VIDYAKUL

Math Chapter 4
Determinants

Chapter 4: Determinants

Exercise 4.1

Q1: Evaluate the determinants.

|||2−54−1|||

Ans:

|||2−54−1||| = 2 (-1) – 4 (-5) = – 2 + 20 **= 18**

Q2: Evaluate the determinant.

(i) |||cosΘsinΘ−sinΘcosΘ|||
(ii) |||x2–x+1x+1x–1x+1|||

Ans:

(i) (cos Θ)(cos Θ) – (-sin Θ) (sin Θ) = $\cos^2$ Θ + $\sin^2$ Θ **= 1**

(ii) = (x^2 – x + 1) (x + 1) – (x – 1) (x + 1)

= x^3 – x^2 + x + x^2 – x + 1 – (x^2 – 1)

= x^3 + 1 – x^2 + 1

= x^3 – x^2 + 2

Q3: If A = [1422] , then show that |2A| = 4 |A|

Ans: The given matrix is A = [1422]
Therefore, 2A = 2[1422]=[2844]
Therefore, L.H.S. = |2A| = |||2844||| = 2 × 4 – 4 × 8 = 8 – 32 = -24

Now, $|A| = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} =$
$1 \times 2 - 2 \times 4 = 2 - 8 = -6$

Therefore, R.H.S. $= 4|A| = 4 \times (-6)$ **= -24**

Therefore, L.H.S. = R.H.S.

Q.4: If A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$**, then show that |3A| = 27|A|.**

Ans: The given matrix is $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$
It can be observed that in the first column, two entries are zero. Thus, we expand along the first column (C_1) for easier calculation.

$|A| = 1\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 0 & 4 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} = 1(4 - 0) - 0 + 0 = 4$
Therefore, $27|A| = 27(4) = 108$ **…(i)**
Now, $3A = 3\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$
Therefore, $|3A| = 3\begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0\begin{vmatrix} 0 & 3 \\ 0 & 12 \end{vmatrix} + 0\begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix}$
$= 3(36 - 0) = 3(36) = 108$ **…(ii)**

From equations (i) and (ii), we have:

$|3A| = 27|A|$

Hence, the given result is proved.

Q5: Evaluate the determinants

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$
(ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$
(iii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$
(iv) $\begin{bmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}$

Ans:

(i) Let A = $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

It can be observed that in the second row, two entries are zero. Thus, we expand along the second row for easier calculations.

|A| = $0\begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 0\begin{vmatrix} 3 & -2 \\ 3 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 3 & -1 \\ 3 & -5 \end{vmatrix}$ = (-15 + 3) = -12

(ii) Let $\begin{bmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{bmatrix}$

By expanding along the first row, we have:

|A| = $3\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} + 4\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$

= 3 (1 + 6) + 4 (1 + 4) + 5 (3 – 2)

= 3 (7) + 4 (5) + 5 (1)

= 21 + 20 + 5 = 46

(iii) Let A = $\begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix}$

By expanding along the first row, we have:

|A| = $0\begin{vmatrix} 0 & -3 \\ 3 & 0 \end{vmatrix} - 1\begin{vmatrix} -1 & -3 \\ -2 & 0 \end{vmatrix} + 2\begin{vmatrix} -1 & 0 \\ -2 & 3 \end{vmatrix}$

= 0 – 1(0 – 6) + 2 (-3 – 0)

= – 1 (-6) + 2 (-3)

= 6 – 6 = 0

(iv) Let A = $\begin{bmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}$

By expanding along the first column, we have:

|A| = $2\begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} - 0\begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 3\begin{vmatrix} -1 & -2 \\ 2 & -1 \end{vmatrix}$

= 2 (0 – 5) – 0 + 3 (1 + 4)

= -10 + 15 = 5

Q6: If A = $\begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$**, find |A|.**

Ans:

Let A = $\begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$

By expanding along the first row, we have:

|A| = 1|||14−3−9|||–1|||25−3−9|||–2|||2514|||

= 1 (-9 + 12) – 1 (-18 + 15) – 2 (8 – 5)

= 1 (3) – 1 (-3) – 2 (3)

= 3 + 3 – 6

= 6 – 6

= 0

Q7: Find values of x, if

(i) |||2241|||=|||2x64x|||
(ii) |||2435|||=|||x2x35|||

Ans:

(i) |||2241|||=|||2x64x|||
⇒ 2 × 1 – 5 × 4 = 2x × x – 6 × 4
⇒ 2 – 20 = $2x^2$ – 24
⇒ $2x^2$ = 6
⇒ x^2 = 3
⇒ x = ± 3–√
(ii) |||2435|||=|||x2x35|||
⇒ 2 × 5 – 3 × 4 = x × 5 – 3 × 2x
⇒ 10 -12 = 5x – 6x
⇒ -2 = -x
⇒ **x = 2**

Q8: If |||x182x|||=|||61826||| **, then x is equal to**
(A) 6

(B) ± 6

(C) – 6

(D) 0

Ans:

|||x182x|||=|||61826|||

⇒ $x^2 – 36 = 36 – 36$
⇒ $x^2 – 36 = 0$
⇒ $x = \pm 6$
Hence, the correct answer is B

Exercise 4.2

Q.1: Using the property of determinants and without expanding, prove that:

||||xyzabcx+ay+bz+c||||=0

Sol:

||||xyzabcx+ay+bz+c||||=||||xyzabcxyz||||+||||xyzabcabc||||=0+0=0

(Here, the two columns of the determinants are identical)

Q2: Using the property of determinants and without expanding, prove that:

||||a–bb–cc–ab–cc–aa–bc–aa–bb–c||||=0

Sol:

||||a–bb–cc–ab–cc–aa–bc–aa–bb–c||||=0

Applying $R_1 \to R_1 + R_2$, we have:

⇒ Δ=||||a–bb–c−(a–c)b–ac–a−(b–a)c–ba–b−(c–b)||||=0

⇒ –||||a–cb–ca–cb–ac–ab–ac–ba–bc–b||||=0

Here, the two rows R_1 and R_3 are identical.

Therefore, Δ=0

Q3: Using the property of determinants and without expanding, prove that:

||||235789657586||||=0

Sol:

⇒ ||||235789657586||||= ||||23578963+272+381+5||||

⇒ ||||235789637281||||+||||235789235||||

⇒ |||||2357899(7)9(8)9(9)|||||+0 **(Two columns are identical)**

9||||235789789||||+0 **(Two columns are identical)**

= 0

Q4: Using the property of determinants and without expanding, prove that:

|||||111bccaaba(b+c)b(c+a)c(a+b)|||||=0

Sol:

Δ=|||||111bccaaba(b+c)b(c+a)c(a+b)|||||=0

By applying $C_3 \to C_3 + C_2$

⇒ Δ=||||111bccaabab+bc+caab+bc+caab+bc+ca||||=0

⇒ (ab+bc+ca)||||111bccaab111||||=0

Here, two columns C_1 and C_3 are proportional.

