

SECTION: GENERAL APTITUDE

1. The board arrived _____ dawn.
(a) on (b) in
(c) at (d) under

Ans. (c)

2. The strategies that the company _____ to sell its products _____ house-to-house marketing.
(a) used, includes (b) uses, including
(c) uses, include (d) use, includes

Ans. (c)

3. It would take one machine 4 hours to complete a production order and another machine 2 hours to complete the same order. If both machines work simultaneously at their respective constant rates, the time taken to complete the same order is _____ hours.
(a) 3/4 (b) 7/3
(c) 4/3 (d) 2/3

Ans. (c)

Sol. Machine one rate = $\frac{W}{4}$

Machine two rate = $\frac{W}{2}$

$$\Rightarrow \left(\frac{W}{4} + \frac{W}{2} \right) t = W$$

$$t = \frac{1}{0.75} = \frac{4}{3} \text{ hours}$$

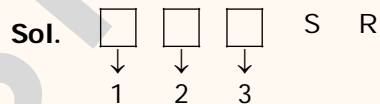
4. When he did not come home, she _____ him lying dead on the roadside somewhere.
(a) notice (b) looked
(c) pictured (d) concluded

Ans. (c)

5. Five different books (P, Q, R, S, T) are to be arranged on a shelf. The books R and S are to be arranged first and second, respectively from the right side of the shelf. The number of different orders in which P, Q, and T may be arranged is

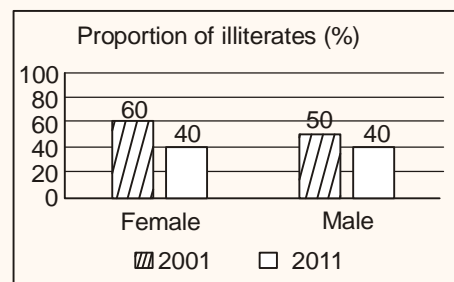
- (a) 12 (b) 6
(c) 2 (d) 120

Ans. (b)

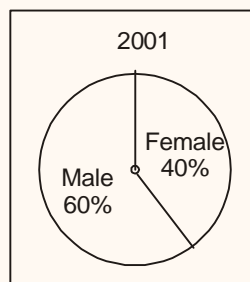


$$\Rightarrow 3 \times 2 \times 1 = 6$$

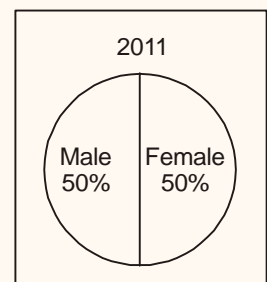
6. The bar graph in Panel (a) shows the proportion of male and female illiterates in 2001 and 2011. The proportions of males and females in 2001 and 2011 are given in Panel (b) and (c), respectively. The total population did not change during this period. The percentage increase in the total number of literates from 2001 to 2011 is _____.



Panel (a)



Panel (b)



Panel (c)



- (a) 35.43 (b) 33.43
(c) 34.43 (d) 30.43

Ans. (d)

Sol. Assuming total no. of population is 100

$$\begin{aligned} \text{Literate in 2001} &= 40 \times \frac{40}{100} + 60 \times \frac{50}{100} \\ &= 16 + 30 \\ &= 46 \end{aligned}$$

$$\begin{aligned} \text{Literate in 2011} &= 50 \times \frac{60}{100} + 50 \times \frac{60}{100} \\ &= 30 + 30 \\ &= 60 \end{aligned}$$

% increase in total no. of literates from 2001

$$\begin{aligned} \text{to 2011} &= \frac{60 - 46}{46} \times 100 \\ &= 30.43\% \end{aligned}$$

7. Five people P, Q, R, S and T gets a promotion and moves to the big office next to the garden. R, who is currently sharing an office with T wants to move to the adjacent office with S, the handsome new intern. Given the floor plan, what is the current locations of Q, R and T? (O = Office, WR = Washroom)

WR	O 1	O 2	O 3	O 4
	P, Q		R	S
Garden				
(a) Manager T	Entry		Teller 1	Teller 2
Garden				

WR	O 1	O 2	O 3	O 4
	P	Q	R	S
Garden				
(b) Manager T	Entry		Teller 1	Teller 2
Garden				

WR	O 1	O 2	O 3	O 4
	P, Q		R	R, S
Garden				
(c) Manager	Entry		Teller 1	Teller 2
Garden				

WR	O 1	O 2	O 3	O 4
	P, Q		R, T	S
Garden				
(d) Manager	Entry		Teller 1	Teller 2
Garden				

Ans. (d)

8. "Indian history was written by British historians – extremely well documented and researched, but not always impartial. History had to serve its purpose. Everything was made subservient to the glory of the Union Jack. Latter-day Indian scholars present a contrary picture."



From the text above, we can infer that :

Indian history written by British historians _____.

- (a) was not well documented and researched and was always biased
- (b) was not well documented and researched and was sometimes biased
- (c) was well documented and researched but was sometimes biased
- (d) was well documented and not researched but was always biased

Ans. (c)

9. Two design consultants, P and Q, started working from 8 AM for a client. The client budgeted a total of USD 3000 for the consultants. P stopped working when the hour hand moved by 210 degrees on the clock. Q stopped working when the hour hand moved by 240 degrees. P took two tea breaks of 15 minutes each during her shift, but took no lunch break. Q took only one lunch break for 20 minutes, but no tea breaks. The market rate for consultants is USD 200 per hour and breaks are not paid. After paying the consultants, the client shall have USD _____ remaining in the budget.

- (a) 166.67 (b) 300.00
- (c) 433.33 (d) 000.00

Ans. (a)

Sol.

<p>P 8 AM 210° → 7 hour $-2 \times \frac{1}{4} = -0.5$ hour working hours = 6.5 hr</p>	<p>Q 8 AM 240° → 8 hour $-\frac{1}{3}$ hour $8 - \frac{1}{3} = \frac{23}{3}$ hour</p>
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$$\text{Payment done} = \left(\frac{13}{2} + \frac{23}{3} \right) \times 200$$

$$\text{USD} = 2833.33 \text{ USD}$$

$$\text{Budget} = 3000 \text{ USD}$$

$$\begin{aligned} \text{Remaining Budget} &= 3000 - 2833.33 \\ &= 166.67 \text{ USD} \end{aligned}$$

10. Four people are standing in a line facing you. They are Rahul, Mathew, Seema and Lohit. One is engineer, one is a doctor, one is teacher and another a dancer. You are told that :

1. Mathew is not standing next to Seema.
 2. There are two people standing between Lohit and the engineer.
 3. Rahul is not a doctor.
 4. The teacher and the dancer are standing next to each other.
 5. Seema is turning to her right to speak to the doctor standing next to her.
- (a) Seema (b) Lohit
(c) Mathew (d) Rahul

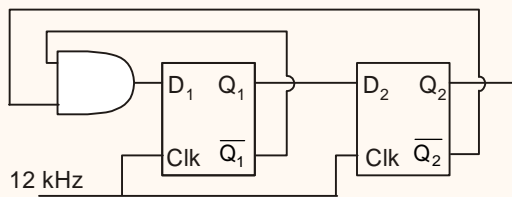
Ans. (c)

	Person	Profession
Sol.	Rohit	Doctor
	Seema	Dancer
	Rahul	Teacher
	Mathew	Engineer

SECTION

ELECTRONICS & COMMUNICATION

1. In the circuit shown, the clock frequency, i.e., the frequency of the Clk signal, is 12 kHz. The frequency of the signal at Q_2 is ___ kHz.



Ans. (4)

Sol. $D_1 = \overline{Q_1} \cdot \overline{Q_2}$
 $D_2 = Q_1$

Present state		Input		Next state	
Q_1	Q_2	D_1	D_2	Q_1^+	Q_2^+
0	0	1	0	1	0
1	0	0	1	0	1
0	1	0	0	0	0

There are only three states.

So, output frequency will be $\frac{12}{3} = 4$ KHz

2. Which one of the following functions is analytic over the entire complex plane?

- (a) $\frac{1}{1-z}$ (b) $e^{1/z}$
(c) $\cos(z)$ (d) $\ln(z)$

Ans. c

Sol. (i) $\frac{1}{1-z}$, not analytic at $z = 1$

(ii) $e^{1/z}$

$$e^{1/z} = \left(\frac{1}{z}\right) + \frac{1}{2!}\left(\frac{1}{z}\right)^2 + \frac{1}{3!}\left(\frac{1}{z}\right)^3 + \dots$$

not analytic at $z = 0$

- (iii) $\cos(z)$

$$\cos(z) = 1 - z^2 + \frac{z^4}{4!} - \frac{z^6}{6!} + \frac{z^8}{8!}$$

analytic for every value of z , so analytic for entire z -plane

- (iv) $\ln(z)$

at $z = 0$, $\ln(z)$ is not analytical.

3. A linear Hamming code is used to map 4-bit messages to 7-bit codewords. The encoder mapping is linear. If the message 0001 is mapped to the codeword 0000111, and the message 0011 is mapped to the codeword 1100110, then the message 0010 is mapped to

- (a) 1100001 (b) 1111111
(c) 1111000 (d) 0010011

Ans. (a)

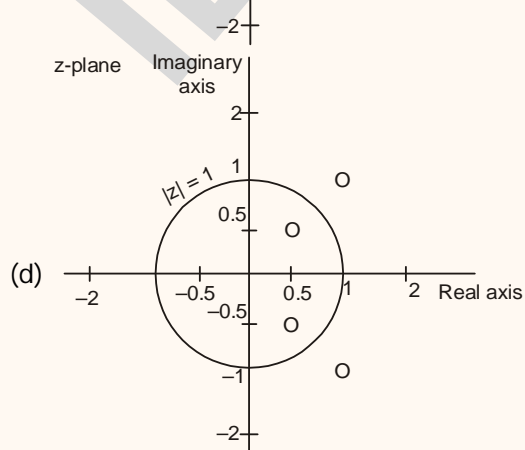
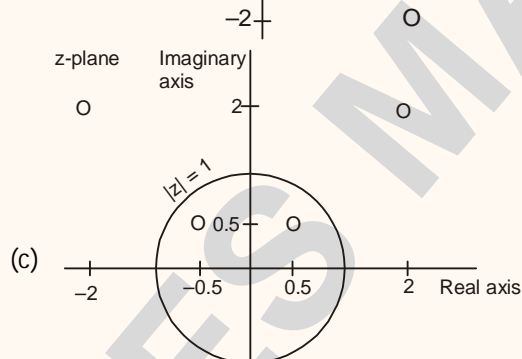
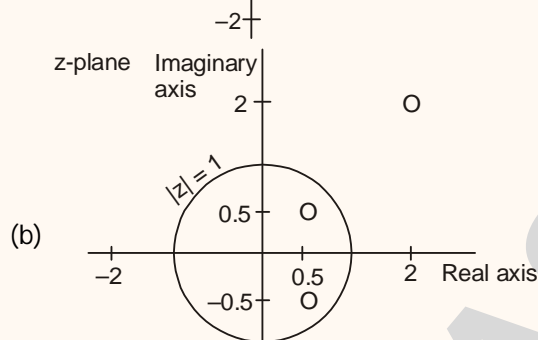
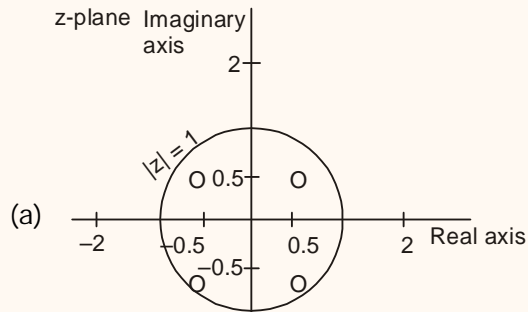
Sol. Linear code is an error correcting code for which any linear combination of code word is also a code word.

	Message	Code word
	0001	0000111
\oplus	0011	1100110
Ex OR	0010	1100001

Combination of two message and code words according to linearity property.

4. Let $H(z)$ be the z -transform of a real-valued discrete-time signal $h[n]$. If $P(z) = H(z)H\left(\frac{1}{z}\right)$

has a zero at $z = \frac{1}{2} + \frac{1}{2}j$, and $P(z)$ has a total of four zeros, which one of the following plots represents all the zeros correctly?



