MULTIPLE CHOICE QUESTIONS

SUB : PHYSICS & CHEMISTRY

1. Experimental investigations show that the intensity of solar radiation is maximum for a wavelength 480 nm in the visible region. Estimate the surface temperature of sun. Given Wein's constant $b = 2.88 \times 10^{-3}$ mK.

(A) 4000 K (B) 6000 K (C) 8000 K (D) 10^6 K

Ans : (B)

Hints : $\lambda_m \times T = b$

$\lambda_m = 480$ nm

$$T = \frac{b}{\lambda_m} = \frac{2.88\times10^{-3}}{480\times10^{-9}} = 6000\ K$$

2. The temperature of an ideal gas is increased from 120 K to 480 K. If at 120 K, the root mean square speed of gas molecules is v, then at 480 K it will be

(A) $4v$ (B) $2v$ (C) $\frac{v}{2}$ (D) $\frac{v}{4}$

Ans : (B)

Hints : $\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$

$$\frac{V_1}{V_2} = \sqrt{\frac{120}{480}} - = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$V_2 = 2v$

3. Two mirrors at an angle θº produce 5 images of a point. The number of images produced when θ is decreased to θº – 30º is

(A) 9 (B) 10 (C) 11 (D) 12

Ans : (C)

Hints : No. of images = 5

$\therefore \theta = 60º$

New angle = θ – 30º = 30º. No of images = $\frac{360º}{30º} - 1 = 11$

4. The radius of the light circle observed by a fish at a depth of 12 meter is (refractive index of water = 4/3)

(A) $36\sqrt{7}$ (B) $\frac{36}{\sqrt{7}}$ (C) $36\sqrt{5}$ (D) $4\sqrt{5}$

Ans : (B)

Hints : $r = \dfrac{h}{\sqrt{\mu^2 - 1}} = \dfrac{12}{\sqrt{\dfrac{16}{9} - 1}} = \dfrac{12 \times 3}{\sqrt{7}} = \dfrac{36}{\sqrt{7}}$

5. In Young's double slit experiment, the fringe width is β. If the entire arrangement is placed in a liquid of refractive index n, the fringe width becomes :

(A) $n\beta$ (B) $\dfrac{\beta}{n+1}$ (C) $\dfrac{\beta}{n-1}$ (D) $\dfrac{\beta}{n}$

Ans : (D)

6. A plano-convex lens (f = 20 cm) is silvered at plane surface. Now focal length will be :

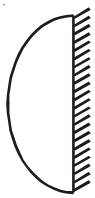
(A) 20 cm (B) 40 cm (C) 30 cm (D) 10 cm

Ans : (D)

Hints : $P = 2P_L + P_M$

$P_M = 0$

$P = \dfrac{1}{f} \times 2 = \dfrac{2}{f}$



$-\dfrac{1}{F} = \dfrac{2}{f}$

$F = -\dfrac{f}{2}$

7. The light beams of intensities in the ratio of 9 : 1 are allowed to interfere. What will be the ratio of the intensities of maxima and minima ?

(A) 3 : 1 (B) 4 : 1 (C) 25 : 9 (D) 81 : 1

Ans : (B)

Hints : $\dfrac{A_1}{A_2} = \dfrac{3}{1}$

$\dfrac{I_{max}}{I_{min}} = \dfrac{16}{4} = \dfrac{4}{1}$

8. If x_1 be the size of the magnified image and x_2 the size of the diminished image in Lens Displacement Method, then the size of the object is :

(A) $\sqrt{x_1 x_2}$ (B) $x_1 x_2$ (C) $x_1^2 x_2$ (D) $x_1 x_2^2$

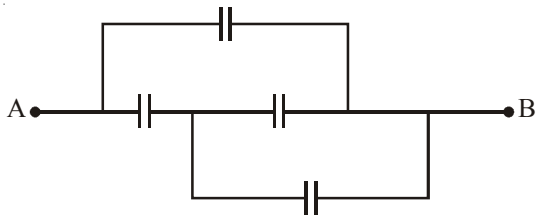
Ans : (A)

9. A point charge +q is placed at the centre of a cube of side L. The electric flux emerging from the cube is

(A) $\dfrac{q}{\varepsilon_0}$ (B) Zero (C) $\dfrac{6qL^2}{\varepsilon_0}$ (D) $\dfrac{q}{6L^2\varepsilon_0}$

Ans : (A)

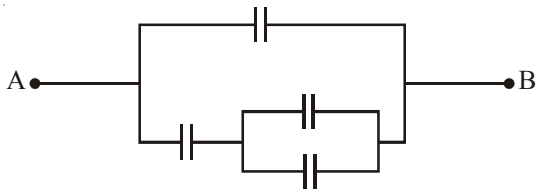
10. In the figure below, the capacitance of each capacitor is 3 μF. The effective capacitance between A and B is :



(A) $\frac{3}{4}\mu F$ (B) 3 μF (C) 6 μF (D) 5 μF

Ans : (D)

Hints :



$\frac{2C}{3} + C = 2 + 3 = 5\mu F$

11. *n* identical droplets are charged to *v* volt each. If they coalesce to form a single drop, then its potential will be

(A) $n^{2/3}v$ (B) $n^{1/3}v$ (C) nv (D) v/n

Ans : (A)

Hints : $n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$\Rightarrow R = rn^{1/3}$

$C_0 = 4\pi\varepsilon_0 r$

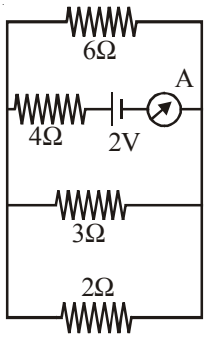
$q_0 = C_0 V = (4\pi\varepsilon_0 r)V$

Capacitance of Bigger drop,

$C = 4\pi\varepsilon_0 R$

So, $V = \frac{nq_0}{C} = \frac{n(4\pi\varepsilon_0 rV)}{4\pi\varepsilon_0 R} = n\left(\frac{r}{R}\right)V = n\left(\frac{1}{n^{1/3}}\right)V = n^{2/3}V$

12. The reading of the ammeter in the following figure will be



(A) 0.8 A (B) 0.6 A (C) 0.4 A (D) 0.2 A

Ans : (C)

Hints : $\frac{1}{R}=\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=\frac{3+2+1}{6}=1\Omega$

$R_{eq}=1+4=5\,\Omega$

$I=\frac{2}{5}=0.4\,A$

13. A wire of resistance R is elongated *n*-fold to make a new uniform wire. The resistance of new wire

(A) nR (B) n^2R (C) $2nR$ (D) $2n^2R$

Ans : (B)

Hints : $R'=n^2R$

14. The ratio of magnetic field and magnetic moment at the centre of a current carrying circular loop is x. When both the current and radius is doubled the ratio will be

(A) $x/8$ (B) $x/4$ (C) $x/2$ (D) $2x$

Ans : (A)

Hints : $B=\frac{\mu_0 I}{2a}$ $M=I(\pi a^2)$

$\frac{B}{M}=\frac{\mu_0 I}{2a}\times\frac{1}{I\pi a^2}=\frac{\mu_0}{2\pi a^3}=x$

Again, Ratio $=\frac{\mu_0}{2\pi(2a)^3}=\frac{1}{8}\left(\frac{\mu_0}{2\pi a^3}\right)=\frac{x}{8}$

15. The current through a coil of self inductance L = 2*m*H is given by $I=t^2e^{-t}$ at time t. How long it will take to make the e m.f. zero?

(A) 1 s (B) 2 s (C) 3 s (D) 4 s

Ans : (B)

Hints : $I=t^2e^{-t}$

$\frac{dI}{dt}=2te^{-t}-e^{-t}t^2=e^{-t}t(2-t)$

$e=-L\frac{dI}{dt}$

$\Rightarrow\frac{dI}{dt}=0\Rightarrow e^{-t}t(2-t)=0$

$t=2$ sec

16. The magnetic flux through a loop of resistance 10 Ω is given by $\phi=5t^2-4t+1$ Weber. How much current is induced in the loop after 0.2 sec ?

(A) 0.4 A (B) 0.2 A (C) 0.04 A (D) 0.02 A

Ans : (B)

Hints : $\phi=5t^2-4t+1$

$\frac{d\phi}{dt}=10t-4$

$I=\frac{e}{R}=\frac{-d\phi/dt}{R}=-\frac{10t-4}{10}$

At $t=0.2$ sec

$I=\frac{-(10\times0.2-4)}{10}=-\frac{(2-4)}{10}=+\frac{2}{10}=+0.2\,A=0.2\,A$

17. The decimal equivalent of the binary number $(11010.101)_2$ is

(A) 9.625 (B) 25.265 (C) 26.625 (D) 26.265

Ans : (C)

Hints : $(11010.101) = 0\times 2^0 + 1\times 2^1 + 0\times 2^2 + 1\times 2^3 + 1\times 2^4 + 1\times 2^{-1} + 0\times 2^{-2} + 1\times 2^{-3} = 2+8+16+\frac{1}{2}+\frac{1}{8} = 26.625$

18. In a common emitter configuration, a transistor has $\beta = 50$ and input resistance 1 kΩ. If the peak value of a.c. input is 0.01 V then the peak value of collector current is

(A) 0.01 μA (B) 0.25 μA (C) 100 μA (D) 500 μA

Ans : (D)

Hints : $\beta = 50 \Rightarrow \beta = \frac{\Delta I_C}{\Delta I_B} \Rightarrow \Delta I_C = \beta \times \Delta I_B$

$$\Delta I_B = \frac{0.01}{10^3} = 10^{-2}\times 10^{-3} = 10^{-5}$$

$\Delta I_C = 50 \times 10^{-5} = 500 \times 10^{-6} = 500\ \mu A$

19. Half-life of a radioactive substance is 20 minute. The time between 20% and 80% decay will be :

(A) 20 min (B) 30 min (C) 40 min (D) 25 min

Ans : (C)

Hints : For 20% decay

$$\frac{80N_0}{100} = N_0 e^{-\lambda t_1} \quad \text{.... (1)}$$

For 80% decay

$$\frac{20N_0}{100} = N_0 e^{-\lambda t_2} \quad \text{... (2)}$$

On dividing

$4 = e^{\lambda (t_2 - t_1)}$

$$2\ln 2 = \frac{\ln 2}{t_{1/2}}(t_2 - t_1)$$

$\Rightarrow t_2 - t_1 = 2 \times 20 = 40$ min

20. The energy released by the fission of one uranium atom is 200 MeV. The number of fissions per second required to produce 3.2 W of power is (Take 1 eV = 1.6×10^{-19} J)

(A) 10^7 (B) 10^{10} (C) 10^{15} (D) 10^{11}

Ans : (D)

Hints : $u = 200$ MeV $= 200 \times 10^6$ eV $= 200 \times 10^6 \times 1.6 \times 10^{-19}$ J

E = 3.2 J

$$\text{No of fissions} = \frac{3.2}{2\times 1.6\times 10^{-11}} = 10^{11}$$

21. A body is projected with a speed u m/s at an angle β with the horizontal. The kinetic energy at the highest point is 3/4th of the initial kinetic energy. The value of β is :

(A) 30° (B) 45° (C) 60° (D) 120°

Ans : (A)

Hints : (K.E.) at maximum height $= \frac{1}{2}m(u^2\cos^2\beta)$

K.E. = K $\cos^2\beta$

Here, $K\cos^2\beta = \frac{3}{4}K$

$\cos\beta = \frac{\sqrt{3}}{2}$

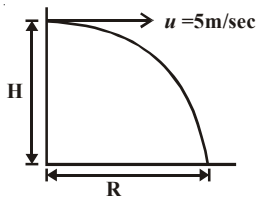
$\beta = 30^\circ$

22. A ball is projected horizontally with a velocity of 5 m/s from the top of a building 19.6 m high. How long will the ball take of hit the ground ?

(A) $\sqrt{2}$ s (B) 2 s (C) $\sqrt{3}$ s (D) 3 s

Ans : (B)

Hints : $T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2\times19.6}{9.8}} = 2\,\text{sec}$



23. A stone falls freely from rest and the total distance covered by it in the last second of its motion equals the distance covered by it in the first three seconds of its motion. The stone remains in the air for

(A) 6 s (B) 5 s (C) 7 s (D) 4 s

Ans : (B)

Hints : $u = 0$

$S_3 = 0 + \frac{1}{2}gt^2 = \frac{1}{2}\times10\times9 = 45$

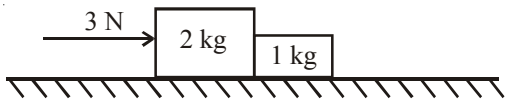
$S_t\text{th} = u + (2t-1)\frac{g}{2}$

$S_t\text{th} = 0 + 5(2t\text{-}1) = 45$

$2t - 1 = 9$

$t = 5$ sec

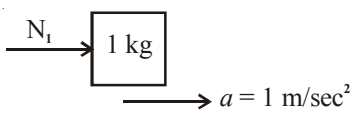
24. Two blocks of 2 kg and 1 kg are in contact on a frictionless table. If a force of 3 N is applied on 2 kg block, then the force of contact between the two blocks will be :



(A) 0 N (B) 1 N (C) 2 N (D) 3 N

Ans : (B)

Hints : Common acceleration $= \frac{3}{3} = 1\,\text{m/sec}^2$



$N_1 = 1$ N

25. If momentum is increased by 20%, then kinetic energy increases by

(A) 48% (B) 44% (C) 40% (D) 36%

Ans : (B)

Hints : $K = \frac{P^2}{2m}$

Here $P' = 1.2\,P$

Hence, $K' = \frac{(1.2P)^2}{2m}$

$K' = 1.44\frac{P^2}{2m}$

$K' = 1.44\,K$ or Percentage increase in $K = 44\%$

26. A boy of mass 40 kg is climbing a vertical pole at a constant speed. If the coefficient of friction between his palms and the pole is 0.8 and $g = 10$ m/s², the horizontal force that he is applying on the pole is

(A) 300 N (B) 400 N (C) 500 N (D) 600 N

Ans : (C)

Hints : Here $\mu = 0.8$

Frictional force $= \mu N_1 = mg$

$N_1 = \frac{mg}{\mu} = \frac{400}{0.8} = 500\text{N}$

27. The value of 'λ' for which the two vectors $\vec{a} = 5\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ are perpendicular to each other is

(A) 2 (B) – 2 (C) 3 (D) – 3

Ans : (C)

Hints : For $\vec{a} \perp \vec{b}$

$\vec{a}.\vec{b} = 0$

i.e., $5 - 2\lambda + 1 = 0$

$\lambda = 3$

28. If $\vec{a} + \vec{b} = \vec{c}$ and $a + b = c$, then the angle included between $\vec{a}$ and $\vec{b}$ is

(A) 90º (B) 180º (C) 120º (D) Zero

Ans : (D)

Hints : Here $\vec{a} + \vec{b} = \vec{c}$ & $c = a + b$

Now, $c = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

$(a + b) = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

$a^2 + b^2 + 2ab = a^2 + b^2 + 2ab\cos\theta$

$\cos\theta = 1, \theta = 0^\circ$

29. The height vertically above the earth's surface at which the acceleration due to gravity becomes 1% of its value at the surface is (R is the radius of the Earth)

