
#420304

Topic: Fundamental Integrals

Find the integral of $\int (4e^{3x}+1)dx$

Solution

$\int (4e^{3x}+1)dx$

$= 4\int e^{3x}dx + \int 1dx$

$= 4\left(\frac{e^{3x}}{3}\right) + x + C$

$= \frac{4}{3}e^{3x} + x + C$

#420641

Topic: Fundamental Integrals

Find the integral of $\int \sec x(\sec x + \tan x)dx$

Solution

$\int \sec x(\sec x + \tan x)dx$

$= \int (\sec^2 x + \sec x\tan x)dx$

$= \int \sec^2 xdx + \int \sec x\tan xdx$

$= \tan x + \sec x + C$

#420643

Topic: Fundamental Integrals

Find the integral of $\int \frac{\sec^2 x}{co\sec^2 x}dx$

Solution

$\int \frac{\sec^2 x}{co\sec^2 x}dx$

$= \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}}dx$

$= \int \frac{\sin^2 x}{\cos^2 x}dx$

$= \int \tan^2 xdx$

$= \int (\sec^2 x - 1)dx$

$= \int \sec^2 xdx - \int 1dx$

$= \tan x - x + C$

#420644

Topic: Fundamental Integrals

Find the integral of $\int \frac{2-3\sin x}{\cos^2 x}dx$

Solution

$\int \frac{2-3\sin x}{\cos^2 x}dx$

$= \int\left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x}\right)dx$

$= \int 2\sec^2 xdx - 3\int \tan x\sec xdx$

$= 2\tan x - 3\sec x + C$

#420653

Topic: Integration by Substitution

Integrate the function $\frac{2x}{1+x^2}$

Solution

$\int \frac{2x}{1+x^2}dx$

Put $1+x^2 = t \Rightarrow 2xdx = dt$

$\Rightarrow \int \frac{2x}{1+x^2}dx = \int \frac{1}{t}dt$

$= \log|t| + C$

$= \log(1+x^2) + C$

#420654

Topic: Integration by Substitution

Integrate the function $\frac{(\log x)^2}{x}$

Solution

$\int \frac{(\log x)^2}{x}dx$

Put $\log|x| = t \Rightarrow \frac{1}{x}dx = dt$

$\Rightarrow \int \frac{(\log|x|)^2}{x}dx = \int t^2 dt$

$= \frac{t^3}{3} + C$

$= \frac{(\log|x|)^3}{3} + C$

#420655

Topic: Integration by Substitution

Integrate the function $\frac{1}{x + x\log x}$

Solution

$\int \frac{1}{x + x\log x}$

$= \int \frac{1}{x(1+\log x)}$

Put $1 + \log x = t \Rightarrow \frac{1}{x}dx = dt$

$\Rightarrow \int \frac{1}{x(1+\log x)}dx = \int \frac{1}{t}dt$

$= \log|t| + C = \log|1+\log x| + C$

#420656

Topic: Integration by Substitution

Integrate the function $\sin x \sin(\cos x)$

Solution

$\int \sin x \cdot \sin(\cos x)$

Put $\cos x = t \Rightarrow -\sin x dx = dt$

$\Rightarrow \int \sin x \cdot \sin(\cos x)dx = -\int \sin t dt$

$= -[-\cos t] + C$

$= \cos t + C = \cos(\cos x) + C$

#420657

Topic: Fundamental Integrals

Integrate the function $\sin(ax+b)\cos(ax+b)$

Solution

$\sin(ax+b)\cos(ax+b) = \dfrac{2\sin(ax+b)\cos(ax+b)}{2} = \dfrac{\sin 2(ax+b)}{2}$

Put $2(ax+b) = t$

$\Rightarrow 2adx = dt$

$=> \int \dfrac{\sin 2(ax+b)}{2}dx = \dfrac{1}{2}\int \dfrac{\sin t dt}{2a}$

$= \dfrac{1}{4a}[-\cos t] + C$

$= \dfrac{-1}{4a}\cos 2(ax+b) + C$

#420663

Topic: Integration by Substitution

Integrate the function $\dfrac{1}{x-\sqrt{x}}$

Solution

$\dfrac{1}{x-\sqrt{x}} = \dfrac{1}{\sqrt{x}(\sqrt{x}-1)}$

Put $(\sqrt{x}-1) = t$

$\Rightarrow \dfrac{1}{2\sqrt{x}}dx = dt$

$= \int \dfrac{1}{\sqrt{x}(\sqrt{x}-1)}dx = \int \dfrac{2}{t}dt$

$= 2\log|t| + C = 2\log|\sqrt{x}-1| + C$

#421126

Topic: Integration by Substitution

Integrate the function $\dfrac{x^2}{(2+3x^3)^3}$

Solution

Put $2+3x^3 = t$

$\therefore 9x^2dx = dt$

$\Rightarrow \int \dfrac{x^2}{(2+3x^3)^3}dx = \dfrac{1}{9}\int \dfrac{dt}{(t)^3}$

$= \dfrac{1}{9}\left[\dfrac{t^{-2}}{-2}\right] + C$

$= \dfrac{-1}{18}\left(\dfrac{1}{t^2}\right) + C$

$= \dfrac{-1}{18(2+3x^3)^2} + C$

#421127

Topic: Integration by Substitution

Integrate the function $\dfrac{1}{x(\log x)^m}, x > 0, m \neq 1$

Solution

Put $\log x = t$

$\therefore \frac{1}{x}dx = dt$

$\Rightarrow \int \frac{1}{x(\log x)^m}dx = \int \frac{dt}{(t)^m}$

$= \left(\frac{t^{-m+1}}{1-m}\right) + C$

$= \frac{(\log x)^{1-m}}{(1-m)} + C$

#421130

Topic: Integration by Substitution

Integrate the function $\frac{x}{e^{x^2}}$

Solution

Put $x^2 = t$

$\therefore 2xdx = dt$

$\Rightarrow \int \frac{x}{e^{x^2}}dx = \frac{1}{2}\int \frac{1}{e^t}dt$

$= \frac{1}{2}\int e^{-t}dt$

$= \frac{1}{2}\left(\frac{e^{-t}}{-1}\right) + C$

$= -\frac{1}{2}e^{-x^2} + C$

$= \frac{-1}{2e^{x^2}} + C$

#421131

Topic: Integration by Substitution

Integrate the function $\frac{e^{tan^{-1}x}}{1+x^2}$

Solution

Put $\tan^{-1}x = t$

$\therefore \frac{1}{1+x^2}dx = dt$

$\Rightarrow \int \frac{e^{\tan^{-1}x}}{1+x^2}dx = \int e^t dt$

$= e^t + C = e^{\tan^{-1}x} + C$

#421132

Topic: Integration by Substitution

Integrate the function $\frac{e^{2x}-1}{e^{2x}+1}$

Solution

$\dfrac{e^{2x}-1}{e^{2x}+1}$

Dividing numerator and denominator by e^{x}, we obtain

$$\frac{\frac{(e^{2x}-1)}{e^{x}}}{\frac{(e^{2x}+1)}{e^{x}}}=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

Put $e^{x}+e^{-x}=t$

$\therefore (e^{x}-e^{-x})dx=dt$

$\Rightarrow \int \dfrac{e^{2x}-1}{e^{2x}+1}dx=\int \dfrac{e^{x}-e^{-x}}{e^{x}+e^{-x}}dx$

$=\int \dfrac{dt}{t}$

$=\log|t|+C=\log|e^{x}+e^{-x}|+C$

#421133

Topic: Integration by Substitution

Integrate the function $\dfrac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

Solution

Let $e^{2x}+e^{-2x}=t$

$\therefore (2e^{2x}-2e^{-2x})dx=dt$

$\Rightarrow 2(e^{2x}-e^{-2x})dx=dt$

$\Rightarrow \int\left(\dfrac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\right)dx=\int \dfrac{dt}{2t}$

$=\dfrac{1}{2}\int \dfrac{1}{t}dt$

$=\dfrac{1}{2}\log|t|+C$

$=\dfrac{1}{2}\log|e^{2x}+e^{-2x}|+C$

#421137

Topic: Integration by Substitution

Integrate the function $\dfrac{\sin^{-1}x}{\sqrt{1-x^{2}}}$

Solution

Let $\sin^{-1}x=t$

$\therefore \dfrac{1}{\sqrt{1-x^{2}}}dx=dt$

$\Rightarrow \int \dfrac{\sin^{-1}x}{\sqrt{1-x^{2}}}dx=\int tdt$

$=\dfrac{t^{2}}{2}+C$

$=\dfrac{(\sin^{-1}x)^{2}}{2}+C$

#421138

Topic: Fundamental Integrals

Integrate the function $\dfrac{2\cos x-3\sin x}{6\cos x+4\sin x}$

Solution

$\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} = \frac{2\cos x - 3\sin x}{2(3\cos x + 2\sin x)}$

Let $3\cos x + 2\sin x = t$

$\therefore (-3\sin x + 2\cos x)dx = dt$

$\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} dx = \int \frac{dt}{2t}$

$= \frac{1}{2}\int \frac{1}{t} dt$

$= \frac{1}{2}\log|t| + C$

$= \frac{1}{2}\log|2\sin x + 3\cos x| + C$

#421139

Topic: Integration by Substitution

Integrate the function $\frac{1}{\cos^2 x(1 - \tan x)^2}$

Solution

$\frac{1}{\cos^2 x(1 - \tan x)^2} = \frac{\sec^2 x}{(1 - \tan x)^2}$

Let $(1 - \tan x) = t$

$\therefore -\sec^2 x dx = dt$

$\Rightarrow \int \frac{\sec^2 x}{(1 - \tan x)^2} dx = \int \frac{-dt}{t^2}$

$= -\int t^{-2} dt$

$= \frac{1}{t} + C$

$= \frac{1}{(1 - \tan x)} + C$

#421140

Topic: Integration by Substitution

Integrate the function $\frac{\cos\sqrt{x}}{\sqrt{x}}$

Solution

Let $\sqrt{x} = t$

$\therefore \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \int \frac{\cos\sqrt{x}}{\sqrt{x}} dx = 2\int \cos t dt$

$= 2\sin t + C$

$= 2\sin\sqrt{x} + C$

#421141

Topic: Integration by Substitution

Integrate the function $\sqrt{\sin 2x}\cos 2x$

Solution

Let $\sin 2x = t$

$\therefore 2\cos 2x dx = dt$

$\Rightarrow \int \sqrt{\sin 2x} \cos 2x dx = \frac{1}{2} \int \sqrt{t} dt$

$= \frac{1}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$

$= \frac{1}{3} t^{\frac{3}{2}} + C$

$= \frac{1}{3}(\sin 2x)^{\frac{3}{2}} + C$

#421142

Topic: Integration by Substitution

Integrate the function $\frac{\cos x}{\sqrt{1 + \sin x}}$

Solution

Put $1 + \sin x = t$

$\therefore \cos x dx = dt$

$\Rightarrow \int \frac{\cos x}{\sqrt{1 + \sin x}} dx = \int \frac{dt}{\sqrt{t}}$

$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= 2\sqrt{t} + C = 2\sqrt{1 + \sin x} + C$

#421143

Topic: Integration by Substitution

Integrate the function $\cot x \log \sin x$

Solution

Put $\log \sin x = t$

$\Rightarrow \frac{1}{\sin x} \cdot \cos x dx = dt$

$\therefore \cot x dx = dt$

$\Rightarrow \int \cos x \log \sin x dx = \int t dt$

$= \frac{t^2}{2} + C$

$= \frac{1}{2}(\log \sin x)^2 + C$

#421144

Topic: Integration by Substitution

Integrate the function $\frac{\sin x}{1 + \cos x}$

Solution

Let $1 + \cos x = t$

$\therefore -\sin x dx = dt$

$\Rightarrow \int \frac{\sin x}{1 + \cos x} dx = \int -\frac{dt}{t}$

$= -\log|t| + C$

$= -\log|1 + \cos x| + C$

#421145

Topic: Integration by Substitution

Integrate the function $\frac{\sin x}{(1+\cos x)^2}$

Solution

Let $1+\cos x = t$

$\therefore\ -\sin x dx = dt$

$\Rightarrow \int \frac{\sin x}{(1+\cos x)^2} dx = \int -\frac{dt}{t^2}$

$= -\int t^{-2} dt$

$= \frac{1}{t} + C$

$= \frac{1}{1+\cos x} + C$

#421146

Topic: Integration by Substitution

Integrate the function $\frac{1}{1+\cot x}$

Solution

Let $I = \int \frac{1}{1+\cot x} dx$

$= \int \frac{\sin x}{\sin x + \cos x} dx$

$= \frac{1}{2}\int \frac{2\sin x}{\sin x + \cos x} dx$

$= \frac{1}{2}\int \frac{(\sin x + \cos x) + (\sin x - \cos x)}{(\sin x + \cos x)} dx$

$= \frac{1}{2}\int 1 dx + \frac{1}{2}\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$

$= \frac{1}{2}(x) + \frac{1}{2}\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

Put $\sin x + \cos x = t \Rightarrow (\cos x - \sin x)dx = dt$

$\therefore\ I = \frac{x}{2} + \frac{1}{2}\int \frac{-(dt)}{t}$

$= \frac{x}{2} - \frac{1}{2}\log|t| + C$

$= \frac{x}{2} - \frac{1}{2}\log|\sin x + \cos x| + C$

#421148

Topic: Integration by Substitution

Integrate the function $\frac{1}{1-\tan x}$

Solution

Let $I = \int \frac{1}{1-\tan x} dx$

$= \int \frac{\cos x}{\cos x - \sin x} dx$

$= \frac{1}{2} \in \frac{2\cos x}{\cos x - \sin x} dx$

$= \frac{1}{2}\int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{(\cos x - \sin x)} dx$

$= \frac{1}{2}\int 1 dx + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x} dx$

$= \frac{x}{2} + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x} dx$

Put $\cos x - \sin x = t \Rightarrow (-\sin x - \cos x)dx = dt$

$\therefore\ I = \frac{x}{2} + \frac{1}{2}\int \frac{-(dt)}{t}$

$= \frac{x}{2} - \frac{1}{2}\log|t| + C$

$= \frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + C$

#421149

Topic: Integration by Substitution

Integrate the function $\frac{\sqrt{\tan x}}{\sin x \cos x}$

Solution

Let $I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$

$= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x} dx$

$= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x} dx$

$= \int \frac{\sec^2 x dx}{\sqrt{\tan x}}$

Put $\tan x = t \Rightarrow \sec^2 x dx = dt$

$\therefore I = \int \frac{dt}{\sqrt{t}}$

$= 2\sqrt{t} + C$

$= 2\sqrt{\tan x} + C$

#421150

Topic: Integration by Substitution

Integrate the function $\frac{(1 + \log x)^2}{x}$

Solution

Put $1 + \log x = t$

$\therefore \frac{1}{x} dx = dt$

$\Rightarrow \int \frac{(1 + \log x)^2}{x} dx = \int t^2 dt$

$= \frac{t^3}{3} + C$

$= \frac{(1 + \log x)^3}{3} + C$

#421151

Topic: Integration by Substitution

Integrate the function $\frac{(x + 1)(x + \log x)^2}{x}$

Solution

$\frac{(x + 1)(x + \log x)^2}{x} = \left(\frac{x + 1}{x}\right)(x + \log x)^2 = \left(1 + \frac{1}{x}\right)(x + \log x)^2$

Put $(x + \log x) = t$

$\therefore \left(1 + \frac{1}{x}\right) dx = dt$

$\Rightarrow \int \left(1 + \frac{1}{x}\right)(x + \log x)^2 dx = \int t^2 dt$

$= \frac{t^3}{3} + C$

$= \frac{1}{3}(x + \log x)^3 + C$

#421152

Topic: Integration by Substitution

Integrate the function $\dfrac{x^3\sin(\tan^{-1}x^4)}{1+x^8}$

Solution

Let $x^4 = t$

$\therefore 4x^3dx = dt$

$\Rightarrow \int \dfrac{x^3\sin(\tan^{-1}x^4)}{1+x^8}dx = \dfrac{1}{4}\int \dfrac{sin(tan^{-1}t)}{1+t^2}dt$ ......(1)

Let $\tan^{-1}t = u$

$\therefore \dfrac{1}{1+t^2}dt = du$

From (1), we obtain

$\int \dfrac{x^3\sin(\tan^{-1}x^4)dx}{1+x^8} = \dfrac{1}{4}\int \sin u du$

$= \dfrac{1}{4}(-\cos u) + C$

$= \dfrac{-1}{4}\cos(\tan^{-1}t) + C$

$= \dfrac{-1}{4}\cos(\tan^{-1}x^4) + C$

#421154

Topic: Integration by Substitution

$\int \dfrac{10x^9 + 10^x\log_{e^{10}}dx}{x^{10}+10^x}$ equals

A $10^x - x^{10} + C$

B $10^x + x^{10} + C$

C $(10^x - x^{10})^{-1} + C$

$\boxed{\text{D}}$ $\log(10^x + x^{10}) + C$

Solution

Let $x^{10} + 10^x = t$

$\therefore (10x^9 + 10^x\log_e 10)dx = dt$

$\Rightarrow \int \dfrac{10x^9 + 10^x\log_e 10}{x^{10}+10^x}dx = \int \dfrac{dt}{t}$

$= \log t + C$

$= \log(10^x + x^{10}) + C$

#421155

Topic: Fundamental Integrals

$\int \dfrac{dx}{\sin^2 x\cos^2 x}$ equals

A $\tan x + \cot x + C$

$\boxed{\text{B}}$ $\tan x - \cot x + C$

C $\tan x\cot x + C$

D $\tan x - \cot 2x + C$

Solution

Let $I = \int \frac{dx}{\sin^2 x \cos^2 x}$

$= \int \frac{1}{\sin^2 x \cos^2 x} dx$

$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$

$= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx$

$= \int \sec^2 x dx + \int co\sec^2 x dx$

$= \tan x - \cot x + C$

#421899

Topic: Fundamental Integrals

Find the integrals of the functions $\sin 3x \cos 4x$

Solution

It is known that, $\sin A \cos B = \frac{1}{2}\{\sin(A+B) + \sin(A-B)\}$

$\therefore \int \sin 3x \cos 4x dx = \frac{1}{2}\int \{\sin(3x+4x) + \sin(3x-4x)\} dx$

$= \frac{1}{2}\int \{\sin 7x + \sin(-x)\} dx$

$= \frac{1}{2}\int \{\sin 7x - \sin x\} dx$

$= \frac{1}{2}\int \sin 7x dx - \frac{1}{2}\int \sin x dx$

$= \frac{1}{2}\left(\frac{-\cos 7x}{7}\right) - \frac{1}{2}(-\cos x) + C$

$= \frac{-\cos 7x}{14} + \frac{\cos x}{2} + C$

#421900

Topic: Fundamental Integrals

Find the integrals of the functions $\cos 2x \cos 4x \cos 6x$

Solution

It is known that, $\cos A \cos B = \frac{1}{2}\{\cos(A+B) + \cos(A-B)\}$

$\therefore \int \cos 2x(\cos 4x \cos 6x) dx = \int \cos 2x\left[\frac{1}{2}\{\cos(4x+6x) + \cos(4x-6x)\}\right] dx$

$= \frac{1}{2}\int \{\cos 2x \cos 10x + \cos 2x \cos(-2x)\} dx$

$= \frac{1}{2}\int \left\{\cos 2x \cos 10x + \cos^2 2x\right\} dx$

$= \frac{1}{2}\int \left[\left\{\frac{1}{2}\cos(2x+10x) + \cos(2x-10x)\right\} + \left(\frac{1+\cos 4x}{2}\right)\right] dx$

$= \frac{1}{4}\int (\cos 12x + \cos 8x + 1 + \cos 4x) dx$

$= \frac{1}{4}\left[\frac{\sin 12x}{12} + \frac{\sin 8x}{8} + x + \frac{\sin 4x}{4}\right] + C$

#421903

Topic: Integration by Substitution

Find the integrals of the functions $\sin^3(2x+1)$

Solution

Let $I = \int \sin^3(2x+1)$

$\Rightarrow \int \sin^3(2x+1)dx = \int \sin^2(2x+1)\cdot\sin(2x+1)dx$

$= \int (1-\cos^2(2x+1))\sin(2x+1)dx$

Put $\cos(2x+1) = t$

$\Rightarrow -2\sin(2x+1)dx = dt$

$\Rightarrow \sin(2x+1)dx = \dfrac{-dt}{2}$

$\Rightarrow I = \dfrac{-1}{2}\int(1-t^2)dt$

$= \dfrac{-1}{2}\left\{t - \dfrac{t^3}{3}\right\}$

$= \dfrac{-1}{2}\left\{\cos(2x+1) - \dfrac{\cos^3(2x+1)}{3}\right\}$

$= \dfrac{-\cos(2x+1)}{2} + \dfrac{\cos^3(2x+1)}{6} + C$

#421906

Topic: Integration by Substitution

Find the integrals of the functions $\sin^3x\cos^3x$

Solution

Let $I = \int \sin^3x\cos^3x\cdot dx$

$= \int \cos^3x\cdot\sin^2x\cdot\sin x\cdot dx$

$= \int \cos^3x(1-\cos^2x)\sin x\cdot dx$

Put $\cos x = t$

$\Rightarrow -\sin x dx = dt$

$\Rightarrow I = -\int t^3(1-t^2)dt$

$= -\int(t^3 - t^5)dt$

$= -\left\{\dfrac{t^4}{4} - \dfrac{t^6}{6}\right\} + C$

$= -\left\{\dfrac{\cos^4x}{4} - \dfrac{\cos^6x}{6}\right\} + C$

$= \dfrac{\cos^6x}{6} - \dfrac{\cos^4x}{4} + C$

#421915

Topic: Fundamental Integrals

Find the integrals of the functions $\sin x\sin 2x\sin 3x$

Solution

It is known that, $\sin A \sin B = \frac{1}{2}\{\cos(A - B) - \cos(A + B)\}$

$\therefore \int \sin x \sin 2x \sin 3x dx = \int [\sin x \cdot \frac{1}{2}\{\cos(2x - 3x) - \cos(2x + 3x)\}]dx$

$= \frac{1}{2}\int (\sin x \cos(-x) - \sin x \cos 5x)dx$

$= \frac{1}{2}\int (\sin x \cos x - \sin x \cos 5x)dx$

$= \frac{1}{2}\int \frac{\sin 2x}{2} dx - \frac{1}{2}\int \sin x \cos 5x dx$

$= \frac{1}{4}\left[\frac{-\cos 2x}{2}\right] - \frac{1}{2}\int\left\{\frac{1}{2}\sin(x + 5x) + \sin(x - 5x)\right\}dx$

$= \frac{-\cos 2x}{8} - \frac{1}{4}\int (\sin 6x + \sin(-4x))dx$

$= \frac{-\cos 2x}{8} - \frac{1}{4}\left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{4}\right] + C$

$= \frac{-\cos 2x}{8} - \frac{1}{8}\left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{2}\right] + C$

$= \frac{1}{8}\left[\frac{\cos 6x}{3} - \frac{\cos 4x}{2} - \cos 2x\right] + C$

#421918

Topic: Fundamental Integrals

Find the integrals of the functions $\sin 4x \sin 8x$

Solution

It is known that, $\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$

$\therefore \int \sin 4x \sin 8x dx = \int\left\{\frac{1}{2}[\cos(4x - 8x) - \cos(4x + 8x)]\right\}dx$

$= \frac{1}{2}\int (\cos(-4x) - \cos 12x)dx$

$= \frac{1}{2}\int (\cos 4x - \cos 12x)dx$

$= \frac{1}{2}\left[\frac{\sin 4x}{4} - \frac{\sin 12x}{12}\right] + C$

#421924

Topic: Fundamental Integrals

Find the integrals of the functions $\frac{1 - \cos x}{1 + \cos x}$

Solution

$\frac{1 - \cos x}{1 + \cos x} = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$

$= \tan^2\frac{x}{2} = \left(\sec^2\frac{x}{2} - 1\right)$

$\therefore \int \frac{1 - \cos x}{1 + \cos x} dx = \int\left(\sec^2\frac{x}{2} - 1\right)dx$

$= \left[\frac{\tan\frac{x}{2}}{\frac{1}{2}} - x\right] + C$

$= 2\tan\frac{x}{2} - x + C$

#421930

Topic: Fundamental Integrals

Find the integrals of the functions $\frac{\cos x}{1+\cos x}$

Solution

$$\frac{\cos x}{1+\cos x}=\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$=\frac{1}{2}\left[1-\tan^2\frac{x}{2}\right]$$

$$\therefore\int\frac{\cos x}{1+\cos x}dx=\frac{1}{2}\int\left(1-\tan^2\frac{x}{2}\right)dx$$

$$=\frac{1}{2}\int\left(1-\sec^2\frac{x}{2}+1\right)dx$$

$$=\frac{1}{2}\int\left(2-\sec^2\frac{x}{2}\right)dx$$

$$=\frac{1}{2}\left[2x-\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]+C$$

$$=x-\tan\frac{x}{2}+C$$

#421941

Topic: Fundamental Integrals

Find the integrals of the functions $\sin^4 x$

Solution

$$\sin^4 x=\sin^2 x\sin^2 x$$

$$=\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)$$

$$=\frac{1}{4}(1-\cos 2x)^2$$

$$=\frac{1}{4}[1+\cos^2 2x-2\cos 2x]$$

$$=\frac{1}{4}\left[1+\left(\frac{1+\cos 4x}{2}\right)-2\cos 2x\right]$$

$$=\frac{1}{4}\left[1+\frac{1}{2}+\frac{1}{2}\cos 4x-2\cos 2x\right]$$

$$=\frac{1}{4}\left[\frac{3}{2}+\frac{1}{2}\cos 4x-2\cos 2x\right]$$

$$\therefore\int\sin^4 x\,dx=\frac{1}{4}\int\left[\frac{3}{2}+\frac{1}{2}\cos 4x-2\cos 2x\right]dx$$

$$=\frac{1}{4}\left[\frac{3}{2}x+\frac{1}{2}\left(\frac{\sin 4x}{4}\right)-\frac{2\sin 2x}{2}\right]+C$$

$$=\frac{1}{8}\left[3x+\frac{\sin 4x}{4}-2\sin 2x\right]+C$$

$$=\frac{3x}{8}-\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+C$$

#421949

Topic: Fundamental Integrals

Find the integrals of the functions $\cos^4 2x$

Solution

$$\cos^4 2x = (\cos^2 2x)^2$$

$$= \left(\frac{1+\cos 4x}{2}\right)^2$$

$$= \frac{1}{4}[1+\cos^2 4x + 2\cos 4x]$$

$$= \frac{1}{4}\left[1+\left(\frac{1+\cos 8x}{2}\right)+2\cos 4x\right]$$

$$= \frac{1}{4}\left[1+\frac{1}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]$$

$$\int cos^4 2x = \frac{1}{4}\int\left[\frac{3}{2}+\frac{\cos 8x}{8}+\frac{\cos 4x}{2}\right)dx$$

$$= \frac{3}{8}x+\frac{\sin 8x}{64}+\frac{\sin 4x}{8}+C$$

#421961

Topic: Fundamental Integrals

Find the integrals of the function $\dfrac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}$