Ans. d

Sol. $P(z) = H(z) H\left(\frac{1}{z}\right)$

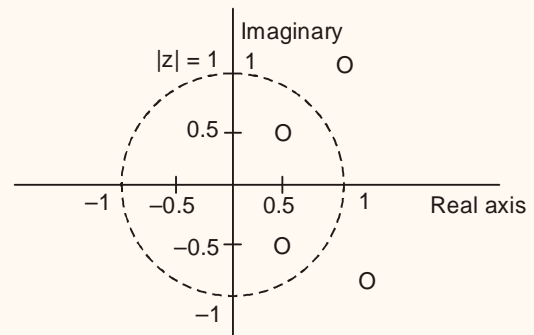
$$P\left(\frac{1}{z}\right) = H\left(\frac{1}{z}\right) H(z)$$

hence $P(z) = P\left(\frac{1}{z}\right)$

if a zero of $P(z)$ at $z = z_0$ than other zero must be at $z = \frac{1}{z_0}$

Given $z_1 = \frac{1}{2} + \frac{1}{2}j$ and other must be $z_2 = \frac{1}{\frac{1}{2} + \frac{1}{2}j}$ two other zeroes.

$$\begin{aligned} z_3 &= \frac{1}{z_1} \\ &= \frac{1}{\frac{1}{2} + \frac{1}{2}j} \\ &= \frac{2}{1+j} \times \left(\frac{1-j}{1-j}\right) \\ &= \frac{2(1-j)}{2} \\ &= 1-j \end{aligned}$$



$$\begin{aligned} z_4 &= \frac{1}{z_2} = \frac{1}{\frac{1}{2} - \frac{1}{2}j} \\ &= \frac{2}{1-j} \times \frac{(1+j)}{(1-j)} \end{aligned}$$

$$= \frac{2}{2}(1+j)$$

$$= 1 + j$$

5. The families of curves represented by the solution of the equation

$$\frac{dy}{dx} = -\left(\frac{x}{y}\right)^n$$

For $n = -1$ and $n = +1$, respectively, are

- (a) Hyperbola and Parabolas
- (b) Circles and Hyperbolas
- (c) Parabolas and Circles
- (d) Hyperbolas and Circles

Ans. d

Sol.
$$\frac{dy}{dx} = -\left(\frac{x}{y}\right)^n$$

for $n = -1$

$$\frac{dy}{dx} = -\left(\frac{x}{y}\right)^{-1}$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

$$\ln y = -\ln(x) + \ln k$$

$$\ln(y) + \ln(x) = \ln(k)$$

$$\ln(xy) = \ln(k)$$

taking log inverse

$$\boxed{xy = k} \text{ Rectangular hyperbola}$$

for $n = 1$

$$\frac{dy}{dx} = -\left(\frac{x}{y}\right)$$

$$\int y \cdot dy = -\int x dx$$

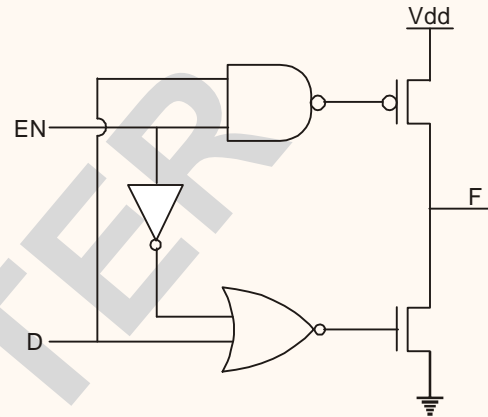
$$\frac{y^2}{2} = -\frac{x^2}{2} + c$$

$$\boxed{x^2 + y^2 = c} \text{ circle.}$$

for $n = -1$; Hyperbolas

for $n = +1$; circles

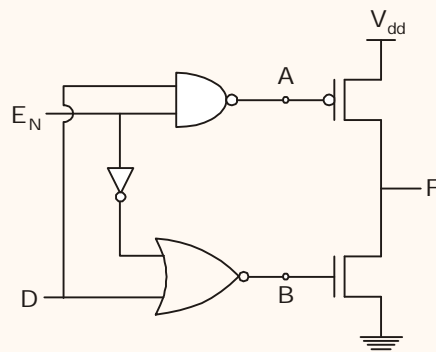
6. In the circuit shown, what are the values of F for $E_N = 0$ and $E_N = 1$, respectively.



- (a) Hi-Z and \bar{D}
- (b) 0 and 1
- (c) 0 and D
- (d) Hi-Z and D

Ans. (d)

Sol.



$$A = \overline{E_N \cdot D}$$

$$B = \overline{\overline{E_N + D}}$$

$$B = E_N \cdot \bar{D}$$

when $E_N = 0$, $A = 1$, $B = 0$

Hence PMOS and NMOS both OFF so circuit in high impedance.

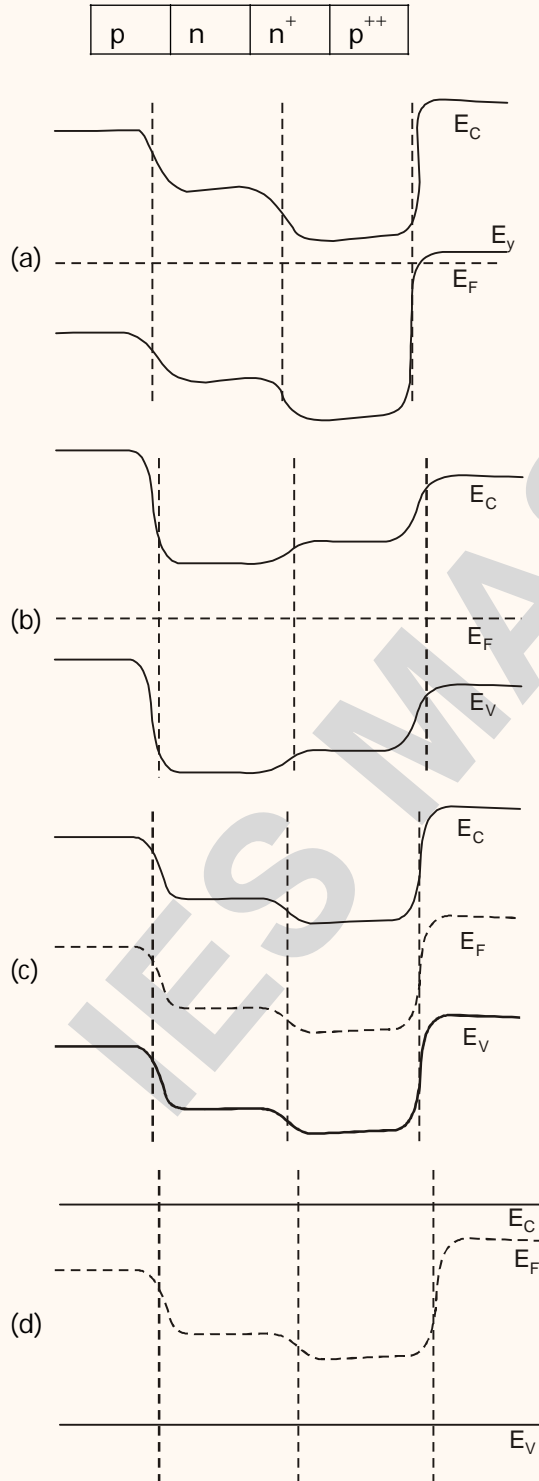
when $E_N = 1$

$$A = \bar{D}, B = \bar{D}$$

Hence output $F = D$

7. Which one of the following options describes correctly the equilibrium band diagram at T

= 300 K of a Silicon pnn⁺p⁺⁺ configuration shown in the figure?



Ans. (a)

Sol. Fermi level for n & p type semiconductor are given by

$$E_{F(n)} = E_c - kT \ln\left(\frac{N_c}{n}\right) \quad \dots(i)$$

$$E_{F(p)} = E_v - kT \ln\left(\frac{N_v}{p}\right) \quad \dots(ii)$$

E_F – Fermi level

E_c – Conduction band energy

E_v – Valence band energy

N_c – Density of conduction states

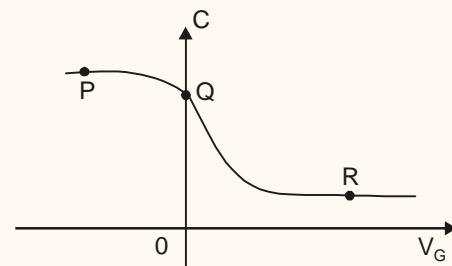
N_v – Density of valence states

n – electron density

p – Hole density

from (i) As n increase $E_{f(n)}$ moves closer to E_c and from (ii), as p increase $E_{f(p)}$ moves closer to E_v . So fermi level must get closer to E_v for p type and to E_c for n type.

8. The figure shows the high-frequency C-V curve of a MOS capacitor (at $T = 300$ K) with $\Phi_m = 0$ V and no oxide charges. The flat-band, inversion, and accumulation conditions are represented, respectively, by the points



(a) Q, R, P

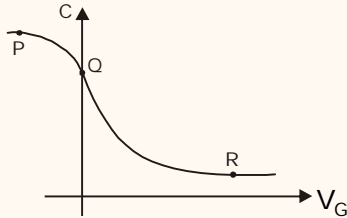
(b) R, P, Q

(c) Q, P, R

(d) P, Q, R

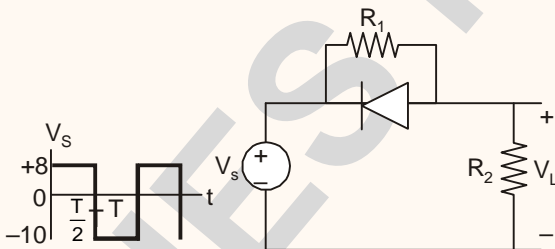
Ans. (a)

Sol. We know that C-V curve is given by



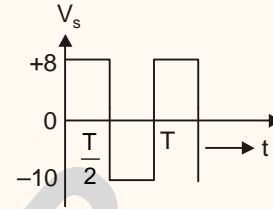
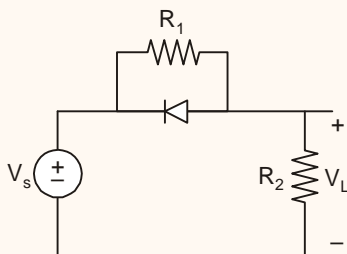
- (i) Flat band \rightarrow Q
- (ii) As V_G increases inversion layer is formed near gate of MOS So
Inversion \rightarrow R
- (iii) If V_{GS} is (-)ve more holes are accumulated near gate in P-type substrate (n-MOSFET)
So Accumulation \rightarrow P

9. In the circuit shown, V_s is a square wave of a period T with maximum and minimum values of 8V and -10V, respectively. Assume that the diode is ideal and $R_1 = R_2 = 50\Omega$. The average value of V_L is _____ volts (rounded off to 1 decimal place).



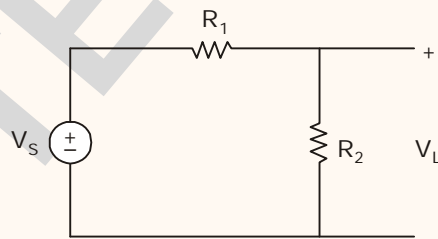
Ans. (-3)

Sol.



$$R_1 = R_2 = 50\Omega$$

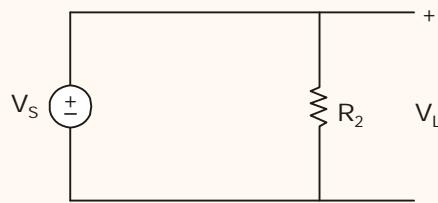
For (+ve) half cycle diode D is OFF hence circuit is



$$V_L = V_s \frac{R_2}{R_1 + R_2}$$

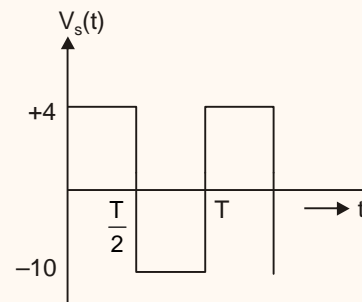
$$V_L = \frac{V_s}{2}$$

For (-ve) half cycle diode is ON



$$V_L = V_s$$

Output $V_L(t)$



$$\text{Average value} = \frac{1}{T} \int_0^T V_s(t) dt$$

$$\begin{aligned}
 &= \frac{1}{T} \left\{ \int_0^{T/2} 4dt + \int_{T/2}^T (-10)dt \right\} \\
 &= \frac{1}{T} \left\{ 4 \cdot \left(\frac{T}{2}\right) - 10 \cdot \left(\frac{T}{2}\right) \right\} \\
 &= \frac{-6}{2} = -3 \text{ volt}
 \end{aligned}$$

10. The value of the contour integral

$$\frac{1}{2\pi j} \oint \left(z + \frac{1}{z}\right)^2 dz$$

evaluated over the unit circle $|z| = 1$ is ____.

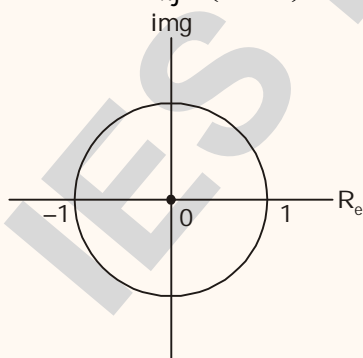
Ans. (0)

Sol. The value of the contour integral

$$\frac{1}{2\pi j} \oint \left(z + \frac{1}{z}\right)^2 dz, |z| = 1$$

$$I = \frac{1}{2\pi j} \oint \left(z + \frac{1}{z}\right)^2 dz$$

$$I = \frac{1}{2\pi j} \oint \left(\frac{z^2 + 1}{z}\right)^2 dz$$



There are two poles at $Z = 0$, (lying inside the contour)

so, by using $\oint \frac{f(z)}{(z-z_0)^n} dz = \frac{1}{(n-1)!} \frac{d^{n-1}}{dz^{n-1}} f(z)$

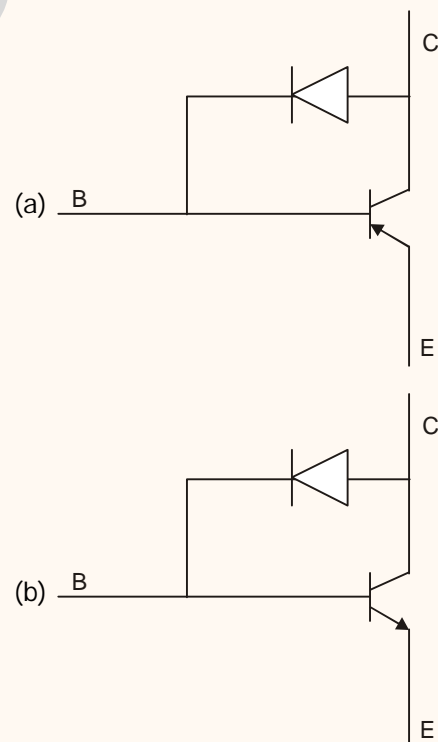
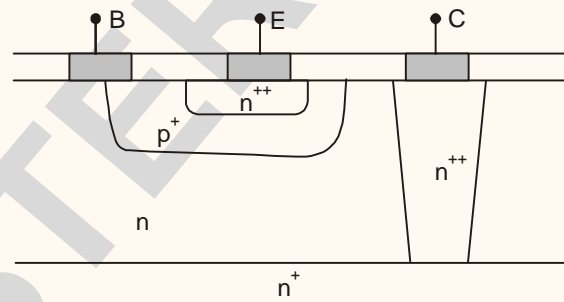
$$I = \left(\frac{1}{2\pi j}\right) \left(\frac{1}{1!}\right) \frac{d}{dz} \left((z^2 + 1)^2\right)$$

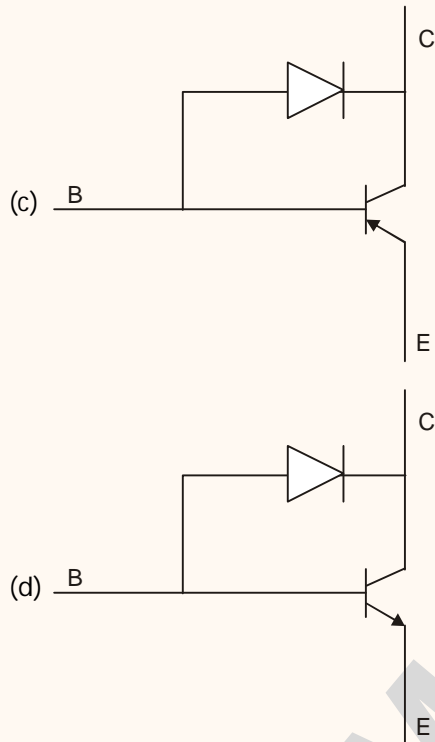
$$I = \frac{1}{2\pi j} (2(z^2 + 1)(2z)) \Big|_{z=0}$$

$$I = 4z(z^2 + 1) \Big|_{z=0}$$

$$I = 0$$

11. The correct circuit representation of the structure shown in the figure is





Ans. (d)

Sol.

