = \frac{1}{2}LI^2$

$E = \frac{1}{2}\times 40 \times 10^{-3} \times 4 = 0.08J$

Sol.17

$E = \frac{LdI}{dt} = 0.1 \times \frac{20}{0.02} = 100V$

Sol.18

$U = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2}(8.85 \times 10^{-12} ?)(48)^2 = 10^{-8} Jm^{-3}$

Sol.19

The apparent wavelength will decrease.

Sol.20

Given depth = $\frac{d}{\mu} + \frac{d'}{\mu'} = \frac{d}{1.414} + \frac{d}{\mu} = \frac{d(\mu+1.414)}{1.414\mu}$

Sol.21

As angle of deviation = $A(\mu - 1)$

And μ_{violet} is more than μ_{red}

Thus blue colour will suffer more deviation than red colour.

Sol.22

$\frac{hc}{\lambda} = W_0 + eV$

$\therefore \frac{h}{e} = \frac{\lambda}{ce}(W_0 + eV) = \frac{6\times10^{-7}}{ce}(W_0 + 0.5)$

Also $\frac{h}{e} = \frac{4\times10^{-7}}{ce}(W_0 + 1.5)$

Equating we get $\frac{h}{e} = 4 \times 10^{-15}$

Sol.23

$n_a = \frac{238-206}{4} = 8$

And $n_\beta = 82 - [92 - (8 \times 2)] = 6$

Sol.24

$\frac{\lambda_{max}}{\lambda_{min}} = \frac{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{\infty}\right)^2}{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{3}\right)^2} = \frac{9}{5}$

Sol.25

Primitives are the intercepts which define the dimensional of a unit cell.

Sol.26

From conservation of energy

$V_2{}^2 = V_1{}^2 + 2gh$

From equation of continuity $A_1V_1 = A_2V_2 \Rightarrow V_2 = \left(\frac{A_1}{A_2}\right)V_1$

$\Rightarrow \frac{A_1{}^2}{A_1{}^2}V_1{}^2 = V_1{}^2 + 2gh \Rightarrow A_2{}^2 = \frac{A_1{}^2V_1{}^2}{V_1{}^2+2gh}$

$\therefore A_2 = \frac{A_1V_1}{\sqrt{V_1{}^2+2gh}} = \frac{(10^{-4})(1.0)}{\sqrt{(1)^2+2(10)(0.15)}} = 5 \times 10^{-5}m^2$

Sol.27

Bulk modulus $B = -\frac{dp}{(dv/v)} = \frac{(1.165-1.01)\times 10^5}{(10/100)} = 1.55 \times 10^5 Pa$

Sol.28

The average translational kinetic energy of an ideal molecule of a gas is given by $\frac{3}{2}KT$ which depends on temperature only. For same temperature, the translational kinetic energy of O_2 and N_2will be equal

Sol.29

The image will be real and between C and O

Sol.30

As $N = N_0e^{-\lambda t}$and $-\frac{dN}{dt} = \lambda N$

The number of nuclei decreases exponentially. So the decay process remains upto infinite time. A given nucleus may decay at any time after $t = 0$

MATHEMATICS

Sol.1

Let A be the set of people speaking Hindi and B donates the set of people speaking English. Now N(A ∪ B) = 50, $n(A) = 35, n(A \cap B) = 25$ We know $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow 50 = 35 + n(B) - 25 \Rightarrow n(B) = 40$ Therefore 40 people speak English.