Solution

$$\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha} = \frac{-2\sin\frac{2x+2\alpha}{2}\sin\frac{2x-2\alpha}{2}}{-2\sin\frac{x+\alpha}{2}\sin\frac{x-\alpha}{2}}, \left[\because \cos C-\cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$= \frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \frac{\left[2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\right]\left[2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)\right]}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= 4\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$= 2\left[\cos\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right)+\cos\left(\frac{x-\alpha}{2}-\frac{x-\alpha}{2}\right)\right]$$

$$= 2[\cos(x)+\cos\alpha]$$

$$= 2\cos x+2\cos\alpha$$

$$\therefore \int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx = \int 2\cos x+2\cos\alpha$$

$$= 2[\sin x+x\cos\alpha]+C$$

#421964

Topic: Integration by Substitution

Find the integrals of the function $\frac{\cos x-\sin x}{1+\sin 2x}$

Solution

$\frac{\cos x-\sin x}{1+\sin 2x}=\frac{\cos x-\sin x}{(\sin^2 x+\cos^2 x)+2\sin x\cos x}$

$=\frac{\cos x-\sin x}{(\sin x+\cos x)^2}$

Put $\sin x+\cos x=t$

$\therefore(\cos x-\sin x)dx=dt$

$\Rightarrow\int\frac{\cos x-\sin x}{1+\sin 2x}dx=\int\frac{\cos x-\sin x}{(\sin x+\cos x)^2}dx$

$=\int\frac{dt}{t^2}=\int t^{-2}dt$

$=-t^{-1}+C=-\frac{1}{t}+C=\frac{-1}{\sin x+\cos x}+C$

#421966

Topic: Integration by Substitution

Find the integrals of the function $\tan^3 2x\sec 2x$

Solution

$\tan^3 2x\sec 2x=\tan^2 2x\tan 2x\sec 2x$

$=(\sec^2 2x-1)\tan 2x\sec 2x$

$=\sec^2 2x\cdot\tan 2x\sec 2x-\tan 2x\sec 2x$

$\therefore\int\tan^3 2x\sec 2xdx=\int\sec^2 2x\tan 2x\sec 2xdx-\int\tan 2x\sec 2xdx$

$=\int\sec^2 2x\tan 2x\sec 2xdx-\frac{\sec 2x}{2}+C$

Let $\sec 2x=t$

$\therefore 2\sec 2x\tan 2xdx=dt$

$\therefore\int\tan^3 2x\sec 2xdx=\frac{1}{2}\int t^2dt-\frac{\sec 2x}{2}+C$

$=\frac{t^3}{6}-\frac{\sec 2x}{2}+C$

$=\frac{(\sec 2x)^3}{6}-\frac{\sec 2x}{2}+C$

#421973

Topic: Integration by Substitution

Find the integrals of the functions $\tan^4 x$

Solution

$\tan^4 x=\tan^2 x\cdot\tan^2 x$

$=(\sec^2 x-1)\tan^2 x=\sec^2 x\tan^2 x-\tan^2 x$

$=\sec^2 x\tan^2 x-(\sec^2 x-1)$

$=\sec^2 x\tan^2 x-\sec^2 x+1$

$\therefore\int\tan^4 xdx=\int\sec^2 x\tan^2 xdx-\int\sec^2 xdx+\int 1\cdot dx$

$=\int\sec^2 x\tan^2 xdx-\tan x+x+C$ ....(i)

Consider $\int\sec^2 x\tan^2 xdx$

Let $\tan x=t\Rightarrow\sec^2 xdx=dt$

$\Rightarrow\int\sec^2 x\tan^2 xdx=\int t^2dt=\frac{t^3}{3}=\frac{\tan^3 x}{3}$

From equation (1), we obtain

$\int\tan^4 xdx=\frac{1}{3}\tan^3 x-\tan x+x+C$

#421975

Topic: Fundamental Integrals

Find the integrals of the functions $\dfrac{\sin^3x+\cos^3x}{\sin^2x\cos^2x}$

Solution

$$\frac{\sin^3x+\cos^3x}{\sin^2x\cos^2x}=\frac{\sin^3x}{\sin^2\cos^2x}+\frac{\cos^3x}{\sin^2x\cos^2x}$$

$$=\frac{\sin x}{\cos^2x}+\frac{\cos x}{\sin^2x}$$

$$=\tan x\sec x+\cot x\,co\sec x$$

$$\therefore\int\frac{\sin^3x+\cos^3x}{\sin^2x\cos^2x}dx=\int(\tan x\sec x+\cot x\,co\sec x)dx$$

$$=\sec x-co\sec x+C$$

#421979

Topic: Fundamental Integrals

Find the integral of the function $\dfrac{\cos2x+2\sin^2x}{\cos^2x}$

Solution

$$\frac{\cos2x+2\sin^2x}{\cos^2x}$$

$$=\frac{\cos2x+(1-\cos2x)}{\cos^2x}$$

$$=\frac{1}{\cos^2x}=\sec^2x$$

$$\therefore\int\frac{\cos2x+2\sin^2x}{\cos^2x}dx=\int\sec^2xdx=\tan x+C$$

#421982

Topic: Integration by Substitution

Find the integral of the function $\dfrac{1}{\sin x\cos^3x}$

Solution

$$\frac{1}{\sin x\cos^3x}=\frac{\sin^2x+\cos^2x}{\sin x\cos^3x}$$

$$=\frac{\sin x}{\cos^3x}+\frac{1}{\sin x\cos x}$$

$$=\tan x\sec^2x+\frac{\frac{1}{\cos^2x}}{\frac{\sin x\cos x}{\cos^2x}}$$

$$=\tan x\sec^2x+\frac{\sec^2x}{\tan x}$$

$$\therefore\int\frac{1}{\sin x\cos^3x}dx=\int\tan x\sec^2xdx+\int\frac{\sec^2x}{\tan x}dx$$

Let $\tan x=t\Rightarrow\sec^2xdx=dt$

$$\Rightarrow\int\frac{1}{\sin x\cos^3x}dx=\int tdt+\int\frac{1}{t}dt$$

$$=\frac{t^2}{2}+\log|t|+C=\frac{1}{2}\tan^2x+\log|\tan x|+C$$

#421991

Topic: Integration by Substitution

Find the integral of the function $\dfrac{\cos2x}{(\cos x+\sin x)^2}$

Solution

$$\frac{\cos 2x}{(\cos x+\sin x)^2}=\frac{\cos 2x}{\cos^2 x+\sin^2 x+2\sin x\cos x}=\frac{\cos 2x}{1+\sin 2x}$$

$$\therefore \int \frac{\cos 2x}{(\cos x+\sin x)^2}dx=\int \frac{\cos 2x}{(1+\sin 2x)}dx$$

Let $1+\sin 2x=t \Rightarrow 2\cos 2x dx=dt$

$$\therefore \int \frac{\cos 2x}{(\cos x+\sin x)^2}dx=\frac{1}{2}\int \frac{1}{t}dt$$

$$=\frac{1}{2}\log|t|+C$$

$$=\frac{1}{2}\log|1+\sin 2x|+C$$

$$=\frac{1}{2}\log|(\sin x+\cos x)^2|+C$$

$$=\log|\sin x+\cos x|+C$$

#422018

Topic: Integration by Substitution

Find the integral of the function $\sin^{-1}(\cos x)$

Solution

Let $\cos x = t$

Then, $\sin x = \sqrt{1 - t^2}$

$\Rightarrow (-\sin x)dx = dt$

$\Rightarrow dx = \frac{-dt}{\sin x}$

$\Rightarrow dx = \frac{-dt}{\sqrt{1 - t^2}}$

$\therefore \int \sin^{-1}(\cos x)dx = \int \sin^{-1} t\left(\frac{-dt}{\sqrt{1 - t^2}}\right)$

$= -\int \frac{\sin^{-1} t}{\sqrt{1 - t^2}} dt$

Let $\sin^{-1} t = u$

$\Rightarrow \frac{1}{\sqrt{1 - t^2}} dt = du$

$\therefore \int \sin^{-1}(\cos x)dx = \int 4du$

$= -\frac{u^2}{2} + C$

$= \frac{-(\sin^1 t)^2}{2} + C$

$= \frac{-[\sin^{-1}(\cos x)]^2}{2} + C$ ....(1)

It is known that,

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

$\therefore \sin^{-1}(\cos x) = \frac{\pi}{2} - \cos^{-1}\cos x = \left(\frac{\pi}{2} - x\right)$

Substituting in equation (1), we obtain

$\int \sin^{-1}(\cos x)dx = \frac{-\left[\frac{\pi}{2} - x\right]^2}{2} + C$

$= -\frac{1}{2}\left(\frac{\pi^2}{2} + x^2 - \pi x\right) + C$

$= -\frac{\pi^2}{8} - \frac{x^2}{2} + \frac{1}{2}\pi x + C$

$= \frac{\pi x}{2} - \frac{x^2}{2} + \left(C - \frac{\pi^2}{8}\right)$

$= \frac{\pi x}{2} - \frac{x^2}{2} + C_1$

#422390

Topic: Fundamental Integrals

Find the integral of the function $\frac{1}{\cos(x - a)\cos(x - b)}$

Solution

$$\frac{1}{\cos(x-a)\cos(x-b)} = \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\frac{[\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)]}{\cos(x-a)\cos(x-b)}$$

$$= \frac{1}{\sin(a-b)}[\tan(x-b)-\tan(x-a)]$$

$$\Rightarrow \int \frac{1}{\cos(x-a)\cos(x-b)}dx = \frac{1}{\sin(a-b)}\int[\tan(x-b)-\tan(x-a)]dx$$

$$= \frac{1}{\sin(a-b)}[-\log|\cos(x-b)|+\log|\cos(x-a)|]$$

$$= \frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right]+C$$

#422392

Topic: Fundamental Integrals

$\int \frac{\sin^2x-\cos^2x}{\sin^2x\cos^2x}dx$ is equal to

[✓ A] $\tan x + \cot x + C$

B $\tan x + cosec x + C$

C $-\tan x + \cot x + C$

D $\tan x + \sec x + C$

Solution

$$\int \frac{\sin^2x-\cos^2x}{\sin^2x\cos^2x}dx = \int\left(\frac{\sin^2x}{\sin^2x\cos^2x}-\frac{\cos^2x}{\sin^2x\cos^2x}\right)dx$$

$$= \int(\sec^2x - cosec^2x)dx$$

$$= \tan x + \cot x + C$$

#422396

Topic: Integration by Substitution

$\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$ equals

A $-cot(ex^x)+C$

[✓ B] $tan(xe^x)+C$

C $tan(e^x)+C$

D $cot(e^x)+C$

Solution

$$\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$$

Let $e^x x = t \Rightarrow (e^x \cdot x + e^x \cdot 1)dx = dt$

$\Rightarrow e^x(x+1)dx = dt$

$$\therefore \int \frac{e^x(1+x)}{\cos^2(e^x x)}dx = \int \frac{dt}{\cos^2t}$$

$$= \int \sec^2t dt = \tan t + C = \tan(e^x \cdot x) + C$$

Hence, the correct Answer is B.

#422401

Topic: Integration by Substitution

Integrate the function $\dfrac{3x^2}{x^6+1}$

Solution

Let $x^3 = t$

$\therefore 3x^2 dx = dt$

$\Rightarrow \int \dfrac{3x^2}{x^6+1} dx = \int \dfrac{dt}{t^2+1}$

$= \tan^{-1} t + C$

$= \tan^{-1}(x^3) + C$

#423065

Topic: Integration using Partial Fractions

Integrate the rational function $\dfrac{3x+5}{x^3-x^2-x+1}$

Solution

$\dfrac{3x+5}{x^3-x^2-x+1} = \dfrac{3x+5}{(x-1)^2(x+1)}$

Let $\dfrac{3x+5}{(x-1)^2(x+1)} = \dfrac{A}{(x-1)} + \dfrac{B}{(x-1)^2} + \dfrac{C}{(x+1)}$

$\Rightarrow 3x+5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$

$\Rightarrow 3x+5 = A(x^2-1) + B(x+1) + C(x^2+1-2x)$ .... (1)

Substituting $x = 1$ in equation (1), we obtain

$B = 4$

Equating the coefficients of x^2 and x, we obtain

$A + C = 0$

$B - 2C = 3$

On solving, we obtain

$A = -\dfrac{1}{2}$ and $C = \dfrac{1}{2}$

$\therefore \dfrac{3x+5}{(x-1)^2(x+1)} = \dfrac{-1}{2(x-1)} + \dfrac{4}{(x-1)^2} + \dfrac{1}{2(x+1)}$

$\Rightarrow \int \dfrac{3x+5}{(x-1)^2(x+1)} dx = -\dfrac{1}{2}\int \dfrac{1}{x-1} dx + 4\int \dfrac{1}{(x-1)^2} dx + \dfrac{1}{2}\int \dfrac{1}{(x+1)} dx$

$= -\dfrac{1}{2}\log|x-1| + 4\left(\dfrac{-1}{x-1}\right) + \dfrac{1}{2}\log|x+1| + C$

$= \dfrac{1}{2}\log\left|\dfrac{x+1}{x-1}\right| - \dfrac{4}{(x-1)} + C$

#423847

Topic: Integration by Substitution

$\int x^2 e^{x^3} dx$ equals

- **A** (boxed) $\dfrac{1}{3}e^{x^3} + C$
- **B** $\dfrac{1}{3}e^{x^2} + C$
- **C** $\dfrac{1}{2}e^{x^3} + C$
- **D** $e^{-x^2} + C$

Solution

Let $I = \int x^2 e^{x^3} dx$

Put $x^3 = t \Rightarrow 3x^2 dx = dt$

$\Rightarrow I = \frac{1}{3}\int e^t dt$

$= \frac{1}{3}(e^t) + C$

$= \frac{1}{3}e^{x^3} + C$

#424638

Topic: Definite Integrals

Evaluate the definite integral $\int_{-1}^{1}(x+1)dx$

Solution

Let $I = \int_{-1}^{1}(x+1)dx$

$\Rightarrow \int(x+1)dx = \frac{x^2}{2} + x = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F(1) - F(-1)$

$= \left(\frac{1}{2}+1\right) - \left(\frac{1}{2}-1\right) = \frac{1}{2} + 1 - \frac{1}{2} + 1 = 2$

#424639

Topic: Definite Integrals

Evaluate the definite integral $\int_{2}^{3}\frac{1}{x}dx$

Solution

Let $I = \int_{2}^{3}\frac{1}{x}dx$

$\Rightarrow \int\frac{1}{x}dx = \log|x| = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F(3) - F(2)$

$= \log|3| - \log|2| = \log\frac{3}{2}$

#424640

Topic: Definite Integrals

Evaluate the definite integral $\int_{1}^{2}(4x^3 - 5x^2 + 6x + 9)dx$

Solution

$\int_{1}^{2}(4x^3 - 5x^2 + 6x + 9)dx$

$= 4\int_{1}^{2}x^3dx - 5\int_{1}^{2}x^2dx + 6\int_{1}^{2}xdx + 9\int_{1}^{2}1dx$

$= 4\left[\frac{x^4}{4}\right]_1^2 - 5\left[\frac{x^3}{3}\right]_1^2 + 6\left[\frac{x^2}{2}\right]_1^2 + 9[x]_1^2$

$= \left[x^4\right]_1^2 - 5\left[\frac{x^3}{3}\right]_1^2 + 3\left[x^2\right]_1^2 + 9[x]_1^2$

$= (16-1) - \frac{5}{3}(8-1) + 3(4-1) + 9(2-1)$

$= 15 - \frac{35}{3} + 9 + 9 = 33 - \frac{35}{3} = \frac{64}{3}$

#425559

Topic: Integration by Substitution

Integrate the function $\frac{1}{x^2(x^4+1)^{\frac{3}{4}}}$

Solution

$$\frac{1}{x^2(x^4+1)^{\frac{3}{4}}}$$

Multiplying and dividing by x^{-3}, we obtain

$$\frac{x^{-3}}{x^2\cdot x^{-3}(x^4+1)^{\frac{3}{4}}} = \frac{x^{-3}(x^4+1)^{\frac{-3}{4}}}{x^2\cdot x^{-3}}$$

$$= \frac{(x^4+1)^{\frac{-3}{4}}}{x^5\cdot(x^4)^{-\frac{3}{4}}}$$

$$= \frac{1}{x^5}\left(\frac{x^4+1}{x^4}\right)^{-\frac{3}{4}}$$

$$= \frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}$$

Let $\frac{1}{x^4} = t \Rightarrow -\frac{4}{x^5}dx = dt \Rightarrow \frac{1}{x^5}dx = -\frac{dt}{4}$

$$\therefore \int\frac{1}{x^2(x^4+1)^{\frac{3}{4}}}dx = \int\frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}dx$$

$$= -\frac{1}{4}\int(1+t)^{-\frac{3}{4}}dt$$

$$= -\frac{1}{4}\left[\frac{(1+t)^{\frac{1}{4}}}{\frac{1}{4}}\right]+C$$

$$= -\frac{1}{4}\frac{\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}}{\frac{1}{4}}+C$$

$$= -\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+C$$

#425574

Topic: Fundamental Integrals

Integrate the function $\frac{\sin x}{\sin(x-a)}$

Solution

Let $x-a=t \Rightarrow dx=dt$

$$\Rightarrow \int\frac{\sin x}{\sin(x-a)}dx = \int\frac{\sin(t-a)}{\sin t}dt$$

$$= \int\frac{\sin t\cos a+\cos t\sin a}{\sin t}dt$$

$$= \int(\cos a+\cot t\sin a)dt$$

$$= t\cos a+\sin a\log|\sin t|+C_1$$

$$= (x-a)\cos a+\sin a\log|\sin(x-a)|+C_1$$

$$= x\cos a+\sin a\log|\sin(x-a)|-a\cos a+C_1$$

$$= \sin a\log|\sin(x-a)|+x\cos a+C$$

#425579

Topic: Fundamental Integrals

Integrate the function $\frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x\cos^2 x}$

Solution

$$\frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x\cos^2 x} = \frac{(\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x)}{\sin^2 x + \cos^2 x - \sin^2 x\cos^2 x - \sin^2 x\cos^2 x}$$

$$= \frac{(\sin^4 x + \cos^4 x)(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)}{(\sin^2 x - \sin^2 x\cos^2 x) + (\cos^2 x - \sin^2 x\cos^2 x)}$$

$$= \frac{(\sin^4 x + \cos^4 x)(\sin^4 x - \cos^2 x)}{\sin^2 x(1 - \cos^2 x) + \cos^2 x(1 - \sin^2 x)}$$

$\displaystyle =\frac {-(\sin^4x+\cos^4x)(\cos^2x-\sin^2x)}{(\sin^4x+\cos^4x)}=-\cos 2x$

$\displaystyle \therefore \int \frac {\sin^8x-\cos^8x}{1-2 \sin^2x \cos^2x}dx=\int -\cos 2 x dx=-\frac {\sin 2x}{2}+C$

#425580

Topic: Fundamental Integrals

Integrate the function $\cfrac {1}{\cos(x+a)\cos (x+b)}$

Solution

$\displaystyle \frac {1}{\cos(x+a)\cos (x+b)}$

Multiplying and dividing by $\sin(a-b)$, we obtain

$\displaystyle\frac {1}{\sin(a-b)}\left [\frac {\sin(a-b)}{\cos (x+a)\cos (x+b)}\right]$

$\displaystyle=\frac {1}{\sin(a-b)}\left [\frac {\sin[(x+a)-(x+b)]}{\cos(x+a)\cos(x+b)}\right]$

$\displaystyle=\frac {1}{\sin(a-b)}\left [\frac {\sin(x+a)\cdot \cos(x+b)-\cos(x+a)\sin(x+b)}{\cos(x+a)\cos (x+b)}\right]$

$\displaystyle=\frac {1}{\sin(a-b)}\left [\frac {\sin(x+a)}{\cos(x+a)}-\frac {\sin(x+b)}{\cos (x+b)}\right]$

$\displaystyle=\frac {1}{\sin(a-b)}\left [\tan(x+a)-\tan(x+b)\right]$

$\displaystyle\therefore \int \frac {1}{\cos(x+a)\cos (x+b)}dx=\frac {1}{\sin(a-b)}\int [\tan(x+a)-\tan (x+b)]dx$

$\displaystyle=\frac {1}{\sin(a-b)}[-\log|\cos(x+a)|+\log|\cos(x+b)|]+C$

$\displaystyle=\frac {1}{\sin(a-b)}\log \left |\frac {\cos(x+b)}{\cos(x+a)}\right |+C$

#425584

Topic: Integration by Substitution

Integrate the function: $\cos^3xe^{\log\sin\ x}$

Solution

$\cos^3xe^{\log\sin\ x}=\cos^3 x \times \sin x$

Let $\cos x=t\Rightarrow -\sin x dx=dt$

$\Rightarrow\displaystyle \int \cos^3x e^{\log\sin\ x}dx=\int \cos^3 x \sin x dx$

$\displaystyle =-\int t^3\cdot dt =-\frac {t^4}{4}+C=-\frac {\cos^4x}{4}+C$

#425585

Topic: Integration by Substitution

Integrate the function: $e^{3 \log x}(x^4+1)^{-1}$

Solution

$\displaystyle e^{3\log\ x}(x^4+1)^{-1}=e^{\log x^3}(x^4+1)^{-1}=\frac {x^3}{(x^4+1)}$

Let $x^4+1=t\Rightarrow 4x^3dx=dt$

$\Rightarrow\displaystyle \int e^{3\log\ x}(x^4+1)^{-1}dx=\int \frac {x^3}{(x^4+1)}dx$

$\displaystyle =\frac {1}{4}\int \frac {dt}{t}=\frac {1}{4}\log|t|+C$

$\displaystyle =\frac {1}{4}\log|x^4+1|+C=\frac {1}{4}\log(x^4+1)+C$

#425586

Topic: Integration by Substitution

Integrate the function f'(ax+b)[f(ax+b)]^n

Solution

f'(ax+b)[f(ax+b)]^n

Let f(ax+b)=t\Rightarrow af'(ax+b)dx=dt

\displaystyle \therefore \int f'(ax+b)[f(ax+b)]^ndx=\frac {1}{a}\int t^ndt

\displaystyle =\frac {1}{a}\left [\frac {t^{n+1}}{n+1}\right]

\displaystyle =\frac {1}{a(n+1)}(f(ax+b))^{n+1}+C

#425587

Topic: Integration by Substitution

Integrate the function \cfrac {1}{\sqrt {\sin^3x \sin (x+\alpha)}}

Solution

\displaystyle \frac {1}{\sqrt {\sin^3x \sin (x+\alpha)}}=\frac {1}{\sqrt {\sin^3x(\sin x \cos \alpha +\cos x \sin \alpha)}}

\displaystyle=\frac {1}{\sqrt {\sin^4 x \cos \alpha+\sin^2 x \cos x \sin \alpha}}

\displaystyle=\frac {1}{\sin^2x\sqrt {\cos\alpha +\cot x \sin \alpha}}

\displaystyle=\frac {co\sec^2x}{\sqrt {\cos\alpha+\cot x \sin \alpha}}

Let \cos\alpha+\cot x sin \alpha=t\Rightarrow -co\sec^2x \sin\alpha dx=dt

\therefore\displaystyle \int \frac {1}{\sin^3x \sin(x+\alpha)}dx=\int \frac {co\sec^2x}{\sqrt {\cos \alpha+\cot x \sin \alpha}}dx

\displaystyle=\frac {-1}{\sin\alpha}\int \frac {dt}{\sqrt t}

\displaystyle=\frac {-1}{\sin\alpha}[2\sqrt t]+C

\displaystyle=\frac {-1}{\sin\alpha}[2\sqrt {\cos \alpha+\cos x \sin \alpha}]+C

\displaystyle=\frac {-2}{\sin\alpha}\sqrt {\cos \alpha+\frac {\cos x \sin \alpha}{\sin x}}+C

\displaystyle=\frac {-2}{\sin\alpha}\sqrt {\frac {\sin x \cos\alpha+\cos x \sin\alpha}{\sin x}}+C

\displaystyle=\frac {-2}{sin\alpha}\sqrt {\frac {\sin (x+\alpha)}{\sin x}}+C

#425679

Topic: Integration by Substitution

\displaystyle \int \frac {\cos 2x}{(\sin x+\cos x)^2}dx is equal to

A \dfrac {-1}{(\sin x+\cos x)}+C

[✓ B] \log |\sin x+\cos x|+C

C \log |\sin x-\cos x|+C

D \dfrac {1}{(\sin x+\cos x)^2}

Solution

Let \displaystyle I=\int \frac {\cos 2x}{(\sin x+\cos x)^2}dx

I=\displaystyle \int \frac {\cos^2x-\sin^2x}{(\cos x+\sin x)^2}dx

\displaystyle =\int \frac {(\cos x+\sin x)(\cos x-\sin x)}{(\cos x+\sin x)^2}dx

\displaystyle =\int \frac {\cos x- \sin x}{\cos+\sin x}dx

Let \cos x+\sin x=t\Rightarrow (\cos x-\sin x)dx=dt

\displaystyle \therefore I=\int \frac {dt}{t}=\log|t|+C

=\log|\cos x+\sin x|+C

Hence, the correct Answer is B.

