IES MASTER

Institute for Engineers (IES/GATE/PSUs)

GATE 2019

CIVIL ENGINEERING

Detailed Solution

EXAM DATE: 10-02-2019

MORNING SESSION (09:30 AM-12:30 PM)

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SECTION: GENERAL APTITUDE

1. They have come a long way in_____trust among the users.

(a) created (b) creating

(c) creation (d) create

Ans. **(b)**

Sol. "creating"

2. The CEO's decision to quit was as shocking to the Board as it was to _____.

(a) myself (b) me

(c) I (d) my

Ans. **(b)**

Sol. "me"

3. The lecture was attended by quite ____students, so the hall was not very_____.

(a) few, quite (b) a few, quite

(c) few, quiet (d) a few, quiet

Ans. **(d)**

Sol. a few, quiet

4. If E = 10; J = 20; O = 30; and T = 40, what will be P + E + S + T?

(a) 82 (b) 164

(c) 120 (d) 51

Ans. **(c)**

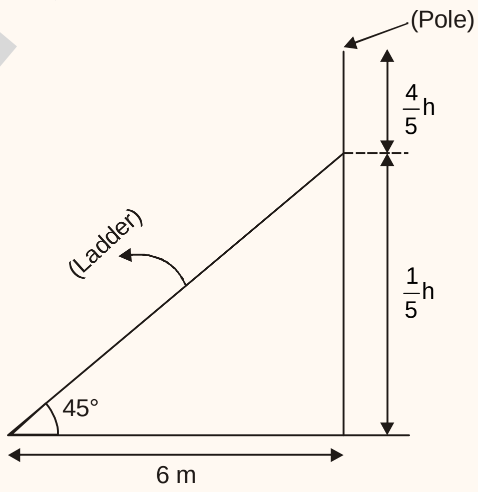
Sol.

$$P = 16 \times 2 = 32$$
$$E = 5 \times 2 = 10$$
$$S = 19 \times 2 = 38$$
$$T = 20 \times 2 = 40$$
$$P + E + S + T = 120$$

5. On a horizontal ground, the base of straight ladder is 6m away from the base of a vertical pole. The ladder makes an angle of 45° to the horizontal. If the ladder is resting at a point located at one-fifth of the height of the pole from the bottom, the height of the pole is _____meter.

Ans. **(30)**

Sol.



$$\tan 45° = \frac{\frac{h}{5}}{6}$$

$$\Rightarrow \quad 1 = \frac{h}{30}$$

$$\Rightarrow \quad \boxed{h = 30\text{ m}}$$

6. P, Q, R, S and T are related and belong to the same family. P is the brother of S. Q is the wife of P. R and T are the children of the siblings P and S respectively. Which one of the following statements is necessarily FALSE?

(a) S is the sister-in-law of Q

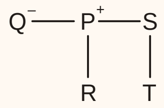
(b) S is the aunt of T

(c) S is the aunt of R

(d) S is the brother of P

Ans. **(b)**

Sol.



'T' is child of 'S'. So option (b) is right.

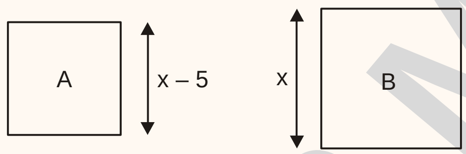
7. A square has sides 5cm smaller than the sides of a second square. The area of the larger square is four times the area of the smaller square. The side of the larger square is _____cm.

(a) 15.10 (b) 18.50

(c) 10.00 (d) 8.50

Ans. **(c)**

Sol.



Given,

$$(\text{Area})_B = 4 \times (\text{Area})_A$$

$\Rightarrow$ $x^2 = 4(x-5)^2$

$\Rightarrow$ $x^2 = 4[x^2 + 25 - 10x]$

$\Rightarrow$ $x^2 = 4x^2 + 100 - 40x$

$\Rightarrow$ $3x^2 - 40x + 100 = 0$

$\Rightarrow$ $3x^2 - 30x - 10x + 100 = 0$

$\Rightarrow$ $3x(x-10) - 10(x-10) = 0$

$\Rightarrow$ $x = 10$ or $x = \frac{10}{3}$

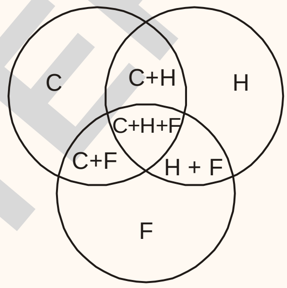
8. In a sports academy of 300 people, 105 play only cricket, 70 play only hockey, 50 play only football, 25 play both cricket and hockey, 15 play both hockey and football and 30 play both cricket and football. The rest of them play all three sports. What is percentage of people who play at least two sports?

(a) 23.30 (b) 50.00

(c) 28.00 (d) 25.00

Ans. **(d)**

Sol.



$$\text{Total} = 300$$
$$C = 105$$
$$H = 70$$
$$F = 50$$
$$C + H = 25$$
$$H + F = 15$$
$$C + F = 30$$
$$C + H + F = 300 - (295) = 5$$

% of people playing at least 25 sports

$$= \frac{25+15+30+5}{300} \times 100$$

$$= \frac{75}{300} \times 100 = 25\%$$

9. The increasing interest in tribal characters might be a mere coincidence, but the timing is of interest. None of this, though, is to say that the tribal hero has arrived in Hindi cinema, or that the new crop of characters represents the acceptance of the tribal character in the industry. The films and characters are too few to be described as a pattern.

Who does the word 'arrived' mean in the paragraph above?

(a) reached a terminus
(b) came to a conclusion
(c) attained a status
(d) went to a place

Ans. **(c)**

10. The new cotton technology, Bollgard-II, with herbicide tolerant traits has developed into a thriving business in India. However, the commercial use of this technology is not legal in India. Notwithstanding that, reports indicate that the herbicide tolerant Bt cotton had been purchased by farmers at an average of Rs 200 more than the control price of ordinary cotton, and planted in 15% of the cotton growing area in the 2017 Kharif season.

Which one of the following statements can be inferred from the given passage?

(a) Farmers want to access the new technology for experimental purposes
(b) Farmers want to access the new technology if India benefits from it
(c) Farmers want to access the new technology by paying high price
(d) Farmers want to access the new technology even if it is not legal

Ans. **(d)**

SECTION: CIVIL ENGINEERING

1. In a soil specimen, the total stress, effective stress, hydraulic gradient and critical hydraulic gradient are σ, σ', i and i_c, respectively. For initiation of quicksand condition, which one of the following statement is TRUE?

(a) $\sigma' \neq 0$ and $i = i_c$ (b) $\sigma = 0$ and $i = i_c$
(c) $\sigma' \neq 0$ and $i \neq i_c$ (d) $\sigma' = 0$ and $i = i_c$

Ans. **(d)**

Sol. During quick sand condition, the effective stress is reduced to zero [i.e, $\sigma' = 0$]

2. Assuming that there is no possibility of shear buckling in the web, the maximum reduction permitted by IS 800-2007 in the (low-shear) design bending strength of a semi-compact steel section due to high shear is

(a) 25%
(b) 50%
(c) governed by the area of the flange
(d) zero

Ans. **(d)**

3. The coefficient of average rolling friction of a road is f_r and its grade is +G%. If the grade of this road is doubled, what will be the percentage change in the braking distance (for the design vehicle to come to stop) measured along the horizontal (assume all other parameters are kept unchanged)?

(a) $\frac{0.02G}{f_r + 0.01G} \times 100$ (b) $\frac{f_r}{f_r + 0.02G} \times 100$
(c) $\frac{0.01G}{f_r + 0.02G} \times 100$ (d) $\frac{2f_r}{f_r + 0.01G} \times 100$

Ans. **(c)**

Sol. **Case I:** Braking distance = $\frac{V^2}{2g(f_r + 0.01G)}$

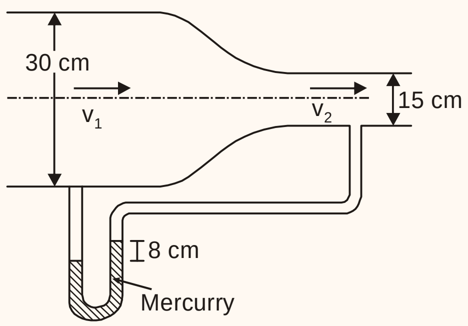
Case II: Braking distance = $\frac{V^2}{2g(f_r + 0.02G)}$

Percentage change

$$= \frac{\frac{V^2}{2g(f_r + 0.01G)} - \frac{V^2}{2g(f_r + 0.02G)}}{\frac{V^2}{2g(f_r + 0.01G)}} \times 100$$

$$= \frac{0.01G}{(f_r + 0.02G)} \times 100$$

4. A circular duct carrying water gradually contracts from a diameter of 30cm to 15cm. The figure (not drawn to scale) shows the arrangement of differential manometer attached to the duct.



When the water flows, the differential manometer shows a deflection of 8cm of mercury (Hg). The values of specific gravity of mercury and water are 13.6 and 1.0, respectively. Consider the acceleration due to gravity, g = 9.81 m/s^2. Assuming frictionless flow, the flow rate (in m^3/s, round off to 3 decimal places) through the duct is _____.

Ans. **(0.081)**

Sol.

$$h = x\left(\frac{G_m}{G_w}-1\right)$$

$$h = 8\left(\frac{13.6}{1}-1\right)$$

$$h = 100.8 \text{ cm} = 1.008 \text{ m}$$

Flow rate

$$Q = \frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2gh}$$

$$A_1 = 4A_2, \quad g = 9.81 \text{ m/sec}^2$$

$$A_2 = \frac{\pi}{4}\times 0.15^2$$

$$= 0.01767 \text{ m}^2$$

$$Q = \frac{4A_2^2}{\sqrt{16A_2^2-A_2^2}}\sqrt{2\times 9.81\times 1.008}$$

$$= \frac{4A_2}{\sqrt{15}}\sqrt{19.777}$$

$$Q = \frac{4\times(0.01767)}{\sqrt{15}}\sqrt{19.777}$$

$$Q = 0.081 \text{ m}^3/\text{sec}$$

5. A concentrated load of 500 kN is applied on an elastic half space. The ratio of the increase in vertical normal stress at depths of 2m and 4m along the point of the loading, as per Boussinesq's theory, would be _______.

Ans. **(4)**

Sol. Boussinesq's theory $= \frac{3Q}{2\pi z^2}\left(\frac{1}{1+\left(\frac{r}{z}\right)^2}\right)^{5/2}$

$$r = 0$$

$$Q_1 = \frac{3Q}{2\pi Z^2}$$

$$Q_1 \propto \frac{1}{z^2}$$

$$\frac{Q_1}{Q_2} = \left(\frac{4}{2}\right)^2 = 4$$

6. A retaining wall of height H with smooth vertical backface supports a backfill inclined at an angle β with the horizontal. The backfill consists of cohesionless soil having angle of internal friction ϕ. If the active lateral thrust acting on the wall is P_a which one of the following statements is TRUE?