(A) 8 R (B) 9 R (C) 10 R (D) 20 R

Ans : (B)

Hints : $g' = \frac{g}{\left(1 + \frac{h}{R}\right)^2} \Rightarrow \frac{g}{100} = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$

$1 + \frac{h}{R} = 10 \Rightarrow \frac{h}{R} = 9,\ h = 9R$

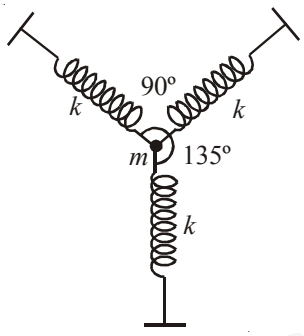
30. The change in the gravitational potential energy when a body of mass m is raised to a height nR above the surface of the Earth is (here R is the radius of the Earth)

(A) $\left(\frac{n}{n+1}\right)mg\mathrm{R}$ (B) $\left(\frac{n}{n-1}\right)mg\mathrm{R}$ (C) $nmg\mathrm{R}$ (D) $\frac{mg\mathrm{R}}{n}$

Ans : (A)

Hints : $\Delta\mathrm{U} = \frac{mgh}{1+\frac{h}{\mathrm{R}}} = \frac{mg\times n\mathrm{R}}{1+\frac{n\mathrm{R}}{\mathrm{R}}} = \frac{nmg\mathrm{R}}{n+1}$

31. A particle of mass m is attached to three identical massless springs of spring constant 'k' as shown in the figure. The time period of vertical oscillation of the particle is



(A) $2\pi\sqrt{\frac{m}{k}}$ (B) $2\pi\sqrt{\frac{m}{2k}}$ (C) $2\pi\sqrt{\frac{m}{3k}}$ (D) $\pi\sqrt{\frac{m}{k}}$

Ans : (B)

Hints : $\mathrm{T} = 2\pi\sqrt{\frac{m}{\mathrm{K_{eq}}}}$

$\mathrm{F} = \mathrm{K}x + 2\mathrm{K}x\cos^2 45$

$\mathrm{K_{eq}}x = \mathrm{K}x + \mathrm{K}x$

$\mathrm{K_{eq}} = 2\mathrm{K}$

32. A spring of force constant k is cut into three equal parts. The force constant of each part would be

(A) $\frac{k}{3}$ (B) $3k$ (C) k (D) $2k$

Ans : (B)

Hints : $\mathrm{K} \propto \frac{1}{l}$

33. A body floats in water with 40% of its volume outside water. When the same body floats in oil, 60% of its volume remains outside oil. The relative density of the oil is

(A) 0.9 (B) 1.2 (C) 1.5 (D) 1.8

Ans : (C)

Hints : Fraction of immersed part $f = \frac{d}{\rho}$

Case-1,

$f = 1 - 0.4 = 0.6$

$0.6 = \frac{d}{1}$

$d = 0.6$

Case-2,

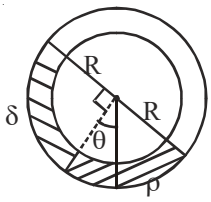
$f = 1 - 0.6 = 0.4$

$$f = \frac{d}{\rho_{oil}}$$

$$0.4 = \frac{0.6}{\rho_{oil}}$$

$\rho_{oil} = 1.5$

34. A uniform long tube is bent into a circle of radius R and it lies in vertical plane. Two liquids of same volume but densities ρ and δ fill half the tube. The angle θ is



(A) $\tan^{-1}\left(\frac{\rho-\delta}{\rho+\delta}\right)$ (B) $\tan^{-1}\frac{\rho}{\delta}$ (C) $\tan^{-1}\frac{\delta}{\rho}$ (D) $\tan^{-1}\left(\frac{\rho+\delta}{\rho-\delta}\right)$

Ans : (A)

Hints : $\delta g R(\cos\theta + \sin\theta) = \rho g R(\cos\theta - \sin\theta)$

$\delta\cos\theta + \delta\sin\theta = \rho\cos\theta - \rho\sin\theta$

$\sin\theta(\delta+\rho) = \cos\theta(\rho-\delta)$

$$\tan\theta = \frac{\rho-\delta}{\rho+\delta}$$

35. Two solid spheres of same metal but of mass M and 8 M fall simultaneously on a viscous liquid and their terminal velocities are v and nv then value of n is

(A) 16 (B) 8 (C) 4 (D) 2

Ans : (C)

Hints : $m = \frac{4}{3}\pi r^3 \times \rho$

$m \propto r^3$

$$\left(\frac{r_1}{r_2}\right)^3 = \frac{1}{8}$$

$$\frac{r_1}{r_2} = \frac{1}{2}$$

$6\pi n r V = \frac{4}{3}\pi r^3 (d = \rho)$

$V \propto r^2$, $\frac{V_1}{V_2} = \frac{1}{4}$

$n = 4$

36. A particle is executing linear simple harmonic motion of amplitude A. At what displacement is the energy of the particle half potential and half kinetic ?

(A) $\frac{A}{4}$ (B) $\frac{A}{2}$ (C) $\frac{A}{\sqrt{2}}$ (D) $\frac{A}{\sqrt{3}}$

Ans : (C)

Hints : Total Energy (E) = $\frac{1}{2}m\omega^2 A^2$

P.E. = $\frac{1}{2}m\omega^2 x^2$

As P.E. = $\frac{E}{2}$

Then, $\frac{1}{2}m\omega^2 A^2 \times \frac{1}{2} = \frac{1}{2}m\omega^2 x^2$

$x^2 = \frac{A^2}{2} \Rightarrow x = \frac{A}{\sqrt{2}}$

37. The equation of a progressive wave is $y = 4\sin(4\pi t - 0.04x + \pi/3)$ where x is in meter and t is in second. The velocity of the wave is

(A) 100π m/s (B) 50π m/s (C) 25π m/s (D) π m/s

Ans : (A)

Hints : Velocity of wave = $\frac{\omega}{K} = \frac{4\pi}{0.04} = 100\pi$ m/sec

38. A longitudinal wave is represented by $x = x_0 \sin 2\pi(nt - x/\lambda)$. The maximum particle velocity will be four times the wave velocity if :

(A) $\lambda = \frac{\pi x_0}{4}$ (B) $\lambda = 2\pi x_0$ (C) $\lambda = \frac{\pi x_0}{2}$ (D) $\lambda = 4\pi x_0$

Ans : (C)

Hints : Maximum particle velocity (V_p) = $A\omega = 2\pi n x_0$

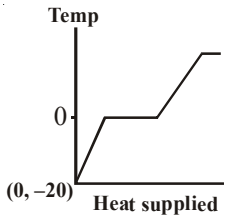
Wave velocity (V_ω) = $n\lambda$

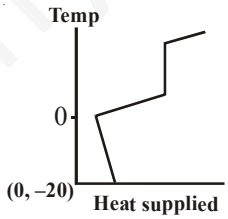
Here, $V_P = 4V_\omega$

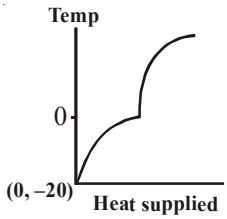
$2\pi n x_0 = 4n\lambda$

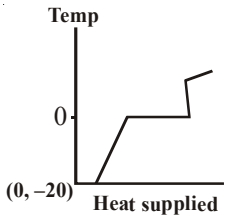
$\lambda = \frac{\pi}{2}x_0$

39. A block of ice at temperature –20 ºC is slowly heated and converted to steam at 100 ºC. Which of the following diagram is most appropriate ?

(A) 

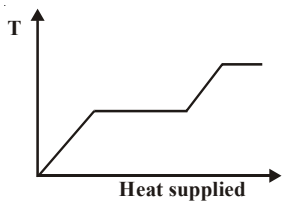
(B) 

(C) 

(D) 

Ans : (A)

Hints :



40. Two black bodies at temperatures 327 ºC and 427 ºC are kept in an evacuated chamber at 27 ºC. The ratio of their rates of loss of heat are :

(A) $\frac{6}{7}$ (B) $\left(\frac{6}{7}\right)^2$ (C) $\left(\frac{6}{7}\right)^3$ (D) $\frac{243}{464}$

Ans : (D)

Hints : Rate of loss of heat $\propto (T^4 - T_0^4)$

$$\frac{E_1}{E_2} = \frac{T_1^4 - T_0^4}{T_2^4 - T_0^4} = \frac{(600)^4 - (300)^4}{(700)^4 - (300)^4} = \frac{6^4 - 3^4}{7^4 - 3^4}$$

$$\frac{E_1}{E_2} = \frac{243}{464}$$

41. At identical temperature and pressure, the rate of diffusion of hydrogen gas is $3\sqrt{3}$ times that of a hydrocarbon having molecular formula C_nH_{2n-2}. What is the value of 'n' ?

(A) 1 (B) 4 (C) 3 (D) 8

Ans : (B)

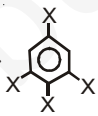
Hints : $\frac{r_{H_2}}{r_{C_nH_{2n-2}}} = \sqrt{\frac{M_{C_nH_{2n-2}}}{M_{H_2}}} = \sqrt{\frac{M_{C_nH_{2n-2}}}{2}}$

$\because \sqrt{\frac{M_{C_nH_{2n-2}}}{2}} = 3\sqrt{3} = \sqrt{27}$

$\Rightarrow M_{C_nH_{2n-2}} = 27 \times 2 = 54$

Hence, $12n + (2n-2) \times 1 = 54 \Rightarrow 14n = 56 \Rightarrow n = 4$

Thus Hydrocarbon is C_4H_6

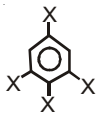
42. Dipole moment of  is 1.5D. The dipole moment of  is

(A) 1.5 D (B) 2.25 D (C) 1D (D) 3D

Ans : (A)

Hints : Given for this molecule  $\mu_1 = 1.5D$

For  $\mu = 0$ (as it is symmetrical)

Hence for  μ will be 1.5D

43. Which of the following thermodynamic relation is correct ?

(A) $dG = VdP - SdT$ (B) $dE = PdV + TdS$ (C) $dH = -VdP + TdS$ (D) $dG = VdP + SdT$

Ans : (A)

Hints : $dG = dH - TdS - SdT$ (as $G = H - TS$)

again, $H = U + PV$

$\therefore dH = dU + PdV + VdP$

& $dU = TdS - PdV$

Thus $dG = (TdS - PdV) + PdV + VdP - TdS - SdT$

$= VdP - SdT$

44. In the hydrolysis of an organic chloride in presence of large excess of water; $RCI + H_2O \rightarrow ROH + HCl$

(A) Molecularity and order of reaction both are 2
(B) Molecularity is 2 but order of reaction is 1
(C) Molecularity is 1 but order of reaction is 2
(D) Molecularity is 1 and order of reaction is also 1

Ans : (B)

Hints : As water used is in large excess.

45. The potential of a hydrogen electrode at pH = 10 is

(A) 0.59 V (B) 0.00 V (C) –0.59 V (D) –0.059

Ans : (C)

Hints : $H^+(pH = 10)|H_2(1atm)|Pt(s)$

Reaction : $2H^+(p^H = 10) + 2e \rightarrow H_2(1\ atm)$

$$E = E^0 - \frac{0.0591}{2}\log\left(\frac{P_{H_2}}{[H^+]^2}\right)$$

$$= 0 - \frac{0.0591}{2}\log\frac{1}{(10^{-10})^2} = -\frac{0.0591}{2}\times 2\log\frac{1}{10^{-10}} = -0.0591\times 10 = -0.591$$

i.e. E = –0.591 V

46. Calculate K_C for the reversible process given below if $K_p = 167$ and $T = 800^0C$

$$CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$$

(A) 1.95 (B) 1.85 (C) 1.89 (D) 1.60

Ans : (C)

Hints : $K_p = K_C(RT)^{\Delta n}$

for eq^n $CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$, $\Delta n = 1$

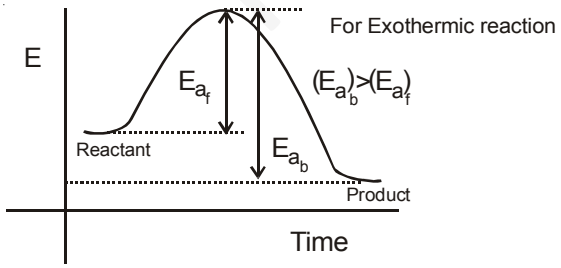
$$K_C = \frac{K_P}{(RT)^{\Delta n}} = \frac{167}{(0.0821\times 1073)^1} = 1.89$$

47. For a reversible chemical reaction where the forward process is exothermic, which of the following statements is correct ?

(A) The backward reaction has higher activation energy than the forward reaction
(B) The backward and the forward processes have the same activation energy
(C) The backward reaction has lower activation energy
(D) No activation anergy is required at all since energy is liberated in the process.

Ans : (A)

Hints :



48. In Sommerfeld's modification of Bohr's theory, the trajectory of an electron in a hydrogen atom is

(A) a perfect ellipse
(B) a closed ellipse – like curve, narrower at the perihelion position and flatter at the aphelion position
(C) a closed loop on spherical surface
(D) a rosette

Ans : (C)

49. In the reaction of sodium thiosulphate with I_2 in aqueous medium the equivalent weight of sodium thiosulphate is equal to
(A) molar mass of sodium thiosulphate
(B) the average of molr masses of $Na_2S_2O_3$ and I_2
(C) half the molar mass of sodium thiosulphate
(D) molar mass of sodium thiosulphate × 2
Ans : (A)

Hints : $2Na_2\overset{+2}{S}_2O_3 + I_2 \longrightarrow Na_2\overset{+2.5}{S}_4O_6 + 2NaI$

n-factor = 1

$$E = \frac{M}{1} = M$$

50. 0.1 (M) HCI and 0.1 (M) H_2SO_4 each of volume 2ml are mixed and the volume is made up to 6 ml by adding 2ml of 0.01 (N) NaCl solution. The pH of the resulting mixture is
(A) 1.17 (B) 1.0 (C) 0.3 (D) log 2 – log 3
Ans : (B)
Hints : Mili moles of $H^+ = 0.1\times2 + 0.1\times2\times2 = 0.6$
Total volume in ml = 6

$$pH = -\log_{10}[H^+] = -\log\left(\frac{0.6}{6}\right) = -\log 0.1 = 1$$

51. The molarity of a NaOH solution by dissolving 4 g of it in 250 ml water is
(A) 0.4 M (B) 0.8 M (C) 0.2 M (D) 0.1 M
Ans : (A)

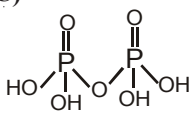
Hints : $\text{Molarity} = \frac{4/40}{250/1000} = 0.4$

52. If a species has 16 protons, 18 electrons and 16 neutrons, find the species and its charge
(A) S^{1-} (B) Si^{2-} (C) P^{3-} (D) S^{2-}
Ans : (D)
Hints : 16p means z = 16
$18e^-$ means , 2 unit negative charge is present.
Hence species is S^{-2}

53. In a periodic table the basic character of oxides
(A) increases from left to right and decreases from top to bottom
(B) decreases from right to left and increases from top to bottom
(C) decreases from left to right and increases from top to bottom
(D) decreases from left to right and increases from bottom to top
Ans : (C)

54. Which one of the following contains P – O – P bond ?
(A) Hypophosphorus acid (B) Phosphorus acid (C) Pyrophosphoric acid (D) Orthophosphoric acid
Ans : (C)

Hints :



55. Which of the following orders regarding ionization energy is correct ?
(A) N > O > F (B) N < O < F (C) N > O < F (D) N < O > F
Ans : (C)
Hints : As IE_1 N > O (because of half filled orbitals of N)
and O < F (because of smaller size of F)

56. Which of the following statements regarding ozone is not correct ?
(A) The Ozone molecule is angular in shape
(B) The Ozone is a resonance hybrid of two structures
(C) The Oxygen– Oxygen bond length in ozone is identical with that of molecular oxygen
(D) Ozone is used as germicide and disinfectant for the purification of air.