#420660

Topic: Integration by Substitution

Integrate the function $x\sqrt{x+2}$

Solution

Put $(x+2) = t$

$\Rightarrow dx = dt$

$\Rightarrow \int x\sqrt{x+2}dx = \int (t-2)\sqrt{t}dt$

$= \int\left(t^{\frac{3}{2}} - 2t^{\frac{1}{2}}\right)dt$

$= \int t^{\frac{3}{2}} - 2\int t^{\frac{1}{2}}dt$

$= \dfrac{t^{\frac{5}{2}}}{\frac{5}{2}} - 2\left(\dfrac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$

$= \dfrac{2}{5}t^{\frac{5}{2}} - \dfrac{4}{3}t^{\frac{3}{2}} + C$

$= \dfrac{2}{5}(x+2)^{\frac{5}{2}} - \dfrac{4}{3}(x+2)^{\frac{3}{2}} + C$

#420687

Topic: Integration by Substitution

Integrate the function $\dfrac{x}{\sqrt{x+4}}, x > 0$

Solution

Let $x+4 = t \Rightarrow dx = dt$

$\therefore \int \dfrac{x}{\sqrt{x+4}}dx = \int \dfrac{(t-4)}{\sqrt{t}}dt$

$= \int\left(\sqrt{t} - \dfrac{4}{\sqrt{t}}\right)dt$

$= \dfrac{t^{\frac{3}{2}}}{\frac{3}{2}} - 4\left(\dfrac{t^{\frac{1}{2}}}{\frac{1}{2}}\right) + C$

$= \dfrac{2}{3}(t)^{\frac{3}{2}} - 8(t)^{\frac{1}{2}} + C$

$= \dfrac{2}{3}t \cdot t^{\frac{1}{2}} - 8t^{\frac{1}{2}} + C$

$= \dfrac{2}{3}t^{\frac{1}{2}}(t-12) + C$

$= \dfrac{2}{3}(x+4)^{\frac{1}{2}}(x+4-12) + C$

$= \dfrac{2}{3}\sqrt{x+4}(x-8) + C$

#421125

Topic: Integration by Substitution

Integrate the function $(x^3-1)^{\frac{1}{3}}x^5$

Solution

Put $x^3 - 1 = t$

$\therefore 3x^2dx = dt$

$\Rightarrow \int (x^3-1)^{\frac{1}{3}} x^5 dx = \int (x^3-1)^{\frac{1}{3}} x^3 \cdot x^2 dx$

$= \int t^{\frac{1}{3}}(t+1)\dfrac{dt}{3}$

$= \dfrac{1}{3}\int \left(t^{\frac{4}{3}} + t^{\frac{1}{3}}\right)dt$

$= \dfrac{1}{3}\left[\dfrac{t^{\frac{7}{3}}}{\frac{7}{3}} + \dfrac{t^{\frac{4}{3}}}{\frac{4}{3}}\right] + C$

$= \dfrac{1}{3}\left[\dfrac{3}{7}t^{\frac{7}{3}} + \dfrac{3}{4}t^{\frac{4}{3}}\right] + C$

$= \dfrac{1}{7}(x^3-1)^{\frac{7}{3}} + \dfrac{1}{4}(x^3-1)^{\frac{4}{3}} + C$

#421129

Topic: Integration by Substitution

Integrate the function e^{2x+3}

Solution

Put $2x + 3 = t$

$\therefore 2dx = dt$

$\Rightarrow e^{2x+3}dx = \dfrac{1}{2}\int e^t dt$

$= \dfrac{1}{2}(e^t) + C$

$= \dfrac{1}{2}e^{(2x+3)} + C$

#421134

Topic: Integration by Substitution

Integrate the function $\tan^2(2x-3)$

Solution

$\tan^2(2x-3) = \sec^2(2x-3) - 1$

Let $2x - 3 = t$

$\therefore 2dx = dt$

$\Rightarrow \int \tan^2(2x-3)dx = \int [(\sec^2(2x-3)) - 1]dx$

$= \dfrac{1}{2}\int (\sec^2 t)dt - \int 1dx$

$= \dfrac{1}{2}\int \sec^2 tdt - \int 1dx$

$= \dfrac{1}{2}\tan t - x + C$

$= \dfrac{1}{2}\tan(2x-3) - x + C$

#421135

Topic: Integration by Substitution

Integrate the function $\sec^2(7-4x)$

Solution

Let $7 - 4x = t$

$\therefore\ -4dx = dt$

$\therefore \int \sec^2(7-4x)dx = \frac{-1}{4}\int sec^2 tdt$

$= \frac{-1}{4}(\tan t) + C$

$= \frac{-1}{4}\tan(7-4x) + C$

#421898

Topic: Integration by Substitution

Find the integrals of the functions $\sin^2(2x+5)$

Solution

$\sin^2(2x+5) = \frac{1-\cos2(2x+5)}{2} = \frac{1-\cos(4x+10)}{2}$

$\Rightarrow \int \sin^2(2x+5)dx = \int \frac{1-\cos(4x+10)}{2}dx$

$= \frac{1}{2}\int 1dx - \frac{1}{2}\int \cos(4x+10)dx$

$= \frac{1}{2}x - \frac{1}{2}\left(\frac{\sin(4x+10)}{4}\right) + C$

$= \frac{1}{2}x - \frac{1}{8}\sin(4x+10) + C$

#422459

Topic: Special Integrals (Irrational Functions)

Integrate the function $\frac{1}{\sqrt{1+4x^2}}$

Solution

Let $2x = t \Rightarrow 2dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{1+4x^2}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{1+t^2}}$

$= \frac{1}{2}[\log|t + \sqrt{t^2+1}|] + C, \left[\because \int \frac{1}{\sqrt{x^2+a^2}}dt = \log|x + \sqrt{x^2+a^2}|\right]$

$= \frac{1}{2}\log|2x + \sqrt{4x^2+1}| + C$

#422469

Topic: Special Integrals (Irrational Functions)

Integrate the function $\frac{1}{\sqrt{(2-x)^2+1}}$

Solution

Let $2 - x = t \Rightarrow\ -dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{(2-x)^2+1}}dx = -\int \frac{1}{\sqrt{t^2+1}}dt$

$= -\log|t + \sqrt{t^2+1}| + C, \left[\because \int \frac{1}{\sqrt{x^2+a^2}}dt = \log|x + \sqrt{x^2+a^2}|\right]$

$= -\log|2 - x + \sqrt{(2-x)^2+1}| + C$

$= \log\left|\frac{1}{(2-x) + \sqrt{x^2-4x+5}}\right| + C$

#422473

Topic: Special Integrals (Irrational Functions)

Integrate the function $\dfrac{1}{\sqrt{9-25x^2}}$

Solution

Let $5x = t$

$\therefore 5dx = dt$

$\Rightarrow \int \dfrac{1}{\sqrt{9-25x^2}}dx = \dfrac{1}{5}\int \dfrac{1}{\sqrt{9-t^2}}dt$

$= \dfrac{1}{5}\sin^{-1}\left(\dfrac{t}{3}\right) + C$

$= \dfrac{1}{5}\sin^{-1}\left(\dfrac{5x}{3}\right) + C$

#422507

Topic: Special Integrals (Irrational Functions)

Integrate the function $\dfrac{x^2}{\sqrt{x^6+a^6}}$

Solution

Let $x^3 = t$

$\Rightarrow 3x^2dx = dt$

$\therefore \int \dfrac{x^2}{\sqrt{x^6+a^6}}dx = \dfrac{1}{3}\int \dfrac{dt}{\sqrt{t^2+(a^3)^2}}$

$= \dfrac{1}{3}\log|t+\sqrt{t^2+a^6}| + C$

$= \dfrac{1}{3}\log|x^3+\sqrt{x^6+a^6}| + C$

#422515

Topic: Special Integrals (Irrational Functions)

Integrate the function $\dfrac{\sec^2x}{\sqrt{\tan^2x+4}}$

Solution

Let $\tan x = t$

$\therefore \sec^2xdx = dt$

$\Rightarrow \int \dfrac{\sec^2x}{\sqrt{\tan^2x+4}}dx = \int \dfrac{dt}{\sqrt{t^2+2^2}}$

$= \log|t+\sqrt{t^2+4}| + C$

$= \log|\tan x+\sqrt{\tan^2x+4}| + C$

#422524

Topic: Special Integrals (Irrational Functions)

Integrate the function $\dfrac{1}{\sqrt{x^2+2x+2}}$

Solution

$$\frac{1}{\sqrt{x^2+2x+2}}dx = \int \frac{1}{\sqrt{(x+1)^2+(1)^2}}dx$$

Let $x+1=t$

$\therefore dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{x^2+2x+2}}dx = \int \frac{1}{\sqrt{t^2+1}}dt$$

$= \log|t+\sqrt{t^2+1}| + C$

$= \log|(x+1)+\sqrt{(x+1)^2+1}| + C$

$= \log|(x+1)+\sqrt{x^2+2x+2}| + C$

#422530

Topic: Special Integrals (Irrational Functions)

Integrate the function $\dfrac{1}{\sqrt{9x^2+6x+5}}$

Solution

$$\int \frac{1}{9x^2+6x+5}dx = \int \frac{1}{\sqrt{(3x+1)^2+(2)^2}}dx$$

Let $(3x+1)=t$

$\therefore 3dx = dt$

$$\Rightarrow \int \frac{1}{(3x+1)^2+(2)^2}dx = \frac{1}{3}\int \frac{1}{t^2+2^2}dt$$

$$= \frac{1}{3}\left[\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)\right] + C$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{3x+1}{2}\right) + C$$

#422542

Topic: Special Integrals (Irrational Functions)

Integrate the function $\dfrac{1}{\sqrt{7-6x-x^2}}$

Solution

$7-6x-x^2 = 7-(x^2+6x+9-9) = 7-(x^2+6x+9-9)$

$= 16-(x^2+6x+9) = 16-(x+3)^2 = (4)^2-(x+3)^2$

$$\therefore \int \frac{1}{\sqrt{7-6x-x^2}}dx = \int \frac{1}{\sqrt{(4)^2-(x+3)^2}}dx$$

Let $x+3=t \Rightarrow dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{(4)^2-(x+3)^2}}dx = \int \frac{1}{\sqrt{(4)^2-(t)^2}}dt$$

$$= \sin^{-1}\left(\frac{t}{4}\right) + C$$

$$= \sin^{-1}\left(\frac{x+3}{4}\right) + C$$

#422554

Topic: Special Integrals (Irrational Functions)

Integrate the function $\dfrac{1}{\sqrt{(x-1)(x-2)}}$

Solution

$(x-1)(x-2) = x^2 - 3x + 2 = x^2 - 3x + \frac{9}{4} - \frac{9}{4} + 2$

$= \left(x - \frac{3}{2}\right)^2 - \frac{1}{4} = \left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$\therefore \int \frac{1}{\sqrt{(x-1)(x-2)}} dx = \int \frac{1}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} dx$

Let $x - \frac{3}{2} = t \Rightarrow dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} dx = \int \frac{1}{\sqrt{t^2 - \left(\frac{1}{2}\right)^2}} dt$

$= \log\left|t \sqrt{t^2 - (\frac{1}{2})^2}\right| + C$

$= \log\left|\left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + 2}\right| + C$

#422568

Topic: Special Integrals (Irrational Functions)

Integrate the function $\frac{1}{\sqrt{8 + 3x - x^2}}$

Solution

$8 + 3x - x^2 = 8 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right)$

$= \frac{41}{4} - \left(x - \frac{3}{2}\right)^2$

$\Rightarrow \int \frac{1}{\sqrt{8 + 3x - x^2}} dx = \int \frac{1}{\sqrt{\frac{41}{4} - \left(x - \frac{3}{2}\right)^2}} dx$

Let $x - \frac{3}{2} = t$

$\therefore dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{\frac{41}{4} - \left(x - \frac{3}{2}\right)^2}} dx = \int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2 - t^2}} dt$

$= \sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{2}}\right) + C$

$= \sin^{-1}\left(\frac{x - \frac{3}{2}}{\frac{\sqrt{41}}{2}}\right) + C$

$= \sin^{-1}\left(\frac{2x - 3}{\sqrt{41}}\right) + C$

#422581

Topic: Special Integrals (Irrational Functions)

Integrate the function $\frac{1}{\sqrt{(x-a)(x-b)}}$

Solution

$(x-a)(x-b) = x^2-(a+b)x+ab = x^2-(a+b)x+\frac{(a+b)^2}{4}-\frac{(a+b)^2}{4}+ab$

$= \left[x-\left(\frac{a+b}{2}\right)\right]^2-\frac{(a-b)^2}{4}$

$\Rightarrow \int \frac{1}{\sqrt{(x-a)(x-b)}}dx = \int \frac{1}{\sqrt{\left\{x-\left(\frac{a+b}{2}\right)\right\}^2-\left(\frac{a-b}{2}\right)^2}}dx$

Let $x-\left(\frac{a+b}{2}\right) = t \Rightarrow dx = dt$

$\Rightarrow \int \frac{1}{\sqrt{\left\{x-\left(\frac{a+b}{2}\right)\right\}^2-\left(\frac{a-b}{2}\right)^2}}dx = \int \frac{1}{\sqrt{t^2-\left(\frac{a-b}{2}\right)^2}}dt$

$= \log\left|t+\sqrt{t^2-\left(\frac{a-b}{2}\right)^2}\right|+C$

$= \log\left\{x-\left(\frac{a+b}{2}\right)\right\}+\sqrt{(x-a)(x-b)}\Big|+C$

#423015

Topic: Special Integrals (Irrational Functions)

$\int \frac{dx}{\sqrt{9x-4x^2}}$ equals

A $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$

[✓ **B**] $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$

C $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$

D $\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{9}\right)+C$

Solution

$$\int \frac{dx}{\sqrt{9x-4x^2}}$$

$$= \int \frac{1}{\sqrt{-4\left(x^2-\frac{9}{4}x\right)}}dx$$

$$= \int \frac{1}{-4\left(x^2-\frac{9}{4}x+\frac{81}{64}-\frac{81}{4}\right)}dx$$

$$= \int \frac{1}{\sqrt{-4\left[\left(x-\frac{9}{8}\right)^2-\left(\frac{9}{8}\right)^2\right]}}dx$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{\left(\frac{9}{8}\right)^2-\left(x-\frac{9}{8}\right)^2}}dx$$

$$= \frac{1}{2}\left[\sin^{-1}\left(\frac{x-\frac{9}{8}}{\frac{9}{8}}\right)\right]+C, \left(\because \int \frac{dy}{\sqrt{a^2-y^2}} = \sin^{-1}\frac{y}{a}+C\right)$$

$$= \frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$$

#423061

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{x}{(x-1)^2(x+2)}$

Solution

Let $\frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$

$\Rightarrow x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$

Substituting $x = 1$, we obtain $B = \frac{1}{3}$

Equating the coefficients of x^2 and constant term, we obtain

$A + C = 0$

$2A + 2B + C = 0$

On solving, we obtain

$A = \frac{2}{9}$ and $C = \frac{-2}{9}$

$$\therefore \frac{x}{(x-1)^2(x+2)} = \frac{2}{9(x-1)} + \frac{1}{3(x-1)^2} - \frac{2}{9(x+2)}$$

$$\Rightarrow \int \frac{x}{(x-1)^2(x+2)}dx = \frac{2}{9}\int\frac{1}{(x-1)}dx + \frac{1}{3}\int\frac{1}{(x-1)^2}dx - \frac{2}{9}\int\frac{1}{(x+2)}dx$$

$$= \frac{2}{9}\log|x-1| + \frac{1}{3}\left(\frac{-1}{x-1}\right) - \frac{2}{9}\log|x+2| + C$$

$$= \frac{2}{9}\log\left|\frac{x-1}{x+2}\right| - \frac{1}{3(x-1)} + C$$

#423213

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{3x-1}{(x+2)^2}$

Solution

Let $\frac{3x-1}{(x+2)^2} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2}$

$\Rightarrow 3x-1 = A(x+2) + B$

Equating the coefficient of x and constant term, we obtain

$A = 3, 2A + B = -1 \Rightarrow B = -7$

$\therefore \frac{3x-1}{(x+2)^2} = \frac{3}{(x+2)} - \frac{7}{(x+2)^2}$

$\Rightarrow \int \frac{3x-1}{(x+2)^2}dx = 3\int \frac{1}{(x+2)}dx - 7\int \frac{x}{(x+2)^2}dx$

$= 3\log|x+2| - 7\left(\frac{-1}{(x+2)}\right) + C$

$= 3\log|x+2| + \frac{7}{(x+2)} + C$

#423277

Topic: Integration by Parts

Integrate the function $x\sin x$

Solution

$\int x\sin x dx$

Using by parts,

$= x\int \sin x dx - \int\left\{\left(\frac{d}{dx}x\right)\int \sin x dx\right\}dx$

$= x(-\cos x) - \int 1\cdot(-\cos x)dx$

$= -x\cos x + \sin x + C$

#423279

Topic: Integration by Parts

Integrate the function $x\sin 3x$

Solution

Let $I = \int x\sin 3x dx$

Using by parts,

$I = x\int \sin 3x dx - \int\left\{\left(\frac{d}{dx}x\right)\int \sin 3x dx\right\}$

$= x\left(\frac{-\cos 3x}{3}\right) - \int 1\cdot\left(\frac{-\cos 3x}{3}\right)dx$

$= \frac{-x\cos 3x}{3} + \frac{1}{3}\int \cos 3x dx$

$= \frac{-x\cos 3x}{3} + \frac{1}{9}\sin 3x + C$

#423280

Topic: Integration by Parts

Integrate the function x^3e^x

Solution

Let $I = \int x^2 e^x dx$

Taking x^2 as first function and e^x as second function and integrating using by parts, we obtain

$$I = x^2 \int e^x dx - \int \left\{ \left(\frac{d}{dx} x^2 \right) \int e^x dx \right\} dx$$

$$= x^2 e^x - \int 2x \cdot e^x dx$$

$$= x^2 e^x - 2\left[x \cdot \int e^x dx - \int \left\{ \left(\frac{d}{dx} x \right) \cdot \int e^x dx \right\} dx \right]$$

$$= x^2 e^x - 2[xe^x - \int e^x dx]$$

$$= x^2 e^x - 2[xe^x - e^x]$$

$$= x^2 e^x - 2xe^x + 2e^x + C$$

$$= e^x(x^2 - 2x + 2) + C$$

#423281

Topic: Integration by Parts

Integrate the function $x \log x$

Solution

Let $I = \int x \log x dx$

Taking $\log x$ as first function and x as second function and using integrating by parts, we obtain

$$I = \log x \int x dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x dx \right\} dx$$

$$= \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \log x}{2} - \int \frac{x}{2} dx$$

$$= \frac{x^2 \log x}{2} - \frac{x^2}{4} + C$$

#423282

Topic: Integration by Parts

Integrate the function $x \log 2x$

Solution

Let $I = \int x \log 2x dx$

Taking $\log 2x$ as first function and x as second function and integrating by parts, we obtain

$$I = \log 2x \int x dx - \int \left\{ \left(\frac{d}{dx} \log 2x \right) \int x dx \right\} dx$$

$$= \log 2x \cdot \frac{x^2}{2} - \int \frac{2}{2x} \cdot \frac{x^2}{2} dt$$

$$= \frac{x^2 \log 2x}{2} - \int \frac{x}{2} dx$$

$$= \frac{x^2 \log 2x}{2} - \frac{x^2}{4} + C$$

#423283

Topic: Integration by Parts

Integrate the function $x^2 \log x$

Solution

Let $I = \int x^2 \log x dx$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain

$$I = \log x \int x^2 dx = \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$$

$$= \log x \left(\frac{x^3}{3} \right) - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \frac{x^3 \log x}{3} - \int \frac{x^2}{3} dx$$

$$= \frac{x^3 \log x}{3} - \frac{x^3}{9} + C$$

#423284

Topic: Integration by Parts

Integrate the function $x \sin^{-1} x$

Solution

Let $I = \int x \sin^{-1} x dx$

Taking $\sin^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \sin^{-1} x \int x dx - \int \left\{ \left(\frac{d}{dx} sin^{-1} x \right) \int x dx \right\} dx$$

$$= \sin^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{\sqrt{1 - x^2}} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{-x^2}{\sqrt{1 - x^2}} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \frac{1 - x^2}{\sqrt{1 - x^2}} - \frac{1}{\sqrt{1 - x^2}} \right\} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \sqrt{1 - x^2} - \frac{1}{\sqrt{1 - x^2}} \right\} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \int \sqrt{1 - x^2} dx - \int \frac{1}{\sqrt{1 - x^2}} dx \right\}$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \left\{ \frac{x}{2} \sqrt{1 - x^2} + \frac{1}{2} \sin^{-1} x - \sin^{-1} x \right\} + C$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{x}{4} \sqrt{1 - x^2} + \frac{1}{4} \sin^{-1} x - \frac{1}{2} \sin^{-1} x + C$$

$$= \frac{1}{4} (2x^2 - 1) \sin^{-1} x + \frac{x}{4} \sqrt{1 - x^2} + C$$

#423286

Topic: Integration by Parts

Integrate the function $x \tan^{-1} x$

Solution

Let $I = \int x\tan^{-1}x dx$

Taking $\tan^{-1}x$ as first function and x as second function and integrating by parts, we obtain

$$I = \tan^{-1}x\int xdx - \int\left\{\left(\frac{d}{dx}tan^{-1}x\right)\int xdx\right\}dx$$

$$= \tan^{-1}x\left(\frac{x^2}{2}\right) - \int\frac{1}{1+x^2}\cdot\frac{x^2}{2}dx$$

$$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\int\frac{x^2}{1+x^2}dx$$

$$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\int\left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right)dx$$

$$= \frac{x^2\tan^{-1}x}{2} - \frac{1}{2}(x - \tan^{-1}x) + C$$

$$= \frac{x}{2}\tan^{-1}x - \frac{x}{2} + \frac{1}{2}\tan^{-1}x + C$$

#423322

Topic: Integration by Parts

Integrate the function $x\cos^{-1}x$

Solution

Let $I = \int x\cos^{-1}x\,dx$

Applying integration by parts, by taking $\cos^{-1}x$ as first function and x as second function

$$I = \cos^{-1}x\int x\,dx - \int\left\{\left(\frac{d}{dx}\cos^{-1}x\right)\int x\,dx\right\}dx$$

$$= \cos^{-1}x\frac{x^2}{2} - \int\frac{-1}{\sqrt{1-x^2}}\cdot\frac{x^2}{2}\,dx$$

$$= \frac{x^2\cos^{-1}x}{2} - \frac{1}{2}\int\frac{1-x^2-1}{\sqrt{1-x^2}}\,dx$$

$$= \frac{x^2\cos^{-1}x}{2} - \frac{1}{2}\int\left\{\sqrt{1-x^2} + \left(\frac{-1}{\sqrt{1-x^2}}\right)\right\}dx$$

$$= \frac{x^2\cos^{-1}x}{2} - \frac{1}{2}\int\sqrt{1-x^2}\,dx - \frac{1}{2}\int\left(\frac{-1}{\sqrt{1-x^2}}\right)dx$$

$$= \frac{x^2\cos^{-1}x}{2} - \frac{1}{2}I_1 - \frac{1}{2}\cos^{-1}x \quad ....(1)$$

where, $I_1 = \int\sqrt{1-x^2}\,dx$

Applying integration by parts

$$\Rightarrow I_1 = \sqrt{1-x^2}\int 1dx - \int\frac{d}{dx}\sqrt{1-x^2}\int 1dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int\frac{-x^2}{\sqrt{1-x^2}}\cdot dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int\frac{-x^2}{\sqrt{1-x^2}}\,dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \int\frac{1-x^2-1}{\sqrt{1-x^2}}\,dx$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \left\{\int\sqrt{1-x^2}\,dx + \int\frac{-dx}{\sqrt{1-x^2}}\right\}$$

$$\Rightarrow I_1 = x\sqrt{1-x^2} - \left\{I_1 + \cos^{-1}x\right\}$$

$$\Rightarrow 2I_1 = x\sqrt{1-x^2} - \cos^{-1}x$$

$$I = \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\cos^{-1}x$$

Substituting in (1), we obtain

$$I = \frac{x^2\cos^{-1}x}{2} - \frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\cos^{-1}x\right) - \frac{1}{2}\cos^{-1}x$$

$$= \frac{(2x^2-1)}{4}\cos^{-1}x - \frac{x}{4}\sqrt{1-x^2} + C$$

#423331

Topic: Integration by Parts

Integrate the function $(\sin^{-1}x)^2$

Solution

Let $I = \int (\sin^{-1}x)^2 \cdot 1dx$

Taking $(\sin^{-1}x)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$I = (\sin^{-1}x)^2 \int 1dx - \int \left\{ \frac{d}{dx}(\sin^{-1}x)^2 \cdot \int 1 \cdot \right\} dx$$

$$= (\sin^{-1}x)^2 \cdot x - \int \frac{2\sin^{-1}x}{\sqrt{1-x^2}} \cdot xdx$$

$$= x(\sin^{-1}x)^2 + \int \sin^{-1}x \cdot \left(\frac{-2x}{\sqrt{1-x^2}} \right) dx$$

$$= x(\sin^{-1}x)^2 + \left[\sin^{-1}x \int \frac{-2x}{\sqrt{1-x^2}} dx - \int \left\{ \left(\frac{d}{dx} \sin^{-1}x \right) \int \frac{-2x}{\sqrt{1-x^2}} dx \right\} dx \right]$$

$$= x(\sin^{-1}x)^2 + \left[\sin^{-1} \cdot 2\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2} dx \right]$$

$$= x(\sin^{-1}x)^2 + 2\sqrt{1-x^2} \sin^{-1}x - \int 2dx$$

$$= x(\sin^{-1}x)^2 + 2\sqrt{1-x^2} \sin^{-1}x - 2x + C$$

#423335

Topic: Integration by Parts

Integrate the function $\frac{x\cos^{-1}x}{\sqrt{1-x^2}}$

Solution

Let $I = \int \frac{x\cos^{-1}x}{\sqrt{1-x^2}} dx$

$$I = \frac{-1}{2} \int \frac{-2x}{\sqrt{1-x^2}} \cdot \cos^{-1}xdx$$

Taking $\cos^{-1}x$ as first function and $\left(\frac{-2x}{\sqrt{1-x^2}} \right)$ as second function and integrating by parts, we obtain $I = \frac{-1}{2} \left[\cos^{-1}x \int \frac{-2x}{\sqrt{1-x^2}} dx - \int \left\{ \left(\frac{d}{dx} \cos^{-1}x \right) \int \frac{-2x}{\sqrt{1-x^2}} dx \right\} dx \right]$

$$= \frac{-1}{2} \left[\cos^{-1}x \cdot 2\sqrt{1-x^2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2} dx \right]$$

$$= \frac{-1}{2} \left[2\sqrt{1-x^2} \cos^{-1}x + \int 2dx \right]$$

$$= \frac{-1}{2} \left[2\sqrt{1-x^2} \cos^{-1}x + 2x \right] + C$$

$$= -[\sqrt{1-x^2} \cos^{-1}x + x] + C$$

#423338

Topic: Integration by Parts

Integrate the function $x\sec^2x$

Solution

Let $I = \int x\sec^2xdx$

Taking x as first function and $\sec^2x$ as second function and integrating by parts, we obtain

$$I = x \int \sec^2xdx - \int \left\{ \left\{ \frac{d}{dx} x \right\} \int \sec^2xdx \right\} dx$$

$$= x\tan x - \int 1 \cdot \tan xdx$$

$$= x\tan x + \log|\cos x| + C$$

#423343

Topic: Integration by Parts

Integrate the function $\tan^{-1}x$

Solution

Let $I = \int 1\cdot\tan^{-1}xdx$

Taking $\tan^{-1}$ as first function and 1 as second function and integrating by parts, we obtain

$$I = \tan^{-1}x\int 1dx - \int\left\{\left(\frac{d}{dx}\tan^{-1}x\right)\int 1\cdot dx\right\}dx$$

$$= \tan^{-1}x\cdot x - \int\frac{1}{1+x^2}\cdot xdx$$

$$= x\tan^{-1}x - \frac{1}{2}\int\frac{2x}{1+x^2}dx$$

$$= x\tan^{-1}x - \frac{1}{2}\log|1+x^2| + C$$

$$= x\tan^{-1}x - \frac{1}{2}\log(1+x^2) + C$$

#423676

Topic: Integration by Parts

Integrate the function $x(\log x)^2$

Solution

$I = \int x(\log x)^2dx$

Taking $(\log x)^2$ as first function and 1 as second function and integrating by part, we obtain

$$I = (\log x)^2\int xdx - \int\left[\left\{\left(\frac{d}{dx}\log x\right)^2\right\}\int xdx\right]dx$$

$$= \frac{x^2}{2}(\log x)^2 - \left[\int 2\log x\cdot\frac{1}{x}\cdot\frac{x^2}{2}dx\right]$$

$$= \frac{x^2}{2}(\log x)^2 - \int x\log xdx$$

Again integrating by parts, we obtain

$$I = \frac{x^2}{2}(\log x)^2 - \left[\log x\int xdx - \int\left\{\left(\frac{d}{dx}\log x\right)\int xdx\right\}dx\right]$$

$$= \frac{x^2}{2}(\log x)^2 - \left[\frac{x^2}{2} - \log x - \int\frac{1}{x}\cdot\frac{x^2}{x}dx\right]$$

$$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{1}{2}\int xdx$$

$$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{x^2}{4} + C$$

#423678

Topic: Integration by Parts

Integrate the function $(x^2+1)\log x$

Solution

Let $I = \int (x^2 + 1)\log x dx = \int x^2 \log x dx + \int \log x dx$

Let $I = I_1 + I_2$ .... (1)