Sol.2

Let $f(x) = \sqrt{-16x - x^2}$

$f(x) isrealif \sqrt{-16x - x^2} isreal$

$\therefore -16x - x^2 \geq 0 \Rightarrow x^2 + 16 \leq 0$

$\Rightarrow x^2 + 16x + 64 \leq 64 \Rightarrow (x+8)^2 \leq (8)^2$

$\Rightarrow |x+8|^2 \Rightarrow (8)^2 \Rightarrow |x+8| \leq 8$

$\Rightarrow -8 \leq x + 8 \leq 8 \Rightarrow -16 \leq x \leq 0$

$\Rightarrow x \in [-16,0]$

$\therefore D_f = [-16, 0]$

Sol.3

$\cos 20^0 \cos 40^0 \cos 60^0 \cos 80^0 = \cos 20^0 \cos 40^0 \left(\frac{1}{2}\right) \cos 80^0$

$= \frac{1}{2} \cos 20^0 \cos 40^0 \cos 80^0 = \frac{1}{4} \cos 20^0 \, [2 \cos 80^0 \cos 40^0]$

$= \frac{1}{4} \cos 20^0 \, [\cos 120^0 + \cos 40^0] \;= \frac{1}{4} \cos 20^0 \left[-\frac{1}{2} + \cos 40^0\right]$

$= -\frac{1}{8} \cos 20^0 + \frac{1}{8} [2 \cos 40^0 \cos 20^0] \;\; = -\frac{1}{8} \cos 20^0 + \frac{1}{8} [\cos 60^0 + \cos 20^0]$

$= -\frac{1}{8} \cos 20^0 + \frac{1}{8} \left[\frac{1}{2} + \cos 20^0\right] = \frac{1}{16}$

Sol.4

Let $-2 - 2i = r(\cos\theta + i \sin\theta)$

$r \cos\theta = -2, r \sin\theta = -2$

Squaring and adding

$r^2(cos^2\theta + sin^2\theta) = 4 + 4$

$\Rightarrow r^2 = 8 \Rightarrow r = 2\sqrt{2}$

$\cos\theta = -\frac{1}{\sqrt{2}}, \sin\theta = -\frac{1}{\sqrt{2}}$

Therefore θ lies in third quadrant

$\Rightarrow \theta = -\pi + \frac{\pi}{4} = -\frac{3\pi}{4}$ Principle value of $-2 - 2i$ is $-\frac{3\pi}{4}$

Sol.5

$3x^2 + 8x < 3$

$\Rightarrow 3x^2 + 8x - 3 < 0$

$\Rightarrow 3x^2 + 9x - x - 3 < 0$

$\Rightarrow 3x(x + 3) - 1(x + 3) < 0$

$\Rightarrow (x + 3)(3x - 1) < 0$

$\Rightarrow (x + 3)\left(x - \frac{1}{3}\right) < 0$

$\Rightarrow -3 < x < \frac{1}{3}$

$\Rightarrow x \in \left(-3, \frac{1}{3}\right)$

∴ Solution se is $\left(-3, \frac{1}{3}\right)$

Sol.6

The given word TRAINGLE

No two vowels comes together, therefore vowels can occupy □ places in □T□R□N□G□L□.

Three vowels I, A, E can be arranged in □marked placed in ${}^6P_3$ ways.

Also five consonants can be arranged among themselves in 5! ways

∴ The required number of words = ${}^6P_3 \times 5! = \frac{6!}{3!3!} \times 5!$

= 120 × 120 = 14400

Sol.7

Number of subjects = 5

Candidate will fail if he fail in one or two or three or four or five subjects

∴ Required number of ways

$= {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$

$= 5 + 10 + 10 + 5 + 1$

= 31

Sol.8

Consider $\left(\frac{x^3}{2}-\frac{2}{x^3}\right)^9$

The number of terms in the expansion = 10

∴ fourth term from the end means seventh term from the beginning

$T_7 = {}^9c_6\left(\frac{x^3}{2}\right)^3\left(-\frac{2}{x^3}\right)^6$

$=\frac{9.87}{3.2.1}\times\frac{x^9}{8}\times\frac{64}{x^{18}}$

$=\frac{672}{x^9}$

Sol.9

Here a = 1. Let r be the common ratio of G.P.