Ans : (C)

Hints : Due to resonance the bond order in ozone is 1.5, hence O – O bond length in O_3 > O – O bond length in O_2

57. P_4O_{10} is the anhydride of

(A) H_3PO_2 (B) H_3PO_3 (C) H_3PO_4 (D) $H_4P_2O_7$

Ans : (C)

Hints : $4H_3PO_4 \longrightarrow P_4O_{10} + 6H_2O$

58. Which of the following metals has the largest abundance in the earth's crust ?

(A) Aluminium (B) Calcium (C) Magnesium (D) Sodium

Ans : (A)

59. Which of the following orbitals will have zero probability of finding the electron in the yz plane ?

(A) P_x (B) P_y (C) P_z (D) d_{yz}

Ans : (A)

Hints : P_x orbital lies along x-axis only.

60. What type of orbital hybridisation is considered on P in PCl_5 ?

(A) sp^3d (B) dsp^3 (C) sp^3d^2 (D) d^2sp^3

Ans : (A)

61. For which element the inertness of the electron pair will not be observed?

(A) Sn (B) Fe (C) Pb (D) In

Ans : (B)

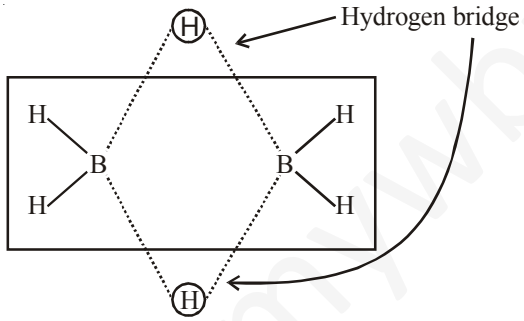
Hints : Inert pair effect is exhibited only by heavy metals of p-block elements

62. In which of the following molecules is hydrogen bridge bond present?

(A) Water (B) Inorganic benzene (C) Diborane (D) Methanol

Ans : (C)

Hints :



63. When a manganous salt is fused with a mixture of KNO_3 and solid NaOH the oxidation number of Mn changes from +2 to

(A) +4 (B) +3 (C) +6 (D) +7

Ans : (C)

Hints : $\overset{(+2)}{Mn^{+2}} + NO_3^- + O\bar{H} \rightarrow \overset{(+6)}{M}\, nO_4^{-2} + H_2O$

64. In hemoglobin the metal ion present is

(A) Fe^{2+} (B) Zn^{2+} (C) Co^{2+} (D) Cu^{2+}

Ans : (A)

65. Ortho-and para-hydrogens have

(A) Identical chemical properties but different physical properties

(B) Identical physical and chemical properties

(C) Identical physical properties but different chemical properties

(D) Different physical and chemical properties

Ans : (A)

66. The bond order of CO molecule is

(A) 2 (B) 2.5 (C) 3 (D) 3.5

Ans : (C)

Hints : $CO \rightarrow \sigma(1S)^2, \sigma^*(1S)^2, \sigma(2S)^2, \sigma(2P_z)^2, \pi(2P_x)^2 = \pi(2P_y)^2, \sigma^*(2S)^2$

$$B.O = \frac{N_b - N_o}{2} = \frac{10-4}{2} = 3$$

67. Vitamin C is

(A) Citric acid (B) Lactic acid (C) Paracetamol (D) Ascorbic acid

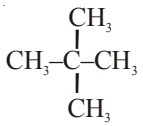
Ans : (D)

68. On mixing an alkane with chlorine and irradiating with ultra-violet light, it forms only one mono-chloro-alkane. The alkane is

(A) Propane (B) Pentane (C) Isopentane (D) Neopentane

Ans : (D)

Hints : Neopentane



contains all hydrogen atom equivalent

69. Keto-enol tautomerism is not observed in

(A) $C_6H_5COC_6H_5$ (B) $C_6H_5COCH{=}CH_2$ (C) $C_6H_5COCH_2COCH_3$ (D) $CH_3COCH_2COCH_3$

Ans : (A) as contains no α - H

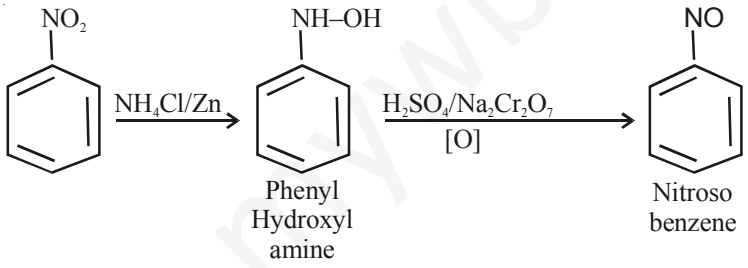
70. What is obtained when nitrobenzene is treated sequentially with (i) NH_4Cl/Zn dust and (ii) $H_2SO_4/Na_2Cr_2O_7$?

(A) meta-chloronitrobenzene (B) para-chloronitrobenzene

(C) nitrosobenzene (D) benzene

Ans : (C)

Hints :



71. Boiling water reacts with $C_6H_5N_2^+Cl^-$ to give

(A) aniline (B) benzylamine (C) phenol (D) benzaldehyde

Ans : (C)

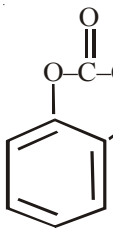
Hints : $C_6H_5N_2^+Cl^- \xrightarrow[(Boil)]{H_2O} C_6H_5OH\ (S_NAr)$

72. Aspirin is

(A) Acetyl salicylic acid (B) Benzoyl salicylic acid

(C) Chloro benzoic acid (D) Anthranilic acid

Ans : (A)

Hints : 
2-Acetoxy benzoic acid

73. $X \xrightarrow{PCl_5} C_2H_5Cl$

$Y \xrightarrow{PCl_5} CH_3COCl$

X and Y are

(A) $(C_2H_5)_2O$ and CH_3CO_2H (B) C_2H_5I and C_2H_5CHO (C) C_2H_5OH and CH_3CO_2H (D) C_2H_5OH and C_2H_5CHO

Ans : (C)

Hints : $C_2H_5OH \xrightarrow{PCl_5} C_2H_5Cl + POCl_3 + HCl$

$CH_3CO_2H \xrightarrow{PCl_5} CH_5COCl + POCl_3 + HCl$

74. Which of the following compounds shows evidence of the strongest hydrogen bonding?

(A) Propan–1–ol (B) Propan–2–ol (C) Propan–1,2–diol (D) Propan–1,2,3–triol

Ans : (D)

Hints : Propan-1,2,3 triol have three polar –OH group.

75. When AgCl is treated with KCN

(A) Ag is precipitated (B) a complex ion is formed

(C) double decomposition takes place (D) no reaction takes place

Ans : (B)

Hints : $AgCl + 2KCN \rightarrow K\left[Ag(CN)_2\right] + KCl$

76. Which one of the following produced when acetone is saturated with HCl gas?

(A) Acetone alcohol (B) Phorone (C) Mesityl oxide (D) Benzene

Ans : (C)

Hints : $2CH_3COCH_3 \xrightarrow{HCl\ gal} \underset{\text{Mesityl oxide}}{CH_3COCH = C\begin{matrix}CH_3\\CH_3\end{matrix}} + H_2O$ [Note : Phorone is formed as minor product]

77. Which one of the following is an example of co-polymer?

(A) Buna–S (B) Teflon (C) PVC (D) Polypropylene

Ans : (A)

Hints : Buna -S is a co-polymer of butadiene and styrene

$$-(CH_2–CH= CH–CH_2–\underset{\displaystyle C_6H_5}{\underset{|}{CH}}– CH_2)_n-$$

78. Identify [A] and [B] in the following

$^{227}_{89}Ac \xrightarrow{-\beta} [A] \xrightarrow{-\alpha} [B] \xrightarrow{-\alpha} Rn$

(A) Po, Rn (B) Th, Po (C) Ra, Th (D) Th, Ra

Ans : (D)

Hints : $^{227}_{89}Ac \xrightarrow{-\beta} {}^{227}_{90}Th \xrightarrow{-\alpha} {}^{223}_{88}Ra$

79. A weak acid of dissociation constant 10^{-5} is being titrated with aqueous NaOH solution. The pH at the point of one-third neutralisation of the acid will be

(A) $5+\log 2-\log 3$ (B) $5-\log 2$ (C) $5-\log 3$ (D) $5-\log 6$

Ans : (B)

Hints : $K_a = 10^{-5} \Rightarrow pK_a = -\log K_a = -\log 10^{-5} = 5$

	HA + NaOH $\longrightarrow$	NaA +	H_2O
Initial	1mole 0	0	0
Final	$(1-1/3)$mole	$1/3$mole	$1/3$mole
	$=2/3$mole	$1/3$mole	

(Assumed weak acid to be monoprotic, since only one dissociation constant value is provided)

Final solution acts as an acidic buffer.

$$\Rightarrow pH = pK_a + \log\frac{[salt]}{[Acid]} \Rightarrow pH = 5 + \log\frac{\frac{1}{3}}{\frac{2}{3}} = 5 + \log\frac{1}{2} \Rightarrow pH = 5 - \log 2$$

80. Radioactivity of a sample (z=22) decreases 90% after 10 years. What will be the half life of the sample?

(A) 5 years (B) 2 years (C) 3 years (D) 10 years

Ans : (C)

Hints : $t = 10$ yrs $\quad t_{\frac{1}{2}} = ?$

$$\lambda = \frac{2.303}{t}\log\frac{N_o}{N_t}$$

Since radioactivity decreases 90% in 10 yrs. $\Rightarrow N_0 = 100$ & $N_t = 10$

$$\text{Thus } \lambda = \frac{2.303}{10}\log\frac{100}{10} \Rightarrow \lambda = \frac{2.303}{10}$$

$$\text{since } t_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{2.303\times\log 2}{\lambda} \Rightarrow t_{\frac{1}{2}} = \frac{2.303\times\log 2}{2.303/10}$$

$$\Rightarrow t_{\frac{1}{2}} = (\log 2)\times 10 \simeq 3 \text{ years}$$

DESCRIPTIVE TYPE QUESTIONS

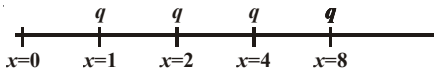
SUB : PHYSICS & CHEMISTRY

1 A circular disc rolls down on an inclined plane without slipping. What fraction of its total energy is translational?

A. $\text{Fraction} = \dfrac{\frac{1}{2}mV^2}{\frac{1}{2}mV^2 + \frac{1}{2}(mK^2)\frac{V^2}{R^2}} = \dfrac{1}{1+\frac{K^2}{R^2}} = \dfrac{1}{1+\frac{1}{2}} = \dfrac{2}{3}$

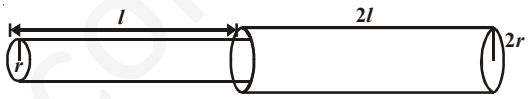
2 An infinite number of charges, each equal to q, are placed along the x-axis at $x = 1$, $x = 2$, $x = 4$, $x = 8$ and so on. What is the potential at $x = 0$ due to this set of charges ?

A. $V = \dfrac{q}{4\pi\varepsilon_0}\left[1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+....\right] = \dfrac{q}{4\pi\varepsilon_0}\dfrac{1}{1-\frac{1}{2}} = \dfrac{2q}{4\pi\varepsilon_0}$



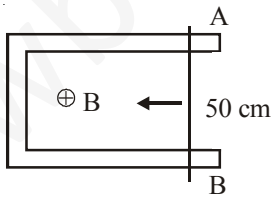
3 A liquid flows through two capillary tubes A and B connected in series. The length and radius of B are twice those of A. What is the ratio of the pressure difference across A to that across B?

A. $Q = \dfrac{\pi P_1 r_1^4}{8nl_1} = \dfrac{\pi P_2 r_2^4}{8nl_2}$



$\dfrac{P_1}{P_2} = \left(\dfrac{r_2}{r_1}\right)^4 \times \dfrac{l_1}{l_2} = \left(\dfrac{2r}{r}\right)^4 \times \dfrac{l}{2l} = 16 \times \dfrac{1}{2} = 8$

4 A 50 cm long conductor AB moves with a speed 4 m/s in a magnetic field B = 0.01 Wb/m^2 as shown. Find the e.m.f. generated and power delivered if resistance of the circuit is 0.1 Ω.



A. e.m.f. (e) = $vBl = 4 \times 0.01 \times 50 \times 10^{-2} = 200 \times 10^{-4} = 2 \times 10^{-2}$ V

Power = $P = \dfrac{e^2}{R} = \dfrac{4\times10^{-4}}{0.1} = 4\times10^{-3}$ watt

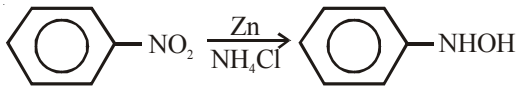
5 An electron is moving with a velocity $\left(2\hat{i}+2\hat{j}\right)$ m/s in an electric field of intensity $\vec{E} = \hat{i}+2\hat{j}-8\hat{k}$ Volt/m and a magnetic field of $\vec{B} = \left(2\hat{j}+3\hat{k}\right)$ tesla. Find the magnitude of force on the electron.

A. $\vec{F} = q(\vec{E}+\vec{V}\times\vec{B}) = (1.6\times10^{-19})(7\hat{i}-4\hat{j}-4\hat{k})$

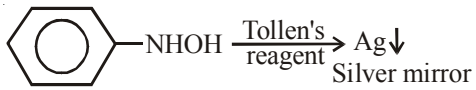
$|\vec{F}| = 1.6\times10^{-19}\times9 = 14.4\times10^{-19}$ N

6. How nitrobenzene is identified using Mulliken-Barker test?

A : Nitrobenzene is reduced using Zn and NH_4Cl in alcohol medium.



The N-phenyl hydroxylamine when reacts with Tollen's reagent gives bright silver miror.



7. Calculate the ratio of the rate of diffusion of oxygen to the rate of diffusion of hydrogen at constant temperature and pressure.

A : $\frac{r_{O_2}}{r_{H_2}} = \sqrt{\frac{2}{32}} = \frac{1}{4}$

8. Why B_2 is paramagnetic whereas C_2 is diamagnetic?

A : For $B_2\,(10\bar{e})$ the MO configuraiton is $(\sigma 1S)^2\left(\sigma^* 1S\right)^2(\sigma 2S)^2\left(\sigma^* 2S\right)^2\left(\pi 2P_x^1 = \pi 2P_y^1\right)$

Due to presence of unpaired electron $\left\{\pi 2P_x^1 = \pi 2P_y^1\right\}$ it shows paramagnetism.