Where, $I_1 = \int x^2 \log x dx$ and $I_2 = \int \log x dx$

$I_1 = \int x^2 \log x dx$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain

$$I_1 = \log x \int x^2 dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$$

$$= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \frac{x^3}{3} \log x - \frac{1}{3} (\int x^2 dx)$$

$$= \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 \text{ .... (2)}$$

$I_2 = \int \log x dx$

Taking $\log x$ as first function and 1 as second function and integrating by part x, we obtain

$$I_1 = \log x \int 1 \cdot dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int 1 \cdot dx \right\}$$

$$= \log x \cdot x - \int \frac{1}{x} \cdot x dx$$

$$= x \log x - \int 1 dx$$

$$= x \log x - x + C_2 \text{ ...... (3)}$$

Using equation (2) and (3) in (1), we obtain

$$I = \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 + x \log x - x + C_2$$

$$= \frac{x^3}{3} \log x - \frac{x^3}{9} + x \log x - x + (C_1 + C_2)$$

$$= \left(\frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + C$$

#423837

Topic: Integration by Parts

Integrate the function $e^{2x} \sin x$

Solution

Let $I = \int e^{2x}\sin x dx$ ....(1)

Integrating by parts, we obtain

$I = \sin x\int e^{2x}dx - \int\left\{\left(\frac{d}{dx}\sin x\right)\int e^{2x}dx\right\}dx$

$\Rightarrow I = \sin x\cdot\frac{x^{2x}}{2} - \int\cos x\cdot\frac{e^{2x}}{2}dx$

$\Rightarrow I = \frac{e^{2x}\sin x}{2} - \frac{1}{2}\int e^{2x}\cos x dx$

Again integrating by parts, we obtain

$I = \frac{e^{2x}\cdot\sin x}{2} - \frac{1}{2}\left[\cos x\int e^{2x}dx - \int\left\{\left(\frac{d}{dx}\cos x\right)\int e^{2x}dx\right\}dx\right]$

$\Rightarrow I = \frac{e^{2x}\cdot\sin x}{2} - \frac{1}{2}\left[\cos x\cdot\frac{e^{2x}}{e} - \int(-\sin x)\frac{e^{2x}}{2}dx\right]$

$\Rightarrow I = \frac{e^{2x}\cdot\sin x}{2} - \frac{1}{2}\left[\frac{e^{2x}\cos x}{2} + \frac{1}{2}\int e^{2x}\sin x dx\right]$

$I = \frac{e^{2x}\cdot\sin x}{2} - \frac{e^{2x}\cos x}{4} - \frac{1}{4}I$ [From (1)]

$\Rightarrow I + \frac{1}{4}I = \frac{e^{2x}\cdot\sin x}{2} - \frac{e^{2x}\cos x}{4}$

$\Rightarrow \frac{5}{4}I = \frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4}$

$\Rightarrow I = \frac{4}{5}\left[\frac{e^{2x}\sin x}{2} - \frac{e^{2x}\cos x}{4}\right] + C$

$\Rightarrow I = \frac{e^{2x}}{5}[2\sin x - \cos x] + C$

#423844

Topic: Integration by Parts

Integrate the function $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Solution

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$

$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$

$\Rightarrow \int\sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = \int 2\theta\cdot\sec^2\theta d\theta = 2\int\theta\cdot\sec^2\theta d\theta$

Integrating by parts, we obtain

$= 2\left[\theta\cdot\int\sec^2\theta d\theta - \int\left\{\left(\frac{d}{d\theta}\theta\right)\int\sec^2\theta d\theta\right\}d\theta\right]$

$= 2[\theta\cdot\tan\theta - \int\tan\theta d\theta]$

$= 2[\theta\tan\theta + \log|\cos\theta|] + C$

$= 2\left[x\tan^{-1}x + \log\left|\frac{1}{\sqrt{1+x^2}}\right|\right] + C$

$= 2x\tan^{-1}x + 2\log(1+x^2)_{\frac{1}{2}} + C$

$= 2x\tan^{-1}x + 2\left[-\frac{1}{2}\log(1+x^2)\right] + C$

$= 2x\tan^{-1}x - \log(1+x^2) + C$

#424708

Topic: Integration by Parts

If $f(x) = \int_0^x t\sin t dt$, then $f'(x)$ is

A $\cos x + x\sin x$

$\boxed{\text{B}}$ $x\sin x$

C $x\cos x$

D $\sin x + x\cos x$

Solution

$f(x) = \int_0^x t\sin t dt$

Integrating by parts, we obtain

$f(x) = t\int_0^x \sin t dt - \int_0^x \left\{\left(\frac{d}{dt}t\right)\int \sin t dt\right\}dt$

$= [t(-\cos t)]_0^x - \int_0^x(-\cos t)dt$

$= [-t\cos t + \sin t]_0^x$

$= -x\cos x + \sin x$

$\Rightarrow f'(x) = -[\{x(-\sin x)\} + \cos x] + \cos x$

$= x\sin x - \cos x + \cos x$

$= x\sin x$

Hence, the correct Answer is B.

#424716

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral $\int_0^{\frac{\pi}{2}} \cos^2 x dx$

Solution

$I = \int_0^{\frac{\pi}{2}} \cos^2 x dx$ .... (1)

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2} - x\right)dx, \left(\because \int_0^0 f(x)dx = \int_0^0 f(a - x)dx\right)$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx$ .... (2)

Adding (1) and (2), we obtain

$2I = \int_0^{\frac{\pi}{2}}(\sin^2 x + \cos^2 x)dx$

$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$

$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$

$\Rightarrow 2I = \frac{\pi}{2}$

$\Rightarrow I = \frac{\pi}{4}$

#424729

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

Solution

Let $I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ....(1)

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx, \left(\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$

$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ .... (2)

(1) + (2) $\Rightarrow 2I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$

$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}1\cdot dx$

\Rightarrow 2I=[x]_0^{\frac {\pi}{2}}

\Rightarrow 2I=\cfrac {\pi}{2}

\Rightarrow I=\cfrac {\pi}{4}

#424739

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral \displaystyle \int_0^{\frac {\pi}{2}}\frac {\sin^{\frac {3}{2}}x dx}{\sin^{\frac {3}{2}}x+\cos^{\frac {3}{2}}x}

Solution

Let \displaystyle I=\int_0^{\frac {\pi}{2}}\frac {\sin^{\frac {3}{2}}x dx}{\sin^{\frac {3}{2}}x+\cos^{\frac {3}{2}}x}dx ....(1)

\Rightarrow\displaystyle I=\int_0^{\frac {\pi}{2}}\frac {\sin^{\frac {3}{2}}\left (\frac {\pi}{2}-x\right)}{\sin^{\frac {3}{2}}\left (\frac {\pi}{2}-x\right)+\cos^{\frac {3}{2}}\left (\frac {\pi}{2}-x\right)}dx, (\because \int_0^af(x)dx=\int_0^af(a-x)dx)

\Rightarrow\displaystyle I=\int_0^{\frac {\pi}{2}}\frac {\cos^{\frac {3}{2}}x}{\sin^{\frac {3}{2}}x+\cos^{\frac {3}{2}}x}dx ....(2)

Adding (1) and (2), we obtain

2I=\displaystyle \int_0^{\frac {\pi}{2}}\frac {\sin^{\frac {3}{2}}x +\cos^{\frac {3}{2}}x}{\sin^{\frac {3}{2}}x+\cos^{\frac {3}{2}}x}dx

\Rightarrow 2I=\int_0^{\frac {\pi}{2}}1\cdot dx

\Rightarrow 2I=[x]_0^{\frac {\pi}{2}}

\Rightarrow 2I=\dfrac {\pi}{2}\Rightarrow I=\dfrac {\pi}{4}

#424752

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral \displaystyle \int_0^{\tfrac {\pi}{2}}\frac {\cos^5xdx}{\sin^5x+\cos^5x}

Solution

Let \displaystyle I=\int_0^{\tfrac {\pi}{2}}\frac {\cos^5x}{\sin^5x+\cos^5x}dx ....(1)

\displaystyle \Rightarrow I=\int_0^{\tfrac {\pi}{2}}\cfrac {\cos^5\left (\cfrac {\pi}{2}-x\right)}{\sin^5\left (\cfrac {\pi}{2}-x\right)+\cos^5\left (\cfrac {\pi}{2}-x\right)}dx ...(\because \int_0^af(x)dx=\int_0^af(a-x)dx)

\Rightarrow\displaystyle I=\int_0^{\tfrac {\pi}{2}}\cfrac {\sin^5x}{\sin^5x+\cos^5x}dx .... (2)

Adding (1) and (2), we obtain

\displaystyle 2I=\int_0^{\tfrac {\pi}{2}}\cfrac {\sin^5x+\cos^5x}{\sin^5x+\cos^5x}dx

\Rightarrow 2I=\int_0^{\tfrac {\pi}{2}}1\cdot dx

\Rightarrow 2I=[x]_0^{\tfrac {\pi}{2}}

\Rightarrow 2I=\cfrac {\pi}{2}\Rightarrow I=\cfrac {\pi}{4}

#424779

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral \displaystyle \int_0^1x(1-x)^ndx

Solution

Let I=\displaystyle \int_0^1x(1-x)^ndx

\therefore I=\displaystyle \int_0^1(1-x)(1-(1-x))^ndx, (\because \displaystyle \int_0^af(x)dx=\int_0^af(a-x)dx)

=\displaystyle \int_0^1(1-x)(x)^ndx

=\displaystyle \int_0^1(x^n-x^{n-1})dx

=\left [\dfrac {x^{n+1}}{n+1}-\dfrac {x^{n+2}}{n+2}\right]_0^1

=\left [\dfrac {1}{n+1}-\dfrac {1}{n+2}\right]

=\dfrac {(n+2)-(n+1)}{(n+1)(n+2)}

=\dfrac {1}{(n+1)(n+2)}

#425479

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral \displaystyle \int_0^{\frac {\pi}{4}}\log (1+\tan x)dx

Solution

Let \displaystyle I=\int_0^{\frac {\pi}{4}}\log (1+\tan x)dx .... (1)

\displaystyle \therefore I=\int_0^{\frac {\pi}{4}}\log \left [1+\tan \left (\frac {\pi}{4}-x\right)\right]dx, \; \left (\because \int_0^af(x)dx=\int_0^af(a-x)dx\right)

\Rightarrow\displaystyle I=\int_0^{\frac {\pi}{4}}\log \left \{1+\frac {\tan\frac {\pi}{4}-\tan x}{1+\tan \frac {\pi}{4}\tan x}\right \}dx

\Rightarrow\displaystyle I=\int_0^{\frac {\pi}{4}}\log \left \{1+\frac {1-\tan x}{1+\tan x}\right \}dx

\Rightarrow\displaystyle I=\int_0^{\frac {\pi}{4}}\log \frac {2}{(1+\tan x)}dx

\Rightarrow\displaystyle I=\int_0^{\frac {\pi}{4}} \log 2 dx-\int_0^{\frac {\pi}{4}}\log (1+\tan x)dx

\Rightarrow\displaystyle I=\int_0^{\frac {\pi}{4}}\log 2dx-I [From (1)]

\Rightarrow\displaystyle 2I=[x\log 2]_0^{\frac {\pi}{4}}

\Rightarrow\displaystyle 2I=\frac {\pi}{4}\log 2\Rightarrow\displaystyle I=\frac {\pi}{8}\log 2

#425480

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral \displaystyle \int_0^2x\sqrt {2-x}dx

Solution

Let \displaystyle I=\int_0^2x\sqrt {2-x}dx

\displaystyle I=\int_0^2(2-x)\sqrt xdx, \; \left(\because\int_0^af(x)dx=\int_0^af(a-x)dx\right)

\displaystyle =\int_0^2\left \{2x^{\frac {1}{2}}-x^{\frac {3}{2}}\right \}dx

\displaystyle =\left [2\left (\frac {x^{\frac {3}{2}}}{\frac {3}{2}}\right)-\frac {x^{\frac {5}{2}}}{\frac {5}{2}}\right]_0^2

\displaystyle=\left [\frac {4}{3}x^{\frac {3}{2}}-\frac {2}{5}x^{\frac {5}{2}}\right]_0^2

\displaystyle=\frac {4}{3}(2)^{\frac {3}{2}}-\frac {2}{5}(2)^{\frac {5}{2}}

\displaystyle=\frac {4\times 2\sqrt 2}{3}-\frac {2}{5}\times 4\sqrt 2

\displaystyle=\frac {8\sqrt 2}{3}-\frac {8\sqrt 2}{5}

\displaystyle=\frac {40\sqrt 2-24\sqrt 2}{15}=\frac {16\sqrt 2}{15}

#425481

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral \displaystyle \int_0^{\tfrac {\pi}{2}}(2 \log \sin x- \log \sin 2x) dx

Solution

Let \displaystyle I=\int_0^{\frac {\pi}{2}}(2 \log \sin x - \log \sin 2x)dx

\displaystyle \Rightarrow I=\int_0^{\frac {\pi}{2}}\left \{2 \log \sin x- \log (2 \sin x \cos x)\right \}dx

\displaystyle \Rightarrow I=\int_0^{\frac {\pi}{2}}\left \{2 \log \sin x - \log \sin x - \log \cos x- \log 2\right \}dx

\Rightarrow\displaystyle I=\int_0^{\frac {\pi}{2}}\left \{\log \sin x- \log \cos x-\log 2\right \}dx .... (1)

It is known that, \displaystyle(\int_o^af(x)dx=\int_0^a f(a-x)dx)

\Rightarrow\displaystyle I=\int_0^{\frac {\pi}{2}}\left \{\log \cos x-\log \sin x-\log 2\right \}dx .... (2)

Adding (1) and (2), we obtain

\displaystyle2I=\int_0^{\frac {\pi}{2}}(-\log 2-\log 2)dx

\Rightarrow\displaystyle 2I=-2 \log 2\int_0^{\frac {\pi}{2}}1\cdot dx

\displaystyle I=-\log 2\left [\frac {\pi}{2}\right]

\Rightarrow\displaystyle I=\frac {\pi}{2}(-\log 2)

\Rightarrow\displaystyle I=\frac {\pi}{2}\left [\log \frac {1}{2}\right]

\Rightarrow\displaystyle I=\frac {\pi}{2}\log \frac {1}{2}

#425482

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral \displaystyle \int_{-\frac {\pi}{2}}^{\frac {\pi}{2}}\sin^2 x dx

Solution

Let \displaystyle I=\int_{-\frac {\pi}{2}}^{\frac {\pi}{2}}\sin^2 x dx

As \sin^2(-x)=(\sin (-x))^2=(- \sin x)^2=\sin^2x,

Therefore, \sin^2x is an even functions.

It is known that if f(x) is an even function, then \displaystyle\int_{-a}^af(x)dx=2\int_0^af(x)dx

\displaystyle\therefore I=2\int_0^{\frac {\pi}{2}}\sin^2 x dx

\displaystyle=2\int_0^{\frac {\pi}{2}}\frac {1-\cos 2x}{2}dx

\displaystyle=\int_0^{\frac {\pi}{2}}(1-\cos 2x)dx

\displaystyle=\left [x-\frac {\sin 2x}{2}\right]_0^{\frac {\pi}{2}}=\frac {\pi}{2}

#425483

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral \displaystyle \int_0^{\pi}\frac {xdx}{1+\sin x}

Solution

Let \displaystyle I=\int_0^{\pi}\frac {xdx}{1+\sin x} ....(1)

\Rightarrow\displaystyle I=\int_0^{\pi}\frac {(\pi-x)}{1+\sin (\pi-x)}dx, \; (\because \int_0^af(x)dx=\int_0^af(a-x)dx)

\Rightarrow\displaystyle I=\int_0^{\pi}\frac {(\pi-x)}{1+\sin x}dx .... (2)

Adding (1) and (2), we obtain

2I\displaystyle=\int_0^{\pi}\frac {\pi}{1+\sin x}dx

\Rightarrow\displaystyle 2I=\pi \int_0^{\pi}\frac {(1-\sin x)}{1+\sin x)(1-\sin x)}dx

\displaystyle\Rightarrow 2I=\pi \int_0^{\pi}\frac {1-\sin x}{\cos^2x}dx

\Rightarrow\displaystyle 2I=\pi \int_0^{\pi}\left \{\sec^2x-\tan x \sec x\right \}dx

\Rightarrow\displaystyle 2I=\pi [\tan x- \sec x]_0^{\pi}

\Rightarrow\displaystyle 2I=\pi [2]\Rightarrow I=\pi

#425484

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral \displaystyle \int_{-\frac {\pi}{2}}^{\frac {\pi}{2}}\sin^7 x dx

Solution

Let \displaystyle I=\int_{-\frac {\pi}{2}}^{\frac {\pi}{2}}\sin^7 x dx .... (1)

As \sin^7(-x)=(\sin (-x))^7=(-\sin x)^7=-\sin^7x,

Therefore, \sin^7x is an odd function.

It is known that, if f(x) is an odd function, then \displaystyle\int_{-a}^af(x)dx=0

\displaystyle\therefore I=\int_{-\frac {\pi}{2}}^{\frac {\pi}{2}}\sin^7 x dx=0

#425485

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral \displaystyle \int_0^{2\pi}\cos^5 x dx

Solution

Let \displaystyle I=\int_0^{2\pi}\cos^5 x dx .... (1)

We know that, \cos^5(2\pi -x)=\cos^5x

Also It is known that,

\displaystyle\int_0^{2a}f(x)dx=\begin{cases}\displaystyle 2\int_0^af(x)dx, \text{if} f(2a-x)=f(x)\\\displaystyle =0 \ \ \ \text{if} \ f(2a-x)=-f(x)\end{cases}

\therefore\displaystyle I=\int_0^{2\pi}\cos^5x dx

\Rightarrow\displaystyle I=2\int_0^{\pi}\cos^5xdx=2(0)=0 \;, [\because\cos^5(\pi -x)=-\cos^5x]

#425489

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral \displaystyle \int_0^{\frac {\pi}{2}}\frac {\sin x-\cos x}{1+\sin x \cos x}dx

Solution

Let \displaystyle I=\int_0^{\frac {\pi}{2}}\frac {\sin x- \cos x}{1+\sin x \cos x}dx .... (1)

\displaystyle\Rightarrow I=\int_0^{\frac {\pi}{2}}\frac {\sin \left (\frac {\pi}{2}-x\right)-\cos \left (\frac {\pi}{2}-x\right)}{1+\sin \left (\frac {\pi}{2}-x\right)\cos \left (\frac {\pi}{2}-x\right)}dx , \;

(\because \int_0^af(x)dx=\int_0^af(a-x)dx)

\Rightarrow\displaystyle I=\int_0^{\frac {\pi}{2}}\frac {\cos x- \sin x}{1+\sin x \cos x}dx ...... (2)

Adding (1) and (2), we obtain

\displaystyle 2I=\int_0^{\frac {\pi}{2}}\frac {0}{1+\sin x \cos x}dx=0

\Rightarrow I=0

#425499

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral \displaystyle \int_0^{\pi}\log (1+\cos x)dx

Solution

Let I=\int_0^{\pi}\log (1+\cos x)dx .... (1)

\Rightarrow I=\int_0^{\pi}\log (1+\cos (\pi-x))dx, \; (\because \int_0^af(x)dx=\int_0^af(a-x)dx)

\Rightarrow I=\int_0^{\pi}\log (1-\cos x)dx .... (2)

Adding (1) and (2), we obtain

2I=\int_0^{\pi}\left \{\log (1+\cos x)+\log (1-\cos x)\right \}dx

\Rightarrow 2I=\int_0^{\pi}\log (1-\cos^2x)dx

\Rightarrow 2I=\int_0^{\pi}\log \sin^2 x dx

\Rightarrow 2I=2\int_0^{\pi} \log \sin x dx

\Rightarrow I=\int_0^{\pi} \log \sin x dx .... (3)

\sin (\pi-x)=\sin x

\therefore I=2\int_0^{\frac {\pi}{2}} \log \sin x dx .... (4)

\Rightarrow I=2\int_0^{\frac {\pi}{2}}\log \sin \left (\dfrac {\pi}{2}-x\right)dx=2\int_0^{\frac {\pi}{2}} \log \cos x dx .... (5)

Adding (4) and (5), we obtain

2I=2\int_0^{\frac {\pi}{2}}(\log \sin x+ \log \cos x)dx

\Rightarrow I=\int_0^{\frac {\pi}{2}} (\log \sin x+ \log \cos x+ \log 2-\log 2)dx

\Rightarrow I=\int_0^{\frac {\pi}{2}} (\log 2 \sin x \cos x-\log 2)dx

\Rightarrow I=\int_0^{\frac {\pi}{2}} \log \sin 2x dx-\int_0^{\frac {\pi}{2}} \log 2 dx

Let 2x=t\Rightarrow 2dx=dt

When x=0, t=0 and when x=\dfrac {\pi}{2}

\therefore I=\dfrac {1}{2}\int_0^{\frac {\pi}{2}}\log \sin t dt-\dfrac {\pi}{2}\log 2

\Rightarrow I=\dfrac {1}{2}I-\dfrac {\pi}{2}\log 2

\Rightarrow \dfrac {I}{2}=-\dfrac {\pi}{2}\log 2

\Rightarrow =-\pi \log 2

#425505

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral \displaystyle \int_0^a\frac {\sqrt x}{\sqrt x+\sqrt {a-x}}dx

Solution

Let \displaystyle I=\int_0^a\frac {\sqrt x}{\sqrt x+\sqrt {a-x}}dx .... (1)

It is known that, \displaystyle\left (\int_0^af(x)dx=\int_0^a f(a-x)dx\right)

\displaystyle I=\int_0^a\frac {\sqrt {a-x}}{\sqrt {a-x}+\sqrt x}dx .... (2)

Adding (1) and (2), we obtain

\displaystyle2I=\int_0^a\frac {\sqrt x+\sqrt {a-x}}{\sqrt x+\sqrt {a-x}}dx

\displaystyle\Rightarrow 2I=\int_0^a1\cdot dx

\Rightarrow \displaystyle2I=[x]_0^a

\Rightarrow 2I=a\displaystyle\Rightarrow I=\frac {a}{2}

#425511

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral \displaystyle \int_0^4|x-1|dx

Solution

I=\displaystyle \int_0^4|x-1|dx

It can be seen that, (x-1)\leq 0 when 0\leq x\leq 1 and (x-1)\geq 0 when 1\leq x\leq 4

I=\displaystyle \int_0^1|x-1|dx+\int_1^4|x-1|dx, \; (\because \int_a^bf(x)=\int_a^cf(x)+\int_c^bf(x))

=\displaystyle \int_0^1-(x-1)dx+\int_0^4(x-1)dx

=\left [x-\displaystyle \frac {x^2}{2}\right]_0^1+\left [\dfrac {x^2}{2}-x\right]_1^4

=1-\dfrac {1}{2}+\dfrac {(4)^2}{2}-4-\dfrac {1}{2}+1

=1-\dfrac {1}{2}+8-4-\dfrac {1}{2}+1=5

#425519

Topic: Properties of Definite Integral

By using the properties of definite integrals, Show that $\displaystyle \int_0^a f(x) g (x) dx=2\int_0^af(x)dx$, if f and g are defined as $f(x)=f(a-x)$ and $g(x)+g(a-x)=4$

Solution

Let $I=\int_0^af(x)g (x)dx$ .... (1)

$\Rightarrow I=\int_0^af(a-x) g (a-x)dx, \; (\because \int_0^af(x)dx=\int_0^a f (a-x)dx)$

$\Rightarrow I=\int_0^a f(x)g(a-x)dx$ .... (2)

Adding (1) and (2), we obtain

$2I=\int_0^a\left \{f(x)g(x)+f(x)g(a-x)\right \{dx$

$\Rightarrow 2I=\int_0^af(x)\left \{g(x)+g(a-x)\right \}dx$

$\Rightarrow 2I=\int_0^af(x)\times 4 dx \;$ using $[g(x)+g(a-x)=4]$

$\Rightarrow I=2\int_0^af(x)dx$

#425523

Topic: Properties of Definite Integral

The value of $\displaystyle \int_{-\frac {\pi}{2}}^{\frac {\pi}{2}}(x^3+x \cos x+\tan^5x+1)dx$

A 0

B 2

[✓ **C**] π

D 1

Solution

Let $\displaystyle I=\int_{-\frac {\pi}{2}}^{\frac {\pi}{2}}(x^3+x \cos x+\tan^5x+1)dx$

$\displaystyle \Rightarrow I=\int_{-\frac {\pi}{2}}^{\frac {\pi}{2}}x^3dx+\int_{-\frac {\pi}{2}}^{\frac {\pi}{2}} x \cos x+\int_{-\frac {\pi}{2}}^{\frac {\pi}{2}}\tan^5 x dx+\int_{-\frac {\pi}{2}}^{\frac {\pi}{2}}1\cdot dx$

It is known that if $f(x)$ is an even function, then $\int_{-a}^af(x)dx=2\int_0^a f(x)dx$ and if $f(x)$ is an odd function, then $\int_{-a}^af(x)dx=0$

$\therefore I=0+0+0+2\int_0^{\frac {\pi}{2}}1\cdot dx$

$\displaystyle =2[x]_0^{\frac {\pi}{2}}=\frac {2\pi}{2}=\pi$

Hence, the correct Answer is C.

#425526

Topic: Properties of Definite Integral

The value of $\displaystyle \int_0^{\frac {\pi}{2}}\log \left (\frac {4+3 \sin x}{4+3 \cos x}\right)dx$ is

A 2

B $\frac {3}{4}$

[✓ **C**] 0

D -2

Solution

Let \displaystyle I=\int_0^{\frac {\pi}{2}}\log \left (\frac {4+3 \sin x}{4+3 \cos x}\right)dx .... (1)

\Rightarrow\displaystyle I=\int_0^{\frac {\pi}{2}}log \left [\frac {4+3\sin \left (\frac {\pi}{2}-x\right)}{4+3 \cos \left (\frac {\pi}{2}-x\right)}\right]dx, \; (\because \int_0^a f(x) dx=\int_0^af(a-x)dx)

\Rightarrow\displaystyle I=\int_0^{\frac {\pi}{2}}\log \left (\frac {4+3 \cos x}{4+3 \sin x}\right)dx .... (2)

Adding (1) and (2), we obtain

2I\displaystyle =\int_0^{\frac {\pi}{2}}\left \{\log \left (\frac {4+3 \sin x}{4+3 \cos x}\right)+\log \left (\frac {4+3 \cos x}{4+3 \sin x}\right)\right \}dx

\Rightarrow\displaystyle 2I=\int_0^{\frac {\pi}{2}}\log \left (\frac {4+3 \sin x}{4+3 \cos x}\times \frac {4+3 \cos x}{4+3 \sin x}\right)dx

\Rightarrow 2I\displaystyle=\int_0^{\frac {\pi}{2}}\log 1 dx=\int_0^{\frac {\pi}{2}}0 dx

\Rightarrow I=0

Hence, the correct Answer is C.