⇒ Δ=0

Q5: Using the property of determinants and without expanding, prove that:

||||b+cc+aa+bq+rr+pp+qy+zz+xx+y||||=2||||abcpqrxyz||||

Sol:

Δ=||||b+cc+aa+bq+rr+pp+qy+zz+xx+y|||| =||||b+cc+aaq+rr+ppy+zz+xx||||+|||| b+cc+abq+rr+pqy+zz+xy||||

= $\Delta_1+\Delta_2$ **……………………….(i)**

Now, Δ_1=||||b+cc+aaq+rr+ppy+zz+xx||||

Applying $R_2 \to R_2 - R_3$, we have:

Δ_1=||||b+ccaq+rrpy+zzx||||

Applying $R_1 \to R_1 - R_2$, we have:

Δ_1=||||bcaqrpyzx||||

Applying $R_1 \leftrightarrow R_3$ and $R_2 \leftrightarrow R_3$ we have:

$\Delta_1=(-1)_2$||||abcpqrxyz||||=||||abcpqrxyz|||| **…………………..(ii)**

Δ_2=||||b+cc+abq+rr+pqy+zz+xy||||

Applying $R_1 \to R_1 - R_3$, we have:

Δ_2=||||cc+abrr+pqzz+xy||||

Applying $R_2 \to R_2 - R_1$, we have:

Δ_2=||||cabrpqzxy||||

Applying $R_1 \leftrightarrow R_2$ and $R_2 \leftrightarrow R_3$ we have:

$\Delta_2=(-1)_2$||||abcpqrxyz||||=||||abcpqrxyz|||| **………………………(iii)**

From (i), (ii) and (iii), we have:

Δ_2=2||||abcpqrxyz||||

Q6: By using properties of determinants, show that:

||||0−aba0c−b−c0||||=0

Sol:

Δ=||||0−aba0c−b−c0||||

Applying $R_1=cR_1$, we have:
Δ=1c||||0−abac0c−bc−c0||||

Applying $R_1 \to R_1 – bR_2$, we have:
Δ=1c||||ab−abac0c0−c0|||| Δ=ac||||b−abc0c0−c0||||

Here, the two rows R_1 and R_3 are identical.
⇒ Δ=0

Q7: By using the properties of determinants, show that:

|||||−a2bacaab−b2cbacbc−c2|||||=4a2b2c2

Sol:

|||||−a2bacaab−b2cbacbc−c2|||||

|||||−a2bacaab−b2cbacbc−c2||||| **(Taking out factors a, b, c from R_1 , R_2 and R_3)**
Δ=a2b2c2||||−1111−1111−1|||| **(Taking out factors a,b,c from C_1, C_2 and C_3)**
Applying $R_2 \to R_2+R_1$ and $R_3 \to R_3+R_1$ we have:
Δ=a2b2c2||||−100102120||||

⇒ Δ=a2b2c2(−1)|||0220|||
⇒ −a2b2c2(0–4)=4a2b2c2

Q8: By using the properties of determinants, show that:

(i) |||||111abca2b2c2|||||=(a–b)(b–c)(c–a)
(ii) ||||1aa31bb31cc3||||=(a–b)(b–c)(c–a)(a+b+c)

Sol:

Let $\Delta=|||||111abca_2b_2c_2|||||$
Applying $R_1 \rightarrow R_1 – R_3$ and $R_2 \rightarrow R_2 – R_3$, we have:
$\Delta=|||||001a–cb–cca_2–c_2b_2–c_2c_2||||| =(c−a)(b–c)|||||001a–cb–cca_2–c_2b_2–c_2c_2|||||$

Applying $R_1 \rightarrow R_1+R_2$ **we have:**
$= (b–c)(c−a)||||00101–cca–bb+cc_2||||$
$= (a–b)(b–c)(c−a)||||00101–cc1b+cc_2||||$
Expanding along C_1 we have:
$\Delta=(a–b)(b–c)(c−a)|0−10b+c|=(a–b)(b–c)(c–a)$

Hence, the given result is proved.

(ii) Let $\Delta=||||1aa_31bb_31cc_3||||$
Applying $C_1 \rightarrow C_1−C_3 and C_2 \rightarrow C_2−C_3$ we have:
$= ||||0a−ca_3−c_30b−cb_3−c_31cc_3||||$
$= ||||0a−c(a−c)(a_2+ac+c_2)0b−c(b−c)(b_2+bc+c_2)1cc_3||||$
$= (c−a)(b−c)||||0−1−(a_2+ac+c_2)01(b_2+bc+c_2)1cc_3||||$
Applying $C_1 \rightarrow C_1+C_2$ **we have:**
$= (c−a)(b−c)||||00(b_2−a_2)+(bc−ac)01(b_2+bc+c_2)1cc_3||||$
$= (a−b)(b−c)(c−a)||||00−(a+b+c)01(b_2+bc+c_2)1cc_3||||$
$= (a−b)(b−c)(c−a)(a+b+c)||||00−101(b_2+bc+c_2)1cc_3||||$
Expanding along C_1 we have:
$\Delta=(a−b)(b−c)(c−a)(a+b+c)(−1)|||011c|||$

$= (a−b)(b−c)(c−a)(c−a)(a+b+c)$
Hence, the given result is proved.

Q9: By using the properties of determinants, show that:

$|||||xyzx_2y_2z_2xyyzzx|||||=(x–y)(y−z)(z−x)(xy+yz+zx)$

Sol:

Let $\Delta=|||||xyzx^2y^2z^2xyyzzx|||||$

Applying $R_2 \to R_2 - R_1 \text{and} R_3 \to R_3 - R_1$ we have:

$\Rightarrow \quad \Delta=|||||xy-xz-xx^2y^2-x^2z^2-x^2xy-yzyzzx-yz-y(z-x)|||||$

= $|||||x-(x-y)z-xx^2-(x-y)(x+y)(z-x)(z+x)yzz(x-y)-y(z-x)|||||$

= $(x-y)(z-x)||||x-11x^2-x-y(z+x)yzz-y||||$

Applying $R_3 \to R_3 + R_2$ **we have:**

$\Rightarrow \quad \Delta=(x-y)(z-x)||||x-10x^2-x-yz-yyzzz-y||||$

= $(x-y)(z-x)(z-y)||||x-10x^2-x-y1yzz1||||$

Expanding along R_3 **we have:**

$\Rightarrow \quad \Delta=[(x-y)(z-x)(z-y)][(-1)|||x-1yzz|||+1|||x-1x^2-x-y|||]$

= (x-y)(z-x)(z-y)[(-xz-zy)+(x^2-xy+x^2)]

= -(x-y)(y-z)(z-x)(xy+yz+zx)

Hence, the given result is proved.

Q10: By using properties of determinants, show that:

(i) $||||x+42x2x2xx+42x2x2xx+4||||=(5x+4)(4-x)^2$

(ii) $||||y+kyyyy+kyyyy+k||||=k^2(3y+k)$

Sol:

(i) $||||x+42x2x2xx+42x2x2xx+4||||$

Applying $R_1 \to R_1 + R_2 + R_3$ we have:

$\Delta=||||5x+42x2x5x+4x+42x5x+42xx+4|||| =(5x+4)||||12x2x1x+42x12xx+4||||$

Applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$ we have:

$\Delta=(5x+4)||||12x2x0-x+4000-x+4||||$

= $(5x+4)(4-x)(4-x)||||12x2x010001||||$

Expanding along C_3 **we have:**

$\Delta=(5x+4)(4-x)^2|||12x01||| =(5x+4)(4-x)^2$

Hence, the given result is proved.

(ii) $||||y+kyyyy+kyyyy+k||||$

Applying $R_1 \rightarrow R_1+R_2+R_3$ **we have:**

$\Delta=||||3y+kyy3y+ky+ky3y+kyy+k||||=(3y+k)||||1yy1y+ky1yy+k||||$

Applying $C_2 \rightarrow C_2-C_1, C_3 \rightarrow C_3-C_1$ **we have:**

$\Delta=(3y+k)||||1yy0k000k||||=k^2(3y+k)||||1yy010001||||$

Expanding along C_3 we have:

$\Delta=k^2(3y+k)|||1y01|||=k^2(3y+k)$

Hence, the given result is proved.