$C_2\,(12\bar{e})$ the MO configuration is $(\sigma 1S)^2\left(\sigma^* 1S\right)^2(\sigma 2S)^2\left(\sigma^* 2S\right)^2\left(\pi 2P_x^2 = \pi 2P_y^2\right)$

No unpaired electrons are there in C_2 $\left\{\pi 2P_x^2 = \pi 2P_y^2\right\}$, hence it shows diamagnetism.

9. Explain briefly the cause of Lanthanoid contraction.

A : On moving in the lanthanid series from left to right successive electrons enter into ante penultimate 4f-subshell which imparts very poor shielding effect (due to its diffused nature), hence effective nuclear charge gradually increases with increase in atomic number. That is why shrinkage is observed on moving through lanthanide series, this is known as lanthanide contraction.

10. Explain why aniline is not as basic as ammonia.

A : In aniline the lone-pair over nitrogen atom is in conjugation with the π-electrons of the benzene ring and it takes part in resonance. That is why availability of lone-pair is not as that as in ammonia. Thus aniline is less basic than ammonia.

by Aakash Institute & Aakash IIT-JEE

MULTIPLE CHOICE QUESTIONS

SUB : BIOLOGY

1. First Genetically modified plant commercially released in India is :
   (A) Golden rice (B) Slow ripening tomato (C) Bt-brinjal (D) Bt-Cotton
   Ans : (D)
   Hints : Bt cotton was developed by MAHYCO (Maharashtra Hybrid Seed Company Limited) in collaboration with Monsanto.
2. Quiescent centre is found in plants at :
   (A) Root tip (B) Cambium (C) Shoot tip (D) Leaf tip
   Ans : (A)
   Hints : It is a zone of low mitotic activity located in the sub-apical region of root.
3. In a DNA molecule distance between two bases is
   (A) 2 nm/20Å (B) 0.2 nm/2Å (C) 3.4 nm / 34 Å (D) 0.34 nm/3.4 Å
   Ans : (D)
   Hints : The distance between two bases is 0.34 nm / 3.4 Å
4. Exine of pollen grain is made up of
   (A) Pectocellulose (B) Ligno cellulose (C) Sporopollenin (D) Pollen Kit
   Ans : (C)
   Hints : Sporopollenin is the product of oxidative polymerisation of carotenoids.
5. When the cell is fully turgid, its
   (A) DPD = OP (B) DPD = Zero (C) WP = TP (D) OP = Zero
   Ans : (B)
   Hints : Since DPD = OP – TP
   In a fully turgid cell, OP = TP
   $\therefore$ DPD = Zero
6. Which one is true for ATP ?
   (A) ATP is prosthetic part of an enzyme (B) ATP is an enzyme
   (C) ATP is organic ions of enzyme (D) ATP is a Co-enzyme
   Ans : (D)
   Hints : ATP is a multifunctional nucleotide which acts as a coenzyme.
7. Root cells of Wheat has 2n = 42 chromosomes. Which one of the following is the basic chromosome number of Wheat ?
   (A) 42 (B) 21 (C) 7 (D) 14
   Ans : (C)

Hints : For wheat, $2n = 6x = 42$

$\therefore x = 7$

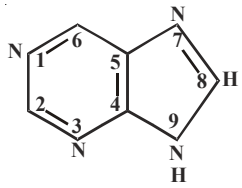
'x' represents basic or genomic number.

8. Purines possess nitrogen at

(A) 1, 2, 4 and 6 position (B) 1, 3, 5 and 7 position

(C) 1, 3, 7 and 9 position (D) 1, 2, 6 and 8 position

Ans : (C)

Hints :  **Purine Base**

9. Thylakoids occur inside

(A) Mitochondria (B) Chloroplast (C) Golgi apparatus (D) Endoplasmic reticulum

Ans : (B)

Hints : Thylakoid occurs in chloroplast.

10. Micropropagation is a technique

(A) for production of true to type plants (B) for production of haploid plant

(C) for production of Somatic hybrids (D) for production of Soma clonal plants

Ans : (A)

Hints : Raising of new plantlets through tissue culture technique producing similar plants (true type plants).

11. Test cross is a cross between

(A) Hybrid × Dominant parent (B) Hybrid × Recessive parent

(C) Hybrid × Hybrid parent (D) Two distantly related species

Ans : (B)

Hints : Test cross - F_1 hybrid is crossed with recessive parent.

12. Mitochondria are semi autonomous as they possess

(A) DNA (B) DNA + RNA

(C) DNA + RNA Ribosomes (D) Protein

Ans : (C)

Hints : Due to presence of 70*s* ribosome, RNA and *ds* circular DNA mitochondria is semiautonomous.

13. Chitin is a

(A) Polysaccharide (B) Nitrogenous polysaccharide

(C) Lipo Protein (D) Protein

Ans : (B)

Hints :

Polymer of N-acetylglucosamine $(C_8H_{13}O_5N)_n$ that forms exoskeleton of arthropods and cell wall of fungi.

14. Balbiani rings are the sites of

(A) DNA replication (B) RNA and protein synthesis

(C) Synthesis of lipids (D) Synthesis of polysaccharides

Ans : (B)

Hints : These rings contain active DNA so RNA and proteins are synthesized here.

15. Which of the cell organelle lacks membrane ?

(A) Mesosome (B) Mitochondria

(C) Ribosome (D) Liposome

Ans : (C)

Hints : Smallest cell organelle without cell membrane is ribosome.

16. Interfacicular cambium is a
(A) Primary meristematic tissue (B) Primordial meristem
(C) Type of Protoderm (D) Secondary meristematic tissue
Ans : (D)
Hints : Parenchymatous cells present between two vascular bundles give rise to interfascicular cambium after dedifferentiation.

17. Cotton fibre is basically a type of
(A) Trichome (B) Scale (C) Dried seed coat (D) Non glandular hair
Ans : (D)
Hints : Cotton fibres are epidermal out growth in form of hairs.

18. Chloroplast dimorphism is a characteristic feature of
(A) Plants with Calvin cycle (B) C_4-Plants
(C) All plants (D) Only in algae
Ans : (B)
Hints : Two types of chloroplast are found in plant having Kranz anatomy

19. In which type of reactions related to plant photosynthesis peroxisomes are involved ?
(A) Glycolate cycle (B) Calvin cycle
(C) Bacterial photosynthesis (D) Glyoxylate cycle
Ans : (A)
Hints : Perosisome perform photorespiration that is also called as glycolate cycle.

20. The term Alpha diversity refers to
(A) Genetic diversity (B) Community & ecosystem diversity
(C) Species diversity (D) Diversity among the plants
Ans : (B)
Hints : Alpha diversity is a type of community or ecosystem diversity

21. How many variable segments are present in the basic structure of antibody molecules ?
(A) One (B) Two (C) Three (D) Four
Ans : (D)
Hints : 2 present in heavy chain and 2 present in light chain.

22. Which one is diaminodicarboxylic amino acid ?
(A) Cystine (B) Lysine (C) Cysteine (D) Aspartic Acid
Ans : (a)
Hints : The chemical formula is $(SCH_2 – CH (NH_2) CO_2H)_2$

23. Which one is the cofactor of carbonic anhydrase ?
(A) Fe (B) Zn (C) Cu (D) Mg
Ans : (B)
Hints : 'Zn' acts as cofactor for carbonic anhydrase

24. Vitamin – D is produced in human body in –
(A) Muscles (B) Nerves (C) Skin (D) Bone-marrow
Ans : (C)
Hints : Vitamin D is synthesized in the skin in presence of sunlight

25. Bacteriophages kill
(A) Fungi (B) Parasites (C) Bacteria (D) Viruses
Ans : (C)
Hints : A virus that is parasite over bacteria is called Bacteriophage

26. What is mitoplast ?
(A) Membraneless mitochondria (B) Another name of mitochondria
(C) Mitochondria without outer membrane (D) Mitochondria without inner membrane
Ans : (C)
Hints : Mitochondria without outer membrane is called as mitoplast.

27. Transposons are –
(A) House - keeping genes (B) Jumping genes
(C) Transporting genes (D) Stationary genes
Ans : (B)

28. Which of the following is not a conjugated protein ?
(A) Peptone (B) Phosphoprotein (C) Lipoprotein (D) Chromoprotein
Ans : (A)
Hints : Peptone is a derived protein. Others are conjugated proteins.

29. The outer covering of cartilage is called
(A) Peritonium (B) Periosteum (C) Endosteum (D) Perichondrium
Ans : (D)
Hints : Perichondrium is the outer covering of cartilage.

30. The blood does not clot inside the body because of :
(A) Oxygenation of blood (B) Movement of blood
(C) Heparin in blood (D) Absence of fibrinogen in blood
Ans : (C)
Hints : Heparin prevent clotting of blood inside the body.

31. Red cell count is carried out by –
(A) Haemocytometer (B) Haemoglobinometer
(C) Sphygmomanometer (D) Electrocardiogram
Ans : (A)
Hints : Blood corpuscle counting is done by this instrument.

32. Rh factor can produce disease
(A) AIDS (B) Turner's Syndrome (C) Erythroblastosis foetalis (D) Sickle - cell anaemia
Ans : (C)
Hints : During second pregnancy it may rupture foetal RBC due to antibody agglutination if the father is Rh^+ ve and the mother is Rh^- ve.

33. Name the hormone that stimulates the secretion of gastric juice
(A) Renin (B) Enterokinase (C) Enterogastrone (D) Gastrin
Ans : (D)
Hints : Gastric glands are activated by this secretion of Argentaffin cell.

34. Bile salts act as activator of which enzyme ?
(A) Pepsinogen (B) Trypsinogen (C) Lipase (D) Pancreatic amylase
Ans : (C)
Hints : Bile salt activates lipase & also emulsifies the fat

35. Heparin is produced by –
(A) Kidney Cells (B) Blood Cells (C) Bone marrow (D) Liver cell
Ans : (D)
Hints : Heparin is produced by liver cells mainly.

36. Which of the following cells produce HCl ?
(A) β-Cell (B) α-Cell (C) Oxyntic Cell (D) Chief Cell
Ans : (C)
Hints : Oxyntric or parietal cell of stomach secretes HCl.

37. Which ribs show "bucket - handle' type of movement ?
(A) Rib No. 1 – 2 (B) Rib No. 3 – 5 (C) Rib No. 6 – 10 (D) Rib No. 11 – 12
Ans : (C)
Hints : The upward and downward movement of the shaft of the rib no 6 - 10 has been likened to raising the handle from the side of a bucket. Therefore, they show bucket handle movement

38. In which of the following subjects the dead space is highest ?
(A) Old man (B) Old woman (C) Young man (D) Young woman
Ans : (A)
Hints : Old man haivng high dead space volume due to low supply of blood to lungs

39. Which one has the thickest wall ?
(A) Right auricle (B) Right Ventricle (C) Left auricle (D) Left ventricle
Ans : (D)
Hints : The thickest wall of heart is found in left ventricle.

40. The cardiac cycle in normal subject is about
(A) 0.5 second (B) 0.8 second (C) 1.0 second (D) 1.2 second
Ans : (B)
Hints : One cardiac cycle is completed in 0.8 sec.

41. What is glycosuria ?
(A) Low amount of sugar in urine (B) Low amount of fat in urine
(C) Average amount of carbohydrate in urine (D) High amount of sugar in urine
Ans : (D)
Hints : Glycosuria is the high amount of sugar in urine mainly due to insulin deficiency.

42. Volume of urine is regulated by –
(A) Aldosterone (B) Aldosterone and testosterone
(C) ADH (D) Aldosterone and ADH
Ans : (D)
Hints : Volume of urine is regulated by Aldosterone and ADH via RAAS involving juxta medullary nephron.

43. Skin is an acessory organ or respiration in –
(A) Human (B) Frogs (C) Rabbit (D) Lizard
Ans : (B)
Hints : Skin is an accessory respiratory organ in amphibians.

44. Name the condition when the concentration of Ketone body increases in urine
(A) Acromegaly (B) Diabetes mellitus (C) Diabetes insipidus (D) Cushing's disease
Ans : (B)
Hints : In diabetes mellitus ketone body synthesis increases due to cellular starvation.

45. Hormone responsible for the secretion of milk after parturition
(A) ICSH (B) Prolactin (C) ACTH (D) LH
Ans : (B)
Hints : Prolactin secreted from pituitary is responsible for secretion of milk after parturition.

46. Endemic goitre is a state of
(A) Increased thyroid function (B) Normal thyroid function
(C) Decreased thyroid function (D) Moderate thyroid function
Ans : (C)
Hints : Endemic goitre is due to low iodine in soil and water in hilly areas.

47. Islets of Langerhans are found in
(A) Anterior Pituitary (B) Kidney Cortex (C) Spleen (D) Endocrine pancreas
Ans : (D)
Hints : Islets of Langerhans are the endocrine part of pancreas.

48. Which of the following is the function of Adrenaline ?
(A) Helps in gastric juice secretion (B) Increases heart rate and blood pressure
(C) Increases blood calcium (D) Helps in milk secretion
Ans : (B)
Hints : Adrenaline is released in stress condition and is responsible for increased heart rate and blood pressure.

49. Which of the following is not related to the autonomic nervous system ?
(A) Peristalsis (B) Digestion (C) Excretion (D) Memory and learning
Ans : (D)
Hints : Autonomic nervous system controls involuntary functions of the visceral organs.

50. Comprehension of spoken and written words take place in the region of
(A) Association Area (B) Motor Area (C) Wernicke's Area (D) Broca's Area
Ans : (C)
Hints : Wernicke's area is responsible for understanding speech.

51. Which one of the following cranial nerves is carrying the nerve fibres originating from the Edinger-Westphal nucleus ?
(A) Oculomotor (B) Trochlear (C) Abducens (D) Vagus
Ans : (A)
Hints : Occulomotor nerve has occulomotor nucleus and Edinger-Westphal nucleus.

52. How many laminae are present in the grey matter of spinal cord ?
(A) Four (B) Six (C) Eight (D) Ten
Ans : (D)
Hints : Rexed, based on the cyto architectural pattern as well as on the density of neuronal packing, identified several groups of arrangement which are 10 in number and now called Rexed laminae.

53. Colour blindness is due to defect in
(A) Cones (B) Rods (C) Rods and cones (D) Rhodopsin
Ans : (A)
Hints : Cones are related with coloured vision.

54. MRI is not allowed in the following conditions except one. Identify the exception.
(A) Presence of pacemaker in the body
(B) Pregnant women
(C) Person suffering from stroke
(D) Presence of metallic plate in the body for treatment of broken bones
Ans : (B)
Hints : It uses no ionizing radiation, but uses a powerful magnetic field to align the nuclear magnetization of Hydrogen atom in water inside body.

55. Which of the following diseases is related to cadmium pollution ?
(A) Minamata (B) Pneumoconiosis (C) Anaemia (D) Itai-itai
Ans : (D)
Hints : Itai-Itai (ouch-ouch disease) is due to Cd poisoning in the drinking water result into skeletal deformity.

56. Percentage composition of Fibroin and Sericin in silk is
(A) 50 : 40 (B) 80 : 20 (C) 30 : 70 (D) 40 : 60
Ans : (B)
Hints : Fibroin is the core silk protein and sericin is the surface gum-like compound.