#425549

Topic: Integration by Substitution

Integrate the function : \displaystyle \frac {1}{x\sqrt {ax-x^2}}

Solution

Let \displaystyle x=\frac {a}{t}\Rightarrow dx=-\frac {a}{t^2}dt

\Rightarrow\displaystyle \int \frac {1}{x\sqrt {ax-x^2}}dx=\int \frac {1}{\frac {a}{t}\sqrt {a\cdot \frac {a}{t}-\left (\frac {a}{t}\right)^2}}\left (-\frac {a}{t^2}dt\right)

\displaystyle=-\int \frac {1}{at}\cdot \frac {1}{\sqrt {\frac {1}{t}-\frac {1}{t^2}}}dt

\displaystyle=-\frac {1}{a}\int \frac {1}{\sqrt {\frac {t^2}{t}-\frac {t^2}{t^2}}}dt

\displaystyle=-\frac {1}{a}\int \frac {1}{\sqrt {t-1}}dt

\displaystyle=-\frac {1}{a}[2\sqrt {t-1}]+C

\displaystyle=-\frac {1}{a}\left [2\sqrt {\frac {a}{x}-1}\right]+C

\displaystyle=-\frac {2}{a}\left (\frac {\sqrt {a-x}}{\sqrt x}\right)+C

\displaystyle=-\frac {2}{a}\left (2\sqrt {\frac {a-x}{x}}\right)+C

#425563

Topic: Integration by Substitution

Integrate the function \dfrac {1}{x^{\frac {1}{2}}+x^{\frac {1}{3}}}

Solution

\dfrac {1}{x^{\frac {1}{2}}+x^{\frac {1}{3}}}=\dfrac {1}{x^{\frac {1}{3}}\left (1+x^{\frac {1}{6}}\right)}

Let x=t^6\Rightarrow dx=6t^5dt

\therefore \displaystyle \int \frac {1}{x^{\frac {1}{2}}+x^{\frac {1}{3}}}dx=\int \frac {1}{x^{\frac {1}{3}}\left (1+x^{\frac {1}{6}}\right)}dx

=\displaystyle \int \frac {6t^5}{t^2(1+t)}dt

=6\displaystyle \int \frac {t^3}{(1+t)}dt

On dividing, we obtain

\displaystyle \int \frac {1}{x^{\frac {1}{2}}+x^{\frac {1}{3}}}dx=6\int \left \{(t^2-t+1)-\frac {1}{1+t}\right \}dt

=6\left [\left (\dfrac {t^3}{3}\right)-\left (\dfrac {t^2}{2}\right)+t-\log |1+t|\right]

=2x^{\frac {1}{2}}-3x^{\frac {1}{3}}+6x^{\frac {1}{6}}-6 \log \left (1+x^{\frac {1}{6}}\right)+C

=2\sqrt x-3x^{\frac {1}{3}}+6x^{\frac {1}{6}}-6 \log \left (1+x^{\frac {1}{6}}\right)+C

#425577

Topic: Special Integrals (Irrational Functions)

Integrate the function \displaystyle \frac {\cos x}{\sqrt {4-\sin^2x}}

Solution

Let \sin x=t\Rightarrow \cos x dx=dt

\displaystyle \Rightarrow \int \frac {\cos x}{\sqrt {4-\sin^2x}}dx=\int \frac {dt}{\sqrt {(2)^2-(t)^2}}

\displaystyle =\sin^{-1}\left (\frac {t}{2}\right)+C

\displaystyle =\sin^{-1}\left (\frac {\sin x}{2}\right)+C

#425581

Topic: Special Integrals (Irrational Functions)

Integrate the function \cfrac {x^3}{\sqrt {1-x^8}}

Solution

Let x^4=t\Rightarrow 4x^3 dx=dt

\Rightarrow\displaystyle \int \frac {x^3}{\sqrt {1-x^8}}dx=\frac {1}{4}\int \frac {dt}{\sqrt {1-t^2}}

\displaystyle =\frac {1}{4}\sin^{-1}t+C

\displaystyle =\frac {1}{4}\sin^{-1}(x^4)+C

#425599

Topic: Integration by Parts

Integrate the function \displaystyle \frac {\sin^{-1}\sqrt x-\cos^{-1}\sqrt x}{\sin^{-1}\sqrt x+\cos^{1-}\sqrt x}x\epsilon [0,1]

Solution

Let I\displaystyle =\int \frac {\sin^{-1}\sqrt x-\cos^{-1}\sqrt x}{\sin^{-1}\sqrt x+\cos^{1-}\sqrt x}dx

It is known that, \sin^{-1}\sqrt x+\cos^{-1}\sqrt x=\frac {\pi}{2}

\Rightarrow I\displaystyle =\int \frac {\left (\frac {\pi}{2}-\cos^{-1}\sqrt x\right)-\cos^{-1}\sqrt x}{\frac {\pi}{2}}dx

\displaystyle=\frac {2}{\pi}\int \left (\frac {\pi}{2}-2\cos^{-1}\sqrt x\right)dx

\displaystyle=\frac {2}{\pi}\cdot \frac {\pi}{2}\int 1\cdot dx-\frac {4}{\pi}\int \cos^{-1}\sqrt x dx

\displaystyle=x-\frac {4}{\pi}\int \cos^{-1}\sqrt xdx .... (1)

Let I_1\displaystyle=\int \cos^{-1}\sqrt xdx

Also, let \sqrt x=t\Rightarrow dx=2t dt

\Rightarrow I_1\displaystyle=2\int \cos^{-1}t\cdot t dt

\displaystyle=2\left [\cos^{-1}t\cdot \frac {t^2}{2}-\int \frac {-1}{\sqrt {1-t^2}}\cdot \frac {t^2}{2}dt\right]

\displaystyle=t^2\cos^{-1}t+\int \frac {t^2}{\sqrt {1-t^2}}dt

\displaystyle=t^2\cos^{-1}t-\int \frac {1-t^2-1}{\sqrt {1-t^2}}dt

\displaystyle=t^2\cos^{-1}t-\int \sqrt {1-t^2}dt+\int \frac {1}{\sqrt {1-t^2}}dt

\displaystyle=t^2\cos^{-1}t-\frac {t}{2}\sqrt {1-t^2}-\frac {1}{2}\sin^{-1}t+\sin^{-1}t

\displaystyle=t^2\cos^{-1}t-\frac {t}{2}\sqrt {1-t^2}+\frac {1}{2}\sin^{-1}t

From equation (1), we obtain

I\displaystyle=x-\frac {4}{\pi}\left [t^2 \cos t-\frac {1}{2}\sqrt {1-t^2}+\frac {1}{2}\sin^{-1}t\right]

\displaystyle=x-\frac {4}{\pi}\left [x\cos^{-1}\sqrt x-\frac {\sqrt x}{2}\sqrt {1-x}+\frac {1}{2}\sin^{-1}\sqrt x\right]

\displaystyle=x-\frac {4}{\pi}\left [x\left (\frac {\pi}{2}-\sin^{-1}\sqrt x\right)-\frac {\sqrt {x-x^{-2}}}{2}+\frac {\pi}{2}\sin^{-1}\sqrt x\right]

=x-2x+\frac {4x}{\pi}\sin^{-1}\sqrt x+\frac {2}{\pi}\sqrt {x-x^2}-\frac {2}{\pi}\sin^{-1}\sqrt x

\displaystyle=-x+\frac {2}{\pi}[(2x-1)\sin^{-1}\sqrt x]+\frac {2}{\pi}\sqrt {x-x^2}+C

\displaystyle=\frac {2(2x-1)}{\pi}\sin^{-1}\sqrt x+\frac {2}{\pi}\sqrt {x-x^2}-x+C

#425600

Topic: Integration by Substitution

Integrate the function \sqrt {\dfrac {1-\sqrt x}{1+\sqrt x}}

Solution

$I=\sqrt {\cfrac {1-\sqrt x}{1+\sqrt x}}$

Put $x=\cos^2\theta$

$\Rightarrow dx=-2 \sin\theta \cos\theta d\theta$

$I=\displaystyle \int \sqrt {\frac {1-\cos\theta}{1+\cos\theta}}(-2 \sin\theta \cos\theta)d \theta$

$=-2\displaystyle \int \sqrt {\frac {2\sin^2\frac {\theta}{2}}{2 \cos^2\frac {\theta}{2}}} (\sin \theta \cos\theta) d\theta$

$=-2\displaystyle \int \frac {\sin\frac {\theta}{2}}{\cos\frac {\theta}{2}}\left (2 \sin\frac {\theta}{2}\cos\frac {\theta}{2}\right)\cos\theta d\theta$

$=-4\displaystyle \int \sin^2\frac {\theta}{2}\cos\theta d\theta$

$=-4 \displaystyle \int \sin^2\frac {\theta}{2}\cdot \left (2 \cos^2\frac {\theta}{2}-1\right)d\theta$

$=-4\displaystyle \int \left (2 \sin^2\frac {\theta}{2}\cos^2\frac {\theta}{2}-\sin^2\frac {\theta}{2}\right)d\theta$

$=-8\displaystyle \int \sin^2\frac {\theta}{2}\cdot \cos^2\frac {\theta}{2}d\theta+4\int \sin^2\frac {\theta}{2}d\theta$

$=-2\displaystyle \int \sin^2 \theta \,d\theta +4 \int \sin^2\frac {\theta}{2}d\theta$

$=-2\displaystyle \int \left (\frac {1-\cos2\theta}{2}\right)d\theta+4\int \frac {1-\cos\theta}{2}d\theta$

$=-2\displaystyle \left [\frac {\theta}{2}-\frac {\sin 2\theta}{4}\right]+4\left [\frac {\theta}{2}-\frac {\sin\theta}{2}\right]+C$

$=-\theta+\cfrac {\sin 2\theta}{2}+2\theta- 2\sin\theta+C$

$=\theta+\cfrac {\sin 2\theta}{2}-2\sin \theta+C$

$=\theta+\cfrac {2 \sin \theta \cos \theta}{2}-2 \sin \theta+C$

$=\theta+\sqrt {1-\cos^2\theta}\cdot \cos \theta-2\sqrt {1-\cos^2\theta}+C$

$=\cos^{-1}\sqrt x+\sqrt {1-x}\cdot \sqrt x-2\sqrt {1-x}+C$

$=-2\sqrt {1-x}+\cos^{-1}\sqrt x+\sqrt {x(1-x)}+C$

$=-2\sqrt {1-x}+cos^{-1}\sqrt x+\sqrt {x-x^2}+C$

#425604

Topic: Integration using Partial Fractions

Integrate the function $\displaystyle \frac {x^2+x+1}{(x+1)^2(x+2)}$

Solution

Let \displaystyle \frac {x^2+x+1}{(x+1)^2(x+2)}=\frac {A}{(x+1)}+\frac {B}{(x+1)^2}+\frac {C}{(x+2)} .... (1)

\Rightarrow x^2+x+1=A(x+1)(x+2)+B(x+2)+C(x^2+2x+1)

\Rightarrow x^2+x+1=A(x^2+3x+2)+B(x+2)+C(x^2+2x+1)

\Rightarrow x^2+x+1=(A+C)x^2+(3A+B+2C)x+(2A+2B+C)

Equating the coefficients of x^2, x,and constant term, we obtain

A + C = 1

3A + B + 2C = 1

2A + 2B + C = 1

On solving these equations, we obtain

A = 2, B = 1, and C = 3

From equation (1), we obtain

\displaystyle\frac {x^2+x+1}{(x+1)^2(x+2)}=\frac {-2}{(x+1)}+\frac {3}{(x+2)}+\frac {1}{(x+1)^2}

\displaystyle\int \frac {x^2+x+1}{(x+1)^2(x+2)}dx=-2\int \frac {1}{x+1}dx+3\int \frac {1}{(x+2)}dx+\int \frac {1}{(x+1)^2}dx

\displaystyle=-2 \log |x+1|+3 \log |x+2|-\frac {1}{(x+1)}+C

#425606

Topic: Integration by Parts

Integrate the function \tan^{-1}\sqrt {\dfrac {1-x}{1+x}}

Solution

\displaystyle I=\int \tan^{-1}\sqrt {\frac {1-x}{1+x}}dx

Let x=\cos\theta \Rightarrow dx=-\sin \theta d\theta

\therefore\displaystyle I=\int \tan^{-1}\sqrt {\frac {1-\cos\theta}{1+\cos\theta}}(-\sin \theta d\theta)

\displaystyle=-\int \tan^{-1}\sqrt {\frac {2 \sin^2\frac {\theta}{2}}{2 \cos^2\frac {\theta}{2}}}\sin\theta d\theta

=-\displaystyle\int \tan^{-1}\tan \frac {\theta}{2}\cdot \sin \theta d \theta

\displaystyle=-\frac {1}{2}\int \theta \cdot \sin \theta d \theta

\displaystyle=-\frac {1}{2}[\theta\cdot (-\cos \theta)-\int 1\cdot (-\cos \theta)d\theta]

\displaystyle=-\frac {1}{2}[-\theta \cos \theta+ \sin\theta]

\displaystyle=\frac {1}{2}\theta \cos \theta-\frac {1}{2}\sin \theta

\displaystyle=\frac {1}{2}\cos^{-1}x\cdot x-\frac {1}{2}\sqrt {1-x^2}+C

\displaystyle=\frac {x}{2}\cos^{-1}x-\frac {1}{2}\sqrt {1-x^2}+C

\displaystyle=\frac {1}{2}(x \cos^{-1}x-\sqrt {1-x^2})+C

#425610

Topic: Integration by Parts

Integrate the function \displaystyle \frac {\sqrt {x^2+1}[\log(x^2+1)-2 \log x]}{x^4}

Solution

\displaystyle \frac {\sqrt {x^2+1}[\log(x^2+1)-2 \log x]}{x^4}=\frac {\sqrt {x^2+1}}{x^4}[\log (x^2+1)-\log x^2]

\displaystyle=\frac {\sqrt {x^2+1}}{x^4}\left [\log \left (\frac {\sqrt {x^2+1}}{x^2}\right)\right]

\displaystyle=\frac {\sqrt {x^2+1}}{x^4}\log \left (1+\frac {1}{x^2}\right)

\displaystyle=\frac {1}{x^3}\sqrt {\frac {\sqrt {x^2+1}}{x^2}}\log \left (1+\frac {1}{x^2}\right)

\displaystyle=\frac {1}{x^3}\sqrt {1+\frac {1}{x^2}}\log \left (1+\frac {1}{x^2}\right)

Let \displaystyle1+\frac {1}{x^2}=t\Rightarrow \frac {-2}{x^3}dx=dt

\therefore\displaystyle I=\int \frac {1}{x^3}\sqrt {1+\frac {1}{x^2}}\log \left (1+\frac {1}{x^2}\right)dx

\displaystyle=-\frac {1}{2}\int \sqrt t \log t dt

\displaystyle=-\frac {1}{2}\int t^{\frac {1}{2}}\cdot \log t dt

Integrating by parts, we obtain

I\displaystyle=-\frac {1}{2}\left [\log t\cdot \int t^{\frac {1}{2}}dt-\left \{\left (\frac {d}{dt}\log t\right)\int t^{\frac {1}{2}}dt\right \}dt\right]

\displaystyle=-\frac {1}{2}\left [\log t\cdot \frac {t^{\frac {3}{2}}}{\frac {3}{2}}-\int \frac {1}{t}\cdot \frac {t^{\frac {3}{2}}}{\frac {3}{2}}dt\right]

\displaystyle=-\frac {1}{2}\left [\frac {2}{3}t^{\frac {3}{2}}\log t-\frac {2}{3}\int t^{\frac {1}{2}}dt\right]

\displaystyle=-\frac {1}{2}\left [\frac {2}{3}t^{\frac {3}{2}}\log t-\frac {4}{9}t^{\frac {3}{2}}\right]

\displaystyle=-\frac {1}{3}t^{\frac {3}{2}}\log t+\frac {2}{9}t^{\frac {3}{2}}

\displaystyle=-\frac {1}{3}t^{\frac {3}{2}}\left [\log t-\frac {2}{3}\right]

\displaystyle=-\frac {1}{3}\left (1+\frac {1}{x^2}\right)^{\frac {3}{2}}\left [\log \left (1+\frac {1}{x^2}\right)-\frac {2}{3}\right]+C

#425641

Topic: Properties of Definite Integral

Evaluate the definite integral \displaystyle \int_0^{\pi}\frac {x \tan x}{\sec x+\tan x}dx

Solution

Let I\displaystyle =\int_0^{\pi}\frac {x \tan x}{\sec x+\tan x}dx .... (1)

I\displaystyle=\int_0^{\pi}\left \{\frac {(\pi-x)\tan (\pi-x)}{\sec(\pi-x)+\tan(\pi-x)}\right \}dx, \; \left (\because \int_0^af(x)dx=\int_0^af(a-x)dx\right)

\Rightarrow\displaystyle I=\int_0^{\pi}\left \{\frac {-(\pi-x)\tan x}{-(\sec x+ \tan x)}\right \}dx

\Rightarrow\displaystyle I=\int_0^{\pi}\frac {(\pi-x)\tan x}{\sec x+ \tan x}dx .... (2)

Adding (1) and (2), we obtain

2I\displaystyle=\int_0^{\pi}\frac {\pi \tan x}{\sec x+\tan x}dx

\Rightarrow 2I\displaystyle=\pi \int_0^{\pi}\frac {\frac {\sin x}{\cos x}}{\frac {1}{\cos x}+\frac {\sin x}{\cos x}}dx

\Rightarrow\displaystyle 2I=\pi \int_0^{\pi}\frac {\sin x+1-1}{1+\sin x}dx

\Rightarrow 2I\displaystyle=\pi \int_0^{\pi}1\cdot dx-\pi \int_0^{\pi}\frac {1}{1+\sin x}dx

\Rightarrow\displaystyle 2I=\pi[x]_0^{\pi}-\pi \int_0^{\pi}\frac {1-\sin x}{\cos^2x}dx

\Rightarrow\displaystyle 2I=\pi^2-\pi \int_0^{\pi}(\sec^2x-\tan x \sec x)dx

\Rightarrow 2I=\pi^2-\pi [\tan x-\sec x]_0^{\pi}

\Rightarrow 2I=\pi^2-\pi [\tan \pi - \sec \pi - \tan 0+\sec 0]

\Rightarrow 2I=\pi^2-\pi [0-(-1)-0+1]

\Rightarrow 2I=\pi^2-2\pi

\Rightarrow 2I=\pi^2(\pi -2)

\Rightarrow I=\dfrac {\pi}{2}(\pi-2)

#425656

Topic: Properties of Definite Integral

Prove \displaystyle \int_{-1}^1x^{17}\cos^4xdx=0

Solution

Let I=\displaystyle \int_{-1}^1x^{17}\cos^4xdx

Also, let f(x)=x^{17}\cos^4x

\Rightarrow f(-x)=(-x)^{17}\cos^4(-x)=-x^{17}\cos^4x=-f(x)

Therefore, f(x) is an odd function.

It is known that if f(x) is an odd function, then \int_{-a}^af(x)dx=0

\therefore I=\displaystyle \int_{-1}^1x^{17}\cos^4xdx=0

Hence, the given result is proved.

#425680

Topic: Properties of Definite Integral

If f(a+b-x)=f(x), then \displaystyle \int_a^bxf(x)dx is equal to

A \dfrac {a+b}{2}\displaystyle \int_a^bf(b-x)dx

B \dfrac {a+b}{2}\displaystyle \int_a^bf(b+x)dx

C \dfrac {b-a}{2}\displaystyle \int_a^bf(x)dx

[✓ **D**] \dfrac {a+b}{2}\displaystyle \int_a^bf(x)dx

Solution

Let I=\displaystyle \int_a^bf(x)dx .... (1)

\therefore I=\displaystyle \int_a^b(a+b-x)f(a+b-x)dx, (\because \int_a^bf(x)dx=\displaystyle \int_a^bf(a+b-x)dx)

\Rightarrow I=\displaystyle \int_a^b(a+b-x)f(x)dx

\Rightarrow I=(a+b)\displaystyle \int_a^bf(x)dx -I [Using (1)]

\Rightarrow I+I=(a+b)\displaystyle \int_a^bf(x)dx

\Rightarrow 2I=(a+b)\displaystyle \int_a^bf(x)dx

\Rightarrow I=\left (\dfrac {a+b}{2}\right)\displaystyle \int_a^bf(x)dx

Hence, the correct Answer is D.

#420280

Topic: Introduction

Find an anti derivative (or integral) of the given function by the method of inspection.

$\sin 2x$

Solution

We know that,

$\frac{d}{dx}(\cos 2x) = -2\sin 2x$

$\Rightarrow \sin 2x = -\frac{1}{2}\frac{d}{dx}(\cos 2x)$

$\therefore \sin 2x = \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$

Therefore, the anti derivative of $\sin 2x$ is $-\frac{1}{2}\cos 2x$

#420287

Topic: Introduction

Find an anti derivative (or integral) of the given function by the method of inspection.

$\cos 3x$

Solution

We know that,

$\frac{d}{dx}(\sin 3x) = 3\cos 3x$

$\therefore \cos 3x = \frac{d}{dx}\left(\frac{1}{3}\sin 3x\right)$

Therefore, the anti derivative of $\cos 3x$ is $\frac{1}{3}\sin 3x$

#420289

Topic: Introduction

Find an anti derivative (or integral) of the given function by the method of inspection.

e^{2x}

Solution

We know that,

$\frac{d}{dx}(e^{2x}) = 2e^{2x}$

$\Rightarrow e^{2x} = \frac{1}{2}\frac{d}{dx}(e^{2x})$

$\therefore e^{2x} = \frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)$

Therefore, the anti derivative of e^{2x} is $\frac{1}{2}e^{2x}$

#420294

Topic: Introduction

Find an anti derivative (or integral) of the given function by the method of inspection.

$(ax+b)^3$

Solution

We know that,

$$\frac{d}{dx}(ax+b)^3 = 3a(ax+b)^2$$

$$\Rightarrow (ax+b)^2 = \frac{1}{3a}\frac{d}{dx}(ax+b)^3$$

$$\therefore (ax+b)^2 = \frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right)$$

Therefore, the anti derivative of $(ax+b)^2$ is $\frac{1}{3a}(ax+b)^3$.

#420298

Topic: Introduction

Find an anti derivative (or integral) of the given function by the method of inspection.

$\sin 2x - 4e^{3x}$

Solution

We know that,

$$\frac{d}{dx}\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right) = \sin 2x - 4e^{3x}$$

Therefore, the anti derivative of $(\sin 2x - 4e^{3x})$ is $\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right)$.

#420311

Topic: Fundamental Integrals

Find the integral of $\int x^2(1-\frac{1}{x^2})dx$

Solution

$$\int x^2(1-\frac{1}{x^2})dx$$

$$= \int (x^2-1)dx$$

$$= \int x^2 dx - \int 1 dx$$

$$= \frac{x^3}{3} - x + C$$

#420315

Topic: Fundamental Integrals

Find the integral of $\int (ax^2+bx+c)dx$

Solution

$$\int (ax^2+bx+c)dx$$

$$= a\int x^2 dx + b\int x dx + c\int 1.dx$$

$$= a\left(\frac{x^3}{3}\right) + b\left(\frac{x^2}{2}\right) + cx + C$$

$$= \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C$$

#420319

Topic: Fundamental Integrals

Find the integral of $\int (2x^2+e^x)dx$

Solution

$\int (2x^2 + e^x)dx$

$= 2\int x^2 dx + \int e^x dx$

$= 2\left(\frac{x^3}{3}\right) + e^x + C$

$= \frac{2}{3}x^3 + e^x + C$

#420320

Topic: Fundamental Integrals

Find the integral of $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$

Solution

$\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$

$= \int \left(x + \frac{1}{x} - 2\right)dx$

$= \int x dx + \int \frac{1}{x}dx - 2\int 1.dx$

$= \frac{x^2}{2} + \log|x| - 2x + C$

#420325

Topic: Fundamental Integrals

Find the integral of $\int \frac{x^3 + 5x^2 - 4}{x^2} dx$

Solution

$\int \frac{x^3 + 5x^2 - 4}{x^2} dx$

$= \int (x + 5 - 4x^{-2})dx$

$= \int x dx + 5\int 1.dx - 4\int x^{-2}dx$

$= \frac{x^2}{2} + 5x - 4\left(\frac{x^{-1}}{-1}\right) + C$

$= \frac{x^2}{2} + 5x + \frac{4}{x} + C$

#420337

Topic: Fundamental Integrals

Find the integral of $\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$

Solution

$\int \frac{x^3 + 3x + 4}{\sqrt{x}} dx$

$= \int (x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 4x^{-\frac{1}{2}})dx$

$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{3(x^{\frac{3}{2}})}{\frac{3}{2}} + \frac{4(x^{\frac{1}{2}})}{\frac{1}{2}} + C$

$= \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + C$

$= \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8\sqrt{x} + C$

#420341

Topic: Fundamental Integrals

Find the integral of $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$

Solution

$\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$

On dividing, we obtain

$= \int (x^2 + 1) dx$

$= \int x^2 dx + \int 1 dx$

$= \frac{x^3}{3} + x + C$

#420343

Topic: Fundamental Integrals

Find the integral of $\int (1 + x)\sqrt{x} dx$

Solution

$\int (1 + x)\sqrt{x} dx$

$= \int (\sqrt{x} - x^{\frac{3}{2}}) dx$

$= \int x^{\frac{1}{2}} dx - \int x^{\frac{3}{2}} dx$

$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$

$= \frac{2}{3} x^{\frac{3}{2}} - \frac{2}{5} x^{\frac{5}{2}} + C$

#420351

Topic: Fundamental Integrals

Find the integral of $\int \sqrt{x}(3x^2 + 2x + 3) dx$

Solution

$\int \sqrt{x}(3x^2 + 2x + 3) dx$

$= \int (3x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}}) dx$

$= 3\int x^{\frac{5}{2}} dx + 2\int x^{\frac{3}{2}} dx + 3\int x^{\frac{1}{2}} dx$

$= 3\left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right) + 2\left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right) + 3\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$

$= \frac{6}{7} x^{\frac{7}{2}} + \frac{4}{5} x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C$

#420354

Topic: Fundamental Integrals

Find the integral of $\int (2x - 3\cos x + e^x) dx$

Solution

$\int (2x - 3\cos x + e^x) dx$

$= 2\int x dx - 3\int \cos x dx + \int e^x dx$

$= \frac{2x^2}{2} - 3(\sin x) + e^x + C$

$= x^2 - 3\sin x + e^x + C$

#420640

Topic: Fundamental Integrals

Find the integral of $\int(2x^2 - 3\sin x + 5\sqrt{x})dx$

Solution

$\int(2x^2 - 3\sin x + 5\sqrt{x})dx$

$= 2\int x^2 dx - 3\int \sin x dx + 5\int x^{\frac{1}{2}} dx$

$= \frac{2x^3}{3} - 3(-\cos x) + 5\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$

$= \frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{\frac{3}{2}} + C$

#420649

Topic: Fundamental Integrals

The anti derivative of $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$ equals

A $\frac{1}{3}x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + C$

B $\frac{2}{3}x^{\frac{2}{3}} + \frac{1}{2}x^2 + C$

[✓ **C**] $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$

D $\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + C$

Solution

$\int\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)dx$

$= \int x^{\frac{1}{2}}dx + \int x^{-\frac{1}{2}}dx$

$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$

#420651

Topic: Fundamental Integrals

If $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$. Then $f(x)$ is

[✓ **A**] $x^4 + \frac{1}{x^3} - \frac{129}{8}$

B $x^3 + \frac{1}{x^4} + \frac{129}{8}$

C $x^4 + \frac{1}{x^3} + \frac{129}{8}$

D $x^3 + \frac{1}{x^4} - \frac{129}{8}$

Solution

We have, $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$

$\therefore f(x) = \int(4x^3 - \frac{3}{x^4})dx$

$f(x) = 4\int x^3 dx - 4\int(x^{-4})dx$

$f(x) = 4\left(\frac{x^4}{4}\right) - 3\left(\frac{x^{-3}}{-3}\right) + C$

$\therefore f(x) = x^4 + \frac{1}{x^3} + C$

Also, $f(2) = 0$

$\therefore f(2) = (2)^4 + \frac{1}{(2)^3} + C = 0$

$\Rightarrow 16 + \frac{1}{8} + C = 0$

$\Rightarrow C = -\left(16 + \frac{1}{8}\right)$

$\Rightarrow C = \frac{-129}{8}$

$\therefore f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8}$

Hence, the correct Answer is A.