- Structure shown is fabrication of npn transistor
 - Emitter highly doped (n^{++})
 - Base lightly doped (p^+)
 - Collector (n^{++})
 - CB junction ($n^{++} - p^+$ junction)
- So Answer d

12. If X and Y are random variables such that $E[2X + Y] = 0$ and $E[X + 2Y] = 33$, then $E[X] + E[Y] = \underline{\hspace{2cm}}$.

Ans. (11)

Sol. $E[2x + y] = 0$
 $E[x + 2y] = 33$
 as we know $E[AX] = AE[x]$
 so $2E[x] + E[y] = 0 \quad \dots(1)$
 $E[x] + 2E[y] = 33$; multiply whole equation by 2
 $2E[x] + 4E[y] = 66 \quad \dots(2)$

Subtracting (2) from (1)

$$-3E[y] = -66$$

$$E[y] = 22$$

and $2E[x] = -22$

hence $E[x] = -11$

so $E[x] + E[y] = -11 + 22 = 11$

13. Let $Y(s)$ be the unit-step response of a causal system having a transfer function

$$G(s) = \frac{3-s}{(s+1)(s+3)}$$

That is, $Y(s) = \frac{G(s)}{s}$. The forced response of the system is

- (a) $u(t) - 2e^{-t}u(t) + e^{-3t}u(t)$
- (b) $2u(t) - 2e^{-t}u(t) + e^{-3t}u(t)$
- (c) $u(t)$
- (d) $2u(t)$

Ans. c

Sol. Total response of system

$$y(s) = \frac{G(s)}{s} = \frac{3-s}{s(s+1)(s+3)}$$

$$y(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$$

$$A = 1, B = -2, C = 1$$

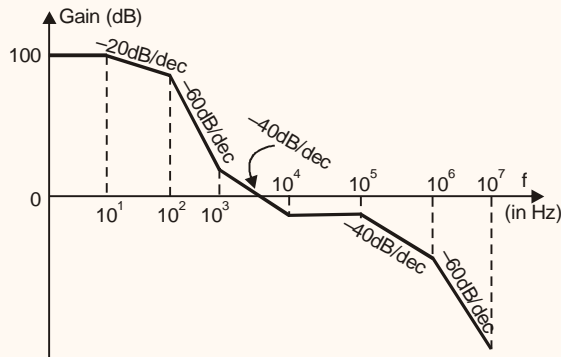
$$y(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{1}{s+3}$$

Taking I.L.T.

$$y(t) = u(t) - 2e^{-t}u(t) + e^{-3t}u(t)$$

Forced response of the system is $u(t)$.

14. For an LTI system, the Bode plot for its gain is as illustrated in the figure shown. The number of system poles N_p and the number of system zeros N_z in the frequency range $1 \text{ Hz} \leq f \leq 10^7 \text{ Hz}$ is



- (a) $N_p = 6, N_z = 3$ (b) $N_p = 4, N_z = 2$
(c) $N_p = 7, N_z = 4$ (d) $N_p = 5, N_z = 2$

Ans. (a)

Sol. When we add one pole, slope becomes -20dB/decade and when we add one zero, slope becomes $+20\text{dB/decade}$

15. Radiation resistance of a small dipole current element of length l at a frequency of 3 GHz is 3 ohms. If the length is changed by 1%, then the percentage change in the radiation resistance, rounded off to two decimal places, is _____ %.

Ans. (2.01)

Sol.

The radiation resistance of small dipole

$$R_{\text{rad}_1} = 80\pi^2 \left(\frac{dl}{\lambda} \right)^2$$

if length is changed by 1% than new length $l_1 = 1.01l$.

$$R_{\text{rad}_2} = 80\pi^2 \left(\frac{1.01l}{\lambda} \right)^2$$

$$R_{\text{rad}_2} = 80\pi^2 \left(\frac{1.021l}{\lambda} \right)^2$$

%change in radiation resistance

$$= \frac{R_{\text{rad}_2} - R_{\text{rad}_1}}{R_{\text{rad}_1}} \times 100$$

$$= \frac{1.0201 - 1}{1} \times 100$$

$$= 2.01\%$$

16. In the table shown, List I and List II, respectively, contain terms appearing on the left-hand side and the right-hand side of Maxwell's equations (in their standard form). Match the left-hand side with the corresponding right-hand side.

List I		List II	
1	$\nabla \cdot \mathbf{D}$	P	0
2	$\nabla \times \mathbf{E}$	Q	ρ
3	$\nabla \cdot \mathbf{B}$	R	$-\frac{\partial \mathbf{B}}{\partial t}$
4	$\nabla \times \mathbf{H}$	S	$\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$

- (a) 1-P, 2-R, 3-Q, 4-S
(b) 1-R, 2-Q, 3-S, 4-P
(c) 1-Q, 2-S, 3-P, 4-R
(d) 1-Q, 2-R, 3-P, 4-S

Ans. d

Sol.

The maxwell eqⁿ (in their standard form)

$$\nabla \cdot \vec{\mathbf{D}} = \rho$$

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\nabla \cdot \vec{\mathbf{B}} = 0$$

$$\nabla \times \vec{\mathbf{H}} = \vec{\mathbf{J}} + \frac{\partial \vec{\mathbf{D}}}{\partial t}$$

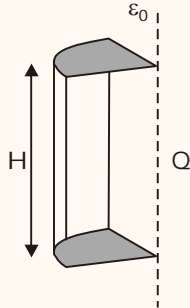
hence 1 \rightarrow Q

2 \rightarrow R

3 \rightarrow P

4 \rightarrow S

17. What is the electric flux ($\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{a}}$) through a quarter-cylinder of height H (as shown in the figure) due to an infinitely long line charge along the axis of the cylinder with a charge density of Q ?



(a) $\frac{HQ}{\epsilon_0}$

(b) $\frac{HQ}{4\epsilon_0}$

(c) $\frac{4H}{Q\epsilon_0}$

(d) $\frac{H\epsilon_0}{4Q}$

Ans. b

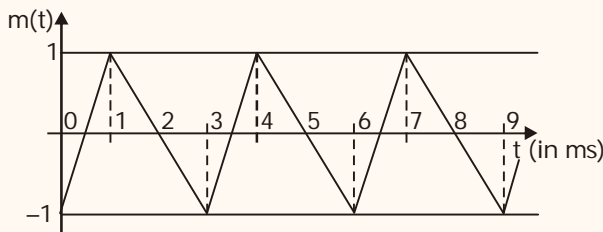
Sol. For quarter-cylinder

$$\int \vec{D} \cdot d\vec{a} = \frac{Q \cdot H}{4}$$

$$\epsilon \int \vec{E} \cdot d\vec{a} = \frac{QH}{4}$$

$$\int \vec{E} \cdot d\vec{a} = \frac{QH}{4\epsilon}$$

18. The baseband signal $m(t)$ shown in the figure is phase-modulated to generate the PM signal $\phi(t) = \cos(2\pi f_c t + km(t))$. The time t on the x-axis in the figure is in milliseconds. If the carrier frequency is $f_c = 50$ kHz and $k = 10\pi$, then the ratio of the minimum instantaneous frequency (in kHz) to the maximum instantaneous frequency (in kHz) is _____ (rounded off to 2 decimal places).



Ans. (0.75)

Sol. $\phi(t) = \cos(2\pi f_c t + km(t))$

$$\theta(t) = 2\pi f_c t + km(t)$$

Instantaneous angular frequency

$$\omega_i = \frac{d\theta(t)}{dt} = 2\pi f_c + k \frac{dm(t)}{dt}$$

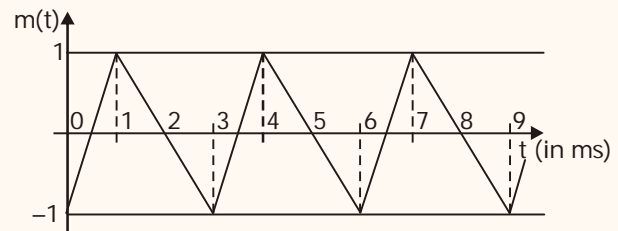
$$2\pi f_i = 2\pi f_c + k \frac{dm(t)}{dt}$$

$$f_i = f_c + \frac{k}{2\pi} \frac{dm(t)}{dt}$$

$$f_i = 50 + \frac{10\pi}{2\pi} \frac{dm(t)}{dt}$$

$$f_i = 50 + 5 \frac{dm(t)}{dt}$$

$$f_{i_{\max}} = 50 + 5 \left. \frac{dm(t)}{dt} \right|_{\max}$$



From figure

$$\left. \frac{dm(t)}{dt} \right|_{\max} = 2$$

$$f_{i_{\max}} = 50 + 5 \times 2 = 60 \text{ kHz}$$

$$f_{i_{\min}} = 50 + 5 \left. \frac{dm(t)}{dt} \right|_{\min}$$

From $m(t)$ figure

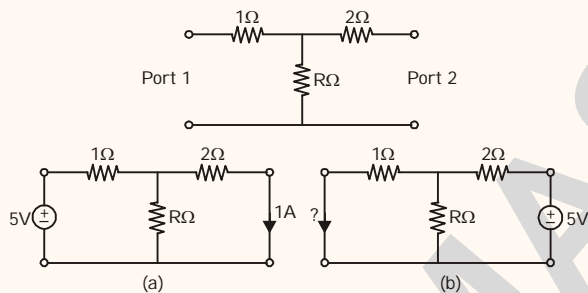
$$\left. \frac{dm(t)}{dt} \right|_{\min} = -1$$

$$f_{i_{\min}} = 50 - 5 = 45 \text{ kHz}$$

$$\frac{f_{i_{\min}}}{f_{i_{\max}}} = \frac{45}{60} = 0.75$$

19. Consider the two-port resistive network shown in the figure. When an excitation of 5V is applied across Port 1, and Port 2 is shorted, the current through the short circuit at Port 2 is measured to be 1 A (see (a) in the figure).

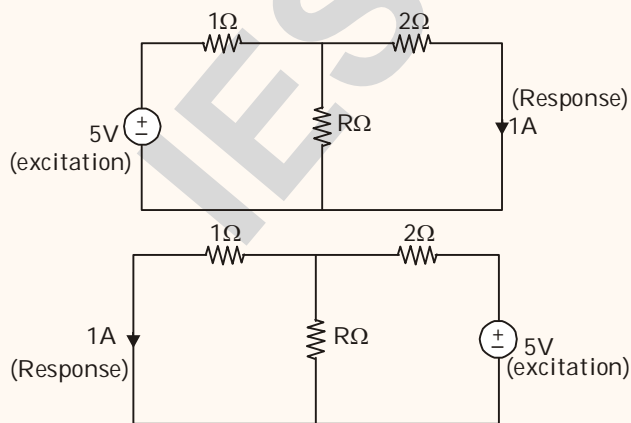
Now, if an excitation of 5V is applied across Port 2, and Port 1 is shorted (see (b) in the figure), what is the current through the short circuit at Port 1?



- (a) 1 A (b) 2.5 A
(c) 2 A (d) 0.5 A

Ans. (a)

Sol.



According to reciprocity theorem

$$\frac{\text{Response}}{\text{Excitation}} = \text{Constant}$$

$$\frac{1}{5} = \frac{I}{5}$$

$$I = 1 \text{ Amp}$$

20. Consider the signal

$$f(t) = 1 + 2\cos(\pi t) + 3\sin\left(\frac{2\pi}{3}t\right) + 4\cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right),$$

where t is in seconds. Its fundamental time period, in seconds, is ____.

Ans. 12

Sol. $f(t) = 1 + 2\cos(\pi t) + 3\sin\left(\frac{2\pi}{3}t\right) + 4\cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)$

Time period of $\cos \pi t$

$$\omega_1 = \pi$$

$$\frac{2\pi}{T_1} = \pi \Rightarrow T_1 = 2$$

Time period $\sin\left(\frac{2\pi}{3}t\right)$

$$\omega_2 = \frac{2\pi}{3}$$

$$\frac{2\pi}{T_2} = \frac{2\pi}{3}$$

$$T_2 = 3$$

Time period $\cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)$

$$\omega_3 = \frac{\pi}{2}$$

$$\frac{2\pi}{T_3} = \frac{\pi}{2}$$

$$T_3 = 4$$

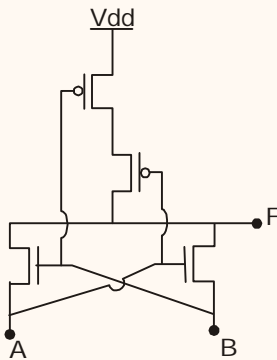
Time period of $f(t)$

$$T = \text{LCM}(T_1, T_2, T_3)$$

$$= \text{LCM}(2, 3, 4)$$

$$= 12$$

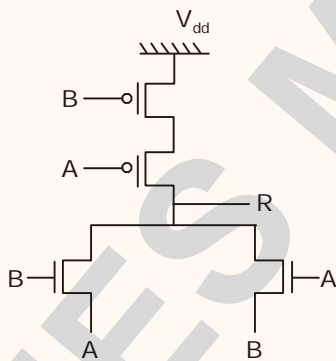
21. In the circuit shown, A and B are the inputs and F is the output. What is the functionality of the circuit?