Q.11: By using properties of determinants, show that:

(i) $||||a–b–c2b2c2ab–c–a2c2a2bc–a–b||||=(a+b+c)^3$

(ii) $||||x+y+2zzzxx+z+2xxyyz+x+2y||||=2(x+y+z)^3$

Sol:

$\Delta=||||a–b–c2b2c2ab–c–a2c2a2bc–a–b||||$

(i)

Applying $R_1 \rightarrow R_1+R_2+R_3$, we have:

$\Delta=||||a+b+c2b2ca+b+cb–c–a2ca+b+c2bc–a–b||||$

$= (a+b+c)||||12b2c1b–c–a2c12bc–a–b||||$

Applying $C_2 \rightarrow C_2–C_1, C_3 \rightarrow C_3–C_1$, we have:

$\Delta=(a+b+c)|||||12b2c0–(a+b+c)000–(a+b+c)|||||$

$= (a+b+c)^3||||12b2c0-1000-1||||$

Expanding along C_3, we have:

$\Delta = (a + b + c)^3 (-1) (-1) = (a + b + c)^3$

Hence, the given result is proved.

$\Delta=||||x+y+2zzzxy+z+2xxyyz+x+2y||||$

(ii) Applying $C_1 \to C_1 + C_2 + C_3$, we have:

Δ=|||||2(x+y+z)2(x+y+z)2(x+y+z)xy+z+2xxyyz+x+2y|||||

= 2(x+y+z)||||111xy+z+2xxyyz+x+2y||||

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we have:

Δ=2(x+y+z)||||100xx+y+z0y0x+y+z||||

= 2(x+y+z)3||||100x10y01||||

Expanding along R_3, we have:

$\Delta = 2(x + y + z)^3 (1) (1 - 0) = 2(x + y + z)^3$

Hence, the given results are proved.

Q.12: By using properties of determinants, show that:

|||||1x2xx1x2x2x1|||||=(1–x3)2

Sol:

Δ=|||||1x2xx1x2x2x1|||||

Applying $R_1 \to R_1 + R_2 + R_3$, we have:

Δ=|||||1+x+x2x2x1+x+x21x21+x+x2x1|||||

= (1+x+x2)||||1x2x11x21x1||||

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$, we have:

Δ=(1+x+x2)||||1x2x01–x2x2–x0x–x21–x||||

= (1+x+x2)(1–x)(1–x)||||1x2x01+x−x0x1||||

= (1–x3)(1–x)||||1x2x01+x−x0x1||||

Expanding along R_1, we have:

Δ=(1–x3)(1–x)(x)|||1+x−xx1|||

$= (1 - x^3)(1 - x)(1 + x + x^2)$

$= (1 - x^3)(1 - x^3)$

$\mathbf{= (1 - x^3)^2}$

Hence, the given result is proved.

Q.13: By using properties of determinants, show that:

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

Sol:

$$\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 + bR_3$ and $R_2 \rightarrow R_2 - aR_3$, we have:

$$\Delta = \begin{vmatrix} 1+a^2+b^2 & 0 & -b(1+a^2+b^2) \\ 0 & 1+a^2+b^2 & a(1+a^2+b^2) \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

$$= (1+a^2+b^2)\begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Expanding along R_1, we have:

$$\Delta = (1+a^2+b^2)^2\left[(1)\begin{vmatrix} 1 & a \\ -2a & 1-a^2-b^2 \end{vmatrix} - b\begin{vmatrix} 0 & 1 \\ 2b & -2a \end{vmatrix}\right]$$

$= (1 + a^2 + b^2)^2 [1 - a^2 - b^2 + 2a^2 - b(-2b)]$

$\mathbf{= (1 + a^2 + b^2)^2 (1 + a^2 + b^2)}$

$\mathbf{= (1 + a^2 + b^2)^3}$

Q.14: By using properties of determinants, show that:

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

Sol:

$$\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Taking out common factors a, b, c from R_1, R_2 and R_3 respectively, we have:

Δ=abc|||||a+1baabb+1bbccc+1c|||||

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$, we have:

Δ=abc|||||a+1b–1a–1ab1b0c01c|||||

Applying $C_1 \rightarrow aC_1$, $C_2 \rightarrow bC_2$, and $C_3 \rightarrow cC_3$, we have:

Δ=abc×1abc||||a2+1−1−1b210c201||||

= ||||a2+1−1−1b210c201||||

Expanding along R_3, we have:

Δ=−1|||b21c20|||+1|||a2+1−1b21|||

$= -1(-c^2) + (a^2 + 1 + b^2) = 1 + a^2 + b^2 + c^2$

Hence, the given result is proved.

Q.15: Choose the correct answer:

Let A be a square matrix of order 3 × 3, KA is equal to

(1). k|A|

(2). k^2|A|

(3). k^3|A|

(4). 3k|A|

Answer:

(3)

A is a square matrix of order 3 × 3

Let A = ||||a1a2a3b1b2b3c1c2c3||||

Then, kA = ||||ka1ka2ka3kb1kb2kb3kc1kc2kc3||||

Therefore, |kA|=||||ka1ka2ka3kb1kb2kb3kc1kc2kc3||||

k3=||||a1a2a3b1b2b3c1c2c3||||(Takingoutcommonfactorskfromeachrow)

$= k^3|A|$

Therefore, $|kA| = k^3|A|$

Hence, the correct answer is C.

Q.16: Which of the following is correct?

(1) Determinant is a square matrix.

(2) Determinant is a number associated to a matrix.

(3) Determinant is a number associated to a square matrix.

(4) None of these answer

Sol:

(3)

We know that to every square matrix, A = [aij] of order n. we can associate a number called the determinant of a square matrix A, where aij = $(i, j)^{th}$ element of A.

Thus, the determinant is a number associated to a square matrix.

Hence, the correct answer is C.

Exercise – 4.3

Q.1: Find area of the triangle with vertices at the point given in each of the following:

(i) (1, 0), (6, 0), (4, 3)

(ii) (2, 7), (1, 1), (10, 8)

(iii) (−2, −3), (3, 2), (−1, −8)

Sol:

(i) The area of the triangle with vertices (1, 0), (6, 0), (4, 3) is given by the relation,

Δ=12||||164003111|||| =12[1(0−3)−0(6−4)+1(18−0)]

=12[−3+18] = 152 **unit²**

(ii) The area of the triangle with vertices (2, 7), (1, 1), (10, 8) is given by the relation,

Δ=12||||2110718111|||| =12[2(1−8)−7(1−10)+1(8−0)]

= 12[−14+63−2]=12[−16+63]=472 unit²

(iii) The area of the triangle with vertices (−2, −3), (3, 2), (−1, −8) is given by the relation,

Δ=12||||−23−1−32−8111|||| =12[−2(2+8)+3(3+1)+1(−24+2)]

=12[−20+12−22]=−302=−15 unit²

Hence, the area of the triangle is |−15|=15 **unit²**

Q.2: Show that points A(a,b+c),B(b,c+a),C(c,a+b) are collinear

Sol:

Area of ΔABC is given by the relation:

Δ=12||||abcb+cc+aa+b111||||

= 12||||ab−ac−ab+ca−ba−b100||||
= 12(a−b)(c−a)||||a−1ab+c1b+c101||||
= 12(a−b)(c−a)||||a−1ab+c1b+c101||||
=0

Thus, the area of the triangle formed by points A, B, and C is zero. Hence, the points A, B, and C are collinear.