57. Which one of the following is used as biological insecticide ?
(A) Tiger beetle (B) Caterpillar (C) Silkmoth (D) Mazra Poka
Ans : (A)
Hints : Caterpillar - larval stage of insects, silkmoth is used in silk culture and Mazra poka is the paddy pest.

58. Which one of the following diseases is spread by Housefly ?
(A) Dengue fever (B) Encephalitis (C) Filariasis (D) Typhoid
Ans : (D)
Hints : Others are spread by mosquito.

59. Water-Vascular' system is found in
(A) Sea-anemone (B) Sea-pen (C) Sea-cucumber (D) Sea-horse
Ans : (C)
Hints : Water vascular system is found in echinoderms.

60. Nutrient enrichment of a lake will cause
(A) Eutrophication (B) Stratification (C) Biomagnification (D) Bioaccumulation
Ans : (A)
Hints : Eutrophication or nutrient enrichment of water body is basically due to excessive presence of nitrates & phosphates.

61. Lichens are decribed as indicator of
(A) Air pollution (B) Water pollution (C) Soil pollution (D) Agriculture productivity
Ans : (A)
Hints : Lichens are indicator plant of air pollution particularly of SO_2

62. Most abundant mineral of animal body is
(A) Iron (B) Sodium (C) Potassium (D) Calcium
Ans : (D)
Hints : Primary component of bones and also present in muscles and blood.

63. Retrogressive metamorphosis occurs in
(A) Hemichordata (B) Cephalochordata (C) Urochordata (D) Vertebrata
Ans : (C)
Hints : Larva is more developed and has notochord and locomotory organ

64. 'Organ of Jacobson' helps in
(A) Touch (B) Vision (C) Smell (D) Hear
Ans : (C)
Hints : Also called vomeronasal organ. It is an olfactory sense organ. Commonly found in reptiles.

65. Cysticercus stage is formed in
(A) *Taenia* (B) *Plasmodium* (C) *Leishmania* (D) *Wuchereria*
Ans : (A)
Hints : Formed in the life-cycle of pork tapeworm (*Taenia solium*)

66. Which one of the following viruses contains both DNA and RNA ?
(A) Cyanophage (B) Herpes Virus (C) Leuko Virus (D) Polio Virus
Ans : (C)
Hints : Lenko virus (a Retro virus) possess both DNA & RNA in life cycle.

67. The hormone responsible for "Fight and Flight" response is
(A) Adrenalin (B) Thyroxine (C) ADH (D) Oxytocin
Ans : (A)
Hints : Fight and flight response is due to adrenlin released from adrenal medulla.

68. Tuberculosis is caused by :
(A) *Mycobacterium sp*. (B) *Aspergillus* sp. (C) *Clostridium* sp. (D) *Vibrio sp.*
Ans : (A)
Hints : T. B. is caused by *Mycobacterium tuberculi.*

69. Which of the following is a catadromous fish ?
(A) *Hilsa sp.* (B) *Mystus sp.* (C) *Anguilla sp.* (D) *Channa sp.*
Ans : (C)
Hints : *Anguilla sp.* (Eel) is a catadromous fish that lives in freshwater and breeds in sea.

70. Which animal of the following belongs to class crustacea ?
(A) Cockroach (B) Cyclops (C) Grasshopper (D) Mosquito
Ans : (B)
Hints : Class crustacea includes *cyclops*. Other options are from class insecta.

71. Radula is found in :
(A) *Pila sp.* (B) *Chiton sp.* (C) *Lamellidens sp.* (D) *Pinctada sp.*
Ans : (A)
Hints : Radula is found in gastropods.

72. The scientific name of Java man is
(A) *Homo habilis* (B) *Homosapiens neandarthalensis*
(C) *Homo erectus erectus* (D) *Australopithecus boisei*
Ans : (C)
Hints : Scientific name *Homo erectus erectus* was given by Ernst Mayr.

73. Which phase comes in between the G 1 and G 2 phases of cell cycle ?
(A) M-phase (B) Go-phase (C) S-phase (D) Interphase
Ans : (C)
Hints : The sequence of Interphase (I-phase) is $G_1 \rightarrow S \rightarrow G_2$

74. How many effective codons are there for the synthesis of twenty amino acids ?
(A) 64 (B) 32 (C) 60 (D) 61
Ans : (D)
Hints : Out of 64 codons, 61 codons code for amino acids & the rest three - UAG, UAA & UGA are stop codons (i.e do not specify any amino acid)

75. Which of the following condition is called monosomic ?
(A) 2n + 1 (B) 2n + 2 (C) n + 1 (D) 2n – 1
Ans : (D)
Hints : Monosomy (2n–1) is a kind of aneuploidy where one chromosome is devoid of its homologue.

76. Chromosome is made up of
(A) DNA + pectin (B) RNA + DNA (C) DNA + Histone (D) Only histone
Ans : (C)
Hints : Chemical composition of a typical chromosome : DNA=40%, Histone = 50%, Non histone = 8.5%, RNA=1.5%

77. Cell division can not be stopped in which phase of the cell cycle ?
(A) G 1-phase (B) G 2-phase (C) S-phase (D) Prophase
Ans : (C)
Hints : The check points are basically present in the interphase.

78. Which of the following is structural subunit of DNA ?
(A) Protein (B) Carbohydrate (C) RNA (D) Nucleotides
Ans : (D)
Hints : DNA is the polymer of deoxyribonucleotides.

79. Cell theory is not applicable for
(A) Bacteria (B) Fungus (C) Algae (D) Virus
Ans : (D)
Hints : Since virus lacks cellular organization so, cell theory is not applicable.

80. The difference between systolic and diastolic pressure in human is
(A) 120 mm Hg (B) 80 mm Hg (C) 40 mm Hg (D) 200 mm Hg
Ans : (C)
Hints : This is called as pulse pressure. Normal systolic pressure = 120 mm Hg
Normal Diastolic pressure = 80 mm Hg

DESCRIPTIVE TYPE QUESTIONS

SUB : BIOLOGY

1. What is Cochlear microphonics ?

   A. It is the electrical potential generated in the hair cells of organ of Corti in response to acoustic stimulation, called as cochlear microphonic.

2. What is axon reflex ?

   A. Axon reflex is a response brought on by peripheral nerve stimulation. It is also known as Hunter reflex reaction as it causes vasodialation and loss of body heat from extremities.

3. What is enterohepatic circulation of bile salt ? Mention its significance .

   A. Enterohepatic recirculation operates between ileum and liver in which bile salts are absorbed from ileum and re-enters into liver for the reutilisation of bile salts.

4. Mention the location and function of juxtaglomerular apparatus .

   A. JGA is found between the vascular pole of the renal corpuscle and the returning DCT of the same nephron.

   Function of JGA : It secretes renin & erythropoietin. Renin controls RAAS and is responsible for osmoregulation.

5. What is telomere ? State its function .

   A. Telomere is a region of repetitive DNA at the end of a chromosome. It protects the end of the chromosome from deterioration.

6. Name two internal characteristic features of class Mammalia.

   A. Internal chracteristic of class mammalia

   – Presence of corpus callosum in brain.

   – Presence of Sertoli cells in testis.

   – Presence of diaphragm.

   – Presence of spongy lungs.

   – Presence of corpus luteum

7. State the advantages of composite fish culture.

   A. Advantage of composite fish culture are

   1. Different type of carps reared in the same pond.

   2. It is economical and highly productive.

   3. Carps reared in different strata of pond habitat utilise different types of food.

8. What is ribophorin ?

   A. Ribophorins are ribosome receptor proteins that aid in the binding 60S subunit of ribosomes to the rough endoplasmic reticulum. Two kinds of Ribophorins are Ribophorin I and Ribophorin II.

9. What is Pro-enzyme ?

A. These are inactive forms of enzymes which are activted in presence of activators.

Pepsinogen $\xrightarrow{HCl}$ Pepsin
(inactive) (active)

10. Name two sulphur containing and two basic amino acids .

A. The sulphur containing amino acids are

– Methionine

– Cysteine

– Cystine

Basic amino acids are :

– Lysine

– Arginine

– Histidine

MULTIPLE CHOICE QUESTIONS

SUB : MATHEMATICS

1. The value of $\frac{\cot x - \tan x}{\cot 2x}$ is

(A) 1 (B) 2 (C) –1 (D) 4

Ans : (B)

Hints : $\frac{\cos^2 x - \sin^2 x}{\sin x \cos x} \times \frac{\sin 2x}{\cos 2x} = \frac{2\cos 2x}{\sin 2x} \times \frac{\sin 2x}{\cos 2x} = 2$

2. The number of points of intersection of $2y = 1$ and $y = \sin x$, in $-2\pi \le x \le 2\pi$ is

(A) 1 (B) 2 (C) $(8)^{1+|\cos x|+|\cos^2|+\infty} = 4^3$ (D) 4

Ans : (D)

Hints : $y = \frac{1}{2} = \sin x$ $\quad -2\pi \le x \le 2\pi$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, -\frac{7\pi}{6}, -\frac{11\pi}{6}$$

No. of sol^n 4

3. Let R be the set of real numbers and the mapping $f : R \to R$ and $g : R \to R$ be defined by $f(x) = 5 - x^2$ and $g(x) = 3x - 4$, then the value of $(fog)(-1)$ is

(A) –44 (B) –54 (C) –32 (D) –64

Ans : (A)

Hints : $f(g(-1)) = f(-3-4) = f(-7) = 5 - 49 = -44$

4. $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 3, 4, 5, 6\}$ are two sets, and function $f : A \to B$ is defined by $f(x) = x + 2\ \forall x \in A$, then the function f is

(A) bijective (B) onto (C) one–one (D) many–one

Ans : (C)

Hints : $f(x) = f(y) \Rightarrow x + 2 = y + 2 \Rightarrow x = y \quad \therefore$ one–one

5. If the matrices $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$, then AB will be

(A) $\begin{bmatrix} 17 & 0 \\ 4 & -2 \end{bmatrix}$ (B) $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ (C) $\begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Ans : (A)

Hints : $AB=\begin{bmatrix}2&1&3\\4&1&0\end{bmatrix}\begin{bmatrix}1&-1\\0&2\\5&0\end{bmatrix}=\begin{bmatrix}17&0\\4&-2\end{bmatrix}$

6. ω is an imaginary cube root of unity and $\begin{vmatrix}x+\omega^2&\omega&1\\\omega&\omega^2&1+x\\1&x+\omega&\omega^2\end{vmatrix}=0$ then one of the values of x is

(A) 1 (B) 0 (C) –1 (D) 2

Ans : (B)

Hints : $\xrightarrow{C_1'\to C_1+C_2+C_3}\begin{vmatrix}x&\omega&1\\x&\omega^2&1+x\\x&x+\omega&\omega^2\end{vmatrix}=x\begin{vmatrix}1&\omega&1\\1&\omega^2&1+x\\1&x+\omega&\omega^2\end{vmatrix}$

$$=x\begin{vmatrix}1&\omega&1\\0&\omega^2-\omega&x\\0&x&\omega^2-1\end{vmatrix}=x\{(\omega^2-\omega)(\omega^2-1)-x^2\}=0 \quad \Rightarrow x=0 \quad \text{One value of } x=0$$

7. If $A=\begin{bmatrix}1&2\\-4&-1\end{bmatrix}$ then A^{-1} is

(A) $\frac{1}{7}\begin{bmatrix}-1&-2\\4&1\end{bmatrix}$ (B) $\frac{1}{7}\begin{bmatrix}1&2\\-4&-1\end{bmatrix}$ (C) $\frac{1}{7}\begin{bmatrix}-1&-2\\4&1\end{bmatrix}$ (D) Does not exist

Ans : Both (A) & (C)

Hints : $|A|=-1+8=7$

$$\text{adj }(A)=\begin{bmatrix}+(-1)&-(2)\\-(-4)&+(1)\end{bmatrix}=\begin{bmatrix}-1&-2\\4&1\end{bmatrix}$$

$$A^{-1}=\frac{1}{7}\begin{bmatrix}-1&-2\\4&1\end{bmatrix}$$ Both (A and C)

8. The value of $\frac{2}{3!}+\frac{4}{5!}+\frac{6}{7!}+\ldots\ldots$ is

(A) $e^{\frac{1}{2}}$ (B) e^{-1} (C) e (D) $e^{-\frac{1}{3}}$

Ans : (B)

Hints : $t_n=\frac{2n}{(2n+1)!}=\frac{2n+1}{(2n+1)!}-\frac{1}{(2n+1)!}=\frac{1}{(2n)!}-\frac{1}{(2n+1)!}$

$$\sum_{n=1}^{\infty}t_n=\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\ldots\ldots\infty=e^{-1}$$

9. If sum of an infinite geometric series is $\frac{4}{5}$ and its 1st term is $\frac{3}{4}$, then its common ratio is

(A) $\frac{7}{16}$ (B) $\frac{9}{16}$ (C) $\frac{1}{9}$ (D) $\frac{7}{9}$

Ans : (A)

Hints : $\frac{a}{1-r}=\frac{4}{3}$ Then $\frac{\frac{3}{4}}{1-r}=\frac{4}{3} \Rightarrow r=1-\frac{9}{16}=\frac{7}{16}$

10. The number of permutations by taking all letters and keeping the vowels of the word COMBINE in the odd places is

(A) 96 (B) 144 (C) 512 (D) 576

Ans : (D)

Hints : Vowels : O, I, E

No. of Odd place : 4

No of ways = ${}^4P_3 \times 4! = 576$

11. If ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^nC_3$, then n is just greater than integer

(A) 5 (B) 6 (C) 4 (D) 7

Ans : (D)

Hints : ${}^{n-1}C_3 + {}^{n-1}C_4 > {}^nC_3$

$\Rightarrow {}^n C_4 > {}^n C_3 \Rightarrow \frac{n!}{4!(n-4)!} > \frac{n!}{3!(n-3)!} \Rightarrow \frac{1}{4} > \frac{1}{(n-3)} \Rightarrow n-3>4 \Rightarrow n>7$

12. If in the expansion of $(a-2b)^n$, the sum of the 5th and 6th term is zero, then the value of $\frac{a}{b}$ is

(A) $\frac{n-4}{5}$ (B) $\frac{2(n-4)}{5}$ (C) $\frac{5}{n-4}$ (D) $\frac{5}{2(n-4)}$

Ans : (B)

Hints : $(a-2b)^n = \sum_{r=0}^{n} {}^nC_r (a)^{n-r}(-2b)^r$

$t_5 + t_6 = 0$

$\Rightarrow {}^n C_4 (a)^{n-4}(-2b)^4 + {}^n C_5 (a)^{n-5}(-2b)^5 = 0 \Rightarrow \frac{n!}{4!(n-4)!}a^{n-4}(-2b)^4 = -\frac{n!}{5!(n-5)!}(a)^{n-5}(-2b)^5$

$\Rightarrow \frac{1}{(n-4)} \times a = \frac{-1}{5}(-2b) \Rightarrow \frac{a}{b} = \frac{2(n-4)}{5}$

13. $\left(2^{3n}-1\right)$ will be divisible by $(\forall n \in N)$

(A) 25 (B) 8 (C) 7 (D) 3

Ans : (C)