- (a) XNOR (b) SRAM Cell
(c) XOR (d) Latch

Ans. (a)

Circuit can be redrawn as:



A	B	R
0	0	1
0	1	0
1	0	0
1	1	1

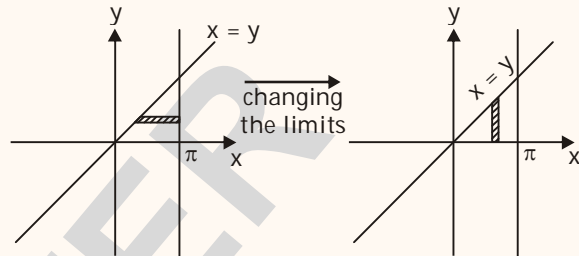
(XNOR gate)

22. The value of the integral $\int_0^{\pi} \int_y^{\pi} \frac{\sin x}{x} dx dy$, is equal to _____.

Ans. (2)

Sol.

$$I = \int_0^{\pi} \int_y^{\pi} \frac{\sin x}{x} \cdot dx \cdot dy$$



After changing the limits

$$I = \int_0^{\pi} \int_0^x \frac{\sin x}{x} \cdot dy \cdot dx$$

$$I = \int_0^{\pi} \frac{\sin x}{x} (y)_0^x \cdot dx$$

$$I = \int_0^{\pi} \frac{\sin x}{x} (x) \cdot dx$$

$$I = \int_0^{\pi} \sin x \cdot dx$$

$$I = (-\cos x)_0^{\pi}$$

$$I = (1 - (-1))$$

$$\boxed{I = 2}$$

23. The number of distinct eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 2 & 3 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

is equal to _____.

Ans. (3)

Sol. $A = \begin{bmatrix} 2 & 2 & 3 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

Given 'A' matrix is an upper triangular matrix so, diagonal elements are its eigen values $\lambda_1 = 2, \lambda_2 = 1, \lambda_3 = 3, \lambda_4 = 2$

so, there are three distinct eigen values of the matrix.

24. Let Z be an exponential random variable with mean 1. That is, the cumulative distribution function of Z is given by

$$F_z(x) = \begin{cases} 1 - e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Then $\Pr(Z > 2 \mid Z > 1)$, rounded off to two decimal places, is equal to _____.

Ans. (0.367)

Sol. Cumulative distribution function of Z is

$$F_z(x) = \begin{cases} 1 - e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

$$\text{Pdf } f_z(x) = \frac{dF_z(x)}{dx}$$

$$f_z(x) = \begin{cases} e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

$$\rho\left(\frac{z > 2}{z > 1}\right) = \frac{\rho((z > 2) \cap (z > 1))}{\rho(z > 1)}$$

$$= \frac{\rho(z > 2)}{\rho(z > 1)}$$

$$\rho(z > 2) = \int_2^{\infty} e^{-x} dx = e^{-2}$$

$$\rho(z > 1) = \int_1^{\infty} e^{-x} dx = e^{-1}$$

$$\text{hence } \rho\left(\frac{z > 2}{z > 1}\right) = \frac{\rho(z > 2)}{\rho(z > 1)} = \frac{e^{-2}}{e^{-1}} = e^{-1} = 0.367$$

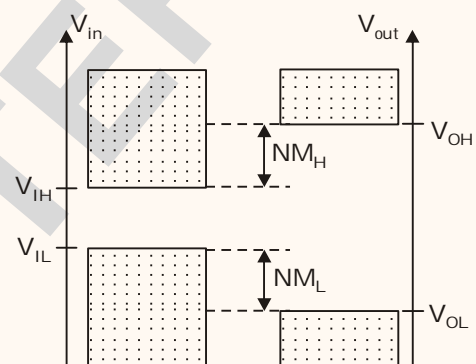
25. A standard CMOS inverter is designed with equal rise and fall times ($\beta_n = \beta_p$). If the width of the pMOS transistor in the inverter is increased, what would be the effect on the

LOW noise margin (NM_L) and the HIGH noise margin NM_H ?

- (a) NM_L increases and NM_H decreases
- (b) NM_L decreases and NM_H increases
- (c) No change in the noise margin
- (d) Both NM_L and NM_H increase

Ans. (a)

Sol.



$$NM_L = V_{IL} - V_{OL}$$

$$NM_H = V_{OH} - V_{IH}$$

For CMOS inverter with equal rise and fall time we have

$$V_{OH} = V_{DD}$$

$$V_{OL} = 0$$

So, $NM_L = V_{IL} - 0 = V_{IL}$

$$NM_H = V_{DD} - V_{IH}$$

$$\text{where } V_{IL} = \frac{2V_{out} + V_{T,p} + \frac{k_n}{k_p} V_{T,n} - V_{DD}}{\left(1 + \frac{k_n}{k_p}\right)}$$

As width pMOS increased k_p increases & V_{IL} increases

So, NM_L increases

$$V_{IH} = \frac{V_{DD} + V_{T,p} + \frac{k_n}{k_p} (2V_{out} + V_{T,n})}{1 + \frac{k_n}{k_p}}$$

V_{IH} increases and hence NM_H decreases.

26. Let $h[n]$ be a length-7 discrete-time finite impulse response filter, given by
 $h[0] = 4, h[1] = 3, h[2] = 2, h[3] = 1,$
 $h[-1] = -3, h[-2] = -2, h[-3] = -1,$

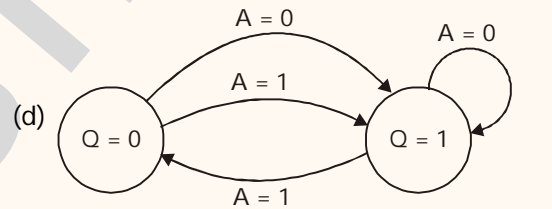
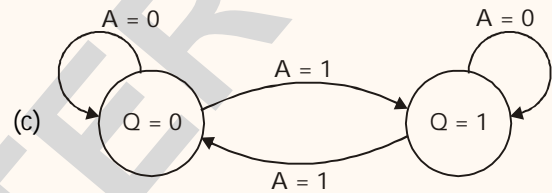
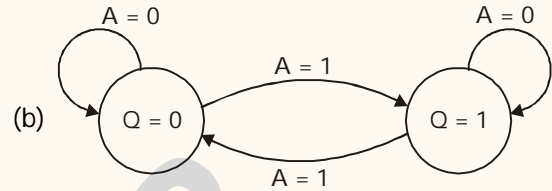
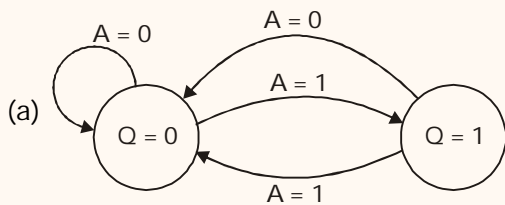
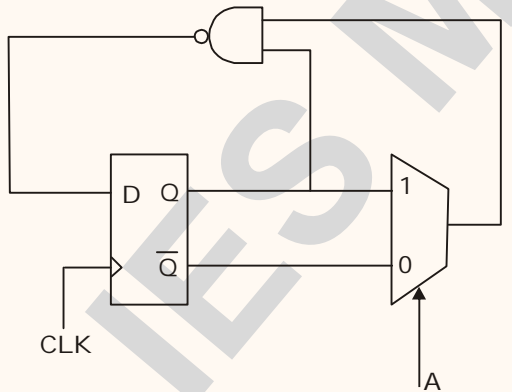
and $h[n]$ is zero for $|n| \geq 4$. A length-3 finite impulse response approximation $g[n]$ of $h[n]$ has to be obtained such that

$$E(h, g) = \int_{-\pi}^{\pi} |H(e^{j\omega}) - G(e^{j\omega})|^2 d\omega$$

is minimized, where $H(e^{j\omega})$ and $G(e^{j\omega})$ are the discrete-time Fourier transforms of $h[n]$ and $g[n]$, respectively. For the filter that minimizes $E(h, g)$, the value of $10g[-1] + g[1]$, rounded off to 2 decimal places, is ____.

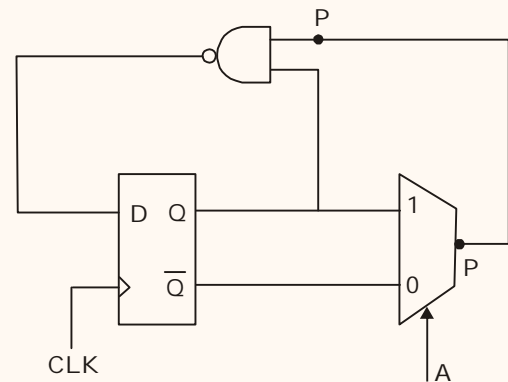
Ans. (*)

27. The state transition diagram for the circuit shown is



Ans. (d)

Sol.



$$\text{Now, } P = \bar{A}\bar{Q} + AQ$$

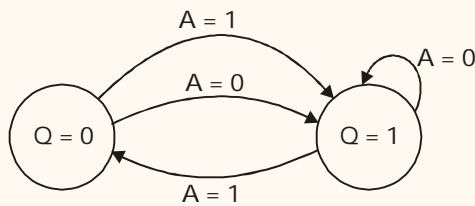
$$P = A \odot Q$$

$$D = \overline{P \cdot \bar{Q}}$$

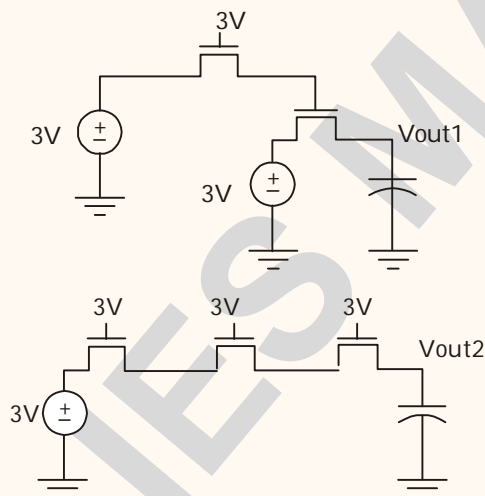
$$D = \overline{(A \odot Q) \cdot \bar{Q}} = \overline{A \odot Q} + \bar{Q}$$

$$D = (A \oplus Q) + \bar{Q}$$

Present state (Q)	Input	Next state (Q ⁺)
0	0	1
0	1	1
1	0	1
1	1	0



28. In the circuits shown, the threshold voltage of each nMOS transistor is 0.6V. Ignoring the effect of channel length modulation and body bias, the value of V_{out1} and V_{out2} , respectively, in volts are



- (a) 2.4 and 1.2 (b) 1.8 and 1.2
(c) 2.4 and 2.4 (d) 1.8 and 2.4

Ans. (d)

Sol. For nMoS at saturation

$$V_D \approx V_G - V_T$$

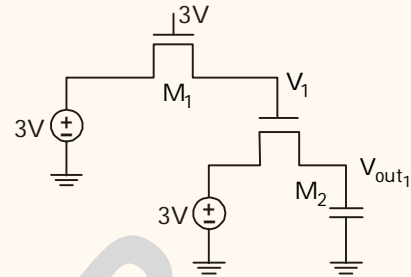


Figure 1

For nMOS 1 (M_1)

$$V_1 \approx 3 - 0.6$$

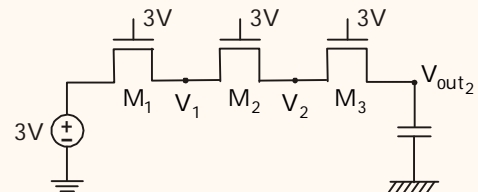
$$V_1 = 2.4$$

For nMOS 2 (M_2)

$$V_{out1} = V_1 - V_T$$

$$= 2.4 - 0.6$$

$$V_{out1} = 1.8$$



From figure (1)

$$\text{For } M_1, V_1 = 3 - 0.6 = 2.4$$

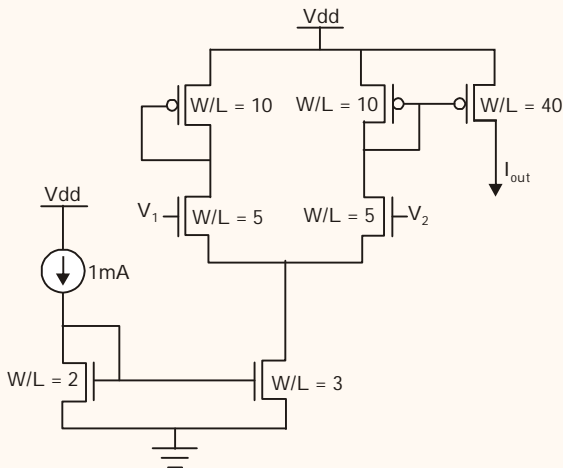
$$\text{For } M_2, V_2 = 3 - 0.6 = 2.4$$

$$\text{For } M_3, V_{out2} = 3 - 0.6 = 2.4$$

$$\text{Hence } V_{out1} = 1.8 \text{ V}$$

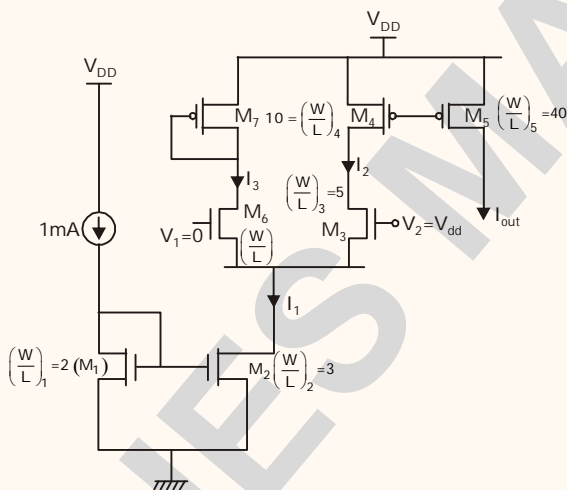
$$V_{out2} = 2.4 \text{ V}$$

29. In the circuit shown, $V_1 = 0$ and $V_2 = V_{dd}$. The other relevant parameters are mentioned in the figure. Ignoring the effect of channel length modulation and the body effect, the value of I_{out} is ___ mA (rounded off to 1 decimal place).