Q.3: Find values of k if area of triangle is 4 square units and vertices are

(i) (k, 0), (4, 0), (0, 2)

(ii) (−2, 0), (0, 4), (0, k)

Sol:

We know that the area of a triangle whose vertices are $(x_1,y_1),(x_2,y_2),\text{and}(x_3,y_3)$ is the absolute value of the determinant (Δ), where
$\Delta=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

It is given that the area of triangle is 4 square units.

Therefore $\Delta=\pm4$

(i) The area of the triangle with vertices (k, 0), (4, 0), (0, 2) is given by the relation:

$$\Delta=\frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} =\frac{1}{2}[k(0-2)-0(4-0)+1(8-0)]$$

$= \frac{1}{2}[-2k+8]=-k+4=-k+4=\pm4$

When −k + 4 = − 4, k = 8.

When −k + 4 = 4, k = 0.

Hence, k = 0, 8.

(ii) The area of the triangle with vertices (−2, 0), (0, 4), (0, k) is given by the relation:

$$\Delta=\frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix} =\frac{1}{2}[-2(4-k)] =k-4$$

i.e. $k-4=\pm4$

When k-4 = − 4, k = 0.

When k-4 = 4, k = 8.

Hence, k = 0, 8.

Q.4: (i) Find equation of line joining (1, 2) and (3, 6) using determinants

(ii) Find equation of line joining (3, 1) and (9, 3) using determinants

Sol:

(i) Let P (x, y) be any point on the line joining points A (1, 2) and B (3, 6). Then, the points A, B, and P are collinear. Therefore, the area of triangle ABP will be zero.

Δ=12||||13x26y111||||=0 =12[1(6−y)−2(3−x)+1(3y−6x)]=0

6-y-6+2x+3y-6x=0

2y-4x=0

y=2x
Hence, the equation of the line joining the given points is y = 2x.
(ii) Let P (x, y) be any point on the line joining points A (3, 1) and B (9, 3). Then, the points A, B, and P are collinear. Therefore, the area of triangle ABP will be zero.

Δ=12||||39x13y111||||=0

= 12[3(3−y)−1(9−x)+1(9y−3x)]=0
9 – 3y – 9 + x + 9y – 3x = 0

6y – 2x = 0

x – 3y = 0

Hence, the equation of the line joining the given points is x − 3y = 0.

Q.5: If area of triangle is 35 square units with vertices (2, −6), (5, 4), and (k, 4). Then k is
A. 12 B. −2 C. −12, −2 D. 12, −2
Sol:

D

The area of the triangle with vertices (2, −6), (5, 4), and (k, 4) is given by the relation:

Δ=12||||25k−644111||||=0 =12[2(4−4)+6(5−k)+1(20−4k)] =12[30−6k+20−4k]

=12[50−10k] **= 25 – 5k**
It is given that the area of the triangle is ±35.

Therefore, we have:

25−5k=±35 5(5−k)=±35 5−k=±7

When 5-k = −7, k = 5+7 = 12.

When 5-k = 7, k = 5-7 = −2.

Hence, k = 12, −2.

The correct Sol: is D.

Exercise 4.4

Q1: Write Minors and Cofactors of the elements of following determinants:

(i) |||20−43|||
(ii) |||abcd|||

Sol:

(i) The given determinant is |||20−43|||
Minor of element a_{ij} is M_{ij} .

Therefore, M_{11} = minor of element a_{11} = 3

M_{12} = minor of element a_{12} = 0

M_{21} = minor of element a_{21} = -4

M_{22} = minor of element a_{22} = 2

Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$.

Therefore, $A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (3) = 3$

$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (0) = 0$

$A_{21} = (– 1)^{2+1} M_{21} = (-1)^3 (-4) = 4$

$A_{22} = (-1)^{2+2} M_{22} (-1)^4 (2) = 2$

(ii) The given determinant is |||abcd|||
Minor of element a_{ij} is M_{ij} .

Therefore, M_{11} = minor of element a_{11} is M_{ij} .

M_{12} = minor of element a_{12} = b

M_{21} = minor of element a_{21} = c

M_{22} = minor of element a_{22} = a

Cofactor of a_{ij} is $A_{ij} = (-1)^{I+j} M_{ij}$.

Therefore, $A_{11} = (-1)^{1+1} M_{11} = (-1)^2 (d) = d$

$\mathbf{A_{12} = (-1)^{1+2} M_{12} = (-1)^3 (b) = -b}$

$\mathbf{A_{21} = (-1)^{2+1} M_{21} = (-1)^3 (c) = -c}$

$\mathbf{A_{22} = (-1)^{2+2} M_{22} = (-1)^4 (a) = a}$

Q2: (i) ||||110010001||||
(ii) ||||1300514−12||||

Sol:

(i) The given determinant is ||||110010001||||
By the definition of minors and cofactors, we have :

M_{11} = minor of a_{11} = |||1001||| = 1
M_{12} = minor of a_{12} = |||0001||| = 0
M_{13} = minor of a_{13} = |||0010||| = 0
M_{21} = minor of a_{21} = |||0001||| = 0
M_{22} = minor of a_{22} = |||1001||| = 1
M_{23} = minor of a_{23} = |||1000||| = 0
M_{31} = minor of a_{31} = |||0100||| = 0
M_{32} = minor of a_{32} = |||1000||| = 0
M_{33} = minor of a_{33} = |||1001||| = 1
A_{11} = cofactor of $a_{11} = (-1)^{1+1}$ $\mathbf{M_{11} = 1}$

A_{12} = cofactor of a_{12} = $(-1)^{1+2}$ $\mathbf{M_{12} = 0}$

A_{13} = cofactor of a_{13} = $(-1)^{1+3}$ $\mathbf{M_{13} = 0}$

A_{21} = cofactor of a_{21} = $(-1)^{2+1}$ $\mathbf{M_{21} = 0}$

A_{22} = cofactor of a_{22} = $(-1)^{2+2}$ $\mathbf{M_{22} = 1}$

A_{23} = cofactor of a_{23} = $(-1)^{2+3}$ $\mathbf{M_{23} = 0}$

A_{31} = cofactor of a_{31} = $(-1)^{3+1}$ $\mathbf{M_{31} = 0}$

A_{32} = cofactor of a_{32} = $(-1)^{3+2}$ $\mathbf{M_{32} = 0}$

A_{33} = cofactor of a_{33} = $(-1)^{3+3}$ $\mathbf{M_{33} = 1}$

(ii) The given determinant is ||||1300514−12||||
By definition of minors and cofactors, we have:

M_{11} = minor of a_{11} = |||51−12||| = 10 + 1 = 11
M_{12} = minor of a_{12} = |||30−12||| = 6 – 0 = 6
M_{13} = minor of a_{13} = |||3051||| = 3 – 0 = 3
M_{21} = minor of a_{21} = |||0142||| = 0 – 4 = -4
M_{22} = minor of a_{22} = |||1042||| = 2 – 0 = 2
M_{23} = minor of a_{23} = |||1001||| = 1 – 0 = 1
M_{31} = minor of a_{31} = |||054−1||| = 0 – 20 = -20
M_{32} = minor of a_{32} = |||134−1||| = -1 – 12 = -13
M_{33} = minor of a_{33} = |||1305||| = 5 – 0 = 5
A_{11} = cofactor of a_{11} = $(-1)^{1+1}$ $\mathbf{M_{11} = 11}$