Hints : $2^{3n} = (8)^n = (1+7)^n = = {}^n C_0 + {}^n C_1 7 + {}^n C_2 7^2 + ... + {}^n C_n 7^n$

$\Rightarrow 2^{3n} - 1 = 7\left[{}^n C_1 + {}^n C_2 7 + ... + {}^n C_n 7^{n-1} \right]$

$\therefore$ divisible by 7

14. Sum of the last 30 coeffivients in the expansion of $(1+x)^{59}$, when expanded in ascending powers of x is

(A) 2^{59} (B) 2^{58} (C) 2^{30} (D) 2^{29}

Ans : (B)

Hints : Total terms = 60

Sum of first 30 terms = $\frac{\text{Sum of all the terms}}{2} = \frac{2^{59}}{2} = 2^{58}$

15. If $(1-x+x^2)^n = a_0 + a_1 x + ... + a_{2n} x^{2n}$ then the value of $a_0 + a_2 + a_4 + ... + a_{2n}$ is

(A) $3^n + \frac{1}{2}$ (B) $3^n - \frac{1}{2}$ (C) $\frac{3^n - 1}{2}$ (D) $\frac{3^n + 1}{2}$

Ans : (D)

Hints : $x = 1$

$$1 = a_0 + a_1 + a_2 + a_3 + ... + a_{2n}$$

$x = -1,\ 3^n = a_0 - a_1 + a_2 - a_3 + ... + a_{2n}$

$$1 + 3^n = 2[a_0 + a_2 + a_4 + ... + a_{2n}]$$

$$\Rightarrow a_0 + a_2 + a_4 + ... + a_{2n} = \frac{1+3^n}{2}$$

16. If α, β be the roots of the quadratic equation $x^2 + x + 1 = 0$ then the equation whose roots are α^{19}, β^7 is

(A) $x^2 - x + 1 = 0$ (B) $x^2 - x - 1 = 0$ (C) $x^2 + x - 1 = 0$ (D) $x^2 + x + 1 = 0$

Ans : (D)

Hints : Roots are ω, ω^2

Let $\alpha = \omega, \beta = \omega^2$

$\alpha^{19} = \omega, \beta^7 = \omega^2$

$\therefore$ Equation remains same i.e. $x^2 + x + 1 = 0$

17. The roots of the quadratic equation $x^2 - 2\sqrt{3}x - 22 = 0$ are :

(A) imaginry (B) real, rational and equal

(C) real, irrational and unequal (D) real, rational and unequal

Ans : (C)

Hints : $x^2 - 2\sqrt{3} - 22 = 0$

$D = 12 + (4 \times 22) > 0$

$\because$ coeffs are irrational,

$$x = \frac{2\sqrt{3} \pm \sqrt{12+88}}{2}$$

$\therefore$ Roots are irrational, real, unequl.

18. The qudratic equation $x^2 + 15|x| + 14 = 0$ has

(A) only positive solutions (B) only negative solutions

(C) no solution (D) both positive and negative solution

Ans : (C)

Hints : $x^2 + 15|x| + 14 > 0\ \forall\ x$

Hence no solution

19. If $z = \frac{4}{1-i}$, then $\bar{z}$ is (where $\bar{z}$ is complex conjugate of z)

(A) $2(1+i)$ (B) $(1+i)$ (C) $\frac{2}{1-i}$ (D) $\frac{4}{1+i}$

Ans : (D)

Hints : $z = \frac{4}{1-i}$

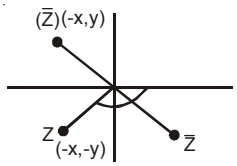
$$\bar{z} = \frac{4}{1+i}$$

20. If $-\pi < \arg(z) < -\frac{\pi}{2}$ then $\arg \bar{z} - \arg(-\bar{z})$ is

(A) π (B) $-\pi$ (C) $\frac{\pi}{2}$ (D) $-\frac{\pi}{2}$

Ans : (A)

Hints :



if $\arg(z) = -\pi + \theta$

$\Rightarrow \arg(\bar{z}) = \pi - \theta$

$\arg(-\bar{z}) = -\theta$

$\arg(\bar{z}) - \arg(-\bar{z}) = \pi - \theta - (-\theta) = \pi - \theta + \theta = \pi$

21. Two dice are tossed once. The probability of getting an even number at the first die or a total of 8 is

(A) $\frac{1}{36}$ (B) $\frac{3}{36}$ (C) $\frac{11}{36}$ (D) $\frac{23}{36}$

Ans : ()

Hints : A = getting even no on 1st dice

B = getting sum 8

So $|A| = 18 \quad |B| = 5 \quad |A \cap B| = 3$

So $P(A \cup B) = \frac{18 + 5 - 3}{36} = \frac{20}{36}$ (No option matches)

22. The probability that at least one of A and B occurs is 0.6 . If A and B occur simultaneously with probability 0.3, then $P(A') + P(B')$ is

(A) 0.9 (B) 0.15 (C) 1.1 (D) 1.2

Ans : (C)

Hints : $P(A \cup B) = 0.6 \qquad P(A) + P(B) = P(A \cup B) + P(A \cap B) = 0.9$

$P(A \cap B) = 0.3 \qquad P(A') + P(B') = 2 - 0.9 = 1.1$

23. The value of $\frac{\log_3 5 \times \log_{25} 27 \times \log_{49} 7}{\log_{81} 3}$ is

(A) 1 (B) 6 (C) $\frac{2}{3}$ (D) 3

Ans : (D)

Hints : $\frac{\left(\frac{\log 5}{\log 3} \times \frac{3 \log 3}{2 \log 5} \times \frac{\log 7}{2 \log 7}\right)}{\left(\frac{\log 3}{4 \log 3}\right)} = 3$

24. In a right-angled triangle, the sides are a, b and c, with c as hypotenuse, and $c-b\neq 1, c+b\neq 1$. Then the value of $(\log_{c+b} a+\log_{c-b} a)/(2\log_{c+b} a\times\log_{c-b} a)$ will be

(A) 2 (B) –1 (C) $\frac{1}{2}$ (D) 1

Ans : (D)

Hints : $c^2 = a^2 + b^2$

$\Rightarrow c^2 - b^2 = a^2$

$$\frac{\frac{\log a}{\log(c+b)}+\frac{\log a}{\log(c-b)}}{\frac{2\log a\times\log a}{\log(c+b)\log(c-b)}}=\frac{\log a(\log(c^2-b^2))}{2\log a\ \log a}=\frac{\log a^2}{\log a^2}=1$$

25. Sum of n terms of the following series $1^3 + 3^3 + 5^3 + 7^3 + ...$ is

(A) $n^2(2n^2-1)$ (B) $n^3(n-1)$ (C) n^3+8n+4 (D) $2n^4+3n^2$

Ans : (A)

Hints : $\sum(2n-1)^3$

$\sum \{(8n^3-3.4n^2+3.2n-1)\}$

$= 2n^2(n+1)^2 - 2n(n+1)(2n+1) + 3n(n+1) - n$

$= 2n^4 + 4n^3 + 2n^2 - 2n[2n^2+3n+1] + 3n^2 + 3n - n$

$= 2n^4 + 4n^3 + 2n^2 - 4n^3 - 6n^2 - 2n + 3n^2 + 3n - n$

$= 2n^4 - n^2$

$= n^2(2n^2-1)$

26. G.. M. and H. M. of two numbers are 10 and 8 respectively. The numbers are :

(A) 5, 20 (B) 4, 25 (C) 2, 50 (D) 1, 100

Ans : (A)

Hints : $\sqrt{ab} = 10 \Rightarrow ab = 100$

$\frac{2ab}{a+b} = 8$

$a + b = 25$

So $a = 5, b = 20$

27. The value of n for which $\frac{x^{n+1}+y^{n+1}}{x^n+y^n}$ is the geometric mean of x and y is

(A) $n=-\frac{1}{2}$ (B) $n=\frac{1}{2}$ (C) $n=1$ (D) $n=-1$

Ans : (A)

Hints : $\frac{x^{n+1}+y^{n+1}}{x^n+y^n}=\sqrt{xy} \Rightarrow x^{n+1}+y^{n+1}=\sqrt{xy}(x^n+y^n)$

$x^{n+\frac{1}{2}}\left(x^{\frac{1}{2}}-y^{\frac{1}{2}}\right)=y^{n+\frac{1}{2}}\left(x^{\frac{1}{2}}-y^{\frac{1}{2}}\right), \left(\frac{x}{y}\right)^{n+\frac{1}{2}}=1 \qquad n=-\frac{1}{2}$

28. If angles A, B and C are in A.P., then $\frac{a+c}{b}$ is equal to

(A) $2\sin\frac{A-C}{2}$ (B) $2\cos\frac{A-C}{2}$ (C) $\cos\frac{A-C}{2}$ (D) $\sin\frac{A-C}{2}$

Ans : (B)

Hints : $2B = A + C$

$$= \frac{\sin A + \sin C}{\sin B} = \frac{2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right)}{\sin B} = \frac{2\sin B}{\sin B}\cos\left(\frac{A-C}{2}\right) = 2\cos\left(\frac{A-C}{2}\right)$$

29. If $\frac{\cos A}{3} = \frac{\cos B}{4} = \frac{1}{5}$, $-\frac{\pi}{2} < A < 0, -\frac{\pi}{2} < B < 0$ then value of $2\sin A + 4\sin B$ is

(A) 4 (B) –2 (C) –4 (D) 0

Ans : (C)

Hints : $\cos A = \frac{3}{5}$ $\sin A = -\frac{4}{5}$

$\cos B = \frac{4}{5}$ $\sin B = -\frac{3}{5}$

$$= 2\left(-\frac{4}{5}\right) + 4\left(-\frac{3}{5}\right) = -\frac{20}{5} = -4$$

30. The value of $\frac{\cot 54^0}{\tan 36^0} + \frac{\tan 20^0}{\cot 70^0}$ is

(A) 0 (B) 2 (C) 3 (D) 1

Ans : (B)

Hints : $\frac{\cot 54^o}{\tan 36^o} + \frac{\tan 20^o}{\cot 70^o} = \frac{\tan 36^o}{\tan 36^o} + \frac{\tan 20^o}{\tan 20^o} = 1 + 1 = 2$

31. If $\sin 6\theta + \sin 4\theta + \sin 2\theta = 0$ then the general value of θ is

(A) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}$ (B) $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{6}$ (C) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{3}$ (D) $\frac{n\pi}{4}, 2n\pi \pm \frac{\pi}{6}$

Ans : (A)

Hints : $2\sin 4\theta \cos 2\theta + \sin 4\theta = 0$

$\sin 4\theta = 0$ $2\cos 2\theta = -1$

$4\theta = n\pi$ $\cos 2\theta = -\frac{1}{2} = \cos\frac{2\pi}{3}$

$\theta = \frac{n\pi}{4}$ $2\theta = 2n\pi \pm \frac{2\pi}{3}$, $\Rightarrow$ $\theta = n\pi \pm \frac{\pi}{3}$

32. In a ΔABC, $2ac\sin\frac{A - B + C}{2}$ is equal to

(A) $a^2 + b^2 - c^2$ (B) $c^2 + a^2 - b^2$ (C) $b^2 - a^2 - c^2$ (D) $c^2 - a^2 - b^2$

Ans : (B)

Hints : $2ac\sin\left(\frac{A+C-B}{2}\right)$ $\left[\frac{A+C}{2} = \frac{\pi}{2} - \frac{B}{2}\right]$, $= 2ac\sin\left(\frac{\pi}{2} - B\right) = 2ac\cos B$ $= a^2 + c^2 - b^2$

33. Value of $\tan^{-1}\left(\frac{\sin 2-1}{\cos 2}\right)$ is

(A) $\frac{\pi}{2}-1$ (B) $1-\frac{\pi}{4}$ (C) $2-\frac{\pi}{2}$ (D) $\frac{\pi}{4}-1$

Ans : (B)

Hints : $\tan^{-1}\left(\frac{\sin 2-1}{\cos 2}\right) = \tan^{-1}\left(\frac{-(\sin 1-\cos 1)^2}{(\cos 1-\sin 1)(\cos 1+\sin 1)}\right) = -\tan^{-1}\left(\frac{\cos 1-\sin 1}{\cos 1+\sin 1}\right) = 1-\frac{\pi}{4}$

34. The straight line 3x+y=9 divides the line segment joining the points (1,3) and (2,7) in the ratio

(A) 3 : 4 externally (B) 3 : 4 internally (C) 4 : 5 internally (D) 5 : 6 externally

Ans : (B)

Hints : $\text{Ratio} = -\frac{3+3-9}{6+7-9} = \frac{3}{4}$ internally

35. If the sum of distances from a point P on two mutually perpendicular straight lines is 1 unit, then the locus of P is

(A) a parabola (B) a circle (C) an ellipse (D) a straight line

Ans : (D)

Hints : $|x|+|y|=1$

36. The straight line $x+y-1=0$ meets the circle $x^2+y^2-6x-8y=0$ at A and B. Then the equation of the circle of which AB is a diameter is

(A) $x^2+y^2-2y-6=0$ (B) $x^2+y^2+2y-6=0$ (C) $2(x^2+y^2)+2y-6=0$ (D) $3(x^2+y^2)+2y-6=0$

Ans : (A)

Hints : $x^2+y^2-6x-8y+\lambda(x+y-1)=0$

$\text{Centre} = \left(3-\frac{\lambda}{2}.4-\frac{\lambda}{2}\right)$ Lie on $x+y-1=0$

$3-\frac{\lambda}{2}+4-\frac{\lambda}{2}-1=0,\ \lambda=6$

$x^2+y^2-6x-8y+6x+6y-6=0;\quad x^2+y^2-2y-6=0$

37. If t_1 and t_2 be the parameters of the end points of a focal chord for the parabola $y^2 = 4ax$, then which one is true?

(A) $t_1t_2=1$ (B) $\frac{t_1}{t_2}=1$ (C) $t_1t_2=-1$ (D) $t_1+t_2=-1$

Ans : (C)

Hints : $t_1t_2=-1$ Fact

38. S and T are the foci of an ellipse and B is end point of the minor axis. If STB is an equilateral triangle, the eccentricity of the ellipse is

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

Ans : (C)

Hints : $\frac{b}{ae}=\sqrt{3};\quad b=\sqrt{3}ae$

$e^2=\frac{a^2-3a^2e^2}{a^2}=1-3e^2;\quad 4e^2=1 \Rightarrow e=\frac{1}{2}$

39. For different values of α, the locus of the point of intersection of the two straight lines $\sqrt{3}x - y - 4\sqrt{3}\alpha = 0$ and $\sqrt{3}\alpha x + \alpha y - 4\sqrt{3} = 0$ is

(A) a hyperbola with eccentricity 2

(B) an ellipse with eccentricity $\sqrt{\frac{2}{3}}$

(C) a hyperbola with eccentricity $\sqrt{\frac{19}{16}}$

(D) an ellipse with eccentricity $\frac{3}{4}$

Ans : (A)

Hints : $\sqrt{3}x - y = 4\sqrt{3}\alpha$.....(1) ; $\sqrt{3}x + y = \frac{4\sqrt{3}}{\alpha}$....(2)

(1) x (2) $\Rightarrow 3x^2 - y^2 = 48 \Rightarrow \frac{x^2}{16} - \frac{y^2}{48} = 1$

$e = \sqrt{\frac{48+16}{16}} = 2$

40. The area of the region bounded by $y^2 = x$ and $y = |x|$ is

(A) $\frac{1}{3}$sq.unit (B) $\frac{1}{6}$sq.unit (C) $\frac{2}{3}$sq.unit (D) 1 sq.unit

Ans : (B)

Hints : $y^2 = x$

$$\int_0^1 \left(\sqrt{x} - x\right)dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^2}{2}\Bigg|_0^1 = \frac{3}{2} - \frac{1}{2} = \frac{4-3}{6} = \frac{1}{6}$$

41. If the displacement, velocity and acceleration of a particle at time, t be x, v and f respectively, then which one is true?