(a) P_a acts at a height H/3 from the base of the wall and at an angle β with the horizontal

(b) P_a acts at a height H/2 from the base of the wall and at an angle ϕ with the horizontal

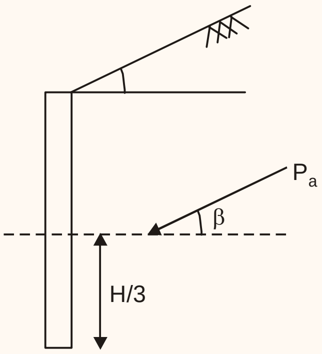
(c) P_a acts at a height H/2 from the base of the wall and at an angle β with the horizontal

(d) P_a acts at a height H/3 from the base of

the wall and at an angle ϕ with the horizontal

Ans. **(a)**

Sol.



active thurst act at a height H/3 from the base of the wall and at an angle equal to backfill inclination.

7. In a rectangular channel, the ratio of the velocity head to the flow depth for critical flow condition, is

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) $\frac{3}{2}$ (d) 2

Ans. **(a)**

Sol. Velocity head for a critical flow

$$= \frac{q^2}{2g\,y_c^2} = \frac{y_c^3}{2y_c} = \frac{1}{2}y_c$$

So, ratio of velocity head to critical flow depth

$$= \frac{1}{2}$$

8. The probability that the annual maximum flood discharge will exceed 25000 m^3/s, at least once in next 5 years is found to be 0.25. The return period of this flood event (in years, round off to 1 decimal place) is _____.

Ans. **(17.9)**

Sol. Probability exceed maximum discharge at least once in next 5 years is given by $= 1\ (1-p)^n$

$\Rightarrow$ $0.25 = 1-(1-P)^n$ [n = 5 year]

$\Rightarrow$ $P = 0.559$

$\Rightarrow$ $\frac{1}{T} = 0.559$

$\Rightarrow$ T = 17.9 year

9. The interior angles of four triangles are given below:

Triangle	Interior Angles
P	85°, 50°, 45°
Q	100°, 55°, 25°
R	100°, 45°, 35°
S	130°, 30°, 20

Which of the triangles are ill-conditioned and should be avoided in Triangulation surveys?

(a) Both P and R (b) Both Q and S

(c) Both P and S (d) Both Q and R

Ans. **(b)**

Sol.

- A triangle is said to be ill condition when angle is less than 30° and more than 120°.

  So, triangle S is ill conditioned.

- For well conditioned of triangulation two angle should not be almost equal.

  So, only triangle Q or triangle R is ill-conditioned

  So, ill-condition S and Q or S and R. option S and Q is given. So option (b) correct.

10. A catchment may be idealised as a rectangle. There are three rain gauges located inside the catchment at arbitrary locations. The average precipitation over the catchment is estimated by two methods: (i) Arithmetic mean (P_A) and (ii) Thiessen polygon (P_T). Which of the following statements is correct?

(a) P_A is always smaller than P_T

(b) There is no definite relationship between P_A and P_T

(c) P_A is always equal to P_T

(d) P_A is always greater than P_T

Ans. (b)

Sol.

- There is no definite relationship between arithmetic mean and Thiessen polygon method.
- Only it can be says that in Thiessen polygon method average value is more uniformly distributed as compared to arithmetic mean.

11. An isolated concrete pavement slab of length L is resting on a frictionless base. The temperature of the top and bottom fibre of the slab are T_t and T_b, respecitvely. Given: the coefficient of thermal expansion = α and the elastic modulus = E. Assuming $T_t > T_b$ and the unit weight of concrete as zero, the maximum thermal stress is calculated as

(a) zero

(b) $E\alpha(T_t - T_b)$

(c) $L\alpha(T_t - T_b)$

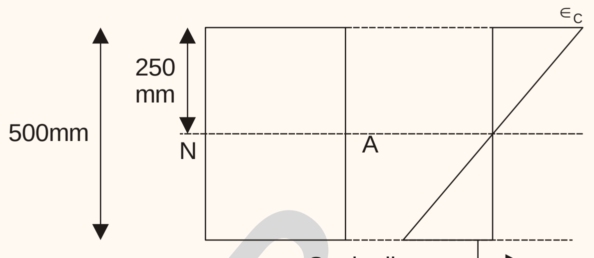
(d) $\dfrac{E\alpha(T_t - T_b)}{2}$

Ans. (a)

12. For a given loading on a rectangular plain concrete beam with an overall depth of 500 mm, the compressive strain and tensile strain developed at the extreme fibers are of the same magnitude of 2.5×10^{-4}. The curvature in the beam cross-section (in m^{-1}, round off to 3 decimal places), is _______.

Ans. (0.001 m^{-1})

Sol.



Strain diagram.

Given, $\in = \in_t = \in_c = 2.5 \times 10^{-4}$

$y = 250 \text{ mm} = 0.25 \text{ m}$

As per flexure formula:

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\Rightarrow \frac{\sigma}{E \times y} = \frac{1}{R} = \text{Curvature of beam cross-section}$$

$$\Rightarrow \left(\frac{1}{R}\right) = \frac{\in}{y} = \left(\frac{2.5 \times 10^{-4}}{0.25}\right) m^{-1}$$

$$= 0.001 \text{ m}^{-1}$$

13. For a small value of h, the Taylor series expansion of f(x + h) is

(a) $f(x) - hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{3}f'''(x) + ...\infty$

(b) $f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + ...\infty$

(c) $f(x) - hf'(x) + \frac{h^2}{2!}f''(x) - \frac{h^3}{3!}f'''(x) + ...\infty$

(d) $f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{3}f'''(x) + ...\infty$

Ans. (b)

Sol. For the small value of h, the Taylor's series expansion of

$$f(x + h) = \frac{f(x)}{0!} + \frac{hf'(x)}{1!} + \frac{h^2}{2!}f''(x)$$

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ESE-2019 Conventional Test Schedule, Civil Engineering

Date		Topic
17th Mar 2019	**N.T. :**	M-1, M-3, M-4, SM-1, SM-3, SM-8
	R.T. :	
24th Mar 2019	**N.T. :**	SA-1, SA-2, SA-5, HY-1, HY-4, HY-5, M-5
	R.T. :	SM-1, M-1
31st Mar 2019	**N.T. :**	DSS-4, DSS-5, FM-1, FM-4, FM-6
	R.T. :	M-3, SA-1, SA-2
07th Apr 2019	**N.T. :**	SA-6, SA-4, SA-3, EE-6, EE-5, EE-4
	R.T. :	FM-4, FM-6, M-1, M-4, M-3, HY-1
14th Apr 2019	**N.T. :**	FM-7, RCC-1, RCC-2, RCC-3, HY-2
	R.T. :	SA-1, SA-2, SM-3, FM-6, EE-6
21st Apr 2019	**N.T. :**	SM-4, DSS-1, DSS-2, DSS-3, RCC-4, RCC-5, RCC-6
	R.T. :	SM-1, SA-3, EE-5
28th Apr 2019	**N.T. :**	SU-1, SU-2, SU-3, SM-2, SM-5, SM-6, SM-7, HY-3, SU-5
	R.T. :	FM-7, RCC-1, RCC-2, RCC-3, HY-1, EE-6
05th May 2019	**N.T. :**	TF-1, TF-2, TF-3, TF-4, FM-5, M-2
	R.T. :	RCC-5, DSS-1, DSS-2, SM-4, M-1, M-3, M-4, FM-4, SA-1
12th May 2019	**N.T. :**	IR-1, IR-2, IR-3, IR-4, EE-7
	R.T. :	SM-5, SM-6, FM-1, EE-5, DSS-3, DSS-4, HY-3, HY-4, HY-5, SU-1, SU-2
19th May 2019	**N.T. :**	CPM-1, CPM-2, EE-1, EE-2, EE-3, SU-4 (Railway & Airport)
	R.T. :	SM-4, FM-5, TF-1, TF-2, FM-7, SA-3, SU-3, SU-5, RCC-5
26th May 2019	**N.T. :**	FM-2, FM-3, FM-8, Building Material, Ports & Harbors/Tunneling
	R.T.	IR-1, IR-2, HY-2, DSS-4, DSS-2, SA-1, SA-2, SA-3, RCC-6, EE-2, FM-6
02nd Jun 2019		Full Length-1 (Test Paper-1 + Test Paper-2)
09th Jun 2019		Full Length-2 (Test Paper-1 + Test Paper-2)
16th Jun 2019		Full Length-3 (Test Paper-1 + Test Paper-2)

Test Type	Timing	Day
Conventional Test	10:00 A.M. to 1:00 P.M.	Sunday
Conventional Full Length Test Paper-1	10:00 A.M. to 1:00 P.M.	Sunday
Conventional Full Length Test Paper-2	02:00 P.M. to 5:00 P.M.	Sunday

Note : The timing of the test may change on certain dates. Prior information will be given in this regard.

***N.T. :** New Topic. ***R.T. :** Revision Topic

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Subject Code Details

Structural Analysis (SA)	SA-1	SA-2	SA-3	SA-4	SA-5	SA-6
	Slope Deflection Method	Moment Distribution Method	Truss, Cables, Arches	Force Method, • Consistent Deformation Method • Method of Least work • Castigliano's Method	Determinacy/ indeterminacy/ stability	Stiffness Matrix Method, Influence Line Diagram/Moving Load, Free and Forced Vibrations, Concepts and use of computer aided design

SOM (M)	M-1	M-2	M-3	M-4	M-5
	Concept of Stress and Strain	Shear Force & Bending Moment, Deflection of Beams	Transformation of Stress & Strains, Theory of Failure, Combined Bending & Torsion/ Combined bending & Transverse shear stress/ combined bending & Axial stress, Torsion	Bending Stress, Shear Stress	Columns, Springs, Thick & Thin Shells, Moment of inertia

RCC & PSC (RCC)	RCC-1	RCC-2	RCC-3	RCC-4	RCC-5		RCC-6
	Working stress Method of RCC Design	Limit State Method	Earthquake resistant structures, Beams (LS, WS), Lintels	Slab-One way, (LS, WS) Staircase	Column, (LS, WS) Water Tanks	Fooling (LS, WS), Retaining Wall	Cement & Concrete, Masonry Structures, PSC-Pre stressed Concrete

Design of Steel Structure (DSS)	DSS-1	DSS-2	DSS-3	DSS-4	DSS-5	DSS-6
	Compression member	Plastic Analysis	Beams	Connections (Direct, Eccentric)	Tension Member	Plate girders, Industrial building

Pert & CPM (CPM)	CPM-1	CPM-2
	Network analysis, Pert, CPM, Crashing, Resource allogaction, Levelling,Smoothing, Rate Analysis	Construction equipments, Engineering Economy,Tendering Process and Contract, Management Estimation and costing, Quality control, Productivity, Opration cost, Land Aquisition

Building Material (BM)	BM-1	BM-2
	Cement, Concrete, Stone, Lime, Glass, Steel	Brick Mortar Timber, Plastics, FRP, Ceramics, Aluminium