From given condition $T_3+T_5=90$

$\Rightarrow (1)r^2+(1)r^4=90$

$\Rightarrow r^4+r^2-90=0$

$\Rightarrow r^4+10r^2-9r^2-90=0$

$\Rightarrow r^2(r^2+10)-9(r^2+10)=0$

$\Rightarrow (r^2-9)(r^2+10)=0$

$\Rightarrow r^2=9,-10$

$\Rightarrow r=\pm 3$

Sol.10

Given vertices are $P(2,-1), Q(-2,3), R(4,5)$

Let S be the mid point of PQ

∴ S is (0,1)

Equation of medium RS is

$y-5=\frac{1-5}{0-4}(x-4)$

$\Rightarrow y-5=x-4\ \Rightarrow x-y+1=0$

Sol.11

Consider the lines

$x - y = 4$ and $2x + 3y = -7$

Solving these lines, we get

$\frac{x}{-7+12} = \frac{y}{-8-7} = \frac{1}{3+2}$

$\therefore x = 1 \; and y = -3$

$\therefore$ Centre of circle is C(1, −3)

Also circle passes through $P(2,4)$

$\therefore$ radius of circle = CP

$= \sqrt{(2-1)^2 + (4+3)^2} = \sqrt{50}$

$\therefore$ Equation of circle is

$(x-1)^2 + (y+3)^2 = (\sqrt{50})^2$

$\Rightarrow x^2 + y^2 - 2x + 6y - 40 = 0$

Sol.12

The foci are on y axis. Let $a < b$

$\therefore$ equation of ellipse is of the focus

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (i)

Length of major axis = 20

Foci are $(0, \pm 5)$

$\therefore be = 5$

Now $b^2e^2 = b^2 - a^2$

$\Rightarrow 25 = 100 - a^2 \Rightarrow a^2 = 75$

Put a^2, b^2 in (i), we get

$\frac{x^2}{75} + \frac{y^2}{100} = 1$, which is required equation of ellipse.

Sol.13

The equation of plane is

$2x + 3y + 5z = 1$ (i)

Let plane (i) divide the joining of $A(1,0,-3)$,

$b(1,5-7)$at P in the ratio $K:1$

$\therefore$ Pis$\left(\frac{K+1}{K+1}, \frac{-5K}{K+1}, \frac{7K-3}{K+1}\right)$

$\therefore$ P lies on plane (i)

$\therefore 2\left(\frac{K+1}{K+1}\right) + 3\left(\frac{-5K}{K+1}\right) + 5\left(\frac{7K-3}{K+1}\right) = 1$

$\Rightarrow 2K + 2 - 15K + 3K - 15 = K + 1$

$\Rightarrow 21K = 14 \Rightarrow K = \frac{2}{3}$

Required ratio = K:1

$= \frac{2}{3}$: 1 = 2: 3

Sol.14

$Lt_{x\to 0} \frac{3^x+3^{-x}-2}{x^2} = Lt_{x\to 0} \frac{3x+\frac{1}{3^x}-2}{x^2}$

$= Lt_{x\to 0} \frac{3^{2x}+1-2.3^x}{3^x.x^2} = Lt_{x\to 0}\left[\left(\frac{3x-1}{x}\right)^2 \times \frac{1}{3^x}\right]$

$= Lt_{x\to 0}\left(\frac{3^x-1}{x}\right)^2 \times Lt_{x\to 0} \frac{1}{3^x} = (\log 3)^2 \times \frac{1}{3^0} = (\log 3)^2 \times 1$

$= (\log 3)^2$

Sol.15

Given digits are 0, 1, 3, 5, 7 Every four digit number freater than 5000 must have either 5 or 7 in the thousand’s place.