A_{12} = cofactor of a_{12} = $(-1)^{1+2}$ $\mathbf{M_{12} = -6}$

A_{13} = cofactor of a_{13} = $(-1)^{1+3}$ $\mathbf{M_{13} = 3}$

A_{21} = cofactor of a_{21} = $(-1)^{2+1}$ $\mathbf{M_{21} = 4}$

A_{22} = cofactor of a_{22} = $(-1)^{2+2}$ $\mathbf{M_{22} = 2}$

A_{23} = cofactor of a_{23} = $(-1)^{2+3}$ $\mathbf{M_{23} = -1}$

A_{31} = cofactor of a_{31} = $(-1)^{3+1}$ $\mathbf{M_{31} = -20}$

A_{32} = cofactor of a_{32} = $(-1)^{3+2}$ $\mathbf{M_{32} = 13}$

A_{33} = cofactor of a_{33} = (-1)$^{3+3}$ **M_{33} = 5**

Q.3: Using Cofactors of elements of second row, evaluate Δ=||||521302813|||| **.**

Sol:

The given determinant is Δ=||||521302813||||
We have:

M_{21} = |||3283||| = 9 – 16 = -7
Therefore, A_{21} = cofactor of a_{21} = (-1)$^{2+1}$ M_{21} = 7

M_{22} = |||5183||| = 15 – 8 = 7
Therefore, A_{22} = cofactor of a_{22} = (-1)$^{2+2}$ M_{22} = 7

M_{23} = |||5132||| = 10 – 3 = 7
Therefore, A_{23} = cofactor of a_{23} = (-1)$^{2+3}$ M_{23} = -7

We know that Δ is equal to the sum of the product of the elements of the second row with their corresponding cofactors.
Therefore, Δ = $a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$ =
2 (7) + 0 (7) + 1 (-7) = 14 – 7 = 7

Q4: Using Cofactors of elements of third column, evaluate Δ=||||111xyzyzzxxy|||| **.**

Sol:

The given determinant is ||||111xyzyzzxxy|||| .
We have:

M_{13} = |||11yz||| = z – y
M_{23} = |||11xz||| = z – x
M_{33} = |||11x||| = y – x
Therefore, A_{13} = cofactor of a_{13} = (-1)$^{1+3}$ M_{13} = (z – y)

A_{23} = cofactor of a_{23} = $(-1)^{2+3} M_{23} = -(z - x) = (x - z)$

A_{33} = cofactor of a_{33} = $(-1)^{3+3} M_{33} = (y - x)$

We know that Δ is equal to the sum of the product of the elements of the second row with their corresponding cofactors.

Therefore, $\Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$

$= yz(z - y) + zx(x - z) + xy(y - x)$

$= yz^2 - y^2z + x^2z + x^2z - xz^2 + xy^2 - x^2y$

$= (x^2z - y^2z) + (yz^2 - xz^2) + (xy^2 - x^2y)$

$= z(x^2 - y^2) + z^2(y - x) + xy(y - x)$

$= z(x - y)(x + y) + z^2(y - x) + xy(y - x)$

$= (x - y)[zx + zy - z^2 - xy]$

$= (x - y)[z(x - z) + y(z - x)]$

$= (x - y)(z - x)[-z + y]$

$= (x - y)(y - z)(z - x)$

Hence, $\Delta = (x - y)(y - z)(z - x)$

Q5: For the matrices A and B, verify that (AB)' = B'A' where

(i) A = $\begin{bmatrix}1\\-4\\3\end{bmatrix}$,

B = $\begin{bmatrix}-1 & 2 & 1\end{bmatrix}$

(ii) A = = $\begin{bmatrix}0\\1\\2\end{bmatrix}$,

B = $\begin{bmatrix}1 & 5 & 7\end{bmatrix}$

Sol:

(i) AB = $\begin{bmatrix}1\\-4\\3\end{bmatrix}\begin{bmatrix}-1 & 2 & 1\end{bmatrix} = \begin{bmatrix}-1 & 4 & -3\\2 & -8 & 6\\1 & -4 & 3\end{bmatrix}$

Therefore, (AB)' = $\begin{bmatrix}-1 & 2 & 1\\4 & -8 & -4\\-3 & 6 & 3\end{bmatrix}$

Now, A' = $\begin{bmatrix}1 & -4 & 3\end{bmatrix}$,

B' = $\begin{bmatrix}-1\\2\\1\end{bmatrix}$

Therefore, B'A'= $\begin{bmatrix}-1\\2\\1\end{bmatrix}\begin{bmatrix}1 & -4 & 3\end{bmatrix} = \begin{bmatrix}-1 & 2 & 1\\4 & -8 & -4\\-3 & 6 & 3\end{bmatrix}$

Hence, we have verified that (AB)’ = B’A’ .

(ii) AB = $\begin{bmatrix} 012 \end{bmatrix} [157] = \begin{bmatrix} 01205100714 \end{bmatrix}$

Therefore, (AB)’ = $\begin{bmatrix} 00015721014 \end{bmatrix}$

Now, A’ = $[012]$,

B’ = $\begin{bmatrix} 157 \end{bmatrix}$

Therefore, B’A’= $\begin{bmatrix} 157 \end{bmatrix} [012] = \begin{bmatrix} 00015721014 \end{bmatrix}$

Hence, we have verified that (AB)’ = B’A’

Exercise 4.5

Q.1: Find the adjoint of each of the matrices

$[2547]$

Sol:

Suppose A = $[2547]$

For X = $[acbd]$

Then adj (X) = $[d–c–ba]$

We have,

$A_{11} = 7$, $A_{12} = –4$, $A_{21} = -5$, $A_{22} = 2$

Therefore, adj (A) = $[7–5–42]$

Q.2: Find the adjoint of each of the matrices

$\begin{bmatrix} 12–2–130251 \end{bmatrix}$

Sol:

Suppose, D = $\left[\begin{matrix}12–2–130251\end{matrix}\right]$

We have,

$D_{11}=\left|\begin{matrix}3051\end{matrix}\right|=3–0=3$ $D_{12}=–\left|\begin{matrix}2–251\end{matrix}\right|=–(2+10)=–12$ $D_{13}=\left|\begin{matrix}2–230\end{matrix}\right|=0+6=6$ $D_{21}=–\left|\begin{matrix}–1021\end{matrix}\right|=–(−1–0)=1$ $D_{22}=\left|\begin{matrix}1–221\end{matrix}\right|=1+4=5$ $D_{23}=–\left|\begin{matrix}1–2–10\end{matrix}\right|=–(0–2)=2$ $D_{31}=\left|\begin{matrix}–1325\end{matrix}\right|=–5–6=–11$ $D_{32}=–\left|\begin{matrix}1225\end{matrix}\right|=–(5–4)=–1$ $D_{33}=\left|\begin{matrix}12–13\end{matrix}\right|=3+2=5$

Hence, adj D = $\left[\begin{matrix}D_{11}D_{21}D_{31}D_{12}D_{22}D_{32}D_{13}D_{23}D_{33}\end{matrix}\right]=\left[\begin{matrix}3–126152–11–15\end{matrix}\right]$

Q.3: Prove whether D (adj D) = (adj D) D = $|D|I$

$D=[2–43–6]$

Sol:

$D=[2–43–6]$

We have,

$|D|=–12–(−12)=–12+12=0$ $|D|I=0[1001]=[0000]$

$D_{11} = – 6$, $D_{12} = 4$, $D_{21} = – 3$, $D_{22} = 2$

adj D = $[–64–32]$

Now,

D (adj D) = $[2–43–6][–64–32]=[–12+1224–24–6+612–12]=[0000]$

And,

(adj D) D = $[–64–32][2–43–6]=[–12+128–8–18+1812–12]=[0000]$

Thus, D (adj D) = (adj D) D = $|D|I$

Q.4: Obtain the inverse of the matrices if it exists

$[24–23]$

Sol:

Suppose D = $\begin{bmatrix} 2 & 4 \\ -2 & 3 \end{bmatrix}$

We have,

$$|D| = 6 + 8 = 14$$

Now,

$D_{11} = 3$, $D_{12} = -4$, $D_{21} = 2$, $D_{22} = 2$

adj D = $\begin{bmatrix} 3 & -4 \\ 2 & 2 \end{bmatrix}$ $D^{-1} = \frac{1}{|D|} = \frac{1}{14}\begin{bmatrix} 3 & -4 \\ 2 & 2 \end{bmatrix}$

Q.5: Obtain the inverse of the matrices if it exists

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 4 & 5 \end{bmatrix}$$

Sol:

Suppose D = $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 4 & 5 \end{bmatrix}$

We have,

$$|D| = 1(10 - 0) - 2(0 - 0) + 3(0 - 0) = 10$$

Now, $D_{11} = 10 - 0 = 10$, $D_{12} = -(0 - 0) = 0$, $D_{13} = 0 - 0 = 0$, $D_{21} = -(10 - 0) = -10$, $D_{22} = 5 - 0 = 5$, $D_{23} = -(0 - 0) = 0$, $D_{31} = 8 - 6 = 2$, $D_{32} = -(4 - 0) = -4$, $D_{33} = 2 - 0 = 2$

adj D = $\begin{bmatrix} 10 & 0 & 0 \\ -10 & 5 & 0 \\ 2 & -4 & 2 \end{bmatrix}$

D^{-1} = $\frac{1}{|D|} adj D = \frac{1}{10}\begin{bmatrix} 10 & 0 & 0 \\ -10 & 5 & 0 \\ 2 & -4 & 2 \end{bmatrix}$

Q.6: Find the inverse of each of the matrices (if it exists).

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$$

Sol:

Let A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

We have,

$$|A| = 1(-\cos^2\alpha - \sin^2\alpha) \quad |A| = -(\cos^2\alpha + \sin^2\alpha) = -1$$

Now,

$A_{11} = -\cos^2\alpha - \sin^2\alpha = -1, A_{12} = 0, A_{13} = 0$ $A_{21} = 0, A_{22} = -\cos\alpha, A_{23} = -\sin\alpha$ $A_{31} = 0, A_{32} = -\sin\alpha, A_{33} = \cos\alpha$

Therefore,

$adjA = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$

Therefore,

$A^{-1} = \frac{1}{|A|}.adjA = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

Q.7: Let $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ **and** $B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$**. Verify that** $(AB)^{-1} = B^{-1}A^{-1}$

Sol:

Let $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$
We have ,

$|A| = 15 - 14 = 1$

Now,

$A_{11} = 5, A_{12} = -2, A_{21} = -7, A_{22} = 3$

Therefore,

$adjA = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

Therefore,

$A^{-1} = \frac{1}{|A|}.adjA = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$

Now, let $adjB = \begin{bmatrix} 6 & 7 \\ 8 & 6 \end{bmatrix}$
We have ,

$|B| = 54 - 56 = -2$

Therefore,

adjB=[9–7–86]

Therefore,

B–1=1|B|.adjB=0.5[9−7−86] [–92724–3]

Now,

B–1A–1=[–92724−3][5−2−73] B–1A–1=[–452–8352+6632+12–492–9][612472872–672]

Then,

⇒ AB=[3275][6779]
⇒ AB=[18+4912+3524+6316+45]
⇒ AB=[67478761]
Therefore, we have AB=[18+4912+3524+6316+45]
Also,

adj(AB)=[61–47–8767]

Therefore, (AB)–1=1|AB|adj(AB)=–12[61–47–8767] . . . . (2)
=[612472872–672]

From (1) and (2), we have: $(AB)^{-1} = B^{-1} A^{-1}$ Hence, the given result is proved.

Q.8: If A=[3−112], show that $A^2 – 5A + 7I = 0$. Hence find A^{-1}

Sol:

A=[3−112]

A 2 = A. A = [3−112][3−112] = A=[9–1−3–23+2–1+4] = [8−553]
Therefore,

$A^2 – 5A + 7I = 0$

= [8−553] – 5 [3−112] + 7 [1001]
= [8−553] – [15–5510] + [7007]
= [–700–7] – [7007] = [0000]
Therefore, $A^2 – 5A + 7I = 0$

Therefore, A.A – 5 A = – 7I

⇒ A.A (A^{-1}) – 5A A^{-1} = – 7I A^{-1} . . . . [post – multiplying by A -1 as |A|≠0]

⇒ A. (A A^{-1}) – 5I = – 7 A^{-1}

⇒ AI – 5I = – 7 A^{-1}

⇒ A^{-1} =–17(A–5I)

⇒ A^{-1} =–17(5I–A)

=17([5005]–[3–112])=17[21−13]

Therefore,

A^{-1} = 17[21−13]

Q.9: Let A be a nonsingular square matrix of order 3 × 3. Then |adjA| **is equal to**

(A) |adjA|

(B) |adjA|2

(C) |adjA|3

(D) 3 |adjA|

Sol:

The correct option is B

(adjA)A=|A|I=⎡⎣⎢|A|000|A|000|A|⎤⎦⎥

⇒ (adjA)A=⎡⎣⎢|A|000|A|000|A|⎤⎦⎥

⇒ (adjA)|A|=|A|3⎡⎣⎢100010001⎤⎦⎥=|A|3(I)

Therefore, |(adjA)|=|A|2

Hence, the correct Sol: is B.

Q.10: If A is an invertible matrix of order 2, then det (A^{-1}) is equal to

(A) det (A)

(B) 1det(A)

(C) 1

(D) 0

Sol:

Since A is an invertible matrix , A -1 exists and A -1 = $1|a|\mathrm{adj}A$
As matrix A is of order 2, let $A=[acbd]$
Then, $|A|=ad–bc and adj A=[d–c–ba]$
Now,

$A^{-1}=1|A|\mathrm{adj}A=\left[d|A|–c|A|–b|A|a|A| \right]$

Therefore,

$||A^{-1}||=\left[d|A|–c|A|–b|A|a|A| \right]=1|A|2[d–c–ba]=1|A|2(ad–bc)=1|A|2.|A|=1|A|$

Therefore,

$\det(A)^{-1}=1\det(A)$

Hence, the correct Sol: is B.

Q.11: Suppose D = $\left[2–11–12–11–12 \right]$, verify that D ^{3} – 6 D^2 + 9D – 4I = 0 and hence find D $^{-1}$

Sol:

D = $\left[2–11–12–11–12 \right]$
D ^{2} = $\left[2–11–12–11–12 \right]\left[2–11–12–11–12 \right]$
$\left[4+1+1–2–2–12+1+2–2–2–11+4+1–1–2–22+1+2–1–2–21+1+4 \right]=\left[6–55–56–55–56 \right]$

D ^{3} = D ^{2} D = $\left[6–55–56–55–56 \right]\left[2–11–12–11–12 \right]=\left[12+5+5–10–6–510+5+6–6–10–55+12+5–5–10–66+5+105–6–105+5+12 \right]=\left[22–2121–2122–2121–2122 \right]$
Now,

$D^3 – 6D^2 + 9D – 4I = 0$

=⎡⎣⎢⎢22–2121–2122–2121–2122⎤⎦⎥⎥–6⎡⎣⎢⎢6–55–56–55–56⎤⎦⎥⎥+9⎡⎣⎢⎢2–11–12–11–12⎤⎦⎥⎥–4⎡⎣⎢⎢100010001⎤⎦⎥⎥
=⎡⎣⎢⎢22–2121–2122–2121–2122⎤⎦⎥⎥–⎡⎣⎢⎢36–3030–3036–3030–3036⎤⎦⎥⎥+⎡⎣⎢⎢18–99–918–99–918⎤⎦⎥⎥–⎡⎣⎢⎢400040004⎤⎦⎥⎥
=⎡⎣⎢⎢40–3030–3040–3030–3040⎤⎦⎥⎥–⎡⎣⎢⎢40–3030–3040–3030–3040⎤⎦⎥⎥=⎡⎣⎢⎢000000000⎤⎦⎥⎥