Environmental (EE)	EE-1	EE-2	EE-3	EE-4	EE-5	EE-6	EE-7
	Characteristics of water, Treatment of water	Distribution of water	Characteristics of Sewage	Disposal of Sewage	Sewer design	Treatment of Sewage	Air Pollution, Noise Pollution, Solid Waste Management, Miscellaneous topics

Fluid Mechanics (FM)	FM-1	FM-2	FM-3	FM-4	FM-5	FM-6	FM-7	FM-8
	Fluid properties, Hydrostatic Pressure, Liquid in relative equilibrium, Buoyancy & Flotation	Fluid Kinematics	Fluid Dynamics, Weirs & Notches	Laminar flow, Turbulent flow, Boundary layer theory, Drag & lift	Flow through Pipes	Open channel flow	Hydraulic Machines, Power House	Modal Analysis & Dimensional Analysis

Soil Mechanics (SM)	SM-1	SM-2	SM-3	SM-4	SM-5	SM-6	SM-7	SM-8
	Classification of Soil, Soil water relationships, index properties of Soil, Compaction of Soil	Effective stress, Seepage, Permeability	Consolidation	Shear Stress/ Vertical Stress	Earth Pressure, Stability of Slopes	Bearing capacity-Shallow Foundation	Deep foundation - Piles	Exploration of Soil, Expansive Soil, Geosynthetics, Ground Modification Techniques

Transportation (TF)	TF-1	TF-2	TF-3	TF-4
	Geometric Design	Pavement Design	Materials, Construction, Maintenance, Hill roads etc.	Traffic Engineering

Surveying (SU)	SU-1	SU-2	SU-3	SU-4	SU-5
	Scale/ Accuracy, Measurements of horizontal distances, Theory of Errors	Angular Measurements, Theodolite	Levelling, Contouring, Curve setting, Measurment of Area & Volume	Triangulation & Traversing Plane tabling, Geology	Photogrammetry, Field Astronomy, GPS, Remote Sensing

Irrigation (IR)	IR-1	IR-2	IR-3	IR-4
	Soil water relationships, irrigation requirements of crops (Duty, Delta)	Design of Canals (Lacey & Kennedy)	Gravity dams	Cross drainage works, Weirs & Barrages, Seepage theory, Canal Falls/ Canal Regulators, Energy dissipators, River training works

Hydrology (HY)	HY-1	HY-2	HY-3	HY-4	HY-5
	Hydrographs	Flood Routing	Ground Water	Evapo-transpiration, Run off	Abstraction from Precipitation, Hydrological Cycle, Precipitation, Stream flow measurement

Railways / Airports / Ports & Harbours / Tunneling

$$+\frac{h^3}{3!}f'''(x)+\cdots\infty$$

$$f(x + h) = f(x)+hf'(x)+\frac{h^2f''(x)}{2!}$$
$$+\frac{h^3}{3!}f'''(x)+\cdots\infty$$

So option (b) is correct

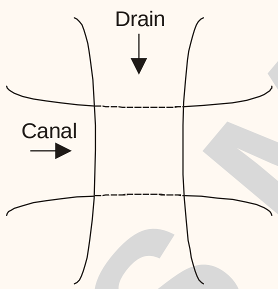
14. If the path of an irrigation canal is below the level of a natural stream, the type of cross-drainage structure provided is

(a) Aqueduct (b) Super passage

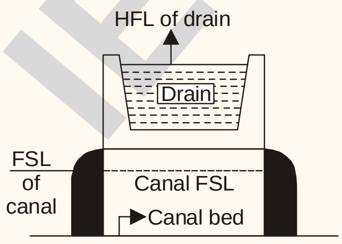
(c) Sluice gate (d) Level crossing

Ans. (b)

Sol. If the path of an irrigation canal is below the bed level of a natural stream, the type of cross-drainage work provided is super passage.

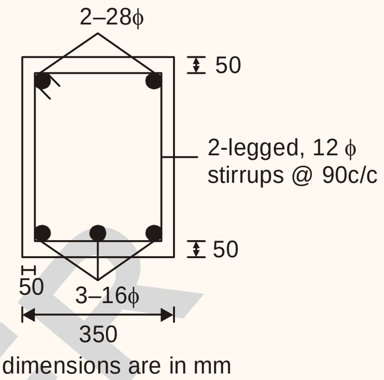


Drain taken ove the canel in a super passage(line plane of crossing)



Typical cross-section of a super passage.

15. In the reinforced beam section shown in the figure (not drawn to scale), the nominal cover provided at the bottom of the beam as per IS 456–2000, is



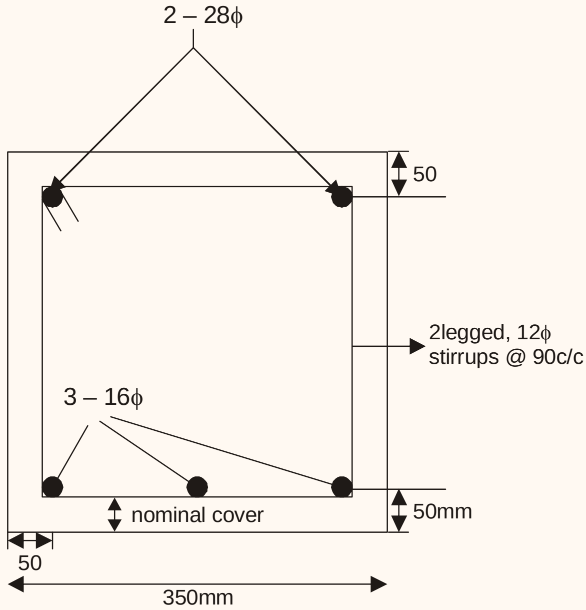
All dimensions are in mm

(a) 36 mm (b) 50 mm

(c) 30 mm (d) 42 mm

Ans. (c)

Sol.



$$\therefore \text{ Nominal cover } = \left(50-\frac{16}{2}-12\right)\text{mm} = 30 \text{ mm}$$

16. Consider a two-dimensional flow through isotropic soil along x-direction and z-direction. If h is the hydraulic head, the Laplace's equation of continuity is expressed as

(a) $\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial x\,\partial z}+\frac{\partial^2 h}{\partial z^2}=0$

(b) $\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial z^2}=0$

(c) $\frac{\partial h}{\partial x}+\frac{\partial h}{\partial x}\frac{\partial h}{\partial z}+\frac{\partial h}{\partial z}=0$

(d) $\frac{\partial h}{\partial x}+\frac{\partial h}{\partial z}=0$

Ans. **(b)**

Sol. $\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial z^2}=0$

For homogeneous isotropic soils, the laplace's equation of continuity is expressed as:

$$\boxed{\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial z^2}=0}$$

17. A soil has specific gravity of its solids equal to 2.65. The mass density of water is 1000 kg/m^3. Considering zero air voids and 10% mositure content of the soil sample, the dry density (in kg/m^3, round off to 1 decimal place) would be _______.

Ans. **(2094.86 kg/m^3)**

Sol.
$$G_s = 2.65$$
$$\rho_w = 1000 \text{ kg/m}^3$$
$$\eta_a = 0$$
$$w = 10\% = 0.10$$
$$\gamma_d = \left[\frac{(1-\eta_a)\,G_s\rho_w}{1+wG_s}\right]$$
$$= \frac{[(1-0)\times 2.65\times 1000]}{1+0.1\times 2.65}$$
$$= 2094.862 \text{ kg/m}^3$$

18. Which one of the following is correct?

(a) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right)=2$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)=1$

(b) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right)=\infty$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)=1$

(c) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right)=1$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)=1$

(d) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right)=2$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)=\infty$

Ans. **(a)**

Sol. $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right)$ $\left(\frac{0}{0}\text{ form}\right)$

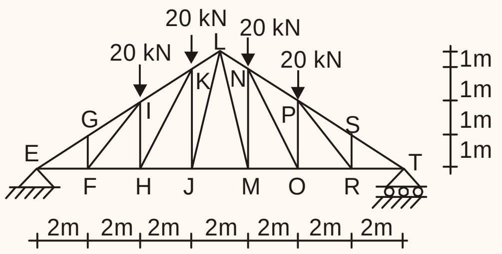
$$= \lim_{x\to 0}\frac{4\cos 4x}{2\sin 2x}=2$$

$$\lim_{x\to 0}\frac{\tan x}{x}\quad\left(\frac{0}{0}\text{ form}\right)$$

$$\lim_{x\to 0}\frac{\sec^2 x}{1} = 1$$

So, option (a) is correct.

19. A plane truss is shown in the figure (not drawn to scale).



Which one of the options contains ONLY zero force members in the truss?

(a) FI, HI, PR, RS
(b) FI, FG, RS, PR
(c) FG, FI, HI, RS
(d) FG, FH, HI, RS

Ans. **(b)**

Sol. Only 4 member having zero force member GF, FI and SR, PR

20. A simple mass-spring oscillatory system consists of a mass *m*, suspended from a spring of stiffness *k*. Considering *z* as the displacement of the system at any time *t*, the equation of motion for the free vibration of the system is $m\ddot{z} + kz = 0$. The natural frequency of the system is

(a) $\sqrt{\frac{k}{m}}$ (b) $\sqrt{\frac{m}{k}}$

(c) $\frac{k}{m}$ (d) $\frac{m}{k}$

Ans. (a)

Sol. For simple harmonic motion

$$m\ddot{z} + kz = 0$$

$$\ddot{z} + \frac{k}{m}z = 0$$

Standard equation is $\frac{d^2x}{dt^2} + w^2x = 0$

$$\Rightarrow \quad w^2 = \frac{k}{m}$$

$$\Rightarrow \quad w = \sqrt{\frac{k}{m}}$$

21. A completely mixed dilute suspension of sand particles having diameters 0.25, 0.35, 0.40, 0.45 and 0.50mm are filled in a transparent glass column of diameter 10 cm and height 2.50 m. The suspension is allowed to settle without any disturbance. It is observed that all particles of diameter 0.35 mm settle to the bottom of the column in 30 s. For the same period of 30s, the percentage removal (round off to integer value) of particles of diameters 0.45 and 0.50 mm from the suspension is ____.

Ans. (100%)

Sol. As we know that settling velocity for discrete particles is given by stokes law as-

$$V_T = \frac{(G-1)\gamma d^2}{18\mu}$$

$$\Rightarrow \quad V_T \propto d^2$$

For 30 second duration if 0.35 mm particle size settles completely then % removal of particle size 0.45 mm and 0.50 mm will be 100% respectively for each. As settling velocity of particle size 0.45 mm and 0.50 mm will be greater than settling velocity of size 0.35 mm ($V_T \propto d^2$).

22. The maximum number of vehicles observed in any five minute period during the peak hour is 160. If the total flow in the peak hour is 1000 vehicles, the five minute peak hour factor (round off to 2 decimal places) is ____.