#420659

Topic: Fundamental Integrals

Integrate the function $\sqrt{ax + b}$

Solution

Put $ax + b = t$

$\Rightarrow adx = dt$

$\Rightarrow dx = \frac{1}{a}dt$

$\Rightarrow \int(ax + b)^{\frac{1}{2}}dx = \frac{1}{a}\int t^{\frac{1}{2}}dt$

$= \frac{1}{a}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$

$= \frac{2}{3a}(ax + b)^{\frac{3}{2}} + C$

#420661

Topic: Fundamental Integrals

Integrate the function $x\sqrt{1 + 2x^2}$

Solution

Put $1 + 2x^2 = t$

$\Rightarrow 4xdx = dt$

$\Rightarrow \int x\sqrt{1 + 2x^2}dx = \int\frac{\sqrt{t}dt}{4}$

$= \frac{1}{4}\int t^{\frac{1}{2}}dt$

$= \frac{1}{4}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$

$= \frac{1}{6}(1 + 2x^2)^{\frac{3}{2}} + C$

#420662

Topic: Fundamental Integrals

Integrate the function $(4x+2)\sqrt{x^2+x+1}$

Solution

Let $x^2+x+1=t$

$\Rightarrow (2x+1)dx=dt$

$\int(4x+2)\sqrt{x^2+x+1}dx$

$=\int 2\sqrt{t}dt$

$=2\int\sqrt{t}dt$

$=2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$

$=\frac{4}{3}(x^2+x+1)^{\frac{1}{2}}+C$

#421128

Topic: Fundamental Integrals

Integrate the function $\frac{x}{9-4x^2}$

Solution

Put $9-4x^2=t$

$\therefore\ -8xdx=dt$

$\Rightarrow\int\frac{x}{9-4x^2}dx=\frac{-1}{8}\int\frac{1}{t}dt$

$=\frac{-1}{8}\log|t|+C$

$=\frac{-1}{8}\log|9-4x^2|+C$

#422477

Topic: Special Integrals (Algebraic Functions)

Integrate the function $\frac{3x}{1+2x^4}$

Solution

Let $\sqrt{2}x^2=t$

$\therefore 2\sqrt{2}xdx=dt$

$\Rightarrow\int\frac{3x}{1+2x^4}dx=\frac{3}{2\sqrt{2}}\int\frac{dt}{1+t^2}$

$=\frac{3}{2\sqrt{2}}[\tan^{-1}t]+C$

$=\frac{3}{2\sqrt{2}}\tan^{-1}(\sqrt{2}x^2)+C$

#422482

Topic: Special Integrals (Algebraic Functions)

Integrate the function: $\frac{x^2}{1-x^6}$

Solution

Let $x^3 = t$

$\therefore 3x^2dx = dt$

$\Rightarrow \int \frac{x^2}{1-x^6}dx = \frac{1}{3}\int \frac{dt}{1-t^2}$

$= \frac{1}{3}\left[\frac{1}{2}\log\left|\frac{1+t}{1-t}\right|\right] + C$

$= \frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right| + C$

#422491

Topic: Fundamental Integrals

Integrate the function $\frac{x-1}{\sqrt{x^2-1}}$

Solution

$\int \frac{x-1}{\sqrt{x^2-1}}dx = \int \frac{x}{\sqrt{x^2-1}}dx - \int \frac{1}{\sqrt{x^2-1}}dx$ ....(1)

For $\int \frac{x}{\sqrt{x^2-1}}dx$, let $x^2 - 1 = t \Rightarrow 2xdx = dt$

$\therefore \int \frac{x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$

$= \frac{1}{2}\int t^{\frac{1}{2}}dt = \frac{1}{2}[2t^{\frac{1}{2}}] = \sqrt{t} = \sqrt{x^2-1}$

From (1), we obtain

$\int \frac{x-1}{\sqrt{x^2-1}}dx = \int \frac{x}{\sqrt{x^2-1}}dx - \int \frac{1}{\sqrt{x^2-1}}dx, \left[\because \int \frac{1}{\sqrt{x^2-a^2}}dt = \log|x + \sqrt{x^2-a^2}|\right]$

$= \sqrt{x^2-1} - \log|x + \sqrt{x^2-1}| + C$

#422620

Topic: Fundamental Integrals

Integrate the function $\frac{4x+1}{\sqrt{2x^2+x-3}}$

Solution

Let $4x + 1 = A\frac{d}{dx}(2x^2 + x - 3) + B$

$\Rightarrow 4x + 1 = A(4x + 1) + B$

$\Rightarrow 4x + 1 = 4Ax + A + B$

Equating the coefficients of x and constant term on both sides, we obtain

$4A = 4 \Rightarrow A = 1$

$A + B = 1 \Rightarrow B = 0$

Let $2x^2 + x - 3 = t$

$\therefore (4x + 1)dx = dt$

$\Rightarrow \int \frac{4x+1}{\sqrt{2x^2+x-3}}dx = \int \frac{1}{\sqrt{t}}dt$

$= 2\sqrt{t} + C = 2\sqrt{2x^2+x-3} + C$

#422891

Topic: Fundamental Integrals

Integrate the function $\frac{x+2}{\sqrt{x^2-1}}$

Solution

Let $x+2=A\frac{d}{dx}(x^2-1)+B$ ......(1)

$\Rightarrow x+2=A(2x)+B$

Equating the coefficients of x and constant term on both sides, we obtain

$2A=1\Rightarrow A=\frac{1}{2}$

$B=2$

From (1), we obtain

$(x+2)=\frac{1}{2}(2x)+2$

Then, $\int\frac{x+2}{\sqrt{x^2-1}}dx=\int\frac{\frac{1}{2}(2x)+2}{\sqrt{x^2-1}}dx$

$=\frac{1}{2}\int\frac{2x}{\sqrt{x^2-1}}dx+\int\frac{2}{\sqrt{x^2-1}}dx$ ......(2)

For $\frac{1}{2}\int\frac{2x}{\sqrt{x^2-1}}dx$, let $x^2-1=t\Rightarrow 2xdx=dt$

$\Rightarrow\frac{1}{2}\int\frac{2x}{\sqrt{x^2-1}}dx=\frac{1}{2}\int\frac{dt}{\sqrt{t}}$

$=\frac{1}{2}[2\sqrt{t}]=\sqrt{t}=\sqrt{x^2-1}$

And, $\int\frac{2}{\sqrt{x^2-1}}dx=2\int\frac{1}{\sqrt{x^2-1}}dx=2\log|x+\sqrt{x^2-1}|$

From equation (2), we obtain

$\int\frac{x+2}{\sqrt{x^2-1}}dx=\sqrt{x^2-1}+2\log|x+\sqrt{x^2-1}|+C$

#422901

Topic: Fundamental Integrals

Integrate the function $\frac{5x-2}{1+2x+3x^2}$

Solution

Let $5x-2=A\frac{d}{dx}(1+2x+3x^2)+B$

$\Rightarrow 5x-2=A(2+6x)+B$

Equating the coefficient of x and constant term on both sides, we obtain

$5=6A\Rightarrow A=\frac{5}{6}$

and $2A+B=-2\Rightarrow B=-\frac{11}{3}$

$\therefore 5x-2=\frac{5}{6}(2+6x)+\left(-\frac{11}{3}\right)$

$\Rightarrow\int\frac{5x-2}{1+2x+3x^2}dx=\int\frac{\frac{5}{6}(2+6x)-\frac{11}{3}}{1+2x+3x^2}dx$

Let $I_1=\int\frac{2+6x}{1+2x+3x^2}dx$ and $I_2=\int\frac{1}{1+2x+3x^2}dx$

$\therefore\int\frac{5x-2}{1+2x+3x^2}dx=\frac{5}{6}I_1-\frac{11}{3}I_2$ .... (1)

$I_1=\int\frac{2+6x}{1+2x+3x^2}dx$

Let $1+2x+3x^2=t$

$\Rightarrow(2+6x)dx=dt$

$\therefore I_1=\int\frac{dt}{t}$

$=\log|t|=\log|1+2x+3x^2|$ .....(2)

$I_2 = \int \frac{1}{1+2x+3x^2} dx$

$1+2x+3x^2+$ can be written as $1+3\left(x^2+\frac{2}{3}x\right)$.

Therefore,

$1+3\left(x^2+\frac{2}{3}x\right)$

$= 1+3\left(x^2+\frac{2}{3}x+\frac{1}{9}-\frac{1}{9}\right)$

$= 1+3\left(x+\frac{1}{3}\right)^2-\frac{1}{3}$

$= \frac{2}{3}+3\left(x+\frac{1}{3}\right)^2$

$= 3\left[\left(x+\frac{1}{3}\right)^2+\frac{2}{9}\right]$

$= 3\left[\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2\right]$

$I_2 = \frac{1}{3}\int \frac{1}{\left[\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2\right]} dx$

$= \frac{1}{3}\left[\frac{1}{\frac{\sqrt{2}}{3}}\tan^{-1}\left(\frac{x+\frac{1}{3}}{\frac{\sqrt{2}}{3}}\right)\right]$

$= \frac{1}{3}\left[\frac{3}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)\right]$

$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)$ ....(3)

Substituting equations (2) and (3) in equation (1), we obtain

$\int \frac{5x-2}{1+2x+3x^2} dx = \frac{5}{6}[\log|1+2x+3x^2|] - \frac{11}{3}\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)\right]+C$

$= \frac{5}{6}\log|1+2x+3x^2| - \frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+C$

#422990

Topic: Fundamental Integrals

Integrate the function $\frac{x+2}{\sqrt{x^2+2x+3}}$

Solution

$\int \frac{(x+2)}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}\int \frac{2(x+2)}{\sqrt{x^2+2x+3}}dx$

$= \frac{1}{2}\int \frac{2x+4}{\sqrt{x^2+2x+3}}dx$

$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx + \frac{1}{2}\int \frac{2}{\sqrt{x^2+2x+3}}dx$

$= \frac{1}{2}\int \frac{2x+2}{\sqrt{x^2+2x+3}}dx + \int \frac{1}{\sqrt{x^2+2x+3}}dx$

Let $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}}dx$ and $I_2 = \int \frac{1}{\sqrt{x^2+2x+3}}dx$

$\therefore \int \frac{x+2}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}I_1 + I_2$ ....(1)

Then, $I_1 = \int \frac{2x+2}{\sqrt{x^2+2x+3}}dx$

Let $x^2+2x+3 = t$

$\Rightarrow (2x+2)dx = dt$

$I_1 = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} = 2\sqrt{x^2+2x+3}$ .... (2)

$I_1 = \int \frac{1}{\sqrt{x^2+2x+3}}dx$

$\Rightarrow x^2+2x+3 = x^2+2x+1+2 = (x+1)^2+(\sqrt{2})^2$

$\therefore I_2 = \int \frac{1}{\sqrt{(x+1)^2+(\sqrt{2})^2}}dx = \log|(x+1)+\sqrt{x^2+2x+3}|$ .... (3)

Using equation (2) and (3) in (1), we obtain

$\int \frac{x+2}{\sqrt{x^2+2x+3}}dx = \frac{1}{2}[2\sqrt{x^2+2x+3}] + \log|(x+1)+\sqrt{x^2+2x+3}| + C$

$= \sqrt{x^2+2x+3} + \log|(x+1)+\sqrt{x^2+2x+3}| + C$

#423013

Topic: Special Integrals (Algebraic Functions)

$\int \frac{dx}{x^2+2x+2}$ equals

A $x\tan^{-1}(x+1)+C$

$\boxed{\text{B}}$ $\tan^{-1}(x+1)+C$

C $(x+1)\tan^{-1}x+C$

D $\tan^{-1}x+C$

Solution

$\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x^2+2x+1)+1}$

$= \int \frac{1}{(x+1)^2+(1)^2}dx$

$= \tan^{-1}(x+1)+C$

#423057

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{x}{(x^2+1)(x-1)}$

Solution

Let $\frac{x}{(x^2+1)(x-1)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{(x-1)}$

$\Rightarrow x = (Ax+B)(x-1) + C(x^2+1)$

$\Rightarrow x = Ax^2 - Ax + Bx - B + Cx^2 + C$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + C = 0$

$-A + B = 1$

$-B + C = 0$

On solving these equations, we obtain

$A = -\frac{1}{2}, B = \frac{1}{2}$, and $C = \frac{1}{2}$

From equation (1), we obtain

$$\therefore \frac{x}{(x^2+1)(x-1)} = \frac{\left(-\frac{1}{2}x + \frac{1}{2}\right)}{x^2+1} + \frac{\frac{1}{2}}{(x-1)}$$

$$\Rightarrow \int \frac{x}{(x^2+1)(x-1)} = -\frac{1}{2}\int \frac{x}{x^2+1}dx + \frac{1}{2}\int \frac{1}{x^2+1}dx + \frac{1}{2}\int \frac{1}{x-1}dx$$

$$= -\frac{1}{4}\int \frac{2x}{x^2+1}dx + \frac{1}{2}\tan^{-1}x + \frac{1}{2}\log|x-1| + C$$

Consider $\int \frac{2x}{x^2+1}dx$, let $(x^2+1) = t \Rightarrow 2xdx = dt$

$$\Rightarrow \int \frac{2x}{x^2+1}dx = \int \frac{dt}{t} = \log|t| = \log|x^2+1|$$

$$\therefore \int \frac{x}{(x^2+1)(x-1)} = -\frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + \frac{1}{2}\log|x-1| + C$$

$$= \frac{1}{2}\log|x-1| - \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x + C$$

#423209

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{2}{(1-x)(1+x^2)}$

Solution

Let $\frac{2}{(1-x)(1+x^2)} = \frac{A}{(1-x)} + \frac{Bx+C}{(1+x^2)}$

$\Rightarrow 2 = A(1+x^2) + (Bx+C)(1-x)$

$\Rightarrow 2 = A + Ax^2 + Bx - Bx^2 + C + Cx$

Equating the coefficient of x^2, x, and constant term, we obtain

$A - B = 0$

$B - C = 0$

$A + C = 2$

On solving these questions, we obtain

$A = 1$, $B = 1$, and $C = 1$

$$\therefore \frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x+1}{1+x^2}$$

$$\Rightarrow \frac{2}{(1-x)(1+x^2)}dx = \int \frac{1}{1-x}dx + \int \frac{x}{1+x^2}dx + \int \frac{1}{1+x^2}dx$$

$$= -\int \frac{1}{x-1}dx + \frac{1}{2}\int \frac{2x}{1+x^2}dx + \int \frac{1}{1+x^2}dx$$

$$= -\log|x-1| + \frac{1}{2}\log|1+x^2| + \tan^{-1}x + C$$

#423220

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{1}{x^4-1}$

Solution

$$\frac{1}{(x^4-1)}=\frac{1}{(x^2-1)(x^2+1)}=\frac{1}{(x+1)(x-1)(1+x^2)}$$

Let $\frac{1}{(x+1)(x-1)(1+x^2)}=\frac{A}{(x+1)}+\frac{B}{(x-1)}+\frac{Cx+D}{(x^2+1)}$

$\Rightarrow 1=A(x-1)(x^2+1)+B(x+1)(x^2+1)+(Cx+D)(x^2+1)$

$\Rightarrow 1=A(x^3+x-x^2-1)+B(x^3+x+x^2+1)+Cx^3+Dx^2-Cx-D$

$\Rightarrow 1=(A+B+C)x^3+(-A+B+D)x^2+(A+B-C)x+(-A+B-D)$

Equating the coefficient of x^3, x^2, x, and constant term, we obtain

$A+B+C=0$

$-A+B+D=0$

$A+B-C=0$

$-A+B-D=1$

On solving these equations, we obtain

$A=-\frac{1}{4}, B=\frac{1}{4}, C=0$, and $D=-\frac{1}{2}$

$\therefore \frac{1}{x^4-1}=\frac{-1}{4(x+1)}+\frac{1}{4(x-1)}-\frac{1}{2(x^2+1)}$

$\Rightarrow \int\frac{1}{x^4-1}dx=-\frac{1}{4}\log|x-1|+\frac{1}{4}\log|x-1|-\frac{1}{2}\tan^{-1}x+C$

$=\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\tan^{-1}x+C$

#423262

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$

Solution

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \frac{(4x^2+10)}{(x^2+3)(x^2+4)}$$

Let $\frac{4x^2+10}{(x^2+3)(x^2+4)} = \frac{Ax+B}{(x^2+3)} + \frac{Cx+D}{(x^2+4)}$

$\Rightarrow 4x^2+10 = (Ax+B)(x^2+4) + (Cx+D)(x^2+3)$

$4x^2+10 = Ax^3+4Ax+Bx^2+4B+Cx^3+3Cx+Dx^2+3D$

$4x^2+10 = (A+C)x^3+(B+D)x^2+(4A+3C)x+(4B+3D)$

Equating the coefficients of x^3, x^2, x, and constant term, we obtain

$A+C=0$

$B+D=4$

$4A+3C=0$

$4B+3D=10$

On solving these equations, we obtain

$A=0$, $B=2$, $C=0$, and $D=6$

$$\therefore \frac{4x^2+10}{(x^2+3)(x^2+4)} = \frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}$$

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \left(\frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}\right)$$

$$\Rightarrow \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx = \int\left\{1 + \frac{2}{(x^2+3)} - \frac{6}{(x^2+4)}\right\}dx$$

$$= \int\left\{1 + \frac{2}{x^2+(\sqrt{3})^2} - \frac{6}{x^2+2^2}\right\}$$

$$= x + 2\left(\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right) - 6\left(\frac{1}{2}\tan^{-1}\frac{x}{2}\right) + C$$

$$= x + \frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - 3\tan^{-1}\frac{x}{2} + C$$

#423274

Topic: Integration using Partial Fractions

$\int \frac{dx}{x(x^2+1)}$ equals

A $\log|x| - \frac{1}{2}\log(x^2+1) + C$

B $\log|x| + \frac{1}{2}\log(x^2+1) + C$

C $-\log|x| + \frac{1}{2}\log(x^2+1) + C$

D $\frac{1}{2}\log|x| + \log(x^2+1) + C$

Solution

Let $\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$\Rightarrow 1 = A(x^2+1) + (Bx+C)x$

Equating the coefficients of x^2, x, the constant term, we obtain

$A + B = 0, C = 0, A = 1$

On solving these equations, we obtain

$A = 1, B = -1,$ and $C = 0$

$\therefore \frac{1}{x(x^2+1)} = \frac{1}{x} + \frac{-x}{x^2+1}$

$\Rightarrow \int \frac{1}{x(x^2+1)} dx = \int \left\{ \frac{1}{x} - \frac{x}{x^2+1} \right\} dx$

$= \log|x| - \frac{1}{2}\log|x^2+1| + C$

#423680

Topic: Special Integrals (Exponential Functions)

Integrate the function $e^x(\sin x + \cos x)$

Solution

Let $I = \int e^x(\sin x + \cos x)dx$

Let $f(x) = \sin x$

$\Rightarrow f'(x) = \cos x$

$\therefore I = \int e^x\left\{f(x) + f'(x)\right\}dx$

$= e^x f(x) + C = e^x \sin x + C$

$\because \int e^x\left\{f(x) + f'(x)\right\}dx = e^x f(x) + C = e^x sinx + C$

#423681

Topic: Special Integrals (Exponential Functions)

Integrate the function $\frac{xe^x}{(1+x)^2}$

Solution

Let $I = \int \frac{xe^x}{(1+x)^2} dx = \int e^x \left\{ \frac{x}{(1+x)^2} \right\} dx$

$= \int e^x \left\{ \frac{1+x-1}{(1+x)^2} \right\} dx$

$= \int e^x \left\{ \frac{1}{1+x} - \frac{1}{(1+x)^2} \right\} dx$

Let $f(x) = \frac{1}{1+x} \Rightarrow f'(x) = \frac{-1}{(1+x)^2}$

$\Rightarrow \int \frac{xe^x}{(1+x)^2} dx = \int e^x\left\{f(x) + f'(x)\right\}dx$

We know that, $\int e^x\left\{f(x) + f'(x)\right\}dx = e^x f(x) + C$

$\therefore I = \frac{e^x}{1+x} + C$

#423682

Topic: Special Integrals (Exponential Functions)

Integrate the function $e^x\left(\frac{1+\sin x}{1+\cos x}\right)$

Solution

Given, $e^x\left(\frac{1+\sin x}{1+\cos x}\right)$

$= e^x\left(\frac{\sin^2\frac{x}{2}+\cos\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$

$= \frac{e^x\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$

$= \frac{1}{2}e^x\cdot\left(\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}}\right)^2$

$= \frac{1}{2}e^x\left(1+\tan\frac{x}{2}\right)^2$

$= \frac{1}{2}e^x\left[1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}\right]$

$= \frac{1}{2}e^x\left[\sec^2\frac{x}{2}+2\tan\frac{x}{2}\right]$

$\frac{e^x(1+\sin x)dx}{(1+\cos x)} = e^x\left[\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right]$ .... (1)

Let $\tan\frac{x}{2} = f(x) \Rightarrow f'(x) = \frac{1}{2}\sec^2\frac{x}{2}$

It is known that, $\int e^x\left\{f(x)+f'(x)\right\}dx = e^xf(x)+C$

From equation (1), we obtain

$\int\frac{e^x(1+\sin x)}{(1+\cos x)}dx = e^x\tan\frac{x}{2}+C$

#423683

Topic: Special Integrals (Exponential Functions)

Integrate the function $e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)$

Solution

Let $I = \int e^x\left[\frac{1}{x}-\frac{1}{x^2}\right]dx$

now let $\frac{1}{x} = f(x) \Rightarrow f'(x) = \frac{-1}{x^2}$

Also we know that, $\int e^x\left\{f(x)+f'(x)\right\}dx = e^xf(x)+C$

$\therefore I = \frac{e^x}{x}+C$

#423684

Topic: Special Integrals (Exponential Functions)

Integrate the function $\frac{(x-3)e^x}{(x-1)^3}$

Solution

Let $I = \int e^x\left\{\frac{x-3}{(x-1)^3}\right\}dx = \int e^x\left\{\frac{x-1-2}{(x-1)^2}\right\}dx$

$= \int e^x\left\{\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}\right\}dx$

Let $f(x) = \frac{1}{(x-1)^2} \Rightarrow f'(x) = \frac{-2}{(x-1)^3}$

Now know that, $\int e^x\left\{f(x) + f'(x)\right\}dx = e^x f(x) + C$

$\therefore I = \frac{e^x}{(x-1)^2} + C$

#423849

Topic: Special Integrals (Exponential Functions)

$\int e^x \sec x(1 + \tan x)dx$

A $e^x \cos x + C$

[✓ B] $e^x \sec x + C$

C $e^x \sin x + C$

D $e^x \tan x + C$

Solution

Let $I = \int e^x \sec x(1 + \tan x)dx = \int e^x(\sec x + \sec x \tan x)dx$

Also, let $\sec x = f(x) \Rightarrow \sec x \tan x = f'(x)$

We know that, $\int e^x\left\{f(x) + f'(x)\right\} = e^x f(x) + C$

$\therefore I = e^x \sec x + C$

#423854

Topic: Special Integrals (Irrational Functions)

Integrate the function $\sqrt{4 - x^2}$

Solution

Let $I = \int \sqrt{4 - x^2}dx = \int \sqrt{(2)^2 - (x)^2}dx$

We know that, $\int \sqrt{a^2 - x^2}dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$\therefore I = \frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2} + C$

$= \frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2} + C$

#423859

Topic: Special Integrals (Irrational Functions)

Integrate the function $\sqrt{1 - 4x^2}$

Solution

Let $I = \int \sqrt{1-4x^2}dx = \int \sqrt{(1)^2-(2x)^2}dx$

Let $2x = t \Rightarrow 2dx = dt$

$\therefore I = \frac{1}{2}\int \sqrt{(1)^2-(t)^2}dt$

We know that, $\int \sqrt{(a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$\Rightarrow I = \frac{1}{2}\left[\frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1}t\right] + C$

$= \frac{t}{4}\sqrt{1-t^2} + \frac{1}{4}\sin^{-1}t + C$

$= \frac{2x}{4}\sqrt{1-4x^2} + \frac{1}{4}\sin^{-1}2x + C$

$= \frac{x}{2}\sqrt{1-4x^2} + \frac{1}{4}\sin^{-1}2x + C$

#423862

Topic: Special Integrals (Irrational Functions)

Integrate the function $\sqrt{x^2+4x+6}$

Solution

$x^2+4x+6 = (x+2)^2+2 = (x+2)^2+(\sqrt{2})^2$

Let $x+2 = t \Rightarrow dx = dt$

Also we know that, $\int \sqrt{a^2+x^2}dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log|x+\sqrt{x^2+a^2}| + C$

$\therefore \int \sqrt{x^2+4x+6}dx = \int \sqrt{(x+2)^2+(\sqrt{2})^2}dx$

$= \int \sqrt{t^2+(\sqrt{2})^2}dt = \frac{t}{2}\sqrt{t^2+2} + \frac{2}{2}\log(t+\sqrt{t^2+2}) + C$

$= \frac{x+2}{2}\sqrt{x^2+4x+6} + \log|x+2+\sqrt{x^2+4x+6}| + C$

#423869

Topic: Special Integrals (Irrational Functions)

Integrate the function $\sqrt{x^2+4x+1}$

Solution

Let $I = \int \sqrt{x^2+4x+6}dx$

$= \int \sqrt{x^2+4x+4+2}dx$

$= \int \sqrt{(x^2+4x+4)+2}dx$

$= \int \sqrt{(x+2)^2+(\sqrt{2})^2}dx$

We know that, $\int \sqrt{x^2+a^2}dx = d\frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log|x+\sqrt{x^2+a^2}| + C$

$\therefore I = \frac{(x+2)}{2}\sqrt{x^2+4x+6} + \frac{2}{2}\log|(x+2)+\sqrt{x^2+4x+6}| + C$

$= \frac{(x+2)}{2}\sqrt{x^2+4x+6} + \log|(x+2)+\sqrt{x^2+4x+6}| + C$

#423872

Topic: Special Integrals (Irrational Functions)