Hence, $D^3 – 6D^2 + 9D – 4I = 0$

Now,

$D^3 – 6D^2 + 9D – 4I = 0$

(DDD) D^{-1} – 6 D + 9D – 4I = 0 [Multiplying by D^{-1} on as |D|I is not equal to 0]

DD (DD^{-1}) – 6D (DD^{-1}) + 9 (DD^{-1}) = 4 ID^{-1}

DDI – 6DI + 9I = 4 D^{-1}

$D^2 – 6D + 9I = 4 D^{-1}$

D^{-1} = 14 ($D^2 – 6D + 9I$)

D^2 – 6D + 9I

=⎡⎣⎢⎢6–55–56–55–56⎤⎦⎥⎥–6⎡⎣⎢⎢2–11–12–11–12⎤⎦⎥⎥+9⎡⎣⎢⎢100010001⎤⎦⎥⎥
=⎡⎣⎢⎢6–55–56–55–56⎤⎦⎥⎥–⎡⎣⎢⎢12–66–612–66–612⎤⎦⎥⎥+⎡⎣⎢⎢900090009⎤⎦⎥⎥=⎡⎣⎢⎢31–1131–113⎤⎦⎥⎥Fromequation(1),wehave,D–1=14⎡⎣⎢⎢31–1131–113⎤⎦⎥⎥

Exercise- 4.6

Q-1: Check the consistency for the system of two equations given below:

a + 3b = 2

2a + 4b = 3

Sol:

As per the data given in the question,

The given system of the two equations is:

a + 3b = 2

2a + 4b = 3

The given system of equations will be written as in the form of MX = N, where

M=[1234],X=[ab]andN=[23]

Thus,

|M| = 1(4) – 3(2) = 3 – 4 = -1 ≠ 0

Hence, M is non- singular.

Thus, M^{-1} exists.

Therefore, the given system of two equations will be consistent.

Q-2. Check the consistency of the system of two equations given below:

a + 4b = 5

2a + 7b = 8

Sol:

The given system of the two equations is:

a + 4b = 5

2a + 7b = 8

The given system of equations will be written as in the form of MX = N, where

M=[1247],X=[ab]andN=[58]

Thus,

|M| = 1(7) – 4(2) = 7 – 8 = -1 ≠ 0

Hence, M is a singular matrix.

Thus,

$(adjM)=[7-2-41]$ $(adjM)N=[7-2-41][58]$

$= [35–32-10+8]=[3-2]\neq 0$

Therefore, the solution for the given system of the equation does not exist. Thus, the given system of the equations will be inconsistent.

Q-3. Check the consistency of the system of three equations given below:

a + b + c = 1

2a + 3b + 2c = 2

pa + pb + 2pc = 4

Sol:

The given system of the two equations is:

a + b + c = 1

2a + 3b + 2c = 2

pa + pb + 2pc = 4

The given system of three equations will be written as in the form MX = N, where

$M=\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ p & p & 2p \end{bmatrix}, X=\begin{bmatrix} a \\ b \\ c \end{bmatrix} and N=\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

|M| = 1(6p – 2p) – 1(4p – 2p) + 1(2p – 3p)

= 1(4p) – 1(2p) + 1(-p)

= 4p – 2p – p

= p ≠ 0

Hence, M is non- singular.

Thus, M^{-1} exists.

Therefore, the given system of two equations will be consistent.

Q-4. Check the consistency of the system of three equations given below:

3a – b – 2c = 2

2b – c = -1

3a – 5b = 3

Sol:

The given system of the two equations is:

3a – b – 2c = 2

2b – c = -1

3a – 5b = 3

The given system of three equations will be written as in the form MX = N, where

$M=\begin{bmatrix}3 & -1 & -2\\ 0 & 2 & -1\\ 3 & -5 & 0\end{bmatrix}, X=\begin{bmatrix}a\\ b\\ c\end{bmatrix} and N=\begin{bmatrix}2\\ -1\\ 3\end{bmatrix}$

|M| = 3[2 × 0 – (-1) × (-5)] – 0[(-1) × 0 – (-2) × (-5)] + 3[(-1) × (-1) – (-1) × (4)]

= 3(0 – 5) – 0 + 3(1 + 4)

= 3(-5) – 0 + 3(5)

= -15 + 15 = 0

Thus,

M is a singular matrix.

Then,

$(adjM)=\begin{bmatrix}-5 & 10 & 5\\ -3 & 6 & 3\\ -6 & 12 & 6\end{bmatrix} (adjM)N=\begin{bmatrix}-5 & 10 & 5\\ -3 & 6 & 3\\ -6 & 12 & 6\end{bmatrix}\begin{bmatrix}2\\ -1\\ 3\end{bmatrix}$

$=\begin{bmatrix}-10-10+15\\ -6-6+9\\ -12-12+18\end{bmatrix}=\begin{bmatrix}-5\\ -3\\ -6\end{bmatrix}\neq 0$

Therefore, the solution for the given system of the equation does not exist. Thus, the given system of the equations will be inconsistent.

Q-5. Solve the following system of the linear equations by suing the matrix method:

5a + 2b = 4

7a + 3b = 5

Sol:

The given system of linear equations will be written as in the form of MX = N, where

M=[5723],X=[ab]andN=[45]

|M| = (5 × 3) – (7 × 2) = 15 – 14 = 1 ≠ 0

Hence, M is non- singular.

Thus, M^{-1} exists.

Now,

M^{-1} = 1|M|(adjM)
⟹ M^{-1} = 11[3−7−25]
Thus,

X = M^{-1} N = [3−7−25][45]
⟹ [ab]=[3−7−25][45]
⟹ [ab]=[3×4−2×5−7×4+5×5]=[12–10−28+25]=[2−3]
⟹ [ab]=[2−3]
Hence, a = 2 and b = -3.

Q-6. Solve the following system of the linear equations by suing the matrix method:

4a – 3b = 3

3a – 5b = 7

Sol:

The given system of linear equations will be written as in the form of MX = N, where

M=[43−3−5],X=[ab]andN=[37]

|M| = [4 × (-5)] – [(-3) × 3] = -20 + 9= -11 ≠ 0

Hence, M is non- singular.

Thus, M^{-1} exists.

Now,

M^{-1} = 1|M|(adjM)
⟹ M^{-1} = –111[−5−334] = 111[53−3−4]
Thus,

X = M^{-1} N = 111[53−3−4][37]
⟹ [ab]=111[53−3−4][37]
⟹ [ab]=111[5×3+(−3)×73×3+(−4)×7]=111[15–219–28]=111[−6−19]=[–611−1911]
⟹ [ab]=[–611−1911]
Hence, a = –611 **and b =** –1911**.**

Q-7. Solve the following system of the linear equations by suing the matrix method:

2a + b + c = 1

a – 2b – c = 32
3b – 5c = 9

Sol:

The given system of linear equations will be written as in the form of MX = N, where
M=⎡⎣⎢2101−231−1−5⎤⎦⎥,X=⎡⎣⎢abc⎤⎦⎥andN=⎡⎣⎢1329⎤⎦⎥

Thus,

|M| = 2[(-2) × (-5) – (-1) × 3] – 1[1 × (-5) – 3 × 1] + 0[(-2) × 1 – 1 × (-1)] = 2(10 + 3) – 1(-5 – 3) + 0

= 2 × 13 – (-8)

= 26 + 8 = 34 ≠ 0

Hence, M is non- singular.