Ans. (0.52)

Sol. Five minute peak hour factor

$$= \frac{\text{Average flow during 1 hour}}{12 \times \text{peak flow during 5 minute}}$$

$$\text{PHF} = \frac{Vav^{60}}{12 \times V_{av}^{5}} = \frac{1000}{12 \times 160} = 0.52$$

23. Which one of the following is secondary pollutant?

(a) Carbon Monoxide

(b) Hydrocarbon

(c) Volatile Organic Carbon (VOC)

(d) Ozone

Ans. (d)

Sol. Ozone is a secondary pollutant

24. An element is subjected to biaxial normal tensile strains of 0.0030 and 0.0020. The normal strain in the plane of maximum shear strain is

(a) 0.0050 (b) Zero

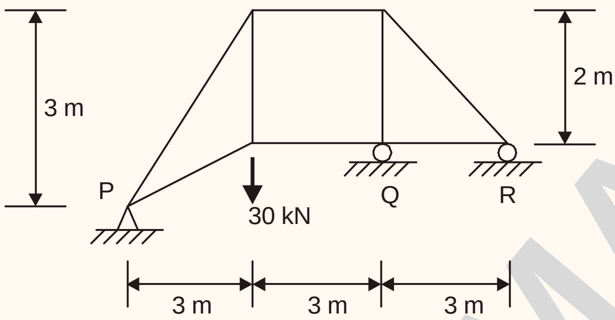
(c) 0.0025 (d) 0.0010

Ans. **(c)**

Sol. $(\epsilon)_{\text{at max shear strain}} = \dfrac{\epsilon_1 + \epsilon_2}{2}$

$$= \frac{0.0030 + 0.002}{2} = 0.0025$$

25. Consider the pin-jointed plane truss shown in the figure (not drawn to scale). Let R_P, R_Q, and R_R denote the vertical reactions (upward positive) applied by the supports at P, Q, and R, respectively, on the truss. The correct combination of (R_P, R_Q, R_R) is represented by

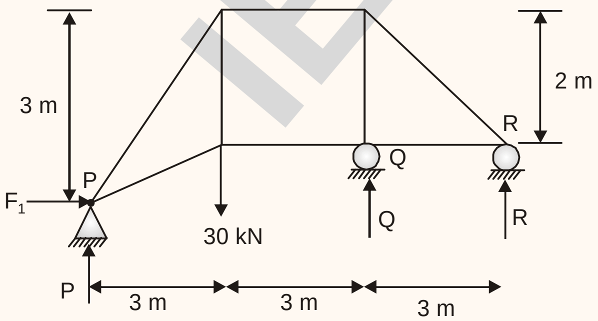


(a) (20, 0, 10) kN (b) (10, 30, – 10) kN

(c) (30, –30, 30) kN (d) (0, 60, – 30) kN

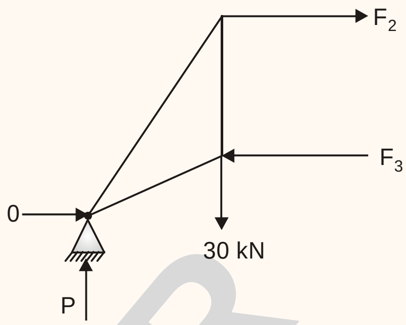
Ans. **(c)**

Sol.



$\Sigma F_H = 0$

$F_1 = 0$ .. (i)



$\Sigma F_V = 0$

$\boxed{P = 30}$ ... (ii)

$\Sigma F_H = 0$

$F_2 = F_3$ ... (iii)

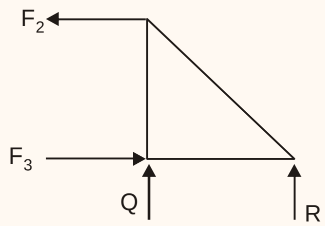
$F_2 \times 3 + 30 \times 3 - F_3 \times 1 = 0$

$F_2 \times 3 - F_2 \times 1 + 90 = 0$

$2F_2 = -90$

$F_2 = -45$

$\Rightarrow$ $F_3 = -45$



$Q + R = 0$ .. (iv)

$F_3 \times 2 + R \times 3 = 0$

$-45 \times 2 + R \times 3 = 0$

$\boxed{R = 30} \Rightarrow \boxed{Q = -30}$

26. For the following statements:

P – The lateral stress in the soil while being tested in an oedometer is always at-rest.

Q – For a perfectly rigid strip footing at deeper depths in a sand deposit, the vertical normal contact stress at the footing edge is greater than that at its centre.

R – The corrections for overburden pressure and dilatancy are not applied to measured SPT-N values in case of clay deposits.

The correct combination of the statements is

(a) P – TRUE; Q – TRUE; R – FALSE

(b) P – TRUE; Q – TRUE; R – TRUE

(c) P – FALSE; Q – FALSE; R – TRUE

(d) P – FALSE; Q – FALSE; R – FALSE

Ans. (b)

27. Tie bars of 12 mm diameter are to be provided in a concrete pavement slab. The working tensile stress of the tie bars is 230 MPa, the average bond strength between a tie bar and concrete is 2 MPa, and the joint gap between the slab is 10mm. Ignoring the loss of bond and the tolerance factor, the design length of the tie bars (in mm, round off to the nearest integer) is ______.

Ans. (700 mm)

Sol. Given:

$$d = 12 \text{ mm}$$

$$\sigma_{st} = 230 \text{ MPa} = 230 \text{ N/mm}^2$$

$$S_b = 2 \text{ MPa} = 2 \text{ N/mm}^2$$

$$t = 10 \text{ mm}$$

$$\text{Length of tie bar} = t + \frac{d\sigma_{st}}{2 \times S_b}$$

$$= 10 + 690$$

$$= 700 \text{ mm}$$

28. Average free flow speed and the jam density observed on a road stretch are 60 km/h and 120 vehicles/km, respectively. For a linear speed-density relationship, the maximum flow on the road stretch (in vehicles/h) is ____.

Ans. (1800)

Sol.

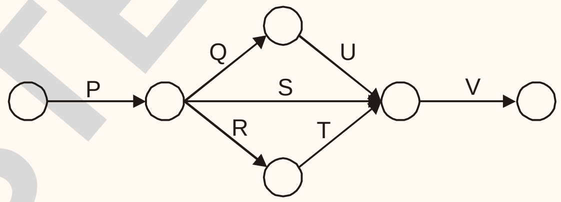
$$V_f = 60 \text{km/h}$$

$$k_J = 120 \text{ Veh/km}$$

$$\therefore \quad q_{max} = \frac{V_f k_J}{4}$$

$$= \frac{60 \times 120}{4} = 1800$$

29. The network of a small construction project awarded to a contractor is shown in the following figure. The normal duration, crash duration, normal cost, and crash cost of all the activities are shown in the table. The indirect cost incurred by the contractor is INR 5000 per day.

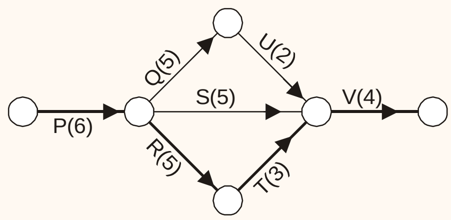


Activity	Normal Duration (days)	Crash Duration (days)	Normal Cost (NR)	Crash Cost (INR)
P	6	4	15000	25000
Q	5	2	6000	12000
R	5	3	8000	9500
S	6	3	7000	10000
T	3	2	6000	9000
U	2	1	4000	6000
V	4	2	20000	28000

If the project is targeted for completion in 16 days, the total cost (in INR) to be incurred by the contractor would be _____

Ans. (149500)

Sol.



Path PRTV is critical path and corresponding normal duration is 18 days.

Activity	t_n	t_c	C_n	C_c	$C_s = \dfrac{C_c - C_n}{t_n - t_c}$
P	6	4	15000	25000	5000
Q	5	2	6000	12000	2000
R	5	3	8000	9500	750
S	6	3	7000	10000	1000
T	3	2	6000	9000	3000
U	2	1	4000	6000	2000
V	4	2	20000	2800	4000

For 18 days:

Direct cost = 66000

Indirect cost = 18 × 5000 = 90000

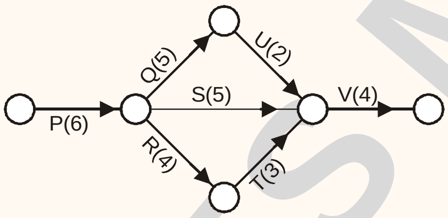
Total project cost = 156000

1st stage crashing :

Crash activity R by 1 day.

New project duration = 17 days.

T.P.C = 156000 + 1 × 750 – 1 × 5000 = 151750



2nd stage crashing :

Crash activity Q & R simultaneously by 1 day.

New project duration = 16 days.

T.P.C = 151750 + 1 × (750 + 2000) – 5000

= 149500

30. A sample of air analysed at 0°C and 1 atm pressure is reported to contain 0.02 ppm (parts per million) of NO_2. Assume the gram molecular mass of NO_2 as 46 and its volume at 0°C and 1 atm pressure as 22.4 litres per mole. The equivalent NO_2 concentration (in microgram per cubic meter, round off to 2 decimal palces) would be _____

Ans. **(41.07 $\mu g/m^3$)**

Sol. 0.02 ppm of NO_2 means $= \dfrac{0.02\ NO_2}{10^6 l \text{ of air}}$

$$= \frac{\frac{0.02}{22.4} \text{ mole } NO_2}{10^6 l \text{ of air}}$$

$$= \frac{\frac{0.02}{22.4} 46 \text{ g}}{10^6 l \text{ of air}}$$

$$= \frac{0.02}{22.4} \times \frac{46 \times 10^3 \text{ mg}}{10^6\ l}$$

$$= \frac{0.02 \times 46}{22.4} \times \frac{10^3 \times 10^3\ \mu g}{10^6 l}$$

$$= 0.04107\ \mu g/l$$

$$= 41.07\ \mu g/m^3$$

31. Traffic on a highway is moving at a rate 360 vehicles per hour at a location. If the number of vehicles arriving on this highway follows Poisson distribution, the probability (round off to 2 decimal places) that the headway between successive vehicles lies between 6 and 10 seconds is _____

Ans. **(0.18)**

Sol. λ = 360 veh/hr

$$= \frac{360}{3600} \frac{\text{veh}}{\text{sec}} = 0.1 \text{veh/sec}$$

$$P(6 \to 10) = \frac{(\lambda t_2)^0 \times e^{-\lambda t_2}}{0!} - \frac{(\lambda t_1)^0 e^{-\lambda t_1}}{0!}$$

$$= \frac{1 \times e^{-0.1 \times 6}}{1} - \frac{(0.1 \times 10)^0 \times e^{-0.1 \times 10}}{1}$$

= 0.18

32. Consider the ordinary differential equation $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0.$ Given the values of y(1) = 0 and y(2) = 2, the value of y(3) (round off to 1 decimal place), is ______

Ans. **(6)**

Sol.