(A) $f = v^3\frac{d^2t}{dx^2}$ (B) $f = -v^3\frac{d^2t}{dx^2}$ (C) $f = v^2\frac{d^2t}{dx^2}$ (D) $f = -v^2\frac{d^2t}{dx^2}$

Ans : (B)

Hints : $\frac{d^2t}{dx^2} = \frac{d\left(\frac{dt}{dx}\right)}{dx} = \frac{d\left(\frac{1}{v}\right)}{dx} = -\frac{1}{v^2}\frac{dv}{dt} \times \frac{1}{v}$

$\Rightarrow f = -v^3 f\frac{d^2t}{dx^2}$

42. The displacement x of a particle at time t is given by $x = At^2 + Bt + C$ where A, B, C are constants and v is velocity of a particle, then the value of $4Ax - v^2$ is

(A) $4AC + B^2$ (B) $4AC - B^2$ (C) $2AC - B^2$ (D) $2AC + B^2$

Ans : (B)

Hints : $x = At^2 + Bt + c$

$v = 2At + B \Rightarrow v^2 = 4A^2t^2 + 4AB\,t + B^2$

$4Ax = 4A^2t^2 + 4AB\,t + 4AC$

$\Rightarrow v^2 - 4ax = B^2 - 4AC$

$\Rightarrow 4Ax - v^2 = 4AC - B^2$

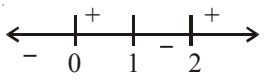
43. For what values of x, the function $f(x) = x^4 - 4x^3 + 4x^2 + 40$ is monotone decreasing?

(A) $0 < x < 1$ (B) $1 < x < 2$ (C) $2 < x < 3$ (D) $4 < x < 5$

Ans : (B)

Hints : $f'(x) = 4x^3 - 12x^2 + 8x = 4x(x^2 - 3x + 2)$

$= 4x(x-1)(x-2)$



$\therefore$ x is decreasing for $x \in (1, 2)$

44. The displacement of a particle at time t is x, where $x = t^4 - kt^3$. If the velocity of the particle at time $t = 2$ is minimum, then

(A) $k = 4$ (B) $k = -4$ (C) $k = 8$ (D) $k = -8$

Ans : (A)

Hints : $\frac{dx}{dt} = 4t^3 - 3kt^2$

$\frac{dv}{dt} = 12t^2 - 6kt$ at $t = 2$

$\Rightarrow \frac{dv}{dt} = 0$, $48 - 12k = 0$; $k = 4$

45. The point in the interval $[0, 2\pi]$, where $f(x) = e^x \sin x$ has maximum slope, is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) π (D) $\frac{3\pi}{2}$

Ans : (B)

Hints : $f'(x) = e^x(\sin x + \cos x)$

$f''(x) = e^x(\sin x + \cos x + \cos x - \sin x) \Rightarrow f''(x) = e^x \cos x = 0$

$\Rightarrow x = \frac{\pi}{2}$

46. The minimum value of $f(x) = e^{\left(x^4 - x^3 + x^2\right)}$ is

(A) e (B) –e (C) 1 (D) –1

Ans : (C)

Hints : $f(x) = e^{(x^4 - x^3 + x^2)}$, $f'(x) = e^{x^4 - x^3 + x^2}$

$e^{x^4 - x^3 + x^2}\left(4x^3 - 3x^2 + 2x\right) x \left(4x^2 - 3x + 2\right)$

$\Rightarrow$ f(x) is decreasing for $x < 0$, increasing for $x > 0$

$\therefore$ Minimum is at $x = 0$ $\therefore f(0) = e^0 = 1$

47. $\int \frac{\log \sqrt{x}}{3x} dx$ is equal to

(A) $\frac{1}{3}\left(\log \sqrt{x}\right)^2 + C$ (B) $\frac{2}{3}\left(\log \sqrt{x}\right)^2 + C$ (C) $\frac{2}{3}(\log x)^2 + C$ (D) $\frac{1}{3}(\log x)^2 + C$

Ans : (A)

Hints : $x = t^2 \Rightarrow \int \frac{\ell nt}{3t^2}(2tdt) = \frac{2}{3}\int \frac{\ell nt}{t} dt = \frac{2}{3}\frac{(\ell nt)^2}{2} + c = \frac{\left(\ell n\sqrt{x}\right)^2}{3} + c$

48. $\int e^x\left(\frac{2}{x}-\frac{2}{x^2}\right)dx$ is equal to

(A) $\frac{e^x}{x}+C$ (B) $\frac{e^x}{2x^2}+C$ (C) $\frac{2e^x}{x}+C$ (D) $\frac{2e^x}{x^2}+C$

Ans : (C)

Hints : $\int e^x\left(\frac{2}{x}-\frac{2}{x^2}\right)dx = 2\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx = \frac{2e^x}{x}+c$

49. The value of the integral $\int\frac{dx}{\left(e^x+e^{-x}\right)^2}$ is

(A) $\frac{1}{2}\left(e^{2x}+1\right)+C$ (B) $\frac{1}{2}\left(e^{-2x}+1\right)+C$ (C) $-\frac{1}{2}\left(e^{2x}+1\right)^{-1}+C$ (D) $\frac{1}{4}\left(e^{2x}-1\right)+C$

Ans : (C)

Hints : $\int\frac{e^{2x}dx}{\left(e^{2x}+1\right)^2}$ $e^x=t$; $e^xdx=dt$

$=\frac{1}{2}\int\frac{2tdt}{\left(t^2+1\right)^2}=\frac{1}{2}\left\{-\frac{1}{\left(t^2+1\right)}\right\}+c=-\frac{1}{2\left(e^{2x}+1\right)}+c$

50. The value of $\underset{x\to0}{Lt}\frac{\sin^2 x+\cos x-1}{x^2}$ is

(A) 1 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) 0

Ans : (B)

Hints : $\underset{x\to0}{Lim}\frac{\sin^2 x+\cos x-1}{x^2}=\underset{x\to0}{Lim}\frac{\cos x-\cos^2 x}{x^2}=\underset{x\to0}{Lim}\left(\frac{1-\cos x}{x^2}\right)\cos x$

$=\underset{x\to0}{Lim}\frac{2\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2\times4}=\frac{1}{2}$

51. The value of $\underset{x\to0}{Lt}\left(\frac{1+5x^2}{1+3x^2}\right)^{\frac{1}{x^2}}$ is

(A) e^2 (B) e (C) $\frac{1}{e}$ (D) $\frac{1}{e^2}$

Ans : (A)

Hints : $\underset{x\to0}{Lim}\left(\frac{1+5x^2}{1+3x^2}\right)^{\frac{1}{x^2}}=e^{\underset{x\to0}{Lim}\frac{1}{x^2}\left(\frac{1+5x^2}{1+3x^2}-1\right)}=e^{\underset{x\to0}{Lim}\frac{2x^2}{x^2\left(1+3x^2\right)}}=e^2$

52. In which of the following functions, Rolle's theorem is applicable?

(A) $f(x) = |x|$ in $-2 \le x \le 2$ (B) $f(x) = \tan x$ in $0 \le x \le \pi$

(C) $f(x) = 1 + (x-2)^{\frac{2}{3}}$ in $1 \le x \le 3$ (D) $f(x) = x(x-2)^2$ in $0 \le x \le 2$

Ans : (D)

Hints : (A) f(x) = |x| not differentiable at x = 0

(B) f(x) = tan x discontinuous at $x = \frac{\pi}{2}$

(C) $f(x) = 1 + (x-2)^{\frac{3}{2}}$ not differentiable at x = 2

(D) $f(x) = x(x-2)^2$ polynomial $\therefore$ differentiable $\forall x \in R$

Hence Rolle's theorem is applicable

53. If $f(5) = 7$ and $f'(5) = 7$ then $\underset{x\to5}{Lt} \frac{xf(5) - 5f(x)}{x-5}$ is given by

(A) 35 (B) –35 (C) 28 (D) –28

Ans : (D)

Hints : $\underset{x\to5}{Lt} \frac{xf(5) - tf(x)}{x-5} = \underset{x\to5}{Lt} \frac{f(5) - 5f'(x)}{1} = f(5) - 5f'(5) = 7 - 5\times7 = -28$

54. If $y = (1+x)(1+x^2)(1+x^4)...(1+x^{2n})$ then the value of $\left(\frac{dy}{dx}\right)_{x=0}$ is

(A) 0 (B) –1 (C) 1 (D) 2

Ans : (C)

Hints : T-log & Differentiate

$\frac{dy}{dx} = y\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + ...\right]$ Put x = 0

$\frac{dy}{dx} = 1$

55. The value of f(0) so that the function $f(x) = \frac{1 - \cos(1 - \cos x)}{x^4}$ is continuous everywhere is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{8}$

Ans : (D)

Hints : $\underset{x\to0}{Lim} \frac{1 - \cos(1 - \cos x)}{x^4}$

$$\underset{x\to0}{Lim} \frac{2\sin^2\left(\frac{2\sin^2\left(\frac{x}{2}\right)}{2}\right)}{x^4} = 2\underset{x\to0}{Lim} \frac{\sin^2\left(\sin^2\left(\frac{x}{2}\right)\right)\left(\sin^2\left(\frac{x}{2}\right)\right)^2}{x^4\left(\sin^2\left(\frac{x}{2}\right)\right)^2} = 2\underset{x\to0}{Lim} \frac{\sin^4\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)^4 2^4} = \frac{1}{2^3} = \frac{1}{8}$$

56. $\int \sqrt{1+\cos x}\,dx$ is equal to

(A) $2\sqrt{2}\cos\frac{x}{2}+C$ (B) $2\sqrt{2}\sin\frac{x}{2}+C$ (C) $\sqrt{2}\cos\frac{x}{2}+C$ (D) $\sqrt{2}\sin\frac{x}{2}+C$

Ans : (B)

Hints : $\int\sqrt{1+\cos x}\,dx=\sqrt{2}\int\cos\left(\frac{x}{2}\right)dx=2\sqrt{2}\sin\left(\frac{x}{2}\right)+c$

57. The function $f(x)=\sec\left[\log\left(x+\sqrt{1+x^2}\right)\right]$ is

(A) odd (B) even (C) neither odd nor even (D) constant

Ans : (B)

Hints : $f(x)=\sec\left(\ell n\left(x+\sqrt{1+x^2}\right)\right)$ =sec (odd function) = even function

$\because$ sec is an even function

58. $\lim\limits_{x\to0}\frac{\sin|x|}{x}$ is equal to

(A) 1 (B) 0 (C) positive infinity (D) does not exist

Ans : (D)

Hints : $\lim\limits_{x\to0}\frac{\sin|x|}{x}$

$LHL=-1\quad RHL=1$

Limit does not exist

59. The co-ordinates of the point on the curve $y=x^2-3x+2$ where the tangent is perpendicular to the straight line $y=x$ are

(A) $(0,2)$ (B) $(1,0)$ (C) $(-1,6)$ (D) $(2,-2)$

Ans : (B)

Hints : $y=x^2-3x+2$

$\frac{dy}{dx}=2x-3=-1\Rightarrow x=1$ at $x=1$, $y=0$

$\therefore$ Point is $(1,0)$

60. The domain of the function $f(x)=\sqrt{\cos^{-1}\left(\frac{1-|x|}{2}\right)}$ is

(A) $(-3,3)$ (B) $[-3,3]$ (C) $(-\infty,-3)\cup(3,\infty)$ (D) $(-\infty,-3]\cup[3,\infty)$

Ans : (B)

Hints : $f(x)=\sqrt{\cos^{-1}\left(\frac{1-|x|}{2}\right)}$

$-1\le\frac{1-|x|}{2}\le1\quad\Rightarrow-2-1\le-|x|\le2-1\Rightarrow-3\le-|x|\le1\quad\Rightarrow-1\le|x|\le3\Rightarrow x\in[-3,3]$

61. If the line $ax+by+c=0$ is a tangent to the curve $xy=4$, then

(A) $a<0, b>0$ (B) $a\le0, b>0$ (C) $a<0, b<0$ (D) $a\le0, b<0$

Ans : (C)

Hints : Slope of line $= -\frac{a}{b}$

$y = \frac{4}{x} = 1,\ \frac{dy}{dx} = -\frac{4}{x^2},\quad -\frac{a}{b} = -\frac{4}{x^2} \Rightarrow \frac{a}{b} = \frac{4}{x^2} > 0$

$a < 0,\ b < 0$

62. If the normal to the curve $y = f(x)$ at the point (3, 4) make an angle $3\pi/4$ with the positive x-axis, then $f'(3)$ is

(A) 1 (B) -1 (C) $-\frac{3}{4}$ (D) $\frac{3}{4}$

Ans : (A)

Hints : $\frac{dy}{dx} = f'(x)$, Slope of normal $= -\frac{1}{f'(x)}$, $-\frac{1}{f'(3)} = \tan\frac{3\pi}{4} = -1$

$f'(3) = 1$

63. The general solution of the different equation $100\frac{d^2y}{dx^2} - 20\frac{dy}{dx} + y = 0$ is

(A) $y = (c_1 + c_2x)e^x$ (B) $y = (c_1 + c_2x)e^{-x}$ (C) $y = (c_1 + c_2x)e^{\frac{x}{10}}$ (D) $y = c_1e^x + c_2e^{-x}$

Ans : (C)

Hints : $100p^2 - 20p + 1 =$

$(10P - 1)^2 = 0,\ P = \frac{1}{10}$

$y = (c_1 + c_2x)e^{\frac{x}{10}}$

64. If $y'' - 3y' + 2y = 0$ where $y(0) = 1$, $y'(0) = 0$, then the value of y at $x = \log, 2$ is

(A) 1 (B) –1 (C) 2 (D) 0

Ans : (D)

Hints : $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

$m^2 - 3m + 2 = 0,\ y = Ae^x + Be^{2x}$

$m = 1,\ m = 2,\ y^1 = Ae^x + 2Be^{2x}$

$y = 0,\ A + B = 1\ \ A + 2B = 0,\ A = 2,\ B = -1$

$y = 2e^x - e^{2x}$

$y = 0$ at $x = \ell n2$

65. The degree of the differential equation $x = 1 + \left(\frac{dy}{dx}\right) + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + ...$

(A) 3 (B) 2 (C) 1 (D) not defined

Ans : (C)

Hints : $x = e\frac{dy}{dx},\ \frac{dy}{dx} = \log_e x$

66. The equation of one of the curves whose slope at any point is equal to y + 2x is

(A) $y = 2\left(e^x + x - 1\right)$ (B) $y = 2\left(e^x - x - 1\right)$ (C) $y = 2\left(e^x - x + 1\right)$ (D) $y = 2\left(e^x + x + 1\right)$