Four digit numbers having 5 in thousands place = 5 × 5 × 5 = 125

Four digit numbers having 7 in thousand place = 5 × 5 × 5 = 125

Total numbers formed = 125 +125 = 250 +Number divisible by 5 must have 0 or 5 in the unit place. $\therefore$ Number divisible by 5= $5 \times 5 + 5 \times 5 + 5 \times 5 + 5 \times 5 = 100$ Required probability $= \frac{100}{250} = \frac{2}{5}$

Sol.16

Let $I = \int \frac{1}{\sqrt{5-4x-2x^2}} dx$

$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\frac{5}{2}-2x-x^2}} dx$

$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\frac{5}{2}-(x^2+2x)}} dx$

$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(\frac{5}{2}+1\right)-(x^2+2x+1)}} dx$

$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(\frac{\sqrt{7}}{2}\right)^2-(x+1)^2}} dx$

$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{x+1}{\sqrt{\frac{7}{2}}}\right) + c$

$= \frac{1}{\sqrt{2}} \sin^{-1}\left\{\frac{\sqrt{2}(x+1)}{\sqrt{7}}\right\} + c$

Sol.17

Let $\Delta = \begin{vmatrix} a+b+c & -c & -b \\ -a & a+b+c & -a \\ -b & -a & -a \end{vmatrix}$

By $C_1 \to C_1 + C_2, C_2 \to C_2 + C_3$

$= \begin{vmatrix} a+b & -(b+c) & -b \\ a+b & b+c & -a \\ -(a+b) & b+c & a+b+c \end{vmatrix}$

$= (a+b)(b+c) \begin{vmatrix} 1 & -1 & -b \\ 1 & 1 & -a \\ -1 & 1 & a+b+c \end{vmatrix}$

$= (a+b)(b+c) \begin{vmatrix} 1 & -1 & -b \\ 0 & 2 & -a+b \\ 0 & 0 & c+a \end{vmatrix}$

$= (a+b)(b+c)[(1)(2)(c+a)]$

$= 2(a+b)(b+c)(c+a)$

Sol.18

Let $y = f(x)$

$\therefore y = \frac{4x+3}{6x-4}$

$\Rightarrow 6xy - 4y = 4x + 3$

$\Rightarrow 6xy - 4y = 4y + 3$

$\Rightarrow x = \frac{4y+3}{6y-3}$

$Rf = set\ of\ all\ real\ number\ except \frac{2}{3} = R\left\{\frac{2}{3}\right\}$

Sol.19

Let $y = \sin^{-1}\frac{1}{2}$ where $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\Rightarrow \sin y = \frac{1}{2}$ where $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\therefore y = \frac{\pi}{6}$

$\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] = \tan^{-1}\left[2\cos\left(2 \times \frac{\pi}{6}\right)\right] = \tan^{-1}\left[2\cos\frac{\pi}{3}\right] = \tan^{-1}\left[2 \times \frac{1}{2}\right] = \tan^{-1}(1) = \frac{\pi}{4}$

Sol.20

Given that $f(x) = \begin{cases} \frac{1 \cos ax}{x \sin x} & x \neq 0 \\ \frac{1}{2} & x = 0 \end{cases}$

$Lt_{x\to 0} f(x) = Lt_{x\to 0} \frac{1-\cos ax}{x \sin x}$

$= Lt_{x\to 0} \frac{1 \cos ax}{x^2 \frac{\sin x}{x}}$

$= Lt_{x\to 0} \frac{1-\cos ax}{x^2} = Lt_{x\to 0} \frac{2sin^2\frac{ax}{2}}{x^2}$

$= \frac{a^2}{2} Lt_{x\to 0} \left(\frac{\sin\frac{ax}{2}}{\frac{ax}{2}}\right)^2 = \frac{a^2}{2}(1) = \frac{a^2}{2}$

Since $f(x)$ is continuous at $x = 0$

$\therefore Lt_{x\to 0} f(x) = f(0)$

$\Rightarrow \frac{a^2}{2} = \frac{1}{2} \Rightarrow a^2 = 1 \ \therefore a = \pm 1$