$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y = 0$$

y(1) = 0 & y(2) = 2

Assume $x = e^t$

$\Rightarrow \frac{dx}{dt} = e^t = x$

$\Rightarrow dx = xdt$

Then $\frac{dy}{dx} = \frac{dy}{xdt}$

$\Rightarrow x\frac{dy}{dt} = \frac{dy}{dt} = Dy \qquad \left(D \equiv \frac{d}{dt}\right)$

$\Rightarrow x^2\frac{d^2y}{dt} = D(D-1)y$

Putting this into ordinary differential equation

$D(D-1)y - 2Dy + 2y = 0$

$[D^2 - 3D + 2] = 0$

Auxillary equation is

$m^2 - 3m + 2 = 0$

$(m-1)(m-2) = 0$

i.e. m = 1 & m = 2

$\therefore$ solution of equation

$y = c_1e^t + c_2e^{2t}$

$\Rightarrow$ Putting e^t as x

$\Rightarrow y = c_1x + c_2x^2$

$\Rightarrow y(1) = 0$

$\Rightarrow c_1 + c_2 = 0$ ...(i)

& y(2) = 2

$\Rightarrow 2c_1 + 4c_2 = 2$

$\Rightarrow c_1 + 2c_2 = 1$ (ii)

(ii) – (i)

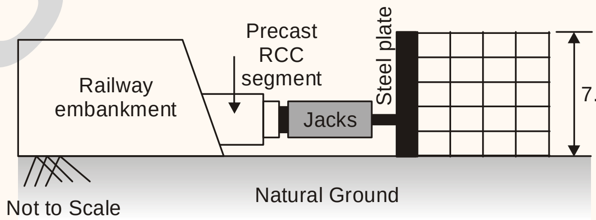
$\Rightarrow c_2 = 1$

$c_1 = -1$

$\Rightarrow y = -x + x^2$

Then $y(3) = -3 + 3^2 = 6$

33. A 3 m × 3 m square precast reinforced concrete segments to be installed by pushing them through an existing railway embankment for making an underpass as shown in the figure. A reaction arrangement using precast PCC blocks placed on the ground is to be made for the jacks.

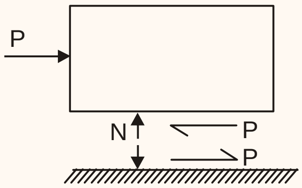


At each stage, the jacks are required to apply a force of 1875 kN to push the segment. The jacks will react against the rigid steel plate placed against the reaction arrangement. The footprint area of reaction arrangement on natural ground are: c = 17 kPa; $\phi = 25°$ and $\gamma = 18\ kN/m^3$. Assuming that the reaction arrangement has rough interface and has the same properties that of soil, the factor of safety (round off to 1 decimal place) against shear failure is ______.

Ans. **(2.0187)**

Sol. FOS against shear failure

$$= \frac{\text{Strength}}{\text{Applied load}} = \frac{(c+\sigma\tan\phi)A}{P}$$



$$\sigma = \frac{N}{A} = \frac{24\frac{kN}{m^3}\times 37.5\,m^2 \times 7.5\,m}{37.5\,m^2}$$

$$= 24 \times 7.5\,kN/m^2$$

$$\Rightarrow \quad FOS = \frac{(c+\sigma\tan\phi)A}{P}$$

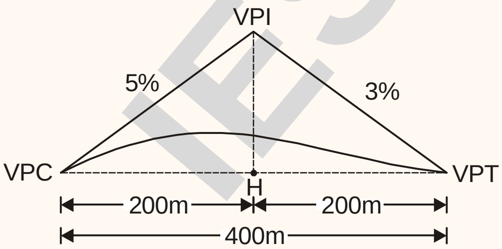
$$= \frac{(17 + 24\times 7.5 \times \tan 25°)\times 37.5}{1875}$$

$$\boxed{FOS = 2.0187}$$

34. A parabolic vertical curve is being designed to join a road of grade + 5% with a road of grade –3%. The length of the vertical curve is 400 m measured along the horizontal. The vertical point of curvature (VPC) is located on the road of grade +5%. The difference in height between VPC and vertical point of intersection (VPI) (in m, round off to the nearest integer) is ____

Ans. **(10 m)**

Sol.



Height difference between

VPI & VPC = 5% of 200 m = 10 m

35. If the section shown in the figure turns from fully-elastic to fully-plastic, the depth of neutral axis (N.A.), $\bar{y}$, decreases by

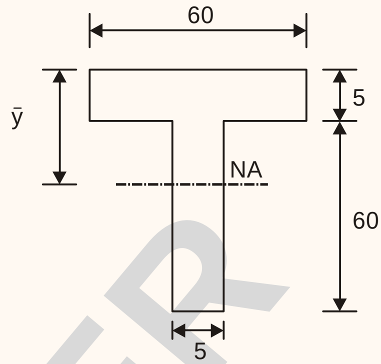


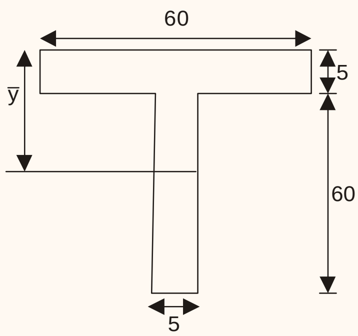
Figure not to scale
All dimensions are in mm

(a) 13.75 mm (b) 10.75 mm

(c) 15.25 mm (d) 12.25 mm

Ans. **(a)**

Sol.



For fully elastic case,

$$\bar{y} = \frac{60\times 5\times \frac{5}{2} + 60\times 5 \times \left(5 + \frac{60}{2}\right)}{60\times 5 + 60\times 5}$$

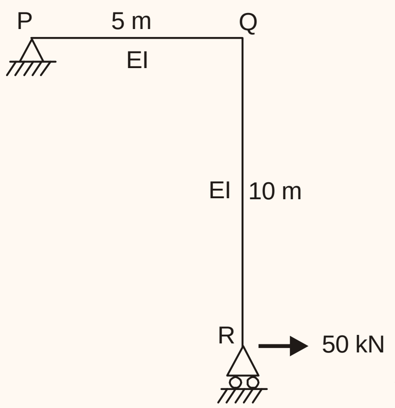
$$= \frac{750 + 10500}{2\times 60 \times 5} = 18.75$$

For fully plastic case,

$\bar{y}$ = Equal area axis = 5

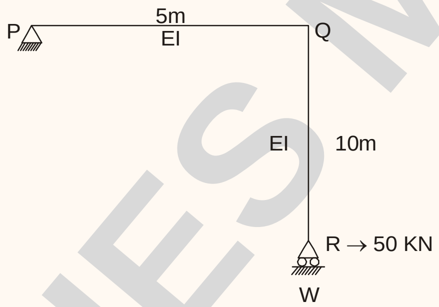
N.A reduces by = 18.75 – 5 = 13.75 mm

36. A portal frame shown in figure (not drawn to scale) has a hinge support at joint P and a roller support at joint R. A point load of 50 kN is acting at joint R in the horizontal direction. The flexural rigidity. EI, of each member is 10^6 kNm^2. Under the applied load, the horizontal displacement (in mm, round off to 1 decimal place) of joint R would be _____



Ans. **(25 mm)**

Sol.



For reaction

$$\sum M_p = 0$$

$$-W \times 5 + 50 \times 10 = 0$$

$$W = \frac{500}{5} = 100kN$$

$$\because \quad \delta = \int \frac{M \cdot m \cdot dx}{EI}$$

When unit load at R is acting in the direction of 50kN load, then reaction at R = 2 (downward)

Number	limit	M	m	EI
PQ	$0-5m$	$-100x+500$	$-2x+10$	10^6
QR	$0-10m$	$50x$	x	10^6

$$\delta = \int_0^5 \frac{(-100x+500)(-2x+10)}{EI}dx + \int_0^{10} \frac{(50x)(x)}{EI}dx$$

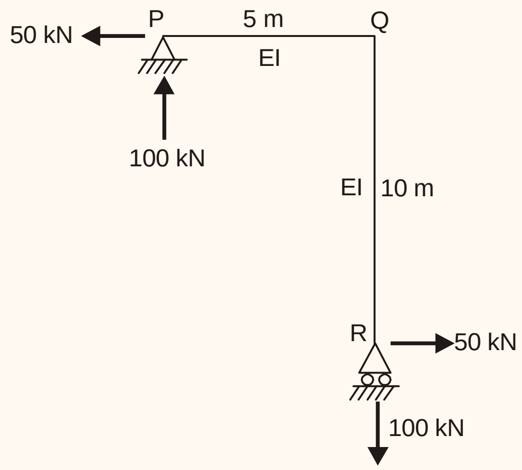
$$= \int_0^5 \frac{(200x^2-1000x-1000x+5000)}{EI}dx + \int_0^{10} \frac{50x^2}{EI}dx$$

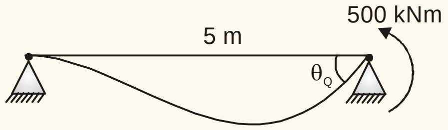
$$= \int_0^5 \frac{200x^2-2000x+5000}{EI}dx + \int_0^{10} \frac{50x^2}{EI}dx$$

$$= \frac{1}{EI}\left[200\times\frac{x^3}{3}-2000\frac{x^2}{2}+5000x\right]_0^5 + \frac{1}{EI}\times 50\left[\frac{x^3}{3}\right]_0^{10}$$

$$= \frac{1}{10^6}\left[\frac{200}{3}\times 125-\frac{2000}{2}\times 25+5000\times 5\right] + \frac{50}{10^6}\times\frac{1}{3}\times 1000 = 25mm$$

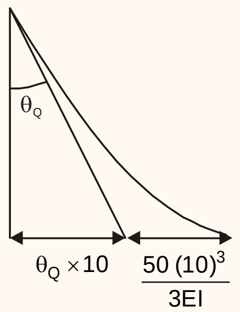
Alternatively:





$$\theta_Q = \frac{500 \times 5 \text{ kNm-m}}{3\,EI}$$

$$\theta_Q = \frac{2500}{3 \times 10^6} \text{ rad}$$



$$\Delta = \frac{10 \times 2500}{3 \times 10^6} + \frac{50(10)^3}{3 \times 10^6}$$

$$= \frac{75000}{3 \times 10^6} \text{ m} = 25 \times 10^{-3} \text{ m} = 25 \text{ mm}$$

37. A box measuring 50 cm × 50 cm × 50 cm is filled to the top with dry coarse aggregate of mass 187.5 kg. The water absorption and specific gravity of the aggregate are 0.5% and 2.5, respectively. The maximum quantity of water (in kg, round off to 2 decimal places) required to fill the box completely is _____

Ans. **(50.94)**

Sol. Volume of the box = 0.5 × 0.5 × 0.5

= 0.125 m^3

Mass of aggregate = 187.5 kg

G_{agg} = 2.5

$$\text{Volume of aggregate} = \frac{187.5}{2.5 \times 1000} = 0.075 \text{ m}^3$$