Ans. (6)

Sol.



When $V_1 = 0$, M_6 is OFF hence $I_3 = 0$ and

$$\frac{I_1}{1\text{mA}} = \frac{(W/L)_2}{(W/L)_1}$$

$$I_1 = \frac{3}{2} \times 1\text{mA} = 1.5 \text{ mA}$$

When $I_3 = 0$,

$$I_1 = I_2 = 1.5 \text{ mA}$$

$$\frac{I_{\text{out}}}{I_2} = \frac{(W/L)_5}{(W/L)_4} = \frac{40}{10}$$

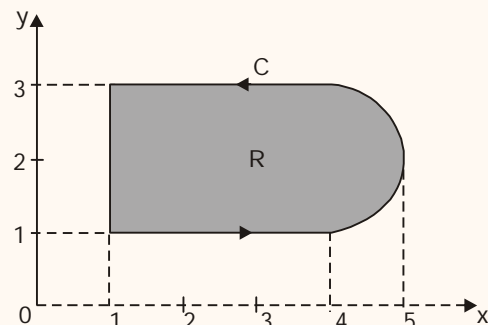
$$\frac{I_{\text{out}}}{I_2} = 4$$

$$I_{\text{out}} = 4 \times 1.5 \text{ mA} = 6 \text{ mA}$$

30. Consider the line integral

$$\int_C (x dy - y dx)$$

the integral being taken in a counter-clockwise direction over the closed curve C that forms the boundary of the region R shown in the figure below. The region R is the area enclosed by the union of a 2×3 rectangle and a semi-circle of radius 1. The line integral evaluates to



(a) $12 + \pi$

(b) $16 + \pi$

(c) $8 + \pi$

(d) $6 + \frac{\pi}{2}$

Ans. a

Sol. $\int_C x dy - y dx$

$$I = \int_C -y dx + x dy$$

$$I = \int_C (-y\hat{i} + x\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$I = \int_c \vec{F} \cdot d\vec{l}$$

By comparison we get

$$\vec{F} = -y\hat{i} + x\hat{j}$$

now By stoke's theorem

$$\int_c \vec{F} \cdot d\vec{l} = \iint_s (\vec{\nabla} \times \vec{F}) \cdot \vec{ds}$$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & x & 0 \end{vmatrix}$$

$$\vec{\nabla} \times \vec{F} = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(1-(-1))$$

$$\vec{\nabla} \times \vec{F} = 2\hat{k}$$

$$\text{and } \vec{ds} = (dx dy)\hat{k}$$

$$\text{so, } \int_c \vec{F} \cdot d\vec{l} = \iint_s (\vec{\nabla} \times \vec{F}) \cdot \vec{ds}$$

$$I = \iint_s (2\hat{k}) \cdot (dx dy)\hat{k}$$

$$I = 2 \iint_s dx dy$$

$$I = 2 \left[(2 \times 3) + \frac{\pi(1)^2}{2} \right]$$

$$I = 12 + \pi$$

31. The dispersion equation of a waveguide, which relates the wavenumber k to the frequency ω is

$$k(\omega) = (1/c)\sqrt{\omega^2 - \omega_0^2}$$

where the speed of light $c = 3 \times 10^8$ m/s, and ω_0 is a constant. If the group velocity is 2×10^8 m/s, then the phase velocity is

- (a) 4.5×10^8 m/s (b) 2×10^8 m/s
(c) 1.5×10^8 m/s (d) 3×10^8 m/s

Ans. (a)

$$\text{Sol. } K(\omega) = \left(\frac{1}{C}\right)\sqrt{\omega^2 - \omega_0^2}$$

$$V_g = 2 \times 10^8 \text{ m/s}$$

$$V_g = \frac{d\omega}{dK(\omega)}$$

$$\text{or } \frac{dk(\omega)}{d\omega} = \frac{1}{V_g}$$

$$\Rightarrow \frac{1 \times 2\omega}{2c\sqrt{\omega^2 - \omega_0^2}} = \frac{1}{V_g}$$

$$\Rightarrow \frac{\omega}{(3 \times 10^8)\sqrt{\omega^2 - \omega_0^2}} = \frac{1}{2 \times 10^8}$$

$$\Rightarrow \sqrt{\omega^2 - \omega_0^2} = \frac{2\omega}{3}$$

$$\therefore V_p = \frac{\omega}{K(\omega)} = \frac{\omega C}{\sqrt{\omega^2 - \omega_0^2}}$$

$$\Rightarrow \frac{\omega C}{(2\omega/3)}$$

$$V_p = \frac{3C}{2} = \frac{3 \times 10^8 \times 3}{2} = 4.5 \times 10^8 \text{ m/s}$$

32. Consider a differentiable function $f(x)$ on the set of real numbers such that $f(-1) = 0$ and $|f'(x)| \leq 2$. Given these conditions, which one of the following inequalities is necessarily true for all $x \in [-2, 2]$?

- (a) $f(x) \leq 2|x+1|$ (b) $f(x) \leq \frac{1}{2}|x+1|$
(c) $f(x) \leq 2|x|$ (d) $f(x) \leq \frac{1}{2}|x|$

Ans. (a)

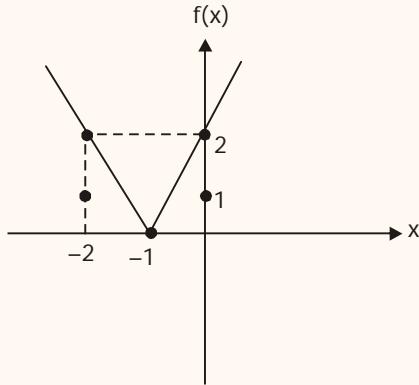
Sol. Conditions given

$$f(-1) = 0$$

$$\text{and } |f'(x)| \leq 2$$

Consider option (1)

$$f(x) = 2|x + 1|$$

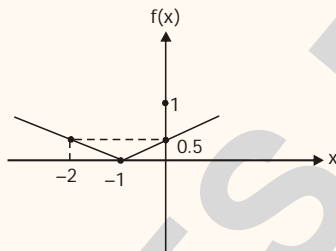


$f(-1) = 0$, satisfied

$|f'(x)| \leq 2$, satisfied

Consider option (2)

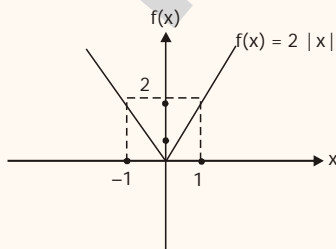
$$f(x) = \frac{1}{2}|x + 1|$$



$f(-1) = 0$, satisfied

If $|f'(x)| \leq 2$, not satisfied

Consider option (3)

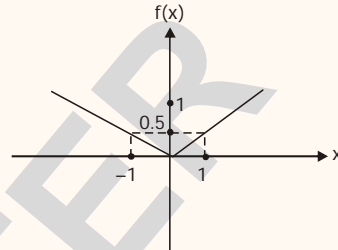


$f(-1) = 0$, not satisfied

$|f'(x)| \leq 2$, satisfied

consider option (4)

$$f(x) = \frac{1}{2}|x|$$



$f(-1) = 0$, not satisfied

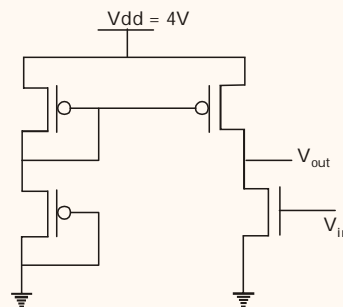
$|f'(x)| \leq 2$, not satisfied

- 33.** In the circuit shown, the threshold voltages of the pMOS ($|V_{tp}|$) and nMOS (V_{tn}) transistors are both equal to 1V. All the transistors have the same output resistance r_{ds} of $6 \text{ M}\Omega$. The other parameters are listed below:

$$\mu_n C_{ox} = 60 \mu\text{A} / \text{V}^2; \left(\frac{W}{L}\right)_{\text{nMOS}} = 5$$

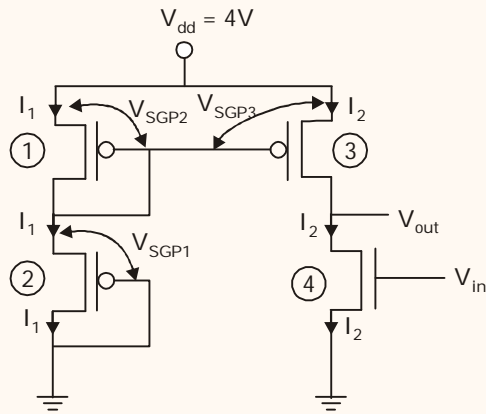
$$\mu_p C_{ox} = 30 \mu\text{A} / \text{V}^2; \left(\frac{W}{L}\right)_{\text{pMOS}} = 10$$

μ_n and μ_p are the carrier mobilities, and C_{ox} is the oxide capacitance per unit area. Ignoring the effect of channel length modulation and body bias, the gain of the circuit is ____ (rounded off to 1 decimal place).



Ans. (-900)

Sol.



$$|V_{tp}| = V_{tn} = 1V, r_{ds} = 6M\Omega$$

$$\mu_n C_{ox} = 60 \mu A / V^2; \left(\frac{W}{L}\right)_{nMOS} = 5$$

$$\mu_p C_{ox} = 30 \mu A / V^2; \left(\frac{W}{L}\right)_{pMOS} = 10$$

\therefore Current through M_1 & M_2 is same i.e. I_1

$$\therefore V_{SGP1} = V_{SGP2} = \frac{V_{DD}}{2} = \frac{4}{2} = 2V$$

\therefore MOS (3) and MOS (1) are in parallel

$$\therefore V_{SGP3} = V_{SGP1} = 2 \text{ volt}$$

$$I_2 = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_P [V_{SGP3} - |V_{tp}|]^2$$

$$= \frac{1}{2} \times 30 \times 10^{-3} [10 [2 - 1]^2]$$

$$I_2 = 0.15mA \text{ or } 150\mu A$$

Same current I_2 will pass through M_4 .

$$g_{m4} = \frac{\partial I_{DS}}{\partial V_{GS}} = K'_n \frac{W}{L} (V_{GS} - V_{tn})$$

$$K'_n = \mu_n C_{ox} = 60 \mu A / V^2$$

$$V_{GS} - V_{tn} = \sqrt{\frac{2I_{DS4}}{K'_n W/L}}$$

$$\therefore I_{DS} = \frac{1}{2} K'_n \frac{W}{L} (V_{GS} - V_t)^2$$

$$\therefore g_{m4} = \sqrt{2I_{DS4}} \sqrt{K'_n W/L}$$

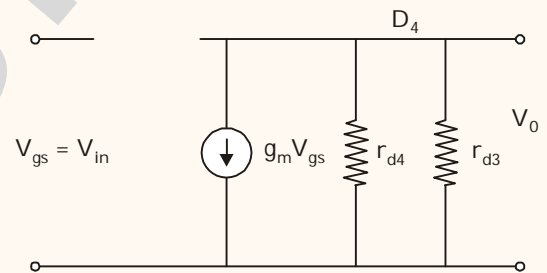
$$I_{DS4} = I_2 = 150\mu A$$

$$g_{m4} = \sqrt{2 \times 150} \sqrt{60 \times 5}$$

$$g_{m4} = 300 \mu A / V$$

Now for gain [AC analysis] $V_{DD} = 0$

MOS-3 is replaced by $r_{d3} = 6M\Omega$ and MOS-4 is replaced by small signal model. [$\therefore M_3$ act as load]



$$\frac{V_0}{V_{in}} = \frac{-g_m V_{gs} r_{d4} \parallel r_{d3}}{V_{gs}} = -g_m r_{d4} \parallel r_{d3}$$

$$= -300 \times 10^{-6} \times [6 \times 10^6 \parallel 6 \times 10^6]$$

$$= -900$$

34. Let the state-space representation of an LTI system be $x(t) = Ax(t) + Bu(t)$, $y(t) = Cx(t) + du(t)$ where A, B, C are matrices, d is a scalar, $u(t)$ is the input to the system, and $y(t)$ is its output. Let $B = [0 \ 0 \ 1]^T$ and $d = 0$. Which one of the following options for A and C will ensure that the transfer function of this LTI system is

$$H(s) = \frac{1}{s^3 + 3s^2 + 3s + 1} ?$$



(a) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$ and $C = [0 \ 0 \ 1]$

(b) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix}$ and $C = [1 \ 0 \ 0]$

(c) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$ and $C = [1 \ 0 \ 0]$

(d) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix}$ and $C = [0 \ 0 \ 1]$

Ans. (c)

Sol. Given transfer function

$$\frac{Y(s)}{U(s)} = H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1}$$

$$\Rightarrow \frac{Y(s)}{X_1(s)} \cdot \frac{X_1(s)}{U(s)} = 1 \times \frac{1}{s^3 + 3s^2 + 2s + 1}$$

$$\therefore \frac{X_1(s)}{U(s)} = \frac{1}{s^3 + 3s^2 + 2s + 1}$$

$$\Rightarrow s^3 X_1(s) + 3s^2 X_1(s) + 2s X_1(s) + X_1(s) = U(s)$$

$$\ddot{x}_1 + 3\dot{x}_1 + 2\dot{x}_1 + x_1 = U(s)$$

let $\dot{x}_1 = x_2$... (i)

$\ddot{x}_1 = \dot{x}_2 = x_3$... (ii)

$\ddot{x}_1 = \dot{x}_3 = \tau$

$\dot{x}_3 = U(s) - 2x_2 - 3x_3 - x_1$... (iii)

By using (i), (ii) & (iii)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U(s)$$

$\dot{x} = AX + BU$... (iv)

and $\frac{Y(s)}{X_1(s)} = 1$

$Y(s) = X_1(s)$

$y = x_1$

$$[y] = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = CX \quad \dots (v)$$

from equation (iv) & (v)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$$

$$C = [1 \ 0 \ 0]$$

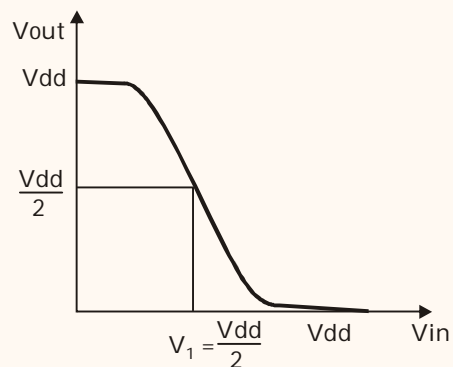
35. A CMOS inverter, designed to have a mid-point voltage V_1 equal to half of V_{dd} , as shown in the figure, has the following parameters :

$V_{dd} = 3V$

$\mu_n C_{ox} = 100 \mu A / V^2$; $V_{tn} = 0.7V$ for nMOS

$\mu_p C_{ox} = 40 \mu A / V^2$; $V_{tp} = 0.9V$ for pMOS

The ratio of $\left(\frac{W}{L}\right)_n$ to $\left(\frac{W}{L}\right)_p$ is equal to _____ (rounded off to 3 decimal places).