Integrate the function $\sqrt{1-4x-x^2}$

Solution

Let $I = \int \sqrt{1 - 4x - x^2}dx$

$= \int \sqrt{1 - (x^2 + 4x + 4 - 4)}dx$

$= \int \sqrt{1 + 4 - (x + 2)^2}dx$

$= \int \sqrt{(\sqrt{5})^2 - (x + 2)^2}dx$

We know that, $\int \sqrt{a^2 - x^2}dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$\therefore\ I = \frac{(x + 2)}{2}\sqrt{1 - 4x - x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x + 2}{\sqrt{5}}\right) + C$

#423875

Topic: Special Integrals (Irrational Functions)

Integrate the function $\sqrt{x^2 + 4x - 5}$

Solution

Let $I = \int \sqrt{x^2 + 4x - 5}dx$

$= \int \sqrt{(x^2 + 4x + 4) - 9}dx$

$= \int \sqrt{(x + 2)^2 - (3)^2}dx$

We know that, $\int \sqrt{x^2 - a^2}dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log|x + \sqrt{x^2 - a^2}| + C$

$\therefore\ I = \frac{(x + 2)}{2}\sqrt{x^2 + 4x - 5} - \frac{9}{2}\log|(x + 2) + \sqrt{x^2 + 4x - 5}| + C$

#423881

Topic: Special Integrals (Irrational Functions)

Integrate the function $\sqrt{1 + 3x - x^2}$

Solution

Let $I = \int \sqrt{1 + 3x - x^2}dx$

$= \int \sqrt{1 - \left(x^2 - 3x + \frac{9}{4} - \frac{9}{4}\right)}dx$

$= \int \sqrt{\left(1 + \frac{9}{4}\right) - \left(x - \frac{3}{2}\right)^2}dx$

$= \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2}dx$

We know that, $\int \sqrt{a^2 - x^2}dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$\therefore\ I = \frac{x - \frac{3}{2}}{2}\sqrt{1 + 3x - x^2} + \frac{3}{4 \times 2}\sin^{-1}\left(\frac{x - \frac{3}{2}}{\frac{\sqrt{13}}{2}}\right) + C$

$= \frac{2x - 3}{4}\sqrt{1 + 3x - x^2} + \frac{13}{8}\sin^{-1}\left(\frac{2x - 3}{\sqrt{13}}\right) + C$

#423889

Topic: Special Integrals (Irrational Functions)

Integrate the function $\sqrt{x^2 + 3x}$

Solution

Let $I = \int \sqrt{x^2 + 3x}dx$

$= \int \sqrt{x^2 + 3x + \frac{9}{4} - \frac{9}{4}}dx$

$= \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2}dx$

We known that, $\int \sqrt{x^2 - a^2}dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log|x + \sqrt{x^2 - a^2}| + C$

$\therefore I = \frac{\left(x + \frac{3}{2}\right)}{2}\sqrt{x^2 + 3x} - \frac{\frac{9}{4}}{2}\log\left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x}\right| + C$

$= \frac{(2x + 3)}{4}\sqrt{x^2 + 3x} - \frac{9}{8}\log\left|\left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x}\right| + C$

#423892

Topic: Special Integrals (Irrational Functions)

Integrate the function $\sqrt{1 + \frac{x^2}{9}}$

Solution

Let $I = \int \sqrt{1 + \frac{x^2}{9}}dx = \frac{1}{3}\int \sqrt{9 + x^2}dx = \frac{1}{3}\sqrt{(3)^2 + x^2}dx$

We know that, $\int \sqrt{x^2 + a^2}dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log|x + \sqrt{x^2 + a^2}| + C$

$\therefore I = \frac{1}{3}\left[\frac{x}{2}\sqrt{x^2 + 9} + \frac{9}{2}\log|x + \sqrt{x^2 + 9}|\right] + C$

$= \frac{x}{6}\sqrt{x^2 + 9} + \frac{3}{2}\log|x + \sqrt{x^2 + 9}| + C$

#424463

Topic: Special Integrals (Irrational Functions)

$\int \sqrt{1 + x^2}dx$ is equal to

$\boxed{\text{A}}$ $\frac{x}{2}\sqrt{1 + x^2} + \frac{1}{2}log| + \sqrt{1 + x^2}| + C$

B $\frac{2}{3}(1 + x^2)^{\frac{2}{3}} + C$

C $\frac{2}{3}x(1 + x^2)^{\frac{3}{2}} + C$

D $\frac{x^2}{2}\sqrt{1 + x^2} + \frac{1}{2}x^2log|x + \sqrt{1 + x^2}| + C$

Solution

We know that, $\int \sqrt{a^2 + x^2}dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\log|x + \sqrt{x^2 + a^2}| + C$

$\therefore \int \sqrt{1 + x^2}dx = \frac{x}{2}\sqrt{1 + x^2} + \frac{1}{2}\log|x + \sqrt{1 + x^2}| + C$

Hence, the correct Answer is A.

#424464

Topic: Special Integrals (Irrational Functions)

$\int \sqrt{x^2 - 8x + 7}dx$ is equal to

A $\frac{1}{2}(x - 4)\sqrt{x^2 - 8x + 7} + 9\log|x - 4 + \sqrt{x^2 - 8x + 7}| + C$

B $\frac{1}{2}(x + 4)\sqrt{x^2 - 8x + 7} + 9\log|x + 4 + \sqrt{x^2 - 8x + 7}| + C$

C $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-3\sqrt{2}\log|x-4+\sqrt{x^2-8x+7}|+C$

$\boxed{\text{D}}$ $\frac{1}{2}(x-4)\sqrt{x^2-8x+7}-\frac{9}{2}\log|x-4+\sqrt{x^2-8x+7}|+C$

Solution

Let $I=\int\sqrt{x^2-8x+7}dx$

$=\int\sqrt{(x^2-8x+16)-9}dx$

$=\int\sqrt{(x-4)^2-(3)^2}dx$

We know that, $\int\sqrt{x^2-a^2}dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log|x+\sqrt{x^2-a^2}|+C$

$\therefore\ I=\frac{(x-4)}{2}\sqrt{x^2-8x+7}-\frac{9}{2}\log|(x-4)+\sqrt{x^2-8x+7}|+C$

Hence, the correct Answer is D.

#424641

Topic: Definite Integrals

Evaluate the definite integral $\int_0^{\frac{x}{4}}\sin 2x dx$

Solution

Let $I=\int_0^{\frac{x}{4}}\sin 2x dx$

$\Rightarrow \int\sin 2x dx=\left(\frac{-\cos 2x}{2}\right)=F(x)$

By second fundamental theorem of calculus, we obtain

$I=F\left(\frac{\pi}{4}\right)-F(0)$

$=-\frac{1}{2}\left[\cos 2\left(\frac{\pi}{4}\right)-\cos 0\right]$

$=-\frac{1}{2}\left[\cos\left(\frac{\pi}{2}\right)-\cos 0\right]$

$=-\frac{1}{2}[0-1]=\frac{1}{2}$

#424642

Topic: Definite Integrals

Evaluate the definite integral $\int_0^{\frac{\pi}{2}}\cos 2x dx$

Solution

Let $I=\int_0^{\frac{\pi}{2}}\cos 2x dx$

$\Rightarrow \int\cos 2x dx=\left(\frac{\sin 2x}{2}\right)=F(x)$

By second fundamental theorem of calculus, we obtain

$I=F\left(\frac{\pi}{2}\right)-F(0)$

$=\frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right)-\sin 0\right]$

$=\frac{1}{2}[\sin\pi-\sin 0]=\frac{1}{2}[0-0]=0$

#424643

Topic: Definite Integrals

Evaluate the definite integral $\int_4^5 e^x dx$

Solution

Let $I = \int_4^5 e^x dx$

$\Rightarrow \int e^x dx = e^x = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F(5) - F(4)$

$= e^5 - e^4$

$= e^4(e-1)$

#424644

Topic: Definite Integrals

Evaluate the definite integral $\int_0^{\frac{\pi}{4}} \tan x dx$

Solution

Let $I = \int_0^{\frac{\pi}{4}} \tan x dx$

$\Rightarrow \int \tan x dx = -\log|\cos x| = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F\left(\frac{\pi}{4}\right) - F(0)$

$= -\log|\cos\frac{\pi}{4}| + \log|\cos 0|$

$= -\log\left|\frac{1}{\sqrt{2}}\right| + \log|1|$

$= -\log(2)^{-\frac{1}{2}} = \frac{1}{2}\log 2$

#424649

Topic: Definite Integrals

Evaluate the definite integral $\int_0^{\frac{\pi}{2}} \cos^2 x dx$

Solution

Let $I = \int_0^{\frac{\pi}{2}} \cos^2 x dx$

$\Rightarrow \int \cos^2 x dx = \int\left(\frac{1+\cos 2x}{2}\right)dx = \frac{x}{2} + \frac{\sin 2x}{4} = \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) = F(x)$

By second fundamental theorem of calculus, we obtain

$I = \left[F\left(\frac{\pi}{2}\right) - F(0)\right]$

$= \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{\sin\pi}{2}\right) - \left(0 + \frac{\sin 0}{2}\right)\right]$

$= \frac{1}{2}\left[\frac{\pi}{2} - 0 - 0 - 0\right] = \frac{\pi}{4}$

#424655

Topic: Definite Integrals

Evaluate the definite integral $\int_0^{\frac{\pi}{4}}(2\sec^2 x + x^3 + 2)dx$

Solution

Let $I = \int_0^{\frac{\pi}{4}}(2\sec^2x + x^3 + 2)dx$

$\Rightarrow \int(2\sec^2x + x^3 + 2)dx = 2\tan x + \frac{x^4}{4} + 2x = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F\left(\frac{\pi}{4}\right) - F(0)$

$= \left\{\left(2\tan\frac{\pi}{4} + \frac{1}{4}\left(\frac{\pi}{4}\right)^4 + 2\left(\frac{\pi}{4}\right)\right) - (2\tan 0 + 0 + 0)\right\}$

$= 2\tan\frac{\pi}{4} + \frac{\pi^4}{4^5} + \frac{\pi}{2}$

$= 2 + \frac{\pi}{2} + \frac{\pi^4}{1024}$

#424656

Topic: Definite Integrals

Evaluate the definite integral $\int_0^{\pi}\left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right)dx$

Solution

Let $I = \int_0^{\pi}\left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right)dx$

$= -\int_0^{\pi}\left(\cos^2\frac{x}{2} - sin^2\frac{x}{2}\right)dx$

$= -\int_0^{\pi}\cos x dx$

$\Rightarrow \int\cos x dx = \sin x = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F(\pi) - F(0)$

$= \sin\pi - \sin 0 = 0$

#424763

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral $\int_{-5}^{5}|x + 2|dx$

Solution

Let $I = \int_{-5}^{5}|x + 2|dx$

It can be seen that $(x + 2) \le 0$ on $[-5, -2]$ and $(x + 2) \ge 0$ on $[-2, 5]$.

$\therefore I = \int_{-5}^{-2} -(x + 2)dx + \int_{-5}^{-2}(x + 2)dx$

$= -\left[\frac{x^2}{2} + 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-5}^{-2}$

$= -\left[\frac{(-2)^2}{2} + 2(-2) - \frac{(-5)^2}{2} - 2(-5)\right] + \left[\frac{(5)^2}{2} + 2(5) - \frac{(-2)^2}{2} - 2(-2)\right]$

$= -\left[-2 - 4 - \frac{25}{2} + 10\right] + \left[\frac{25}{2} + 10 - 2 + 4\right]$

$= -2 + 4 + \frac{25}{2} - 10 + \frac{25}{2} + 10 - 2 + 4 = 29$

#424771

Topic: Properties of Definite Integral

By using the properties of definite integrals, evaluate the integral $\int_2^8|x - 5|dx$

Solution

Let $I = \int_2^8 | x - 5 | \, dx$

It can be seen that $(x - 5) \le 0$ on $[2, 5]$ and $(x - 5) \ge 0$ on $[5, 8]$.

$\therefore \; I = \int_2^5 -(x - 5)dx + \int_1^8 (x - 5)dx$

$$= -\left[\frac{x^2}{2} - 5x\right]_2^5 + \left[\frac{x^2}{2} - 5x\right]_2^8$$

$$= -\left[\frac{25}{2} - 25 - 2 + 10\right] + \left[32 - 40 - \frac{25}{2} + 25\right]$$

$= 9$

#425545

Topic: Fundamental Integrals

Integrate the function : $\dfrac{1}{\sqrt{x + a} + \sqrt{(x + b)}}$

Solution

$$\frac{1}{\sqrt{x + a} + \sqrt{(x + b)}} = \frac{1}{\sqrt{x + a} + \sqrt{(x + b)}} \times \frac{\sqrt{x + a} - \sqrt{x + b}}{\sqrt{x + a} - \sqrt{x + b}}$$

$$= \frac{\sqrt{x + a} - \sqrt{x + b}}{(x + a) - (x + b)} = \frac{(\sqrt{x + a} - \sqrt{x + b})}{a - b}$$

$$\therefore \int \frac{1}{\sqrt{x - a} - \sqrt{x + b}} dx = \frac{1}{a - b} \int (\sqrt{x + a} - \sqrt{x + b}) dx$$

$$= \frac{1}{(a - b)} \left[\frac{(x + a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x + b)^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$= \frac{2}{3(a - b)} \left[(x + a)^{\frac{3}{2}} - (x + b)^{\frac{3}{2}}\right] + C$$

#425573

Topic: Integration using Partial Fractions

Integrate the function $\dfrac{5x}{(x + 1)(x^2 + 9)}$

Solution

Let $\dfrac{5x}{(x+1)(x^2+9)} = \dfrac{A}{(x+1)} + \dfrac{Bx+C}{(x^2+9)}$ .... (1)

$\Rightarrow 5x = A(x^2+9) + (Bx+C)(x+1)$

$\Rightarrow 5x = Ax^2 + 9A + Bx^2 + Bx + Cx + C$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + B = 0$

$B + C = 5$

$9A + C = 0$

On solving these equations, we obtain

$A = -\dfrac{1}{2}$, $B = \dfrac{1}{2}$, and $C = \dfrac{9}{2}$

From equation (1), we obtain

$$\frac{5x}{(x+1)(x^2+9)} = \frac{-1}{2(x+1)} + \frac{\frac{x}{2}+\frac{9}{2}}{(x^2+9)}$$

$$\int \frac{5x}{(x+1)(x^2+9)}dx = \int\left\{\frac{-1}{2(x+1)} + \frac{(x+9)}{2(x^2+9)}\right\}dx$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{2}\int\frac{x}{x^2+9}dx + \frac{9}{2}\int\frac{1}{x^2+9}dx$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\int\frac{2x}{x^2+9}dx + \frac{9}{2}\int\frac{1}{x^2+9}dx$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\log|x^2+9| + \frac{9}{2}\cdot\frac{1}{3}\tan^{-1}\frac{x}{3}$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\log(x^2+9) + \frac{3}{2}\tan^{-1}\frac{x}{3} + C$$

#425576

Topic: Fundamental Integrals

Integrate the function $\dfrac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}$

Solution

$$\frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} = \frac{e^{4\log x}(e^{\log x}-1)}{e^{2\log x}(e^{\log x}-1)}$$

$$= e^{2\log x} = e^{\log x^2} = x^2$$

$$\therefore \int\frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}dx = \int x^2 dx = \frac{x^3}{3} + C$$

#425582

Topic: Special Integrals (Algebraic Functions)

Integrate the function $\dfrac{e^x}{(1+e^x)(2+e^x)}$

Solution

$$\frac{e^x}{(1+e^x)(2+e^x)}$$

Let $e^x = t \Rightarrow e^x dx = dt$

$$\Rightarrow \int\frac{e^x}{(1+e^x)(2+e^x)}dx = \int\frac{dt}{(t+1)(t+2)}$$

$$= \int\left[\frac{1}{(t+1)} - \frac{1}{(t+2)}\right]dt$$

$$= \log|t+1| - \log|t+2| + C$$

$$= \log\left|\frac{t+1}{t+2}\right| + C$$

$$= \log\left|\frac{1+e^x}{2+e^x}\right| + C$$

#425583

Topic: Integration using Partial Fractions

Integrate the function $\frac{1}{(x^2+1)(x^2+4)}$

Solution

$$\frac{1}{(x^2+1)(x^2+4)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+4)}$$

$\Rightarrow 1 = (Ax+B)(x^2+4) + (Cx+D)(x^2+1)$

$\Rightarrow 1 = Ax^3 + 4Ax + Bx^2 + Cx^3 + Cx + Dx^2 + D$

Equating the coefficients of x^3, x^2, x, and constant term, we obtain

$A + C = 0$

$B + D = 0$

$4A + C = 0$

$4B + D = 1$

On solving these equations, we obtain

$A = 0, B = \frac{1}{3}, C = 0$, and $D = -\frac{1}{3}$

From equation (1), we obtain

$$\frac{1}{(x^2+1)(x^2+4)} = \frac{1}{3(x^2+1)} - \frac{1}{3(x^2+4)}$$

$$\therefore \int \frac{1}{(x^2+1)(x^2+4)}dx = \frac{1}{3}\int \frac{1}{x^2+1}dx - \frac{1}{3}\int \frac{1}{x^2+4}dx$$

$$= \frac{1}{3}\tan^{-1}x - \frac{1}{3}\cdot\frac{1}{2}\tan^{-1}\frac{x}{2} + C$$

$$= \frac{1}{3}\tan^{-1}x - \frac{1}{6}\tan^{-1}\frac{x}{2} + C$$

#425601

Topic: Special Integrals (Exponential Functions)

Integrate the function $\frac{2+\sin 2x}{1+\cos 2x}e^x$

Solution

$$I = \int\left(\frac{2+\sin 2x}{1+\cos 2x}\right)e^x$$

$$= \int\left(\frac{2+2\sin x\cos x}{2\cos^2 x}\right)e^x$$

$$= \int\left(\frac{1+\sin x\cos x}{\cos^2 x}\right)e^x$$

$$= \int(\sec^2 x + \tan x)e^x$$

Let $f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$

$\therefore I = \int(f(x) + f'(x)]e^x dx$

$= e^x f(x) + C$

$= e^x \tan x + C$

#425653

Topic: Changing Variable

Prove $\int_1^3 \frac{dx}{x^2(x+1)} = \frac{2}{3} + \log\frac{2}{3}$

Solution

Let $I = \int_1^3 \frac{dx}{x^2(x+1)}$

Also, let $\frac{1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$

$\Rightarrow 1 = Ax(x+1) + B(x+1) + C(x^2)$

$\Rightarrow 1 = Ax^2 + Ax + Bx + B + Cx^2$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + C = 0$

$A + B = 0$

$B = 1$

On solving these equations, we obtain

$A = 1$, $C = 1$, and $B = 1$

$\therefore \frac{1}{x^2(x+1)} = \frac{-1}{x} + \frac{1}{x^2} + \frac{1}{(x+1)}$

$\Rightarrow I = \int_1^3 \left\{ -\frac{1}{x} + \frac{1}{x^2} + \frac{1}{(x+1)} \right\} dx$

$= \left[-\log x - \frac{1}{x} + \log(x+1) \right]_1^3$

$= \left[\log\left(\frac{x+1}{x}\right) - \frac{1}{x} \right]_1^3$

$= \log\left(\frac{4}{3}\right) - \frac{1}{3} - \log\left(\frac{2}{1}\right) + 1$

$= \log 4 - \log 3 - \log 2 + \frac{2}{3}$

$= \log 2 - \log 3 + \frac{2}{3}$

$= \log\left(\frac{2}{3}\right) + \frac{2}{3}$

Hence, the given result is proved.

#425678

Topic: Special Integrals (Algebraic Functions)

$\int \frac{dx}{e^x + e^{-x}}$ is equal to

A ☑	$\tan^{-1}(e^x) + C$
B	$\tan^{-1}(e^{-x}) + C$
C	$\log(e^x - e^{-x}) + C$
D	$\log(e^x + e^{-x}) + C$

Solution

Let $I = \int \frac{dx}{e^x + e^{-x}} dx = \int \frac{e^x}{e^{2x} + 1} dx$

Also, let $e^x = t \Rightarrow e^x dx = dt$

$\therefore I = \int \frac{dt}{1 + t^2}$

$= \tan^{-1} t + C$

$= \tan^{-1}(e^x) + C$

Hence, the correct Answer is A.

#425681

Topic: Definite Integrals

The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$ is

A 1

$\boxed{\textbf{B}}$ 0

C −1

D $\frac{\pi}{4}$

Solution

Let $I = \int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$

$\Rightarrow I = \int_0^1 \tan^{-1}\left(\frac{x-(1-x)}{1+x(1-x)}\right)dx$

$\Rightarrow I = \int_0^1 [\tan^{-1}x - \tan^{-1}(1-x)]dx$ .... (1)

$\Rightarrow I = \int_0^1 [\tan^{-1}(1-x) - \tan^{-1}(1-1+x)]dx$

$\Rightarrow I = \int_0^1 [\tan^{-1}(1-x) - \tan^{-1}(x)]dx$

$\Rightarrow I = \int_0^1 [\tan^{-1}(1-x) - \tan^{-1}(x)]dx$ .... (2)

Adding (1) and (2), we obtain

$2I = \int_0^1 (\tan^{-1}x + \tan^{-1}(1-x) - \tan^{-1}(1-x) - \tan^{-1}x)dx = 0$

$\Rightarrow I = 0$

Hence, the correct Answer is B.

#422974

Topic: Integration using Partial Fractions

Integrate the function $\frac{6x+7}{\sqrt{(x-5)(x-4)}}$

Solution

$\frac{6x+7}{\sqrt{(x-5)(x-4)}} = \frac{6x+7}{\sqrt{x^2-9x+20}}$

Let $6x+7 = A\frac{d}{dx}(x^2-9x+20)+B$

$\Rightarrow 6x+7 = A(2x-9)+B$

Equating the coefficients of x and constant term, we obtain

$2A = 6 \Rightarrow A = 3$

& $-9A+B = 7 \Rightarrow B = 34$

$\therefore 6x+7 = 3(2x-9)+34$

$\Rightarrow \int\frac{6x+7}{\sqrt{x^2-9x+20}} = \int\frac{3(2x-9)+34}{\sqrt{x^2-9x+20}}dx$

$= 3\int\frac{2x-9}{\sqrt{x^2-9x+20}}dx + 34\int\frac{1}{\sqrt{x^2-9x+20}}dx$

Let $I_1 = \int\frac{2x-9}{\sqrt{x^2-9x+20}}dx$ and $I_2 = \frac{1}{\sqrt{x^2-9x+20}}dx$

$\therefore \int\frac{6x+7}{\sqrt{x^2-9x+20}} = 3I_1 + 34I_2$ ....(1)

Then,

$I_1 = \int\frac{2x-9}{\sqrt{x^2-9x+20}}dx$

Let $x^2-9x+20 = t$

$\Rightarrow (2x-9)dx = dt$

$\Rightarrow I_1 = \frac{dt}{\sqrt{t}}$

$I_1 = 2\sqrt{t}$

$I_1 = 2\sqrt{x^2-9x+20}$ ......(2)

and $I_2 = \int\frac{1}{\sqrt{x^2-9x+20}}dx$

$x^2-9x+20$ can be written as $x^2-9x+20+\frac{81}{4}-\frac{81}{4}$

Therefore,

$x^2-9x+20+\frac{81}{4}-\frac{81}{4}$

$= \left(x-\frac{9}{2}\right)^2-\frac{1}{4}$

$= \left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2$

$\Rightarrow I_2 = \int\frac{1}{\sqrt{\left(x-\frac{9}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx$

$\Rightarrow I_2 = \log\left|\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right|$ ....(3)

Substituting equations (2) and (3) in (1), we obtain

$\int\frac{6x+7}{\sqrt{x^2-9x+20}}dx = 3[2\sqrt{x^29x+20}] + 34\log\left[\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right]+C$

$= 6\sqrt{x^2-9x+20} + 34\log[2\sqrt{x^29x+20}] + 34\log\left[\left(x-\frac{9}{2}\right)+\sqrt{x^2-9x+20}\right]+C$

#422984

Topic: Integration using Partial Fractions

Integrate the function $\frac{x+2}{\sqrt{4x-x^2}}$

Solution

Let $x+2 = A\frac{d}{dx}(4x-x^2)+B$

$\Rightarrow x+2 = A(4-2x)+B$

Equating the coefficients of x and constant term on both sides, we obtain

$-2A = 1 \Rightarrow A = \frac{1}{2}$

& $4A+B = 2 \Rightarrow B = 4$

$\Rightarrow (x+2) = -\frac{1}{2}(4-2x)+4$

$\therefore \int \frac{x+2}{\sqrt{4x-x^2}}dx = \int \frac{-\frac{1}{2}(4-2x)+4}{\sqrt{4x-x^2}}dx$

$= -\frac{1}{2}\int \frac{4-2x}{\sqrt{4x-x^2}}dx + 4\int \frac{1}{\sqrt{4x-x^2}}dx$

Let $I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}}dx$ and $I_2 = \int \frac{1}{\sqrt{4x-x^2}}dx$

$\therefore \int \frac{x+2}{\sqrt{4x-x^2}}dx = -\frac{1}{2}I_1 + 4I_2$ .... (1)

Then, $I_1 = \int \frac{4-2x}{\sqrt{4x-x^2}}dx$

Let $4x-x^2 = t$

$\Rightarrow (4-2x)dx = dt$

$\Rightarrow I_1 = \int \frac{1}{\sqrt{t}}dt = 2\sqrt{t} = 2\sqrt{4x-x^2}$ .... (2)

$I_2 = \int \frac{1}{\sqrt{4x-x^2}}dx$

$\Rightarrow 4x-x^2 = -(-4x+x^2)$

$= (-4x+x^2+4-4)$

$= 4-(x-2)^2$

$= (2)^2-(x-2)^2$

$\therefore I_2 = \int \frac{1}{\sqrt{(2)^2-(x-2)^2}}dx = \sin^{-1}\left(\frac{x-2}{2}\right)$ .... (3)

Using equations (2) and (3) in (1), we obtain

$\int \frac{x+2}{\sqrt{4x-x^2}}dx = -\frac{1}{2}(2\sqrt{4x-x^2}) + 4\sin^{-1}\left(\frac{x-2}{2}\right)+C$

$= -\sqrt{4x-x^2} + 4\sin^{-1}\left(\frac{x-2}{2}\right)+C$

#422999

Topic: Integration using Partial Fractions

Integrate the function $\frac{x+3}{x^2-2x-5}$

Solution

Let $(x+3) = A\frac{d}{dx}(x^2 - 2x - 5) + B$

$(x+3) = A(2x-2) + B$

Equating the coefficients of x and constant term on both sides, we obtain

$2A = 2 \Rightarrow A = \frac{1}{2}$

& $-2A + B = 3 \Rightarrow B = 4$

$\therefore (x+3) = \frac{1}{2}(2x-2) + 4$

$\Rightarrow \int \frac{x+2}{x^2-2x-5}dx = \int \frac{\frac{1}{2}(2x-2)+4}{x^2-2x-5}dx$