Volume of empty space = 0.125 – 0.075

= 0.05 m^3

Water absorption = 0.5%

Volume of water absorbed

$$= \frac{0.5}{100} \times \frac{187.5}{1000} = 9.375 \times 10^{-4}$$

Total volume of water that can be filled

= $9.375 \times 10^{-4} + 0.05$

= 0.0509 m^3

Mass of water = 50.94 kg

38. A wastewater is to be disinfected with 35mg/L of chlorine to obtain 99% kill of micro-organisms. The number of micro-organisms remaining alive (N_t) at time t, is modelled by $N_t = N_0 e^{-kt}$, where N_0 is number of micro-organisms at t = 0, and k is the rate of kill. The wastewater flow rate is $36 m^3/h$, and k = 0.23 min^{-1}. If the depth and width of the chlorination tank are 1.5 m and 1.0m, respectively, the length of the tank (in m, round off to 2 decimal places) is _____

Ans. **(8.0089 m)**

Sol. For 99% kill of mircoorganision

$$\eta = \frac{N_0 - N_t}{N_0} = 0.99$$

$$N_t = 0.01\, N_0$$

$$N_0 e^{-kt} = 0.01 N_0$$

$$-Kt\ell ne = \ell n 0.01$$

$$-0.23t = -4.605$$

$$t = 20.022 \text{ min}$$

Volume of tank req. = Q.t

$$= 36 \frac{m^3}{hr} \times 20.02 \text{ min}$$

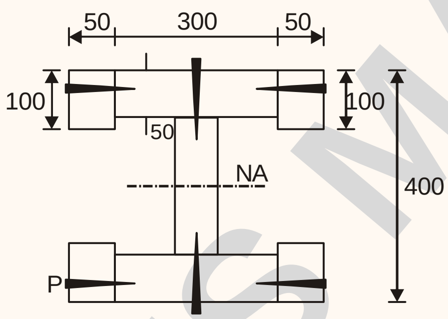
$$= \frac{36 \times 20.02}{60} \text{ m}^3$$

$$= 12.012 \text{ m}^3$$

$$\text{length} = \frac{V}{\text{depth} \times \text{width}}$$

$$= \frac{12.012}{1.5 \times 1} = 8.0089\text{m}$$

39. The cross-section of a built-up wooden beam as shown in the figure (not drawn to scale) is subjected to a vertical shear force of 8kN. The beam is symmetrical about the neutral axis (N.A.) shown, and the moment of inertia about N.A. is $1.5 \times 10^9 \text{mm}^4$. Considering that the nails at the location P are spaced longitudinally (along the length of the beam) at 60 mm, each of the nails at P will be subjected to the shear force of



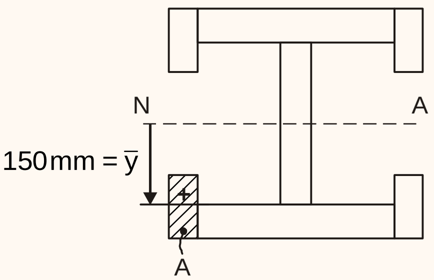
All dimensions are in mm

(a) 60 N (b) 120 N

(c) 240 N (d) 480 N

Ans. (c)

Sol.



$$\text{Shear force in nail at P} = \frac{VA\bar{y}}{I} \times \text{pitch}$$

$$= \frac{8000 \times 100 \times 50 \times 150 \text{Nmm}^3}{1.5 \times 10^9 \text{mm}^4} \times 60\text{mm}$$

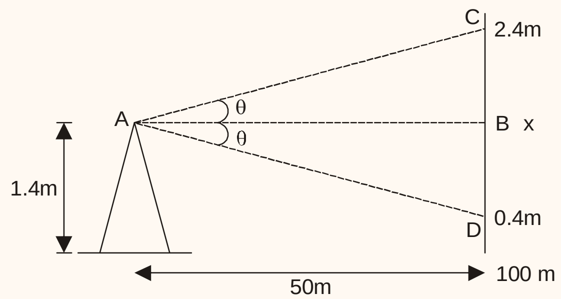
= 240 N

So shear force in nail = 22 × 60 = 1320 N

40. A staff is placed on a benchmark (BM) of reduced level (RL) 100.000 m and a theodolite is placed at a horizontal distance of 50m from the BM to measure the vertical angles. The measured vertical angles from the horizontal at the staff readings of 0.400m and 2.400 m are found to be the same. Taking the height of the instrument as 1.400 m, the RL (in m) of the theodolite station is _____

Ans. (100 m)

Sol.



$$\tan\theta = \frac{2.4 - x}{50} = \frac{x - 0.4}{50}$$

2x = 2.8

x = 1.4m

H.O.I = 100 + 1.4

= 101.4m

RL of theodelite station = 101.4 – theodelite hight

= 101.4 – 1.4

= 100m

41. A 0.80 m deep bed of sand filter (length 4m and width 3m) is made of uniform particles (diameter = 0.40 mm, specific gravity = 2.65, shape factor = 0.85) with bed porosity of 0.4. the bed has to be backwashed at a flow rate of 3.60 m^3/min. During backwashing, if the terminal settling velocity of sand particles is 0.05 m/s, the expanded bed depth (in m, round off to 2 decimal places) is ______

Ans. **(1.2075 m)**

Sol.

$$n_{ex} = \left(\frac{V_B}{V_t}\right)^{0.22}$$

$$V_B = \frac{3.6}{4\times3\times60} = 5 \times 10^{-3} \text{ m/sec}$$

$$n_{ex} = \left(\frac{5\times10^{-3}}{0.05}\right)^{0.22}$$

$$\boxed{n_{ex} = 0.6025}$$

then $L_{ex}(1 - n_{ex}) = L(1 - n)$

$\Rightarrow L_{ex}(1 - 0.6025) = 0.8 \times (1 - 0.4)$

$$\boxed{L_{ex} = 1.2075\text{m}}$$

42. A reinforced concrete circular pile of 12m length and 0.6 m diameter is embedded in stiff clay which has an undrained unit cohesion of 110 kN/m^2. The adhesion factor is 0.5. The Net Ultimate Pullout (Uplift) Load for the pile (in kN, round off to 1 decimal place is) is _____

Ans. **(1244.07)**

Sol. Pull out load = $\alpha C_u \cdot \ell \cdot p$

p = perimeter

ℓ = length

$= 0.5 \times 110 \times 12 \times \pi(0.6)$

= 1244.07 kN

43. A survey line was measured to be 285.5m with a tape having a nominal length of 30m. On checking, the true length of the tape was found to be 0.05 m too short. If the line lay on a slope of 1 in 10, the reduced length (horizontal length) of the line for plotting of survey work would be

(a) 285.0 m (b) 284.5 m

(c) 285.6 m (d) 283.6 m

Ans. **(d)**

Sol. Measured length = 285.5 m

Nominal length of tape = 30 m

Slope = 1 in 10

The tape is 0.05 m too short

Actual length of tape = 30 – 0.05 = 29.95 m

Actual length measured

$$= \frac{\text{Actaul length of tape}}{\text{Nominal length of tape}} \times \text{Measured length}$$

$$= \frac{29.95}{30} \times 285.5$$

$$= 285.024\text{m}$$

Now slope correction = $\dfrac{-h^2}{2L}$

$\because$ Slope = 1 in 10

$\Rightarrow h = \dfrac{1}{10} \times 285.024$

$\Rightarrow h = 28.5024$ m

$\Rightarrow$ Slope correction $= \dfrac{-(28.5024)^2}{2\times285.024}$

$= -1.42512$m

$\Rightarrow$ Length to be plotted

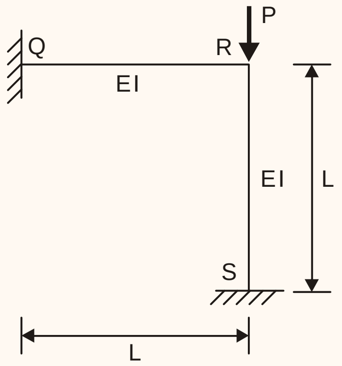
= Actual length measured + correction

$= 285.024 + (-1.42512)$

= 283.599 m

Hence, option (d) is correct.

44. The rigid-joined plane frame QRS shown in the figure is subjected to a load P at the joint R. Let the axial deformation in the frame be neglected. If the support S undergoes a settlement of $\Delta = \frac{PL^3}{\beta EI}$, the vertical reaction at the support S will become zero when β is equal to

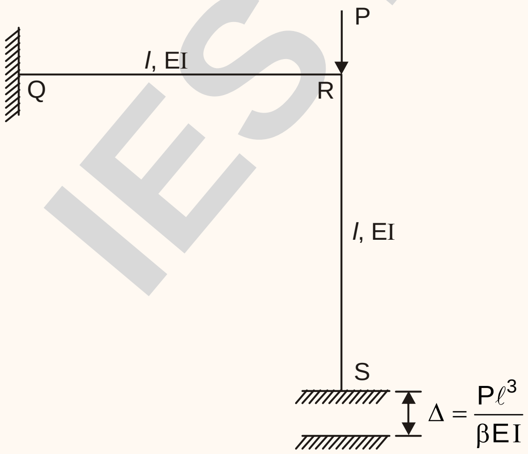


(a) 3.0 (b) 7.5

(c) 0.1 (d) 48.0

Ans. **(7.5)**

Sol.



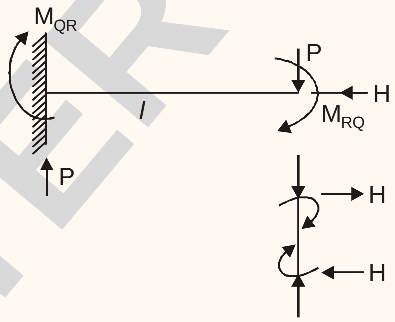
Using slope deflection method,

$$M_{QR} = \frac{2EI}{\ell}\left(\theta_R - \frac{3\Delta}{\ell}\right)$$

$$M_{RQ} = \frac{2EI}{\ell}\left(2\theta_R - \frac{3\Delta}{\ell}\right)$$

$$M_{RS} = \frac{2EI}{\ell}(2\theta_R)$$

If reaction at S is equal to zero



$$M_{RQ} + M_{QR} + P\ell = 0$$

$$\frac{6EI\theta_R}{\ell} - \frac{12EI\Delta}{\ell^2} + P\ell = 0$$

$$\frac{6EI\theta_R}{\ell} - \frac{12EI}{\ell^2} \times \frac{P\ell^3}{\beta EI} + P\ell = 0$$

$$\frac{6EI\theta_R}{\ell} - \frac{12P\ell}{\beta} + P\ell \quad \text{...(i)}$$

From equilibrium of joint

$$M_{RQ} + M_{RS} = 0$$

$$\frac{8EI\theta_R}{\ell} - \frac{6EI\Delta}{\ell^2} = 0$$

$$\frac{6EI\theta_R}{\ell} = \frac{6}{8}\left(\frac{6EI}{\ell^2} \times \frac{P\ell^3}{\beta EI}\right)$$

$$\frac{6EI\theta_R}{\ell} = \frac{36P\ell}{8\beta} \quad \text{...(ii)}$$

$\Rightarrow$ From (i) & (ii)

$$\frac{36P\ell}{8\beta} - \frac{96P\ell}{8\beta} + P\ell = 0$$

$$-\frac{60P\ell}{8\beta} + P\ell = 0$$

$\Rightarrow \quad 8\beta = 60$

$\beta = \frac{60}{8} = 7.5$

45. Which one of the following is NOT a correct statement?

(a) The function $\sqrt[x]{x}, (x > 0)$, has the global minima at x = e

(b) The function $\sqrt[x]{x}, (x > 0)$, has the global maxima at x = e

(c) The function x^3 has neither global minima nor global maxima

(d) The function |x| has the global minima at x = 0

Ans. **(a)**

Sol. $y = x^{1/x}$

$\ln y = \frac{1}{x}\ln x$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{x}\left(\frac{1}{x}\right) + \ln x.\left(\frac{-1}{x^2}\right)$

$\frac{dy}{dx} = x^{1/x} \times \frac{1}{x^2}(1-\ln x)$

For x > 0; $\frac{dy}{dx} = 0$

$\Rightarrow \quad x = e$

Thus point x = e is the critical point for $y = x^{1/x}$

Now at x = e, $\frac{dy}{dx}$ changes its sign from (+ve) to (-ve). Thus point (x = e) is point of global maxima.