Thus, M^{-1} exists.

Now,

$M_{11} = 13$, $M_{12} = 5$, $M_{13} = 3$

$M_{21} = 8$, $M_{22} = -10$, $M_{23} = -6$

$M_{31} = 1$, $M_{32} = 3$, $M_{33} = -5$

Thus,

$M^{-1} = \frac{1}{|M|}(\mathrm{adj} M) = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$

Thus,

$X = M^{-1} N = \frac{1}{|M|}(\mathrm{adj} M) = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 13\times1+8\times\frac{3}{2}+1\times9 \\ 5\times1+(-10)\times\frac{3}{2}+3\times9 \\ 3\times1+(-6)\times\frac{3}{2}+(-5)\times9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 13+12+9 \\ 5-15+27 \\ 3-9-45 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ -\frac{3}{2} \end{bmatrix}$

Therefore, a = 1, b = $\frac{1}{2}$ **and c =** $-\frac{3}{2}$

Q-8. Solve the following system of the linear equations by suing the matrix method:

a – b + c = 4

2a + b – 3c = 0

a + b + c = 2

Sol:

The given system of linear equations will be written as in the form of MX = N, where

$M = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \text{and} N = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

Thus,

$|M| = 1[1 \times 1 - (-3) \times 1] - (-1)[2 \times 1 - (-3) \times 1] + 1[2 \times 1 - 1 \times 1] = 2(1 + 3) + 1(2 + 3) + 1(2 - 1)$

$= 1 \times 4 + 1 \times 5 + 1 \times 1$

$= 4 + 5 + 1 = 10 \neq 0$

Hence, M is non- singular.

Thus, M^{-1} exists.

Now,

$M_{11} = 4$, $M_{12} = -5$, $M_{13} = 1$

$M_{21} = 2$, $M_{22} = 0$, $M_{23} = -2$

$M_{31} = 2$, $M_{32} = 5$, $M_{33} = 3$

Thus,

$M^{-1} = \frac{1}{|M|}(adj M) = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

Thus,

$X = M^{-1} N = \frac{1}{|M|}(adj M) = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 \times 4 + 2 \times 0 + 2 \times 2 \\ -5 \times 4 + 0 \times 0 + 5 \times 2 \\ 1 \times 4 + (-2) \times 0 + 3 \times 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16 + 0 + 4 \\ -20 + 0 + 10 \\ 4 + 0 + 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

Therefore, a = 2, b = -1 and c = 1

Q-9. If from equations,

2a – 3b + 5c = 11, 3a + 2b – 4c = -5 and a + b – 2c = -3

M = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

Find M^{-1}. By using M^{-1}, solve the system of the linear equations.

Sol:

$M = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$|M| = 2[\, 2 \times (-2) - 1 \times (-4)\,] + 3[\, 3 \times (-2) - 1 \times (-4)\,] + 5[\, 3 \times 1 - 2 \times 1\,]$

$= 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2)$

= 2 × 0 + 3 × (-2) + 5 × 1

= 0 – 6 + 5 = -1 ≠ 0

Hence, M is non- singular.

Thus, M^{-1} exists.

Now,

$M_{11} = 0$, $M_{12} = 2$, $M_{13} = 1$

$M_{21} = -1$, $M_{22} = -9$, $M_{23} = -5$

$M_{31} = 2$, $M_{32} = 23$, $M_{33} = 13$

Thus,

$$M^{-1} = \frac{1}{|M|}(adjM) = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of linear equations will be written as in the form of MX = N, where

$$M = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \text{ and } N = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

The solution of the given system of linear equations will be given by X = M^{-1} N.

$$\Rightarrow X = M^{-1} N = \frac{1}{|M|}(adjM) = \begin{bmatrix} 4231-3215-4-2 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0\times11+1\times(-5)+(-2)\times(-3) \\ -2\times11+9\times(-5)+(-23)\times(-3) \\ (-1)\times11+5\times(-5)+(-13)\times(-3) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Therefore, a = 1, b = 2 and c = 3.

Q-10. The price of 4 kg of onion, 3 kg of wheat and 2 kg of rice is Rs 60. The price of 2 kg of onion, 4 kg of wheat and 6 kg of rice is Rs 90. The price of 6 kg of onion, 2 kg of wheat and 3 kg of rice is Rs 70. What is the price of each of the items (per kg)? Use matrix method to find the price.

Sol:

Let us consider the cost of onions, wheat, and rice per kg be given by Rs a, Rs b,and Rs c, respectively.

Thus, the given situation will be represented by the system of the equations as:

$4a + 3b + 2c = 60$

$2a + 4b + 6c = 90$

$6a + 2b + 3c = 70$

The given system of the equations will be written as in the form of MX = N, where

$M=\begin{bmatrix}4 & 3 & 2\\ 2 & 4 & 6\\ 6 & 2 & 3\end{bmatrix}, X=\begin{bmatrix}a\\ b\\ c\end{bmatrix} \text{and} N=\begin{bmatrix}60\\ 90\\ 70\end{bmatrix}$

Thus,

$|M| = 4[\, 4 \times 3 - 6 \times 2 \,] - 3[\, 2 \times 3 - 6 \times 6 \,] + 2[\, 2 \times 2 - 6 \times 4 \,] = 2(\, 12 - 12 \,) - 3(\, 6 - 36 \,) + 2(\, 4 - 24 \,)$

$= 0 + 90 - 40 = 50 \neq 0$

Hence, M is non- singular.

Thus, M^{-1} exists.

Now,

$M_{11} = 0, M_{12} = 30, M_{13} = -20$

$M_{21} = -5, M_{22} = 0, M_{23} = 10$

$M_{31} = 10, M_{32} = -20, M_{33} = 10$

$M^{-1} = \frac{1}{|M|}(\operatorname{adj} M) = \frac{1}{50}\begin{bmatrix}0 & -5 & 10\\ 30 & 0 & -20\\ -20 & 10 & 10\end{bmatrix}$

Thus,

$X = M^{-1} N = \frac{1}{|M|}(\operatorname{adj} M) = \frac{1}{50}\begin{bmatrix}0 & -5 & 10\\ 30 & 0 & -20\\ -20 & 10 & 10\end{bmatrix}\begin{bmatrix}60\\ 90\\ 70\end{bmatrix}$

$\Rightarrow \begin{bmatrix}a\\ b\\ c\end{bmatrix} = \frac{1}{50}\begin{bmatrix}0\times 60+(-5)\times 90+10\times 70\\ 30\times 60+0\times 90+(-20)\times 70\\ (-20)\times 60+10\times 90+10\times 70\end{bmatrix}$

$\Rightarrow \begin{bmatrix}a\\ b\\ c\end{bmatrix} = \frac{1}{50}\begin{bmatrix}0-450+700\\ 1800+0-1400\\ -1200+900+700\end{bmatrix}$

$\Rightarrow \begin{bmatrix}a\\ b\\ c\end{bmatrix} = \frac{1}{50}\begin{bmatrix}250\\ 400\\ 400\end{bmatrix}$

$\Rightarrow \begin{bmatrix}a\\ b\\ c\end{bmatrix} = \begin{bmatrix}5\\ 8\\ 8\end{bmatrix}$

Therefore, a = 5, b = 8 and c = 8

Thus, the price for onions is Rs. 5 per kg, the price for wheat is Rs. 8 per kg and the price for the rice is Rs. 8 per kg.