Sol.21

Given $y = a^{x^{a^{x^{\ldots\ldots\ldots\ldots\infty}}}}$

$\therefore y = a^{(x^y)}$

Log y = log $a^{(x^y)}$

$\Rightarrow \log y = x^y \log a$

$\Rightarrow \log(\log y) = \log(x^y \log a)$

$\Rightarrow \log(\log y) = \log x^y + \log(\log a)$

$\Rightarrow \log(\log y) = y \log x + \log(\log a)$

Differentiate w.r.t. x, we get

$\frac{1}{\log y} \frac{1}{y} \frac{dy}{dx} = y.\frac{1}{x} + \log x.\frac{dy}{dx}$

$\Rightarrow \left(\frac{1}{y \log y} - \log x\right) \frac{dy}{dx} = \frac{y}{x}$

$\Rightarrow \left(\frac{1 - y \log x \log y}{y \log y}\right) \frac{dy}{dx} = \frac{y}{z}$

$\Rightarrow \frac{dy}{dx} = \frac{y^2 \log y}{x(1 - y \log y \log x)}$

Sol.22

The equation of the curve is $y = 5x^2 - 2x^3$ (i)

$\frac{dy}{dx} = 10x - 6x^2$

$m_1 = 10x - 6x^2$,wherem_1 is slope of tangent. At (x, y) slope of line y = $10x - 6x^2$

Let m_2 be slope of line y= $4x + 5$ (ii)

$\therefore m_2$ = 4 Because tangent is parallel to line (ii)

$\therefore m_1 = m_2$

$\Rightarrow 10x - 6x^2 = 4$

$\Rightarrow 3x^2 - 5x + 2 = 0$

$\Rightarrow x = 1, \frac{2}{3}$

When $x = 1, from (i)\ we\ get\ y = 3 \therefore$ Required point is (1, 3)

Sol.23

Let $f(x) = \sin x + \cos x$

$f'(x) = \cos x - \sin x$

$f'(x) = 0 \Rightarrow \cos x - \sin x = 0$

$\Rightarrow \sin x = \cos x \Rightarrow \tan x = 1$

$\Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4} \because x \in [0, 2\pi]$

Now $f(0) = \sin 0 + \cos 0 = 0 + 1 = 1$

$f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$

$f\left(\frac{5\pi}{4}\right) = \sin\frac{5\pi}{4} + \cos\frac{5\pi}{4}$

$= \sin\left(\pi + \frac{\pi}{4}\right) + \cos\left(\pi + \frac{\pi}{4}\right)$

$= -\sin\frac{\pi}{4} - \cos\frac{\pi}{4}$

$= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2}$

$f(2\pi) = \sin 2\pi + \cos 2\pi = 0 + 1 = 1$

$\therefore$ maximum value of $f(x)$ is $\sqrt{2}$

Sol.24

Let $I = \int \frac{dx}{1+\sin x+\cos x}$

Put $tan = \frac{x}{2} = t \; or \frac{x}{2} = \tan^{-1} t$

Or $x = 2\tan^{-1} t$

$\therefore dx = \frac{2}{1+t^2} dt$

Sin $x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \frac{1-t^2}{1+t^2}$

cos$x = \frac{1-tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}} = \frac{1-t^2}{1+t^2}$

$I = \int \frac{\frac{2}{1+t^2}}{1+\frac{2}{1+t^2}+\frac{1-t^2}{1+t^2}} dt = \int \frac{2}{1+t^2+2t+1-t^2} dt = 2\int \frac{dt}{2t+2} = \int \frac{dt}{t+1} = \log|t+1| + c = \log\left|\tan\frac{x}{2} + 1\right| + c$

Sol.25

Let $I = \int_0^{\pi/4} \sec x \sqrt{\frac{1-\sin x}{1+\sin x}}\, dx$

$= \int_0^{\pi/4} \sec x \sqrt{\frac{1-\sin x}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}}\, dx$

$= \int_0^{\pi/4} \frac{1}{\cos x} \sqrt{\frac{(1-\sin x)^2}{1-sin^2 x}}\, dx$

$= \int_0^{\pi/4} \frac{1-\sin x}{cos^2 x}\, dx$

$= \int_0^{\pi/4} \left(\frac{1}{cos^2 x} - \frac{1}{\cos x} \times \frac{\sin x}{\cos x}\right)\, dx$