- $y = x^3$ has neither global minma nor global maxima, it only have saddle point at x = 0
- y | x |; attains its minimum value at x = 0; so x = 0 is the global minima for y = f(x)

46. A rectangular open channel has a width of 5m and a bed slope of 0.001. For a uniform flow of depth 2m, the velocity is 2m/s. The Manning's roughness coefficient for the channel is

(a) 0.033 (b) 0.050

(c) 0.002 (d) 0.017

Ans. **(0.017)**

Sol. For a rectangular channel

Width of channel = 5 m

Depth of flow = 2 m

Bed slope = 0.001

Velocity V = 2 m/sec.

From manning's

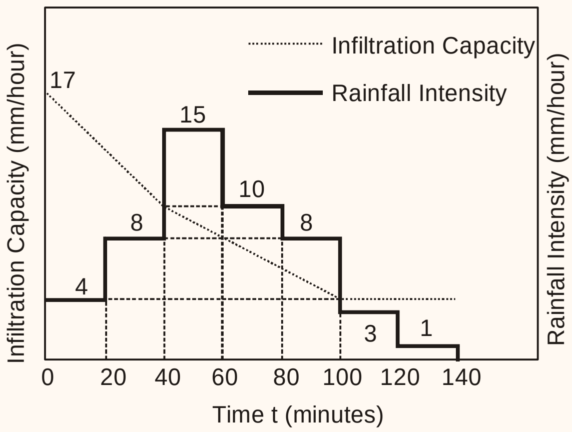
$V = \frac{1}{n}R^{2/3}S^{1/2}$

Where $R = \frac{A}{P} = \frac{5\times 2}{5+2\times 2} = 1.111\,m$

$\Rightarrow \quad 2 = \frac{1}{n}\times(1.111)^{2/3}\times(0.001)^{1/2}$

$\Rightarrow \quad n = 0.017$

47. The hyetograph of a storm event of duration 140 minutes is shown in the figure.



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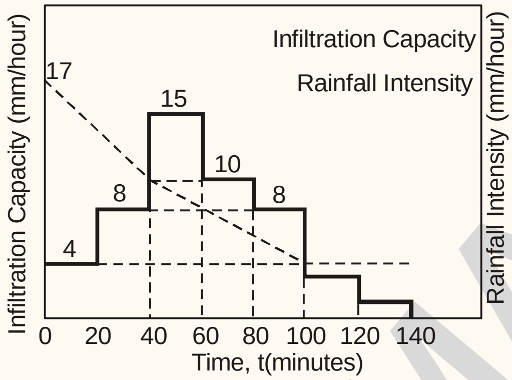
The infiltration capacity at the start of this event (t = 0) is 17mm/hour, which linearly decreases to 10 mm/hour after 40 minutes duration. As the event progresses, the infiltration rate further drops down linearly to attain a value of 4mm/hour at t = 100 minutes and remains constant thereafter till the end of the storm event. The value of the infiltration index, ϕ (in mm/hour, round off to 2 decimal places), is _______

Ans. **(7.25 mm/hr)**

Sol.

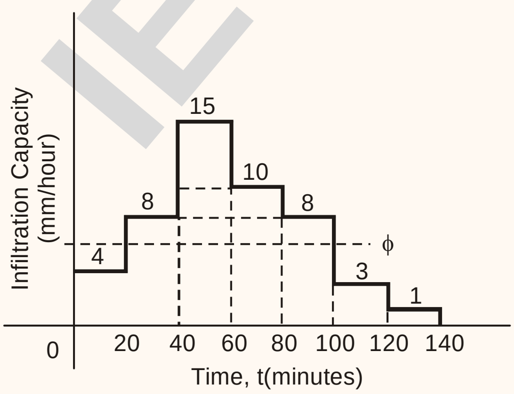


Depth of infiltration = Area of hyetograph above Horton's curve

$$= \left(15\times\frac{20}{60}+10\times\frac{20}{60}+8\times\frac{20}{60}\right)-\frac{10+4}{2}\times\frac{60}{60}$$

= 4 mm

Now, assuming $4\le\phi\le8$



$$(8-\phi)\times\frac{20}{60}+(15-\phi)\times\frac{20}{60}+(10-\phi)\times\frac{20}{60}$$

$$+(8-\phi)\times\frac{20}{60} = 4$$

$$41-4\phi=12$$

$\Rightarrow \phi$ = 7.25 mm/hr **Ans.**

48. Consider a laminar flow in the x-direction between two infinite parallel plates (Couette flow). The lower plate is stationary and the upper plate is moving with a velocity of 1 cm/s in the x-direction. The distance between the plates is 5mm and the dynamic viscosity of the fluid is 0.01 N-s/m^2. If the shear stress on the lower plate is zero, the pressure gradient, $\frac{\partial p}{\partial x}$, (in N/m^2 per m, round off to 1 decimal place) is______

Ans. **(8 N/m^2/m)**

Sol. Given data ;

Velocity of plate, V = 1 cm/sec

Distance between the late = 5 mm

Dynamic viscosity of fluid = 0.01 N-S/m^2

Shear stress at lower plate = 0

Pressure gradient $\frac{\partial P}{\partial x}=?$

We know that, in case of couette flow, shear stress (τ) is given by

$$\tau = \frac{\mu V}{B}+\left(-\frac{\partial P}{\partial x}\right)\left(\frac{B}{2}-y\right)$$

At lower plate, y = 0; $\tau=0$ [Given]

$$0 = \frac{0.01\times0.01}{0.005}-\left(\frac{\partial P}{\partial x}\right)\left[\frac{0.005}{2}-0\right]$$

$$\frac{\partial P}{\partial x} = 8 \text{ N/m}^2 \text{ per m}$$

49. A granular soil has a saturated unit weight of 20 kN/m^3 and an effective angle of shearing resistance of 30°. The unit weight of water is

9.81 kN/m^3. A slope is to be made on this soil deposit in which the seepage occurs parallel to the slope up to the free surface. Under this seepage condition for a factor of safety of 1.5, the safe slope angle (in degree, round off to 1 decimal place) would be ____

Ans. **(11.0953°)**

Sol.

$$\gamma_{sat} = 20 KN/m^2$$

$$\phi = 30°$$

$$\gamma_w = 9.81 \ KN/m^2$$

$$FOS = 1.5$$

We know that

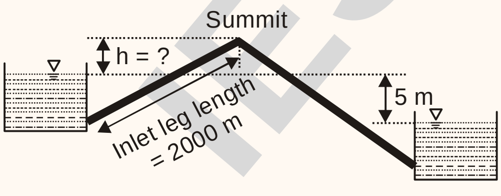
$$FOS = \frac{\gamma_{sub}}{\gamma_{sat}} \times \frac{\tan\phi}{\tan i}$$

[i = safe slope angle]

$$1.5 = \frac{20-9.81}{20} \times \frac{\tan 30}{\tan i}$$

$$\Rightarrow \quad i = 11.0953$$

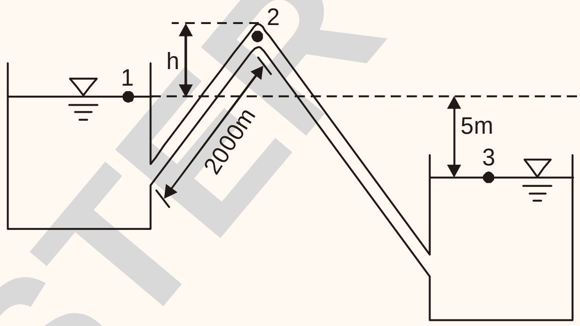
50. Two water reservoirs are connected by a siphon (running full) of total length 5000 m and diameter of 0.10 m, as shown below (figure not drawn to scale).



The inlet leg length of the siphon to its summit is 2000 m. The difference in the water surface levels of the two reservoirs is 5m. Assume the permissible minimum absolute pressure at the summit of siphon to be 2.5m of water when running full. Given: friction factor f = 0.02 throughout, atmospheric pressure = 10.3 m of water, and acceleration due to gravity g = 9.81 m/s^2. Considering only major loss using Darcy-Weisbach equation the maximum height of the summit of siphon from the water level of upper reservoir, h (in m round off to 1 decimal place) is _____

Ans. **(5.8 m)**

Sol. Given data :



$$d = 0.1 \ m$$

Length of siphon = 5000 m

Length of siphon upto summit = 2000 m

Friction Factor, f = 0.02

Acceleration due to gravity, g = 9.81 m/sec^2

Applying Energy equation between point 1 and 3 to get

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3 + h_{f(1-3)}$$

$$10.3 + 0 + Z_1 = 10.3 + 0 + Z_3 + \frac{flQ^2}{12.1d^5}$$

[From Darcy Weisback equation $h_f = \frac{flQ^2}{12.1d^5}$

$$\Rightarrow \quad 5 = \frac{0.02 \times 5000 \times Q^2}{12.1 \times (.1)^5}$$

$[\because \ Z_1 - Z_3 = 5m]$

$$\Rightarrow \quad Q = 2.4597 \times 10^{-3} \ m^3/sec$$

Now applying energy equation between 1 and 2 to get

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_{f(1-2)}$$

$$10.3+0+Z_1=\frac{P_2}{\gamma}+\frac{Q^2}{2ga^2}+Z_2+\frac{fl_{(1-2)}Q^2}{12.1d^5}$$

$$\Rightarrow \quad 10.3-(Z_2-Z_1)=2.5+\frac{(2.4597\times10^{-3})^2}{2\times9.81\times\frac{\pi}{4}\times0.1^2}$$

$$+\frac{0.02\times2000\times(2.4597\times10^{-3})^2}{12.1\times(0.1)^5}$$

$\Rightarrow 10.3 - h = 4.5$ m

$\Rightarrow \quad h = 5.8$ m

51. Sedimentation basin in a water treatment plant is designed for a flow rate of 0.2 m^3/s. The basin is rectangular with a length of 32m, width of 8m and depth of 4m. Assume that the settling velocity of these particles is governed by the Stokes' law. Given: density of the particles = 2.5 g/cm^3; density of water = 1 g/cm^3; dynamic viscosity of water = 0.01 g/(cm.s); gravitatinal acceleration = 980 cm/s^2. If the incoming water contains particles of diameter $25\,\mu m$ (spherical and uniform) the removal efficiency of these particles is