Ans. (0.225)

Sol. At threshold $V_{in} = V_{out} = V_{th}$

Using current equation, we get

$$V_{th} = \frac{V_{T,n} + \sqrt{\frac{K_P}{K_n}} (V_{DD} + V_{T,P})}{1 + \sqrt{\frac{K_P}{K_n}}}$$

For $V_{th} = \frac{V_{DD}}{2}$ we get

$$\left(\frac{K_n}{K_p}\right) = \left(\frac{\frac{V_{DD}}{2} + V_{T,P}}{\frac{V_{DD}}{2} - V_{T,n}}\right)^2$$

$$\frac{\mu_n C_{ox} \left(\frac{W}{L}\right)_n}{\mu_p C_{ox} \left(\frac{W}{L}\right)_p} = \left(\frac{1.5 + (-0.9)}{1.5 - 0.7}\right)^2$$

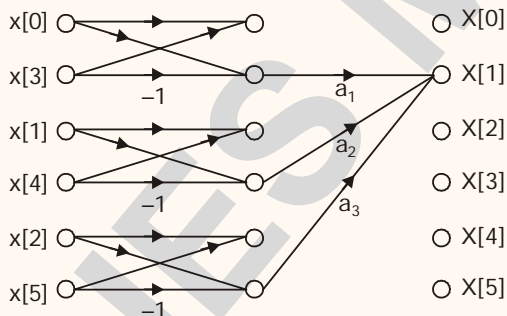
$$\frac{100(W/L)_n}{40(W/L)_p} = \left(\frac{0.6}{0.8}\right)^2$$

$$\frac{(W/L)_n}{(W/L)_p} = \frac{4}{10} \times \frac{36}{64} = 0.225$$

36. Consider a six-point decimation-in-time Fast Fourier Transform (FFT) algorithm, for which the signal-flow graph corresponding to $X[1]$ is shown in the figure. Let

$W_6 = \exp\left(-\frac{j2\pi}{6}\right)$. In the figure, what should

be the values of the coefficients a_1, a_2, a_3 in terms of W_6 so that $X[1]$ is obtained correctly?



- (a) $a_1 = 1, a_2 = W_6, a_3 = W_6^2$
 (b) $a_1 = 1, a_2 = W_6^2, a_3 = W_6$
 (c) $a_1 = -1, a_2 = W_6^2, a_3 = W_6$
 (d) $a_1 = -1, a_2 = W_6, a_3 = W_6^2$

Ans. a

Sol. Six point DFT of $x(n)$

$$X(k) = \sum_{n=0}^5 x(n)W_6^{kn}$$

$$X(1) = x(0) \cdot W_6^0 + x(1) W_6^1 + x(2) W_6^2 + x(3) W_6^3 + x(4) W_6^4 + x(5) W_6^5$$

$$x(1) = x(0) \cdot 1 + [x(1) - x(4)]W_6^1 + [x(2) - x(5)]W_6^2 + x(3)W_6^3$$

$$\left[\begin{aligned} \because W_6^4 &= W_6^3 - W_6^1 = -W_6^1 \text{ and } W_6^5 \\ &= W_6^2 \cdot W_6^3 = -W_6^2 \end{aligned} \right]$$

from signal-flow graph

$$X(1) = a_1[x(0) - x(3)] + [x(1) - x(4)]a_2 + [x(2) - x(5)]a_3$$

on comparing

$$a_1 = 1, a_2 = W_6^1, a_3 = W_6^2$$

37. A rectangular waveguide of width w and height h has cut-off frequencies for TE_{10} and TE_{11} modes in the ratio 1 : 2. The aspect ratio w/h , rounded off to two decimal place, is

Ans. 1.732

Sol. $f_c = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

Cut-off frequency for TE_{10}

$$f_{c1} = \frac{c}{2W}$$

Cut off frequency for TE_{11}

$$f_{c2} = \frac{c}{2} \sqrt{\left(\frac{1}{W}\right)^2 + \left(\frac{1}{h}\right)^2}$$

$$\frac{f_{c1}}{f_{c2}} = \frac{\frac{c}{2} \cdot \frac{1}{W}}{\frac{c}{2} \sqrt{\left(\frac{1}{W}\right)^2 + \left(\frac{1}{h}\right)^2}}$$

$$\frac{1}{2} = \frac{\frac{1}{W}}{\sqrt{\left(\frac{1}{W}\right)^2 + \left(\frac{1}{h}\right)^2}} \quad \left[\because \text{Given } \frac{f_{c1}}{f_{c2}} = \frac{1}{2} \right]$$

$$\left(\frac{1}{W}\right)^2 + \left(\frac{1}{h}\right)^2 = 4\left(\frac{1}{W}\right)^2$$

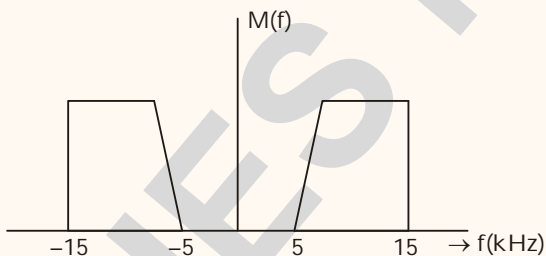
$$\left(\frac{1}{h}\right)^2 = 3\left(\frac{1}{W}\right)^2$$

$$\frac{W}{h} = \sqrt{3} = 1.732$$

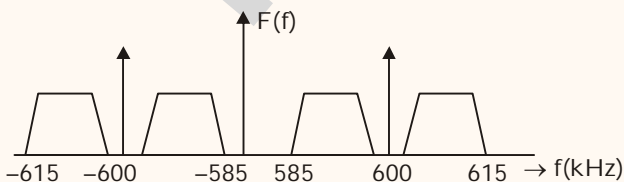
38. A voice signal $m(t)$ is in the frequency range 5 KHz. The signal is amplitude-modulated to generate an AM signal $f(t) = A(1 + m(t)) \cos 2\pi f_c t$, where $f_c = 600$ kHz. The AM signal $f(t)$ is to be digitized and archived. This is done by first sampling $f(t)$ at 1.2 times the Nyquist frequency, and then quantizing each sample using a 256-level quantizer. Finally, each quantized sample is binary coded using K bits, where K is the minimum number of bits required for the encoding. The rate, in Megabits per second (rounded off to 2 decimal places), of the resulting stream of coded bits is ____ Mbps.

Ans. (0.5904)

Sol. $m(t)$ is band limited 5kHz to 15 kHz
Let



After modulation



It is a case of band pass sampling

$$\text{Nyquist frequency} = \frac{2f_H}{k}$$

where $k = \left\lceil \frac{f_H}{BW} \right\rceil$

$$k = \left\lceil \frac{615}{30} \right\rceil = \lceil 20.5 \rceil$$

Integer part of 20.5 is 20 hence

$$k = 20$$

$$\text{Nyquist frequency} = \frac{2 \times 615}{20} = 61.5 \text{ kHz}$$

$$\begin{aligned} \text{Sampling frequency} &= 1.2 \times \text{Nyquist frequency} \\ &= 1.2 \times 61.5 \\ &= 73.8 \text{ kHz} \end{aligned}$$

$$\text{Quantization level} = 256$$

$$2^n = 256$$

$$n = 8$$

$$\text{The rate} = n f_s$$

$$= 8 \times 73.8$$

$$= 590.4 \text{ kbps}$$

$$= 0.5904 \text{ Mbps}$$

39. The quantum efficiency (η) and responsivity (R) at a wavelength λ (in μm) in a p-i-n photodetector are related by

$$(a) R = \frac{\eta \times \lambda}{1.24}$$

$$(b) R = \frac{1.24}{\eta \times \lambda}$$

$$(c) R = \frac{\lambda}{\eta \times 1.24}$$

$$(d) R = \frac{1.24 \times \lambda}{\eta}$$

Ans. (a)

Sol. Responsivity (R) = $\frac{\text{Photon current}}{\text{Optical power}} = \frac{I_p}{P_{\text{opt}}}$

$$R = \frac{e r_e}{h \nu r_p} = \frac{e}{h \nu} (\eta)$$

where quantum efficiency (η) = $\frac{r_e}{r_p}$

$$= \frac{e^- (\text{s}) \text{ generation rate}}{\text{incident photon rate}}$$

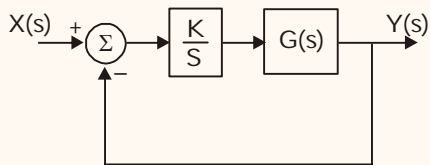
$$R = \frac{\eta e}{h \nu} \left[\because h \nu = \frac{1.24 (\text{eV})}{\lambda (\mu\text{m})} \right]$$

$$R = \frac{\eta \lambda}{1.24}$$

40. Consider a unity feedback system, as in the figure shown, with an integral compensator $\frac{k}{s}$ and open-loop transfer function

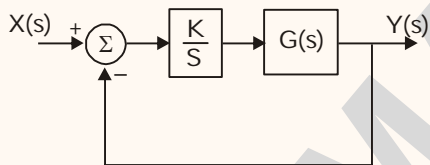
$$G(s) = \frac{1}{s^2 + 3s + 2}$$

where $k > 0$. The positive value of K for which there are exactly two poles of the unity feedback system on the $j\omega$ axis is equal to _____ (rounded off to two decimal places).



Ans. (6)

Sol.



$$G(s) = \frac{1}{s^2 + 3s + 2}$$

Characteristic equation :

$$1 + G'(s)H(s) = 0 \quad \left[G'(s) = \frac{k}{s}G(s) \right]$$

$$1 + \frac{k}{s(s^2 + 3s + 2)} = 0$$

$$s^3 + 3s^2 + 2s + k = 0$$

Using routh array:

s^3	1	2
s^2	3	k
s^1	$\frac{6-k}{3}$	
s^0	k	

$$k > 0$$

For two poles on $j\omega$ -axis

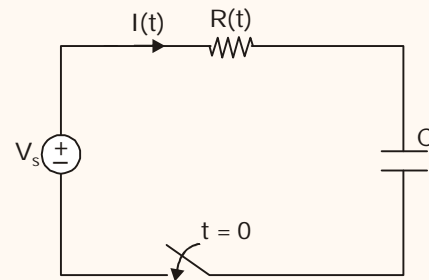
$$\frac{6-k}{3} = 0$$

$$k = 6$$

41. The RC circuit shown below has a variable resistance $R(t)$ given by the following expression:

$$R(t) = R_0 \left(1 - \frac{t}{T} \right) \text{ for } 0 \leq t < T$$

where $R_0 = 1\Omega$, and $C = 1F$. We are also given that $T = 3R_0C$ and the source voltage is $V_s = 1V$. If the current at time $t = 0$ is $1A$, then the current $I(t)$, in amperes, at time $t = T/2$ is _____ (rounded off to 2 decimal places).



Ans. (0.25)

Sol.

Applying KCL, we get

$$C \cdot \frac{dV(t)}{dt} + \frac{V(t) - V_s}{R(t)} = 0$$

$$\Rightarrow \frac{dV(t)}{dt} + \frac{V(t) - 1}{\left(1 - \frac{t}{3}\right)} = 0$$

$$\Rightarrow \frac{dV(t)}{V(t) - 1} = -\frac{dt}{1 - \left(\frac{t}{3}\right)}$$

$$\Rightarrow \frac{dV(t)}{V(t) - 1} = \frac{3dt}{(t - 3)}$$

Integrating on both sides, we get

$$\ln[V(t) - 1] = 3\ln[t - 3] + \ln k$$

$$\Rightarrow \ln[V(t) - 1] - \ln[t - 3]^3 = \ln k$$

$$\Rightarrow \ln \left[\frac{V(t) - 1}{(t - 3)^3} \right] = \ln k$$

$$\Rightarrow V(t) - 1 = K \cdot (t - 3)^3$$

$$\Rightarrow \boxed{V(t) = 1 + K \cdot (t - 3)^3}$$

$$i(t) = C \cdot \frac{dV(t)}{dt} = \frac{dV(t)}{dt} = 3K(t - 3)^2$$

$$i(t = 0) = 3K \cdot (-3)^2 = 27K = 1$$

$$\Rightarrow K = \frac{1}{27}$$

$$\text{So, } i(t = 1.5) = 3K(1.5 - 3)^2$$

$$= 3K \cdot \frac{9}{4} = 3 \times \frac{1}{27} \times \frac{9}{4}$$

$$= \frac{1}{4} \text{ A}$$

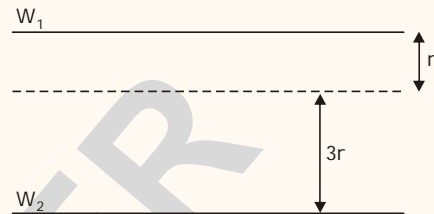
42. Two identical copper wires W1 and W2, placed in parallel as shown in the figure, carry currents I and $2I$, respectively, in opposite directions. If the two wires are separated by a distance of $4r$, then the magnitude field \vec{B} between the wires at a distance r from W1 is



- (a) $\frac{\mu_0 I}{6\pi r}$ (b) $\frac{\mu_0 I^2}{2\pi r^2}$
(c) $\frac{6\mu_0 I}{5\pi r}$ (d) $\frac{5\mu_0 I}{6\pi r}$

Ans. d

Sol.