Ans : (B)

Hints : $\frac{dy}{dx} = y + 2x$ Put $y + 2x = z \Rightarrow \frac{dy}{dx} + z = \frac{dz}{dx}$

$\frac{dz}{dx} - 2 = z, \quad \frac{dz}{dx} = z + 2 \Rightarrow \int \frac{dz}{z+2} = \int dx$

$\log(z+2) = x + c, \ \log(y + 2x + 2) = x + c$

$y + 2x + 2 = x + c, \ y = 2(e^x - x - 1)$

67. Solution of the differential equation $xdy - ydx = 0$ represents a

(A) parabola (B) circle (C) hyperbola (D) straight line

Ans : (D)

Hints : $x.dy - y.dx = 0 \Rightarrow xdy = ydx$

$\frac{dy}{y} = \frac{dx}{x} \Rightarrow \log y = \log x + \log c$

$y = xc$

68. The value of the integral $\int_0^{\pi/2} \sin^5 x dx$ is

(A) $\frac{4}{15}$ (B) $\frac{8}{5}$ (C) $\frac{8}{15}$ (D) $\frac{4}{5}$

Ans : (C)

Hints : $I = \int_0^{\frac{\pi}{2}} \sin^4 x \ dx \quad \cos x = f, \ \sin dx = dt$

$= -\int_1^0 (1 - t^2)^2 \, dt = \int_0^1 \left(t^4 - 2t^2 + 1\right) dt$

$= \frac{1}{5}\left(t^5\right)_0^1 - \frac{2}{3}(t^3)_0^1 + (t)_0^1 = = \frac{1}{5} - \frac{2}{5} + 1 = \frac{3 - 10 + 15}{15} = \frac{8}{15}$

69. If $\frac{d}{dx}\{f(x)\} = g(x)$, then $\int_a^b f(x)g(x)dx$ is equal to

(A) $\frac{1}{2}\left[f^2(b) - f^2(a)\right]$ (B) $\frac{1}{2}\left[g^2(b) - g^2(a)\right]$ (C) $f(b) - f(a)$ (D) $\frac{1}{2}\left[f(b^2) - f(a^2)\right]$

Ans : (A)

Hints : $f(x) = \int g(x)dx$

$\int_a^b f(x).g(x).dx = \left(f(x)\, f(x)\right)_a^b - \int_a^b g(x)\, f(x)dx$

I II

$I = f^2(b) - f^n(a)^{-1}$

$I = \frac{1}{2}\left(f^2(b) - f^2(a)\right)$

70. If $I_1 = \int_0^{3\pi} f(\cos^2 x)dx$ and $I_2 = \int_0^{\pi} f(\cos^2 x)dx$, then

(A) $I_1 = I_2$ (B) $3I_1 = I_2$ (C) $I_1 = 3I_2$ (D) $I_1 = 5I_2$

Ans : (C)

Hints : $I_1 = 3\int_0^{\pi} f(\cos^2 x)dx = 3I$ [period is π]

71. The value of $I = \int_{-\pi/2}^{\pi/2} |\sin x|\,dx$ is

(A) 0 (B) 2 (C) -2 (D) $-2 < I < 2$

Ans : (B)

Hints : $I = 2\int_0^{\frac{\pi}{2}} \sin x\ dx = 2(1) = 2$

72. If $I = \int_0^1 \frac{dx}{1+x^{\pi/2}}$, then

(A) $\log_e 2 < 1 < \pi/4$ (B) $\log_e 2 > 1$ (C) $I = \pi/4$ (D) $I = \log_e 2$

Ans : (A)

Hints : $x^2 < x^{\frac{\pi}{2}} < x$, $1 + x^2 < 1 + x^{\frac{\pi}{2}} < 1 + x$

$\frac{1}{1+x^2} > \frac{1}{1+x^{\frac{\pi}{2}}} > \frac{1}{1+x}$

$\frac{\pi}{4} > I > (\log(1+x)),\ \frac{\pi}{4} > I > \log 2$

73. The area enclosed by $y = 3x - 5$, $y = 0$, $x = 3$ and $x = 5$ is

(A) 12 sq. units (B) 13 sq. units (C) $13\frac{1}{2}$ sq. units (D) 14 sq. units

Ans : (D)

Hints : $A = \int_3^5 (3x - 5)dx$

$= \frac{3}{2}(x^2)_3^5 - 5(x)_3^5, \quad = \frac{3}{2}[25 - 9] - 5(5 - 3)$

$\frac{3}{2}.16 - 5(2) = 24 - 10 = 14$

74. The area bounded by the parabolas $y = 4x^2$, $y = \frac{x^2}{9}$ and the line $y = 2$ is

(A) $\frac{5\sqrt{2}}{3}$ sq. units (B) $\frac{10\sqrt{2}}{3}$ sq. units (C) $\frac{15\sqrt{2}}{3}$ sq. units (D) $\frac{20\sqrt{2}}{3}$ sq. units

Ans : (D)

Hints : $y = 4x^2$ ..... (i)

$y = \frac{x^2}{4}$ ..... (ii)

$A = \int_r^2 \left[\frac{\sqrt{y}}{2} - 3\sqrt{y}\right] dy = \left(\frac{1}{2} - 3\right)\int_0^2 \sqrt{y}\,dy$

$= \left(\frac{-\sqrt{y}}{2}\right)\frac{5}{3}(y^{3/2})_0^2 = -\frac{5}{3}\left(2\sqrt{2} - 0\right)$

$= \left|-\frac{\sqrt{2}}{3}\right| = \frac{10\sqrt{2}}{3}$, Area of bounded figure $= 2A = \frac{20\sqrt{2}}{3}$

75. The equation of normal of $x^2 + y^2 - 2x + 4y - 5 = 0$ at (2, 1) is

(A) $y = 3x - 5$ (B) $2y = 3x - 4$ (C) $y = 3x + 4$ (D) $y = x + 1$

Ans : (A)

Hints : $0(1, -2)$ A(2, 1)

Slope $A \rightarrow \frac{y-1}{-2-1} = \frac{x-2}{1-2}$, $\frac{y-1}{-3} = \frac{x-2}{-1} = 1$, $y - 1 = 3(x - 2)$

$y = 3x - 5$

76. If the three points (3q, 0), (0, 3p) and (1, 1) are collinear then which one is true ?

(A) $\frac{1}{p} + \frac{1}{q} = 1$ (B) $\frac{1}{p} + \frac{1}{q} = 1$ (C) $\frac{1}{p} + \frac{1}{q} = 3$ (D) $\frac{1}{p} + \frac{3}{q} = 1$

Ans : (C)

Hints : A(3q, 0) B (0, 3p) C (11)

Slope = 1 AC = 5 log BC

$\frac{1-0}{1-3q} = \frac{1-3p}{1-0} = 3$, $\frac{1}{1-3q} = \frac{1-3p}{1}$

$1 = (1 - 3p)(1 - 3q)$, $1 = 1 - 3q - 3p + 9pq$

$\Rightarrow 3p + 3q = 9\,pq$, $\frac{1}{q} + \frac{1}{p} = 3$

77. The equations $y = \pm\sqrt{3x}$, $y = 1$ are the sides of

(A) an equilateral triangle (B) a right angled triangle (C) an isosceles triangle (D) an obtuse angled triangle

Ans : (A)

Hints : $y = \tan 60^\circ x$, $y = -\tan 60^\circ x$

$y = 1$, equilateral

78. The equations of the lines through (1, 1) and making angles of 45° with the line $x + y = 0$ are

(A) $x - 1 = 0, x - y = 0$ (B) $x - y = 0, y - 1 = 0$

(C) $x + y - 2 = 0, y - 1 = 0$ (D) $x - 1 = 0, y - 1 = 0$

Ans : (D)

Hints : $m = 1$, $y - 1 = \frac{m \pm \tan 45}{1 \mp m \tan 45}(x - 1)$, $y - 1 = \frac{(-1) \pm 1}{1 \pm 1}(x - 1)$

$y = 1$, $x = 1$

79. In a triangle PQR, $\angle R = \pi/2$. If $\tan\left(\frac{p}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are roots of $ax^2 + bx + c = 0$, where $a \neq 0$, then which one is true ?

(A) $c = a + b$ (B) $a = b + c$ (C) $b = a + c$ (D) $b = c$

Ans : (A)

Hints : $\frac{P}{2} + \frac{Q}{2} = \frac{\pi}{2} - \frac{P}{2} = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

$\tan\left(\frac{\rho}{2} + \frac{Q}{2}\right) = 1$, $\frac{-b/a}{1 - c/a} = 1 \Rightarrow \frac{-b}{a-c} = 1$

$-b = a - c \Rightarrow a + b = c$

80. The value of $\frac{\sin 55^0 - \cos 55^0}{\sin 10^0}$ is

(A) $\frac{1}{\sqrt{2}}$ (B) 2 (C) 1 (D) $\sqrt{2}$

Ans : (D)

Hints : $\frac{\sin 55 - \sin 35}{\sin 10} = \frac{2\cos 45 . \sin 10}{\sin 10} = \sqrt{2}$

DESCRIPTIVE TYPE QUESTIONS

SUB : MATHEMATICS

1. Prove that the equation $\cos 2x + a \sin x = 2a - 7$ possesses a solution if $2 \le a \le 6$.

   A. $\Rightarrow \cos 2x + a \sin x = 2a - 7$

   $\Rightarrow 2\sin^2 x - a\sin x + (2a - 8) = 0$

   Since $\sin x \in IR$, $\sin x = \dfrac{a \pm (a-8)}{4}, = \dfrac{a-4}{2}, \; 2 \; -1 \le \sin x \le 1$

   $\therefore$ Given equation has solution of $2 \le a \le 6$.

2. Find the values of x, $(-\pi < x < \pi, x \ne 0)$ satisfying the equation, $8^{1+|\cos x|+|\cos^2 x|+ \quad \infty} = 4^3$

   A. $(8)^{1+|\cos x|+|\cos^2|+ \quad \infty} = 4^3$

   $\Rightarrow 8^{\frac{1}{1-|\cos x|}} = 2^6, \quad \Rightarrow \dfrac{3}{1-|\cos x|} = 6 \Rightarrow \cos = \pm\dfrac{1}{2}$

   $\Rightarrow x = \dfrac{\pi}{3}, -\dfrac{\pi}{3}, \dfrac{2\pi}{3}, -\dfrac{2\pi}{3}$

3. Prove that the centre of the smallest circle passing through origin and whose centre lies on $y = x + 1$ is $\left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$

   A. Let centre be $c(h, h+1)$, $0(0, 0)$

   $r = oc = \sqrt{h^2 + (h+1)^2} = \sqrt{2h^2 + 2h + 1}$

   $= \sqrt{2\left(h + \dfrac{1}{2}\right)^2 + \dfrac{1}{2}}$ for min radius r, $h + \dfrac{1}{2} = 0, \; h = -\dfrac{1}{2}$

   Centre $\left(-\dfrac{1}{2}, \dfrac{1}{2}\right)$

4. Prove by induction that for all $n \in N$, $n^2 + n$ is an even integer $(n \ge 1)$

   A. $x = 1, \; x^2 + x = 2$ is an even integer

   Let for $n = k$, $k^2 + k$ is even

   Now for $n = k + 1$, $(k + 1)^2 + (k + 1) - (k^2 + k)$

   $= k^2 + 2k + 1 + k + 1 - k^2 - k \quad = 2k + 2$ which is even integer also $k^2 + k$ is even integer

   Hence $(k + 1)^2 + (k + 1)$ ia also an even integer

Hence $n^2 + n$ is even integer for all $n \in N$.

5. If A, B are two square matrices such that AB = A and BA = B, then prove that $B^2 = B$

A. $B^2 = B.B = (BA)B = B(AB) = B(A) = BA = B$ (Proved)

6. If N = n! ($n \in N, n > 2$), then find $\lim_{N\to\infty}\left[(\log_2 N)^{-1} + (\log_3 N)^{-1} + ... + (\log_n N)^{-1}\right]$

A. $\lim_{N\to\infty}\left[\log_N 2 + \log_N 3 + ... + \log_N n\right]$

$= \lim_{N\to\infty} \log_N (2.3.....n) = \lim_{N\to\infty} \log_{n!}^{n!} \quad [\because N = n!] = \lim_{N\to\infty} 1 = 1$

7. Use the formula $\lim_{x\to 0} \frac{a^x - 1}{x} = \log_e a$, to compute $\lim_{x\to 0} \frac{2^x - 1}{\sqrt{1+x} - 1}$

A. $\lim_{x\to 0} \frac{2^x - 1}{\sqrt{1+x} - 1}$

$= \lim_{x\to 0}\left(\frac{2^x - 1}{x}\right) \times \lim_{x\to 0}\left(\sqrt{1+x} + 1\right)$

$= = \log_e 2 \times 2 = \log_e 4$

8. If $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ prove that, $x\sqrt{1-y^2} + y\sqrt{1-x^2} = A$ where A is constant

A. $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$

$\Rightarrow \frac{dy}{\sqrt{1-y^2}} = -\frac{dx}{\sqrt{1-x^2}} \Rightarrow \sin^{-1} y = -\sin^{-1} x + c$ [c is a constant]

$\Rightarrow \sin^{-1}x + \sin^{-1}y = c$

$= \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right] = c$ where A is a $x\sqrt{1-y^2} + y\sqrt{1-x^2} = \sin c = A$ constant

9. Evaluate the following integral $\int_{-1}^{2} |\, x \sin \pi x \,| dx$

A. $I = \int_{-1}^{2} |x \sin \pi x| dx = \int_{-1}^{1} |\, x \sin \pi x \,| \, dx + \int_{1}^{2} |\, x.\sin \pi x \,| \, dx$

$= 2\int_{0}^{1} |\, x \sin \pi x \,| \, dx + \int_{1}^{2} |\, x.\sin \pi x \,| \, dx$

$= 2\int_{0}^{1} x.\sin \pi x dx - \int_{1}^{2} x.\sin \pi x dx = 2I_1 - I_2$

$I_1 = \int_{0}^{1} x \sin \pi \, x dx = -x \frac{\cos \pi x}{\pi} + \int \frac{\cos \pi x}{\pi} \, dx$

$= -x\frac{\cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2}\Big|_0^1 = \frac{1}{\pi}$

$I_2 = \int_{1}^{2} x \sin \pi x \, dx = -x\frac{\cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2}\Big]_1^2 = \frac{-2}{\pi} + 0 + \left(-\frac{1}{\pi}\right)$

$= -\frac{3}{\pi}$ So, $2I_1 - I_2 = \frac{2}{\pi} + \frac{3}{\pi} = \frac{5}{\pi}$

10. If $f(a) = 2, f'(a) = 1, g(a) = -1$ and $g'(a) = 2$, find the value of $\lim_{x \to a} \frac{g(a)f(a) - g(a)f(x)}{x - a}$.

A. $\lim_{x \to a} \frac{g'(a)f(a) - g(a)f'(x)}{1}$ [using L′ Hospital Rule]

$= g'(a)\, f(a) - g(a)\, f'(a)$

$= (2)(2) - (-1)\,(1) = 4 + 1 = 5$