(a) 100% (b) 65%

(c) 78% (d) 51%

Ans. (b)

Sol. Given :

Flow rate = 0.2 m^3/sec

Dimension of tank = 32m × 8 m × 4 m

Density of particles = 2.5 g/cc

Density of water = 1 g/cc

Dynamic viscosity of water = 0.01 g/cm-S

Diameter of particle = 25 μm

We know that

Over flow rate of tank $(V_s) = \frac{0.2}{32\times8}$

$= 7.8125\times10^{-4}$ m/sec

And settling velocity of particle (v_s),

$$v_s = \frac{(\gamma_s-\gamma_w)d^2}{18\mu}$$

$$v_s = \frac{(2.5-1)\times9.81\times(25\times10^{-6})^2}{18\times0.01\times10^{-4}}$$

$v_s = 5.1094 \times 10^{-4}$ m/sec

Now, % removal efficiency $= \frac{v_s}{V_s}\times100$

$$= \frac{5.1094\times10^{-4}}{7.8125\times10^{-4}}\times100$$

$= 65.4\%$

Hence option (b) is correct.

52. A square footing of 4m side is placed at 1 m depth in a sand deposit. The dry unit weight (γ) of sand is 15 kN/m^3. This footing has an ultimate bearing capacity of 600 kPa. Consider the depth factors; $d_q = d_\gamma = 1.0$ and the bearing capacity factor: $N_\gamma = 18.75$. This footing is placed at a depth of 2m in the same soil deposit. For a factor of safety of 3.0 per Terzaghi's theory, the safe bearing capacity (in kPa) of this footing would be ____

Ans. (270 kPa)

Sol.

Side of square footing = 4 m

Depth of footing = 1 m

Unit weight of soil = 15 KN/m^3

Ultimate bearing capacity = 600 KPa

Depth factors, $d_q = d_\gamma = 1$

$N_\gamma = 18.75$

According to terzaghi, the ultimate bearing capacity of square footing is given as

At depth of footing = 1 m

$$q_u = 1.3CN_C + qN_q d_q + 04B\gamma N_\gamma d_\gamma$$

For sand, C = 0, q = γD_f = $15 \times 1 = 15 KN/m^2$

600 = 0 + 15 × N_q × 1 + 0.4×4×15×18.75×1

$\Rightarrow$ N_q = 10

Now at depth of footing at 2m

$$q_u = 1.3CN_C + qN_q + 0.4B\gamma N_\gamma d_\gamma$$

$$q_u = 0 + (2 \times 15) 10 \times 1 + 0.4 \times 4 \times 15 \times 18.75 \times 1$$

$$q_u = 750 \text{ KPa}$$

$\because$ We know that

$$q_{nu} = q_u - \gamma D_f$$

$$q_{nu} = 750 - 15 \times 2$$

$$q_{nu} = 720 \text{ KPa}$$

and safe bearing capacity q_{safe}

$$q_{safe} = \frac{q_{nu}}{FOS} + \gamma D_f$$

$$= \frac{720}{3} + 15 \times 2$$

$$= 270 \text{ KPa}$$

53. Consider two functions: $x = \Psi \ln \phi$ and $y = \phi \ln \Psi$. Which one of the following is the correct expression for $\frac{\partial \Psi}{\partial x}$?

(a) $\frac{x \ln \phi}{\ln \phi \ln \Psi - 1}$ (b) $\frac{\ln \phi}{\ln \phi \ln \Psi - 1}$

(c) $\frac{\ln \Psi}{\ln \phi \ln \Psi - 1}$ (d) $\frac{x \ln \Psi}{\ln \phi \ln \Psi - 1}$

Ans. **(c)**

Sol. $x = \psi \ln \phi \Rightarrow \psi = \frac{x}{\ln \phi}$ ...(i)

$$y = \phi \ln \psi \Rightarrow \phi = \frac{y}{\ln \psi}$$

Putting value of ϕ in (i)

$$\psi = \frac{x}{\ln\left(\frac{y}{\ln \psi}\right)} = \frac{x}{\ln y - \ln(\ln \psi)} \quad \text{...(ii)}$$

Assuming y constant and differentiating ψ w.r.t. x.

$$\frac{\partial \psi}{\partial x} = \frac{(\ln y - \ln(\ln \psi)) \cdot 1 - x\left(0 - \frac{1}{\ln \psi} \cdot \frac{1}{\psi} \cdot \frac{\partial \psi}{\partial x}\right)}{(\ln y - \ln(\ln \psi))^2} \quad \text{...(iii)}$$

Puttting value of $(\ln y - \ln(\ln \psi)) = \frac{x}{\psi}$ from (ii) in equation (iii)

$$\frac{\partial \psi}{\partial x} = \frac{\frac{x}{\psi} + x \times \frac{1}{\psi \ln \psi} \cdot \frac{\partial \psi}{\partial x}}{\left(\frac{x}{\psi}\right)^2}$$

$$\Rightarrow \frac{\partial \psi}{\partial x} = \frac{1 + \frac{1}{\ln \psi} \frac{\partial \psi}{\partial x}}{\left(\frac{x}{\psi}\right)}$$

$$\Rightarrow \frac{x}{\psi} \frac{\partial \psi}{\partial x} = 1 + \frac{1}{\ln \psi} \frac{\partial \psi}{\partial x}$$

$$\Rightarrow \frac{\partial \psi}{\partial x}\left(\frac{x}{\psi} - \frac{1}{\ln \psi}\right) = 1$$

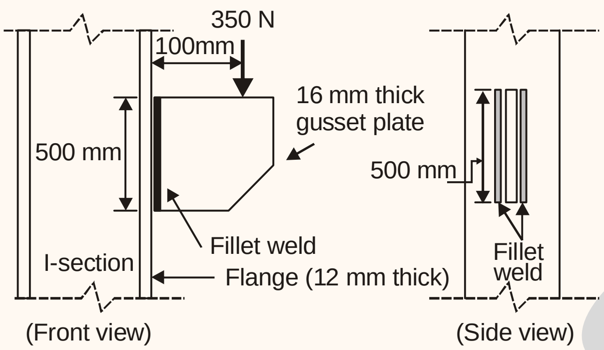
$$\Rightarrow \frac{\partial \psi}{\partial x} = \frac{1}{\frac{x}{\psi} - \frac{1}{\ln \psi}} = \frac{\psi \ln \psi}{x \ln \psi - \psi}$$

$$\Rightarrow \frac{\partial \psi}{\partial x} = \frac{\psi \ln \psi}{\psi \ln \phi \ln \psi - \psi}$$

(replacing x by ψ in ϕ)

$$\Rightarrow \boxed{\frac{\partial \psi}{\partial x} = \frac{\ln \psi}{\ln \phi \ln \psi - 1}}$$

54. A 16 mm thick gusset plate is connected to the 12mm thick flange plate of an I-section using fillet welds on both sides as shwon in the figure (not drawn to scale). The gusset plate is subjected to a point load of 350 kN acting at a distance of 100 mm from the flange plate. Size of fillet weld is 10 mm.



The maximum resultant stress (in MPa, round off to 1 decimal place) on the fillet weld along the vertical plane would be ____

Ans. **(105.36 N/mm²)**

Sol. Given Data :

Thickness of gusset plate (t) = 16 mm

Point load (P) = 350 KN

Eccentricity (e) = 100 mm

Direct shear stress, $q = \frac{P}{2ht}$

$$q = \frac{350\times10^3}{2\times500\times10\times0.7}$$

$$q = 50 \text{ N/mm}^2$$

And bending stress on the extreme edge of weld (f)

$$f = \frac{M}{Z} = \frac{3P.e}{th^2}$$

$$f = \frac{3\times350\times10^3\times100}{0.7\times10\times500^2}$$

$\Rightarrow$ $f = 60 \text{ N/mm}^2$

For checking the safety

Resultant Stress, $F_r = \sqrt{f^2+3q^2}$

$$= 105.36 \text{ N/mm}^2$$

55. A one-dimensional domain is discretized into N sub-domains of width Δx with node numbers i = 0, 1, 2, 3, ..., N. If the time scale is discretized in steps of Δt, the forward-time and centered-space finite difference approximation at i^{th} node and n^{th} time step, for the partial differential equation $\frac{\partial v}{\partial t} = \beta\frac{\partial^2 v}{\partial x^2}$ is

(a) $\frac{v_i^{(n)} - v_i^{(n-1)}}{2\Delta t} = \beta\left[\frac{v_{i+1}^{(n)} - 2v_i^{(n)} + v_{i-1}^{(n)}}{2\Delta x}\right]$

(b) $\frac{v_i^{(n)} - v_i^{(n-1)}}{\Delta t} = \beta\left[\frac{v_{i+1}^{(n)} - 2v_i^{(n)} + v_{i-1}^{(n)}}{(\Delta x)^2}\right]$

(c) $\frac{v_{i+1}^{(n+1)} - v_i^{(n)}}{\Delta t} = \beta\left[\frac{v_{i+1}^{(n)} - 2v_i^{(n)} + v_{i-1}^{(n)}}{2\Delta x}\right]$

(d) $\frac{v_i^{(n+1)} - v_i^{(n)}}{\Delta t} = \beta\left[\frac{v_{i+1}^{(n)} - 2v_i^{(n)} + v_{i-1}^{(n)}}{(\Delta x)^2}\right]$

Ans. **(d)**

Sol. Given differential equation

$$\frac{\partial v}{\partial t} = \beta\frac{\partial^2 v}{\partial x^2}$$

$$\frac{\partial v}{\partial t} = \frac{v_i^{(n+1)} - v_i^{(n)}}{(\Delta t)} \quad ...(i)$$

Using forward time finite difference

$$\frac{\partial^2 v}{\partial x^2} = \frac{v_{i+1}^{(n)} - 2v_i^{(n)} + v_{i-1}^{n}}{(\Delta x)^2} \quad ...(ii)$$

Using centred space finite difference

$$\frac{\partial^2 v}{\partial x^2} = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

Putting (i) and (ii) in PDE

$$\frac{v_i^{(n+1)} - v_i^{(n)}}{(\Delta t)} = \beta\left(\frac{v_{i+1}^{(n)} - 2v_i^{(n)} + v_{i-1}^{(n)}}{\Delta x^2}\right)$$

So, option (d) is correct.