When current in W_1 and W_2 in opposite direction hence at r distance from W_1 is in same direction hence

$$B = B_1 + B_2$$

magnetic field due to W_1

$$B_1 = \frac{\mu_0 I}{2\pi r}$$

Magnetic field due to W_2

$$B_2 = \frac{\mu}{2\pi} \frac{2I}{3r}$$

Total magnetic field

$$B = B_1 + B_2$$

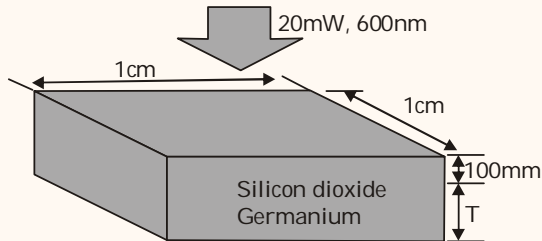
$$= \frac{\mu}{2\pi} \frac{I}{r} + \frac{\mu}{2\pi} \frac{2I}{3r}$$

$$= \frac{\mu_0 I}{2\pi r} \left[1 + \frac{2}{3} \right]$$

$$B = \frac{5\mu_0 I}{6\pi r}$$

43. A Germanium sample of dimensions $1 \text{ cm} \times 1 \text{ cm}$ is illuminated with a 20 mW , 600 nm laser light source as shown in the figure. The illuminated sample surface has a 100 nm of lossless Silicon dioxide layer that reflects one-fourth of the incident light. From the remaining light, one-third of the power is reflected from the Silicon dioxide-Germanium interface, one-third is absorbed in the Germanium layer, and one-third is transmitted through the other side of the sample. If the absorption coefficient of Germanium at 600 nm is $3 \times 10^4 \text{ cm}^{-1}$ and the bandgap is 0.66 eV , the thickness of the

Germanium layer, rounded off to 3 decimal places is, _____ μm .



Ans. (0.231)

Sol.

Power incident $p_i = 20\text{mW}$

Power reflected from $\text{SiO}_2 = \frac{p_i}{4}$... (i)

Power reflected from $\text{SiO}_2 - \text{Ge}$ interface

$$= \frac{1}{3} \left(p_i - \frac{p_i}{4} \right) = \frac{p_i}{4} \quad \dots \text{(ii)}$$

Power absorbed in Ge = $\frac{1}{3} \left(p_i - \frac{p_i}{4} \right) = \frac{p_i}{4}$... (iii)

Power transmitted through sample = $\frac{1}{3} \left(p_i - \frac{p_i}{4} \right)$

$$= \frac{p_i}{4} \quad \dots \text{(iv)}$$

Power entered in Ge sample = (iii) + (iv) = $\frac{2p_i}{4}$

Power transmitted through thickness (t) = $\frac{p_i}{4}$

$$p_t = p_0 e^{-\alpha t}$$

$$\frac{p_i}{4} = \frac{2p_i}{4} e^{-\alpha t} \Rightarrow \alpha t = \ln 2$$

$$t = \frac{\ln 2}{2} = \frac{\ln 2}{3 \times 10^4} (\text{cm}) = 0.231 \times 10^{-4} \text{ cm}$$

$$t = 0.231 \times 10^{-6} \text{ m} \quad \boxed{t = 0.231 \mu\text{m}}$$

44. In an ideal pn junction with an ideality factor of 1 at $T = 300 \text{ K}$, the magnitude of the reverse-bias voltage required to reach 75% of its reverse saturation current, rounded off

to 2 decimal places, is _____ mV.

[$k = 1.38 \times 10^{-23} \text{ JK}^{-1}$, $h = 6.625 \times 10^{-34} \text{ J-s}$,
 $q = 1.602 \times 10^{-19} \text{ C}$]

Ans. (35.821 mV)

Sol. For ideal diode

$$V_t = \frac{KT}{q} = \frac{1.38 \times 10^{-23} \times 300}{1.602 \times 10^{-19}} = 0.02584 \text{ volt}$$

For diode

$$I = I_s \left(e^{\frac{V_{BE}}{V_T}} - 1 \right)$$

given $I = -0.75 I_s$

$$-0.75 I_s = I_s \left(e^{\frac{V_{BE}}{V_T}} - 1 \right)$$

$$e^{\frac{V_{BE}}{V_T}} = 0.25$$

$$V_{BE} = -0.02584 \times 1.386$$

$$V_{BE} = -0.035821$$

$$|V_{BE}| = 35.821 \text{ mV}$$

45. A single bit, equally likely to be 0 and 1, is to be sent across an additive white Gaussian noise (AWGN) channel with power spectral density $N_0/2$. Binary signaling, with $0 \rightarrow p(t)$ and $1 \rightarrow q(t)$, is used for the transmission, along with an optimal receiver that minimizes the bit-error probability.

Let $\phi_1(t), \phi_2(t)$ form an ortho-normal signal set.

If we choose $p(t) = \phi_1(t)$ and $q(t) = -\phi_1(t)$, we would obtain a certain bit-error probability P_b .

If we keep $p(t) = \phi_1(t)$, but take $q(t) = \sqrt{E} \phi_2(t)$, for what value of E would we obtain the same bit-error probability P_b ?

- (a) 0 (b) 1
(c) 3 (d) 2



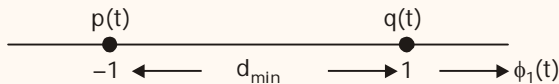
Ans. (c)

Sol. Bits 0 and 1 are equally likely

$$p(0) = \frac{1}{2} = p(1)$$

when $p(t) = \phi_1(t)$ and $q(t) = -\phi_1(t)$

Constellation diagram



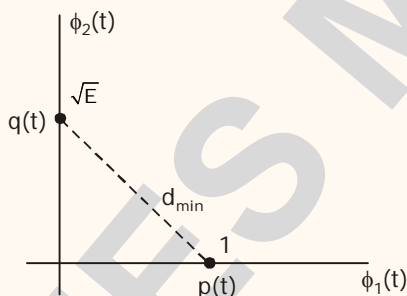
Probability of error

$$P_{e1} = Q\left(\sqrt{\frac{d_{\min}^2}{2N_0}}\right)$$

$$d_{\min} = 2$$

$$P_{e1} = Q\left(\sqrt{\frac{4}{2N_0}}\right)$$

When $p(t) = \phi_1(t)$ and $q(t) = \sqrt{E} \phi_2(t)$



$$d_{\min} = \sqrt{E+1}$$

$$P_{e2} = Q\left(\sqrt{\frac{d_{\min}^2}{2N_0}}\right)$$

$$P_{e2} = Q\left(\sqrt{\frac{E+1}{2N_0}}\right)$$

Given, $P_{e1} = P_{e2}$

$$4 = E+1 \Rightarrow E = 3$$

46. Consider the homogeneous ordinary differential equation :

$$x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 3y = 0, \quad x > 0$$

with $y(x)$ as a general solution. Given that

$$y(1) = 1 \text{ and } y(2) = 14$$

the value of $y(1.5)$, rounded off to two decimal places, is _____.

Ans. (5.25)

Sol. Consider the homogeneous ordinary differential equation

$$x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 3y = 0 \quad \dots(i)$$

Assuming, $x = e^z$, $\frac{d}{dz} = \theta$, $z = \ln x$

equation (i) can be written as

$$[(\theta)(\theta-1) - 3\theta + 3]y = 0$$

$$[\theta^2 - \theta - 3\theta + 3]y = 0$$

$$\theta^2 - 4\theta + 3 = 0$$

$$\theta = 1, 3$$

$$\text{so, } y = C_1 e^z + C_2 e^{3z}$$

$$(z = \ln x)$$

$$y = C_1 e^{\ln x} + C_2 e^{3 \ln x}$$

$$y = C_1 x + C_2 x^3$$

Putting initial conditions $y(1) = 1$ and $y(2) = 14$

$$\text{so, } C_1 + C_2 = 1 \quad \dots(ii)$$

$$\& 2C_1 + 8C_2 = 14 \quad \dots(iii)$$

Solving (ii) & (iii) we get

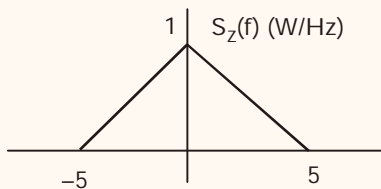
$$C_1 = -1 \text{ and } C_2 = 2$$

$$\text{so, } y = -x + 2x^3$$

$$y(1.5) = -1.5 + 2(1.5)^3$$

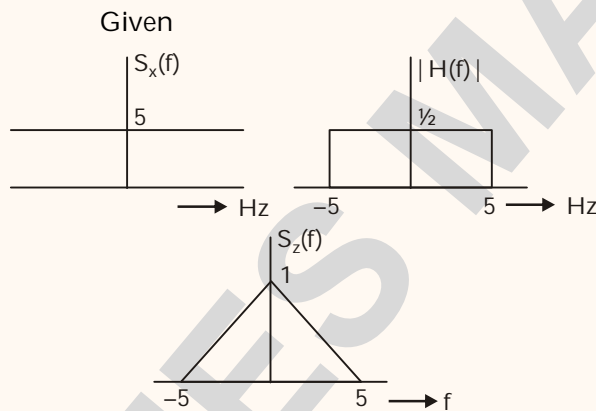
$$y(1.5) = 5.25$$

47. Let a random process $Y(t)$ be described as $Y(t) = h(t) * X(t) + Z(t)$, where $X(t)$ is a white noise process with power spectral density $S_x(f) = 5 \text{ W/Hz}$. The filter $h(t)$ has a magnitude response given by $|H(f)| = 0.5$ for $-5 \leq f \leq 5$, and zero elsewhere. $Z(t)$ is a stationary random process, uncorrelated with $X(t)$, with power spectral density as shown in the figure. The power in $Y(t)$, in watts, is equal to ____ W (rounded off to two decimal places).



Ans. (17.5)

Sol. $y(t) = h(t) * x(t) + z(t)$



Power spectral density of $y(t)$

$$S_y(f) = |H(f)|^2 S_x(f) + S_z(f)$$

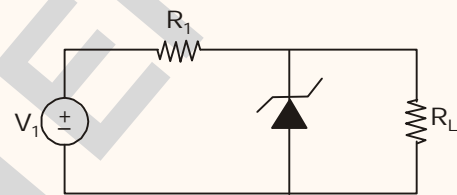
$$\text{Power in } y(t) = \int_{-\infty}^{\infty} S_y(f) df$$

$$= \int_{-\infty}^{\infty} |H(f)|^2 S_x(f) df + \int_{-\infty}^{\infty} S_z(f) df$$

$$= \int_{-5}^5 \left(\frac{1}{2}\right)^2 \cdot 5 df + \frac{1}{2} \times 10 \times 1$$

$$= 12.5 + 5 = 17.5 \text{ W}$$

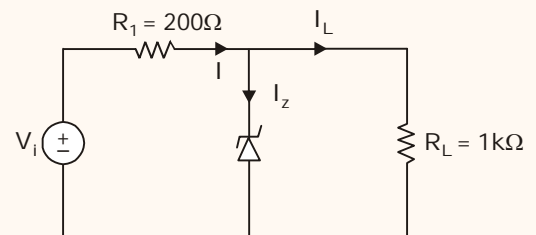
48. In the circuit shown, the breakdown voltage and the maximum current of the Zener diode are 20V and 60 mA, respectively. The values of R_1 and R_L are 200Ω and $1 \text{ k}\Omega$, respectively. What is the range of V_i that will maintain the Zener diode in the 'on' state?



- (a) 20V to 28V (b) 18V to 24V
(c) 22V to 34V (d) 24V to 36V

Ans. (d)

Sol.



$$V_{i \min} \times \frac{R_L}{R_L + R_1} = V_2$$

$$V_{i \min} = \frac{12}{10} \times 20$$

$$= 24 \text{ volt}$$

When diode is in breakdown

$$I = I_L + I_{z \max}$$

$$I = \frac{20}{1 \text{ k}\Omega} + 60$$

$$= 20 + 60$$

$$= 80 \text{ mA}$$

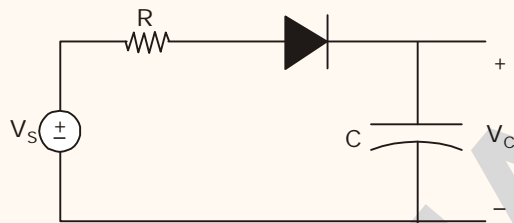
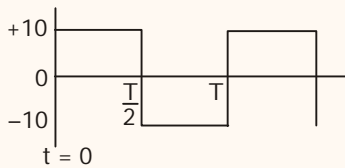
$$V_{i \max} = 200 \times 80 \times 10^{-3} + 20$$

$$= 16 + 20$$

$$= 36 \text{ volt}$$

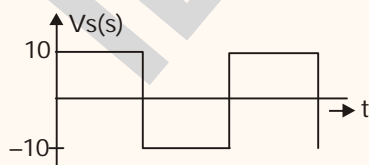
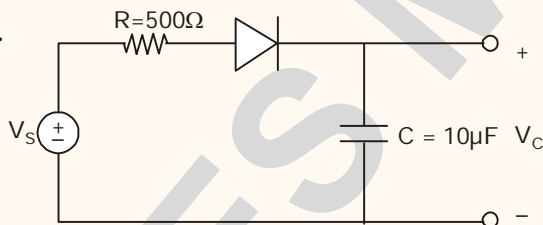
$$24 \text{ V} < V_i < 36 \text{ volt}$$

49. In the circuit shown, V_s is a 10V square wave of period, $T = 4$ ms with $R = 500\Omega$ and $C = 10\mu\text{F}$. The capacitor is initially uncharged at $t = 0$, and the diode is assumed to be ideal. The voltage across the capacitor (V_c) at 3 ms is equal to ___ volts (rounded off to one decimal place).