$= \int_0^{\pi/4} (sec^2 x - \sec x \tan x)\, dx$

$= [\tan x - \sec x]_0^{\pi/4}$

$= \left(\tan\frac{\pi}{4} - \sec\frac{\pi}{4}\right) - (\tan 0 - \sec 0)$

$= \left(1 - \sqrt{2}\right) - (0 - 1)$

$= 1 - \sqrt{2} + 1 = 2 - \sqrt{2}$

Sol.26

The given differential equation is

$\frac{dy}{dx} + 2y \tan x = \sin x \ (i)$

$P = 2 \tan x, Q = \sin x$

$I.F. = e^{\int \mathrm{P\,dx}} = (\cos x)^{-2} = \frac{1}{cos^2 x}$

Solution of (i) is

$y \frac{1}{cos^2 x} = \int \sin x \frac{1}{cos^2 x} dx + c$

$y\, sec^2 x = \int tan\ x \sec x\ dx + c$

$y\, sec^2 x = sec\ x + c$

$y = \cos x + c\, cos^2 x$

Sol.27

Here $\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}, \vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$

$\therefore\ |\vec{b}| = \sqrt{4 + 36 + 9} = 7$

$\vec{a}, \vec{b} = 2\lambda + 6 + 12 = 2\lambda + 18$

Scalar projection of $\vec{a}\ on\ \vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$

$\therefore 4 = \frac{2\lambda+18}{7}$

$\Rightarrow 28 = 2\lambda + 18 \Rightarrow 2\lambda = 10$

$= \lambda = 5$

Sol.28

The equation of given lines are

$\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ (i)

and$\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$ (ii)

Any line through $P(-1,3,-2)$ is

$\frac{x+1}{l} = \frac{y-3}{m} = \frac{z+2}{n}$ (iii)

Where l, m, n are direction ratios of the line.

Since line (iii) is perpendicular to (i) and (ii)

$\therefore l + 2m + 3n = 0$(iv)

And $-3l + 2m + 5n = 0$ (v)

Solving (iv) and (v), we get

$\frac{l}{10-6} = \frac{m}{-9-5} = \frac{n}{2+5}$

$\Rightarrow \frac{l}{4} = \frac{m}{-14} = \frac{n}{8}$

$\Rightarrow \frac{l}{2} = \frac{m}{-7} = \frac{n}{4}$

From (iii), the equation of line is $\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$

Sol.29

Let S denotes the success and F denotes the failure.

$\therefore P(S)=\frac{1}{6}, P(F)=\frac{5}{6}$

P (A wins in the first throw) $= P(S)=\frac{1}{6}$

P(A wind in the third throw) $= P(FFS)$

$= P(F)P(F)P(S)$

$= \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)=\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)$

P(A wins in the fifth throw) $= P(FFFFS)$

$= P(F)P(F)P(F)P(F)P(S)$

$= \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)$

$= \left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)$

And so on

Therefore P (A wins)

$= \frac{1}{6}+\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)+\cdots$

$= \frac{\frac{1}{6}}{1-\frac{25}{36}}=\frac{6}{11}$

Sol.30

Let p denotes the probability of not getting a head and q the probability of not getting a head. Then

$p=\frac{1}{2}, q=\frac{1}{2}$ Probability of getting at least six heads when 8 coins are thrown simultaneously

$= P(6)+P(7)+P(8)$

${}^8C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^2+{}^8C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^1+{}^8C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^0$

$= \left(\frac{1}{2}\right)^8\left[{}^8C_6+{}^8C_7+{}^8C_8\right]$

$= \frac{1}{256}\left[\frac{8\times7}{1\times2}+\frac{8}{1}+1\right]=\frac{1}{256}[28+8+1]=\frac{37}{256}$