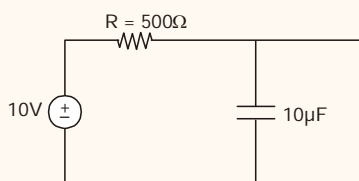


Ans. (3.29)

Sol.



For $0 < t < T/2$ or $0 < t < 2\text{ms}$ diode is ON



$$V_c(t) = 10 - 10e^{-200t}$$

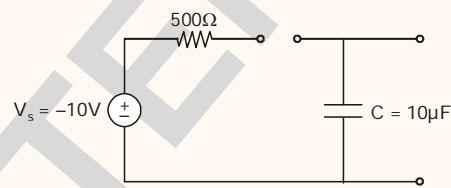
at $t = 2\text{m sec.}$

$$V_c = 10 - 10e^{-0.4}$$

$$V_c = 3.29$$

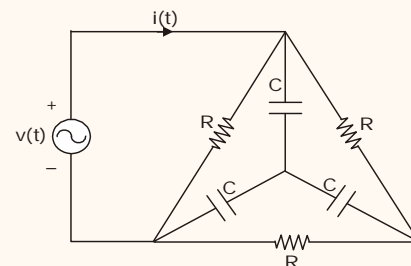
when $2\text{ms} < t < 4$ ms

diode is OFF capacitor has no discharge path



hence $V_c(t = 3\text{m sec}) = 3.29$ volt.

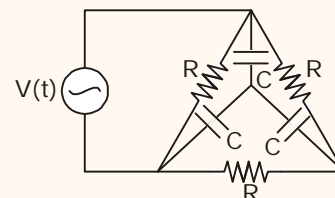
50. In the circuit shown, if $v(t) = 2\sin(1000t)$ volts, $R = 1\text{ k}\Omega$ and $C = 1\mu\text{F}$, then the steady-state current $i(t)$, in milliamperes (mA), is



- (a) $3\sin(1000t) + \cos(1000t)$
(b) $2\sin(100t) + 2\cos(1000t)$
(c) $\sin(100t) + \cos(1000t)$
(d) $\sin(1000t) + 3\cos(1000t)$

Ans. (a)

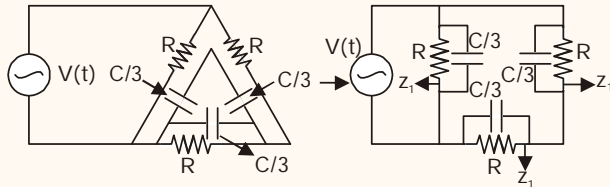
Sol.



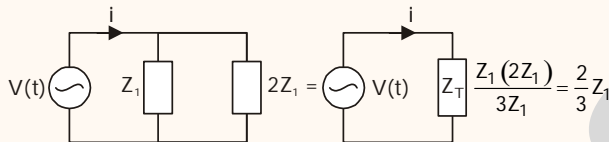
$$\omega = 1000$$

$$C = 1\mu\text{F}$$

$$R = 1k\Omega$$



$$Z_1 = \frac{R \times \frac{1}{j\omega C/3}}{R + \frac{1}{j\omega C/3}} = \frac{R}{1 + j\omega R \frac{C}{3}}$$



$$Z_T = \frac{2}{3} Z_1 = \frac{2}{3} \left[\frac{R}{1 + j\omega RC/3} \right]$$

$$= \frac{2}{3} \times \frac{10^3}{1 + j10^3 \frac{10^3 \times 10^{-6}}{3}}$$

$$Z_T = \frac{2}{3} \times \frac{10^3}{(1 + j/3)}$$

$$= \frac{2}{3} \times \frac{3 \times 10^3}{(3 + j)} = \frac{2 \times 10^3}{3 + j}$$

$$i = \frac{V(t)}{Z_T} = \frac{2 \sin(1000t)}{\frac{2 \times 10^3}{3 + j}}$$

$$= [(3 + j) \sin(1000t)]$$

$$i(t) = [3 \sin 1000t + j \sin 1000t] \text{ mA}$$

$$i(t) = [3 \sin 1000t + 1 \angle 90^\circ (\sin 1000t)] \text{ mA}$$

$$= [3 \sin 1000t + \sin(1000t + 90^\circ)] \text{ mA}$$

$$i(t) = [3 \sin(1000t) + \cos 1000t] \text{ mA}$$

51. Consider a long-channel MOSFET with a channel length $1 \mu\text{m}$ and width $10 \mu\text{m}$. The device parameters are acceptor concentration $N_A = 5 \times 10^{16} \text{ cm}^{-3}$, electron mobility $\mu_n = 800 \text{ cm}^2/\text{V-s}$, oxide capacitance/area $C_{ox} = 3.45 \times 10^{-7} \text{ F/cm}^2$, threshold voltage $V_T = 0.7 \text{ V}$. The drain saturation current (I_{Dsat}) for a gate voltage of 5 V is ____ mA (rounded off to two decimal places).

$$[\epsilon_0 = 8.854 \times 10^{-14} \text{ F/cm}, \epsilon_{Si} = 11.9]$$

Ans. (25.51)

Sol. Given

$$L = 1 \mu\text{m}, W = 10 \mu\text{m}$$

$$N_A = 5 \times 10^{16} \text{ cm}^{-3}$$

$$\mu_n = 800 \frac{\text{cm}^2}{\text{V-s}}$$

$$C_{ox} = 3.45 \times 10^{-7} \frac{\text{F}}{\text{cm}^2}$$

$$V_T = 0.7 \text{ V}; V_G = 5 \text{ V}$$

$$I_{Dsat} = \frac{\mu_n C_{ox} W}{2L} (V_{GS} - V_T)^2$$

$$= \frac{800 \times 3.45 \times 10^{-7} \times 10}{2 \times 1} (5 - 0.7)^2$$

$$= 13.8 \times 10^{-4} \times (4.3)^2$$

$$I_{Dsat} = 25.516 \text{ mA}$$

52. It is desired to find a three-tap causal filter which gives zero signal as an output to an input of the form

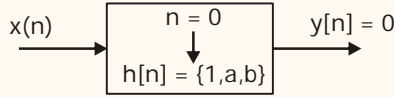
$$x[n] = c_1 \exp\left(-\frac{j\pi n}{2}\right) + c_2 \exp\left(\frac{j\pi n}{2}\right)$$

where c_1 and c_2 are real numbers. The desired three-tap filter is given by

$$h[0] = 1, h[1] = a, h[2] = b$$

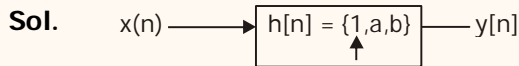
and $h[n] = 0$ for $n < 0$ or $n > 2$

What are the values of the filter taps a and b if the output is $y[n] = 0$ for all n , when $x[n]$ is as given above?



- (a) $a = -1, b = 1$ (b) $a = 0, b = 1$
(c) $a = 1, b = 1$ (d) $a = 0, b = -1$

Ans. b



Let fourier transform of $h(n)$ is $H(\omega)$

$$H(\omega) = 1 + ae^{j\omega} + be^{j2\omega}$$

for input

$$x(n) = c_1 e^{-j\left(\frac{n\pi}{2}\right)} + c_2 e^{j\left(\frac{n\pi}{2}\right)}$$

for output

$$y(n) = c_1 \left| H\left(\omega = -\frac{\pi}{2}\right) \right| e^{-j\frac{n\pi}{2}} + c_2 \left| H\left(\omega = \frac{\pi}{2}\right) \right| e^{j\frac{n\pi}{2}}$$

for $y(n) = 0$

$$\left| H\left(\omega = \frac{\pi}{2}\right) \right| = 0$$

$$1 + ae^{-j\frac{\pi}{2}} + be^{-j\pi} = 0$$

$$1 - aj - b = 0$$

$$(1 - b) - aj = 0 \quad \dots(1)$$

and $\left| H\left(\omega = \frac{\pi}{2}\right) \right| = 0$

$$1 + ae^{j\frac{\pi}{2}} + be^{j\pi} = 0$$

$$1 + aj - b = 0$$

$$(1 - b) + aj = 0 \quad \dots(2)$$

from (1) and (2)

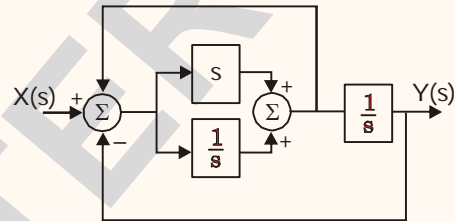
$$1 - b = 0$$

$$b = 1$$

$$a = 0$$

- 53.** The block diagram of a system is illustrated in the figure shown, where $X(s)$ is the input and $Y(s)$ is the output. The transfer function

$$H(s) = \frac{Y(s)}{X(s)} \text{ is}$$



(a) $H(s) = \frac{s+1}{s^2+s+1}$

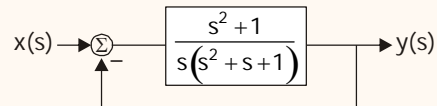
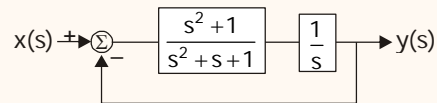
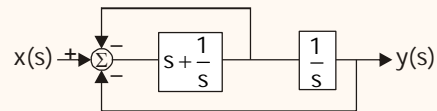
(b) $H(s) = \frac{s^2+1}{s^3+2s^2+s+1}$

(c) $H(s) = \frac{s^2+1}{2s^2+1}$

(d) $H(s) = \frac{s^2+1}{s^3+s^2+s+1}$

Ans. (b)

Sol.



Transfer function

$$\frac{Y(s)}{X(s)} = \frac{s^2+1}{s^3+s^2+s} \cdot \frac{1}{1 + \frac{s^2+1}{s^3+s^2+s}}$$

$$H(s) = \frac{s^2 + 1}{s^3 + 2s^2 + s + 1}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s^2 + 1}{s^3 + 2s^2 + s + 1}$$

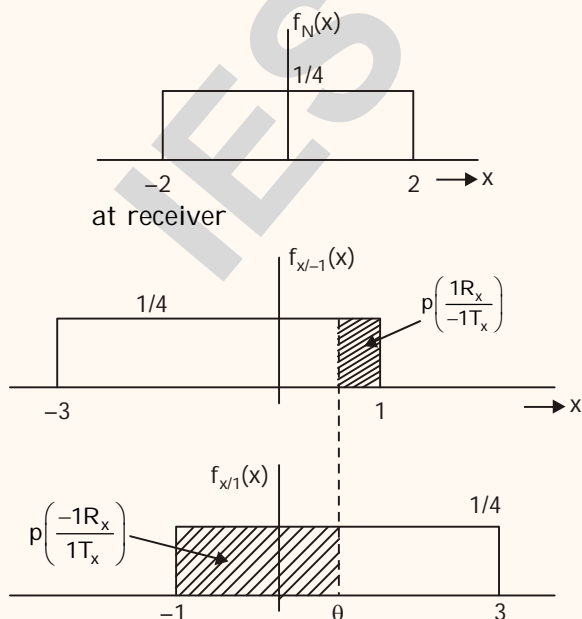
54. A random variable X takes values -1 and $+1$ with probabilities 0.2 and 0.8 , respectively. It is transmitted across a channel which adds noise N , so that the random variable at the channel output is $Y = X + N$. The noise N is independent of X , and is uniformly distributed over the interval $[-2, 2]$. The receiver makes a decision

$$\hat{X} = \begin{cases} -1, & \text{if } Y \leq \theta \\ +1, & \text{if } Y > \theta \end{cases}$$

where the threshold $\theta \in [-1, 1]$ is chosen so as to minimize the probability of error $P_r[\hat{X} \neq X]$. The minimum probability of error, rounded off to 1 decimal place, is _____.

Ans. (0.1)

Sol. $p(x = -1) = 0.2$
 $p(x = 1) = 0.8$
and pdf of noise



$$p_e = p(x = -1)p\left(\frac{1R_x}{-1T_x}\right) + p(x = 1)p\left(\frac{-1}{1T_x}\right)$$

$$p_e = 0.2 \frac{1}{4}(1 - \theta) + 0.8 \times \frac{1}{4}(1 + \theta)$$

For min. p_e

$$\frac{dp_e}{d\theta} = 0$$

$$p_e = -\frac{0.2}{4} + \frac{0.8}{4} = \frac{0.6}{4} = 0.1$$

55. Consider a causal second-order system with the transfer function

$$G(s) = \frac{1}{1 + 2s + s^2}$$

with a unit-step $R(s) = \frac{1}{s}$ as an input. Let

$C(s)$ be the corresponding output. The time taken by the system output $c(t)$ to reach 94% of its steady-state value $\lim_{t \rightarrow \infty} c(t)$, rounded off to two decimal places, is

- (a) 3.89 (b) 2.81
(c) 5.25 (d) 4.50

Ans. (d)

Sol. $G(s) = \frac{1}{s^2 + 2s + 1}$

$$R(s) = \frac{1}{s}$$

$$\frac{C(s)}{R(s)} = G(s)$$

$$C(s) = \frac{1}{s} \frac{1}{(s^2 + 2s + 1)}$$

$$C(s) = \frac{1}{s(s+1)^2}$$

Applying partial fraction



$$\frac{1}{s(s+1)^2} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+1)^2}$$

$$1 = A(s+1)^2 + Bs(s+1) + Cs$$

Now comparing coefficients on both sides

$$A = 1, B = -1, C = -1$$

$$C(s) = \frac{1}{s} - \frac{1}{(s+1)} - \frac{1}{(s+1)^2}$$

Now applying inverse Laplace transform

$$C(t) = 1 - e^{-t} - te^{-t}$$

$$\text{Steady state value } \lim_{t \rightarrow \infty} c(t) = 1 - e^{-\infty} - \infty e^{-\infty}$$

$$= 1$$

$$\text{Now, } 1 \times \frac{94}{100} = 1 - e^{-t} - te^{-t}$$

$$0.94 = 1 - e^{-t} - te^{-t}$$

From trial and error method

$$t = 4.5$$