$= \frac{1}{2}\int \frac{2x-2}{x^2-2x-5}dx + 4\int \frac{1}{x^2-2x-5}dx$

Let $I_1 = \int \frac{2x-2}{x^2-2x-5}dx$ and $I_2 = \int \frac{1}{x^2-2x-5}dx$

$\therefore \int \frac{x+3}{(x^2-2x-5)}dx = \frac{1}{2}I_1 + 4I_2$ ...... (1)

Then, $I_1 = \int \frac{2x-2}{x^2-2x-5}dx$

Let $x^2 - 2x - 5 = t$

$\Rightarrow (2x-2)dx = dt$

$\Rightarrow I_1 = \int \frac{dt}{t} = \log|t| = \log|x^2-2x-5|$ .... (2)

$I_2 = \int \frac{1}{x^2-2x-5}dx$

$= \int \frac{1}{(x^2-2x-5)-6}dx$

$= \int \frac{1}{(x-1)^2+(\sqrt{6})^2}dx$

$= \frac{1}{2\sqrt{6}}\log\left(\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right)$ ....(3)

Substituting (2) and (3) in (1), we obtain

$$\int \frac{x+3}{x^2-2x-5}dx = \frac{1}{2}\log|x^2-2x-5| + \frac{4}{2\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$

$$= \frac{1}{2}\log|x^2-2x-5| + \frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right| + C$$

#423008

Topic: Integration using Partial Fractions

Integrate the function $\frac{5x+3}{\sqrt{x^2+4x+10}}$

Solution

Let $5x + 3 = A\frac{d}{dx}(x^2 + 4x + 10) + B$

$\Rightarrow 5x + 3 = A(2x + 4) + B$

Equating the coefficients of x and constant term, we obtain

$2A = 5 \Rightarrow A = \frac{5}{2}$

& $4A + B = 3 \Rightarrow B = -7$

$\therefore 5x + 3 = \frac{5}{2}(2x + 4) - 7$

$\Rightarrow \int \frac{5x + 3}{\sqrt{x^2 + 4x + 10}} dx = \int \frac{\frac{5}{2}(2x + 4) - 7}{\sqrt{x^2 + 4x + 10}} dx$

$= \frac{5}{2}\int \frac{2x + 4}{\sqrt{x^2 + 4x + 10}} dx - 7\int \frac{1}{\sqrt{x^2 + 4x + 10}} dx$

Let $I_1 = \int \frac{2x + 4}{\sqrt{x^2 + 4x + 10}} dx$ and $I_2 = \int \frac{1}{\sqrt{x^2 + 4x + 10}} dx$

$\therefore \int \frac{5x + 3}{\sqrt{x^2 + 4x + 10}} dx = \frac{5}{2}I_1 - 7I_2$ .... (1)

Then, $I_1 = \int \frac{2x + 4}{\sqrt{x^2 + 4x + 10}} dx$

Let $x^2 + 4x + 10 = t$

$\therefore (2x + 4)dx = dt$

$\Rightarrow I_1 = \int \frac{dt}{t} = 2\sqrt{t} = 2\sqrt{x^2 + 4x + 10}$ ......(2)

$I_2 = \int \frac{1}{\sqrt{x^2 + 4x + 10}} dx$

$= \int \frac{1}{\sqrt{(x^2 + 4x + 4) + 6}} dx$

$= \int \frac{1}{(x + 2)^2 + (\sqrt{6})^2} dx$

$= \log|(x + 2)\sqrt{x^2 + 4x + 10}|$ ....(3)

Using equation (2) and (3) in (1), we obtain

$\int \frac{5x + 3}{\sqrt{x^2 + 4x + 10}} dx = \frac{5}{2}[2\sqrt{x^2 + 4x + 10}] - 7\log|(x + 2) + \sqrt{x^2 + 4x + 10}| + C$

$= 5\sqrt{x^2 + 4x + 10} - 7\log|(x + 2) + \sqrt{x^2 + 4x + 10}| + C$

#423019

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{x}{(x + 1)(x + 2)}$

Solution

Let $\frac{x}{(x + 1)(x + 2)} = \frac{A}{(x + 1)} + \frac{B}{(x + 2)}$

$\Rightarrow x = A(x + 2) + B(x + 1)$

Equating the coefficients of x and constant term, we obtain

$A + B = 1, 2A + B = 0$

On solving, we obtain

$A = -1$ and $B = 2$

$\therefore \frac{x}{(x + 1)(x + 2)} dx = \frac{-1}{(x + 1)} + \frac{2}{(x + 2)}$

$\Rightarrow \int \frac{x}{(x + 1)(x + 2)} dx = \int \frac{-1}{(x + 1)} + \frac{2}{(x + 2)} dx$

$= -\log|x + 1| + 2\log|x + 2| + C$

$= \log(x + 2)^2 - \log|x + 1| + C$

$= \log\frac{(x + 2)^2}{(x + 1)} + C$

#423021

Topic: Integration using Partial Fractions

Integrate the rational function: $\frac{1}{x^2-9}$

Solution

Let $\frac{1}{(x+3)(x-3)}=\frac{A}{(x+3)}+\frac{B}{(x-3)}$

$\Rightarrow 1=A(x-3)+B(x+3)$

Equating the coefficients of x and constant term, we obtain

$A+B=0, 3A+3B=1$

On solving, we obtain

$A=-\frac{1}{6}$ and $B=\frac{1}{6}$

$\therefore \frac{1}{(x+3)(x-3)}=\frac{-1}{6(x+3)}+\frac{1}{6(x-3)}$

$\Rightarrow \int\frac{1}{(x^2-9)}dx=\int\left(\frac{-1}{6(x+3)}+\frac{1}{6(x-3)}\right)dx$

$=-\frac{1}{6}\log|x+3|+\frac{1}{6}\log|x-3|+C$

$=\frac{1}{6}\log\left|\frac{(x-3)}{(x+3)}\right|+C$

#423038

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{3x-1}{(x-1)(x-2)(x-3)}$

Solution

Let $\frac{3x-1}{(x-1)(x-2)(x-3)}=\frac{A}{(x-1)}+\frac{B}{(x-2)}+\frac{C}{(x-3)}$

$\Rightarrow 3x-1=A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)$ ....(1)

Substituting $x=1, 2,$ and 3 respectively in equation (1), we obtain $A=1, B=5,$ and $C=4$

$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)}=\frac{1}{(x-1)}-\frac{5}{(x-2)}+\frac{4}{(x-3)}$

$\Rightarrow \int\frac{3x-1}{(x-1)(x-2)(x-3)}dx=\int\left\{\frac{1}{(x-1)}-\frac{5}{(x-2)}+\frac{4}{(x-3)}\right\}dx$

$=\log|x-1|-5\log|x-2|+4\log|x-3|+C$

#423042

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{x}{(x-1)(x-2)(x-3)}$

Solution

Let $\frac{x}{(x-1)(x-2)(x-3)}=\frac{A}{(x-1)}+\frac{B}{(x-2)}+\frac{C}{(x-3)}$

$\Rightarrow x=A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)$ .... (1)

Substituting $x=1, 2,$ and 3 respectively in equation (1), we obtain

$A=\frac{1}{2}, B=-2,$ and $C=\frac{3}{2}$

$\therefore \frac{x}{(x-1)(x-2)(x-3)}=\frac{1}{2(x-1)}-\frac{2}{(x-2)}+\frac{3}{(x-3)}$

$\Rightarrow \int\frac{x}{(x-1)(x-2)(x-3)}dx=\int\left\{\frac{1}{2(x-1)}-\frac{2}{(x-2)}+\frac{3}{(x-3)}\right\}dx$

$=\frac{1}{2}\log|x-1|-2\log|x-2|+\frac{3}{2}\log|x-3|+C$

#423044

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{2x}{x^2+3x+2}$

Solution

Let $\frac{2x}{x^2+3x+2}=\frac{A}{(x+1)}+\frac{B}{(x+2)}$

$\Rightarrow 2x=A(x+2)+B(x+1)$ .... (1)

Substituting $x=1$ and 2 in equation (1), we obtain

$A=2$ and $B=4$

$\therefore \frac{2x}{(x+1)(x+2)}=\frac{-2}{(x+1)}+\frac{4}{(x+2)}$

$\Rightarrow \int\frac{2x}{(x+1)(x+2)}dx=\int\left\{\frac{4}{(x+2)}-\frac{2}{(x+1)}\right\}dx$

$=4\log|x+2|-2\log|x+1|+C$

#423046

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{1-x^2}{x(1-2x)}$

Solution

$\frac{1-x^2}{x(1-2x)}=\frac{1}{2}+\frac{1}{2}\left(\frac{2-x}{x(1-2x)}\right)$

Let $\frac{2-x}{x(1-2x)}=\frac{A}{x}+\frac{B}{(1-2x)}$

$\Rightarrow (2-x)=A(1-2x)+Bx$ ....(1)

Substituting $x=0$ and $\frac{1}{2}$ in equation (1), we obtain

$A=2$ and $B=3$

$\therefore \frac{2-x}{x(1-2x)}=\frac{2}{x}+\frac{3}{1-2x}$

Substituting in equation (1), we obtain

$\frac{1-x^2}{x(1-2x)}=\frac{1}{2}+\frac{1}{2}\left\{\frac{2}{x}+\frac{3}{(1-2x)}\right\}$

$\Rightarrow \int\frac{1-x^2}{x(1-2x)}dx=\int\left\{\frac{1}{2}+\frac{1}{2}\left(\frac{2}{x}+\frac{3}{1-2x}\right)\right\}dx$

$=\frac{x}{2}+\log|x|+\frac{3}{2(-2)}\log|1-2x|+C$

$=\frac{x}{2}+\log|x|-\frac{3}{4}\log|1-2x|+C$

#423068

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{2x-3}{(x^2-1)(2x+3)}$

Solution

$$\frac{2x-3}{(x^2-1)(2x+3)} = \frac{2x-3}{(x+1)(x-1)(2x+3)}$$

Let $\frac{2x-3}{(x+1)(x-2)(2x+3)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(2x+3)}$

$\Rightarrow (2x-3) = A(x-1)(2x+3) + B(x+1)(2x+3) + C(x+1)(x-1)$

$\Rightarrow (2x-3) = A(2x^2+x-3) + B(2x^2+5x+3) + C(x^2-1)$

$\Rightarrow (2x-3) = (2A+2B+C)x^2 + (A+5B)x + (-3A+3B-C)$

Equating the coefficients of x^2 and x and constant term, we obtain

$B = -\frac{1}{10}, A = \frac{5}{2}$, and $C = -\frac{24}{5}$

$\therefore \frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{5}{2(x+1)} - \frac{1}{10(x-1)} - \frac{24}{5(2x+3)}$

$\Rightarrow \int \frac{2x-3}{(x^2-1)(2x+3)}dx = \frac{5}{2}\int \frac{1}{(x+1)}dx - \frac{1}{10}\int \frac{1}{x-1}dx - \frac{24}{5}\int \frac{1}{(2x+3)}dx$

$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{24}{5\times 2}\log|2x+3|$

$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{12}{5}\log|2x+3| + C$

#423071

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{5x}{(x+1)(x^2-4)}$

Solution

$$\frac{5x}{(x+1)(x^2-4)} = \frac{5x}{(x+1)(x+2)(x-2)}$$

Let $\frac{5x}{(x+1)(x+2)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)} + \frac{C}{(x-2)}$

$\Rightarrow 5x = A(x+2)(x-2) + B(x+1)(x-2) + C(x+1)(x+2)$ .... (1)

Substituting $x = 1, 2,$ and -2 respectively in equation (1), we obtain

$A = \frac{5}{3}, B = -\frac{5}{2}$, and $C = \frac{5}{6}$

$\therefore \frac{5x}{(x+1)(x+2)(x-2)} = \frac{5}{3(x+1)} - \frac{5}{2(x+2)} + \frac{5}{6(x-2)}$

$\Rightarrow \int \frac{5x}{(x+1)(x^2-4)}dx = \frac{5}{3}\int \frac{1}{(x+1)}dx - \frac{5}{2}\int \frac{1}{(x+2)}dx + \frac{5}{6}\int \frac{1}{(x-2)}dx$

$= \frac{5}{3}\log|x+1| - \frac{5}{2}\log|x+2| + \frac{5}{6}\log|x-2| + C$

#423208

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{x^3+x+1}{x^2-1}$

Solution

$\frac{x^3+x+1}{x^2-1} = x + \frac{2x+1}{x^2-1}$

Let $\frac{2x+1}{x^2-1} = \frac{A}{(x+1)} + \frac{B}{(x-1)}$

$\Rightarrow 2x+1 = A(x-1) + B(x+1)$ .... (1)

Substituting $x = 1$ and -1 in equation (1), we obtain

$A = \frac{1}{2}$ and $B = \frac{3}{2}$

$\therefore \frac{x^3+x+1}{x^2-1} = x + \frac{1}{2(x+1)} + \frac{3}{2(x-1)}$

$\Rightarrow \int \frac{x^3+x+1}{x^2-1} dx = \int x dx + \frac{1}{2}\int \frac{1}{x+1} dx + \frac{3}{2}\int \frac{1}{(x-1)} dx$

$= \frac{x^2}{2} + \frac{1}{2}\log|x+1| + \frac{3}{2}\log|x-1| + C$

#423228

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{1}{x(x^n+1)}$

Solution

$\frac{1}{x(x^n+1)}$

Multiply numerator and denominator by x^{n-1}

$\frac{1}{x(x^n+1)} = \frac{x^{n-1}}{x^{n-1}x(x^n+1)} = \frac{x^{n-1}}{x^n(x^n+1)}$

Let $x^n = t \Rightarrow x^{n-1}dx = dt$

$\therefore \int \frac{1}{x(x^n+1)} dx = \int \frac{x^{n-1}}{x^n(x^n+1)} dx = \frac{1}{n}\int \frac{1}{t(t+1)} dt$

Let $\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{(t+1)}$

$\Rightarrow 1 = A(1+t) + Bt$ .... (1)

Substituting $t = 0$, and -1 respectively in equation (1), we obtain

$A = 1$ and $B = -1$

$\therefore \frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{(1+t)}$

$\Rightarrow \int \frac{1}{x(x^n+1)} dx = \frac{1}{n}\int\left\{\frac{1}{t} - \frac{1}{(t+1)}\right\} dx$

$= \frac{1}{n}[\log|t| - \log|t+1|] + C$

$= -\frac{1}{n}[\log|x^n| - \log|x^n+1|] + C$

$= \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + C$

#423261

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{\cos x}{(1-\sin x)(2-\sin x)}$

Solution

Let $\sin x = t \Rightarrow \cos x dx = dt$

$$\therefore \int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx = \int \frac{dt}{(1-t)(2-t)}$$

Let $\frac{1}{(1-t)(2-t)} = \frac{A}{(1-t)} + \frac{B}{(2-t)}$

$\Rightarrow 1 = A(2-t) + B(1-t)$ .... (1)

Substituting $t = 2$ and then $t = 1$ in equation (1), we obtain

$A = 1$ and $B = -1$

$$\therefore \frac{1}{(1-t)(2-t)} = \frac{1}{(1-t)} - \frac{1}{(2-t)}$$

$$\Rightarrow \int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx = \int \left\{\frac{1}{1-t} - \frac{1}{(2-t)}\right\} dt$$

$$= -\log|1-t| + \log|2-t| + C$$

$$= \log\left|\frac{2-t}{1-t}\right| + C$$

$$= \log\left|\frac{2-\sin x}{1-\sin x}\right| + C$$

#423264

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{2x}{(x^2+1)(x^2+3)}$

Solution

Let $x^2 = t \Rightarrow 2xdx = dt$

$$\therefore \int \frac{2x}{(x^2+1)(x^2+3)} dx = \int \frac{dt}{(t+1)(t+3)} \text{ ....(1)}$$

Let $\frac{1}{(t+1)(t+3)} = \frac{A}{(t+1)} + \frac{B}{(t+3)}$

$\Rightarrow 1 = A(t+3) + b(t+1)$ .... (1)

Substituting $t = -3$ and $t = -1$ in equation (1) we obtain

$A = \frac{1}{2}$ and $B = -\frac{1}{2}$

$$\therefore \frac{1}{(t+1)(t+3)} = \frac{1}{2(t+1)} - \frac{1}{2(t+3)}$$

$$\Rightarrow \int \frac{2x}{(x^2+1)(x^2+3)} dx = \int \left\{\frac{1}{2(t+1)} - \frac{1}{2(t+3)}\right\} dt$$

$$= \frac{1}{2}\log|(t+1)| - \frac{1}{2}\log|t+3| + C$$

$$= \frac{1}{2}\log\left|\frac{t+1}{t+3}\right| + C$$

$$= \frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right| + C$$

#423266

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{1}{x(x^4-1)}$

Solution

$\frac{1}{x(x^4-1)}$

Multiplying numerator and denominator by x^3, we obtain

$\frac{1}{x(x^4-1)} = \frac{x^3}{x^4(x^4-1)}$

$\therefore \int \frac{1}{x(x^4-1)}dx = \int \frac{x^3}{x^4(x^4-1)}dx$

Let $x^4 = t \Rightarrow 4x^3dx = dt$

$\therefore \int \frac{x^3}{x^4(x^4-1)}dx = \frac{1}{4}\int \frac{dt}{t(t-1)}$

Let $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{(t-1)}$

$\Rightarrow 1 = A(t-1) + Bt$ .... (1)

Substituting $= 0$ and 1 in (1), we obtain

$A = -1$ and $B = 1$

$\Rightarrow \frac{1}{t(t-1)} = \frac{-1}{t} + \frac{t}{t-1}$

$\Rightarrow \int \frac{1}{x(x^4-1)}dx = \frac{1}{4}\int\left\{\frac{-1}{t} + \frac{1}{t-1}\right\}dt$

$= \frac{1}{4}[-\log|t| + \log|t-1|] + C$

$= \frac{1}{4}\log\left|\frac{t-1}{t}\right| + C$

$= \frac{1}{4}\log\left|\frac{x^4-1}{x^4}\right| + C$

#423268

Topic: Integration using Partial Fractions

Integrate the rational function $\frac{1}{(e^x-1)}$

Solution

$\frac{1}{(e^x-1)}$

Let $e^x = t \Rightarrow e^xdx = dt$

$\Rightarrow \int \frac{1}{e^x-1}dx = \int \frac{1}{t-1} \times \frac{dt}{t} = \int \frac{1}{t(t-1)}dt$

Let $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$

$1 = A(t-1) + Bt$ .... (1)

Substituting $t = 1$ and $t = 0$ in equation (1), we obtain

$A = -1$ and $B = 1$

$\therefore \frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$

$\Rightarrow \int \frac{1}{t(t-1)}dt = \log\left|\frac{t-1}{t}\right| + C$

$= \log\left|\frac{e^x-1}{e^x}\right| + C$

#423271

Topic: Integration using Partial Fractions

$\int \frac{xdx}{(x-1)(x-2)}$ equals

A $\log\left|\frac{(x-1)^2}{x-2}\right| + C$

$\boxed{B}$ $\log\left|\frac{(x-2)^2}{x-1}\right|+C$

C $\log\left|\left(\frac{x-1}{x-2}\right)^2\right|+C$

D $\log|(x-1)(x-2)|+C$

Solution

Let $\frac{x}{(x-1)(x-2)}=\frac{A}{(x-1)}+\frac{B}{(x-2)}$

$\Rightarrow x=A(x-2)+B(x-1)$ .... (1)

Substituting $x=1$ and 2 in (1), we obtain

$A=-1$ and $B=2$

$\therefore \frac{x}{(x-1)(x-2)}=-\frac{1}{(x-1)}+\frac{2}{(x-2)}$

$\Rightarrow \int\frac{x}{(x-1)(x-2)}dx=\int\left\{\frac{-1}{(x-1)}+\frac{2}{(x-2)}\right\}dx$

$=-\log|x-1|+2\log|x-2|+C$

$=\log\left|\frac{(x-2)^2}{x-1}\right|+C$

#424466

Topic: Definite Integral as a Limit of Sum

Evaluate the given definite integrals as limit of sums:

$\int_a^b xdx$

Solution

We know that,

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}[f(a) + f(a+h) + ... + f(a+(n-)h)]'$$

where $h = \frac{b-a}{n}$

Here, $a = a$, $b = b$, and $f(x) = x$

$$\therefore \int_a^b xdx = (b-a)\lim_{n\to\infty}\frac{1}{n}[a + (a+h)....(a+2h)....a+(n-1)h]$$

$$= (b-a)\lim_{n\to\infty}\frac{1}{n}[(a + a + \text{ntimes} a + ... + a) + (h + 2h + 3h + ... + (n-1)h)]$$

$$= (b-a)\lim_{n\to\infty}\frac{1}{n}[na + h(1 + 2 + 3 + ... + (n-1))]$$

$$= (b-a)\lim_{n\to\infty}\frac{1}{n}\left[na + h\left\{\frac{(n-1)(n)}{2}\right\}\right]$$

$$= (b-a)\lim_{n\to\infty}\frac{1}{n}\left(na + \frac{n(n-1)h}{2}\right]$$

$$= (b-a)\lim_{n\to\infty}\frac{n}{n}\left[a + \frac{(n-1)h}{2}\right]$$

$$= (b-a)\lim_{n\to\infty}\left[a + \frac{(n-1)h}{2}\right]$$

$$= (b-a)\lim_{n\to\infty}\left[a + \frac{(n-1)(b-a)}{2n}\right]$$

$$= (b-a)\lim_{n\to\infty}\left[a + \frac{(1-\frac{1}{n})(b-a)}{2}\right]$$

$$= (b-a)\left[a + \frac{(b-a)}{2}\right]$$

$$= (b-a)\left[\frac{2a + b - a}{2}\right]$$

$$= \frac{(b-a)(b+a)}{2}$$

$$= \frac{1}{2}(b^2 - a^2)$$

#424467

Topic: Definite Integral as a Limit of Sum

Evaluate the given definite integrals as limit of sums:

$$\int_0^5 (x+1)dx$$

Solution

Let $I = \int_0^5 (x+1)dx$

It is known that,

$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}[f(a)+f(a+h)....f(a+(n-1)h]$, where $h = \frac{b-a}{n}$

Here, $a = 0$, $b = 5$, and $f(x) = (x+1)$

$$\Rightarrow h = \frac{5-0}{n} = \frac{5}{n}$$

$$\therefore \int_0^5(x+1)dx = (5-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+...+f\left((n-1)\frac{5}{n}\right)\right]$$

$$= 5\lim_{n\to\infty}\frac{1}{n}\left[1+\left(\frac{5}{n}+1\right)+...\left\{1+\left(\frac{5(n+1)}{n}\right)\right\}\right]$$

$$= 5\lim_{n\to\infty}\frac{1}{n}\left[(1+1+1 ntimes.....1)+\left[\frac{5}{n}+2\cdot\frac{5}{n}+3\cdot\frac{5}{n}+...(n-1)\frac{5}{n}\right]\right]$$

$$= 5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\{1+2+3....(n-1)\}\right]$$

$$= 5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\cdot\frac{(n-1)n}{2}\right]$$

$$= 5\lim_{n\to\infty}\frac{1}{n}\left[n+\frac{5(n-1)}{2}\right]$$

$$= 5\lim_{n\to\infty}\left[1+\frac{5}{2}\left(1+\frac{1}{n}\right)\right]$$

$$= 5\left[1+\frac{5}{2}\right]$$

$$= 5\left[\frac{7}{2}\right] = \frac{35}{2}$$

#424468

Topic: Definite Integral as a Limit of Sum

Evaluate the given definite integrals as limit of sums:

$\int_2^3 x^2 dx$

Solution

It is known that,

$$\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}[f(a)+f(a+h)+f(a+2h)......f\{a+(n-1)h\}]'$$

where $h = \frac{b-a}{n}$

Here, $a = 2$, $b = 3$, and $f(x) = x^2$

$$\therefore \int_2^3 x^2 dx = (3-2\lim_{n\to\infty}\frac{1}{n}\left[f(2)+f\left(2+\frac{1}{n}\right)+f\left(2+\frac{2}{n}\right)....f\left\{2+(n-1)\frac{1}{n}\right\}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[(2)^2+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+....\left(2+\frac{(n-1)}{n}\right)^2\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[2^2+\left\{2^2+\left(\frac{1}{n}\right)^2+2.2\cdot\frac{1}{n}\right\}+...+\left\{(2)^2+\frac{(n-1)^2}{n^2}+2.2\cdot\frac{(n-1)}{n}\right\}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[(2^2+......ntimes+2^2)+\left\{\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2+.....+\left(\frac{n-1}{n}\right)^2\right\}+2.2\cdot\left\{\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+....+\frac{(n-1)}{n}\right\}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[4n+\frac{1}{n^2}\left\{1^2+2^2+3^2....+(n-1)^2\right\}+\frac{4}{n}\{1+2+...+(n-1)\}\right]$$

=\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [4n+\frac {1}{n^2}\left \{\frac {n(n-1)(2n-1)}{6}\right \}+\frac {4}{n}\left \{\frac {n(n-1)}{2}\right \}\right]

=\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [4n+\frac {n\left (1-\frac {1}{n}\right)\left (2-\frac {1}{n}\right)}{6}+\frac {4n-4}{2}\right]

=\displaystyle \lim_{n\rightarrow \infty}\left [4+\frac {1}{6}\left (1-\frac {1}{n}\right)\left (2-\frac {1}{n}\right)+2-\frac {2}{n}\right]

\displaystyle =4+\frac {2}{6}+2=\frac {19}{3}

#424469

Topic: Definite Integral as a Limit of Sum

Evaluate the given definite integrals as limit of sums:

\displaystyle \int_1^4(x^2-x)dx

Solution

Let I=\int_1^4(x^2-x)dx

=\int_1^4x^2dx-\int_1^4xdx

Let I=I_1-I_2, where I_1=\int_1^4x^2dx and I_2=\int_1^4x dx ....(1)

It is known that,

\int_a^bf(x)dx=(b-a)\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}[f(a)+f(a+h)+........f(a+(n-1)h], where h=\dfrac {b-a}{n}

For I_1=\int_1^4x^2dx,

a=1, b=4, and f(x)=x^2

\therefore h=\dfrac {4-1}{n}=\dfrac {3}{n}

I_1=\int_1^4dx=(4-1)\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}[f(1)+f(1+h)+.....+f(1+(n-1)h)]

=3\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [I^2+\left (1+\frac {3}{n}\right)^2+\left (1+2\cdot \frac {3}{n}\right)^2+...\left (1+\frac {(n-1)3}{n}\right)^2\right]

=3\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [1^2+\left \{1^2+\left (\frac {3}{n}\right)^2+2\cdot \frac {3}{n}\right \}+....+\left \{1^2+\left (\frac {(n-1)3}{n}\right)^2+\frac {2\cdot (n-1)\cdot 3}{n}\right \}\right]

=3\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [(1^2+\underset {n times}{....}+1^2)+\left (\frac {3}{n}\right)^2\left \{1^2+2^2+....+(n-1)^2\right \}+2\cdot \frac {3}{n}\left \{1+2+....+(n-1)\right \}\right]

=3\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [n+\frac {9}{n^2}\left \{\frac {(n-1)(n)(2n-1)}{6}\right \}+\frac {6}{n}\left \{\frac {(n-1)(n)}{2}\right \}\right]

=3\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [n+\frac {9n}{6}\left (1-\frac {1}{n}\right)\left (2-\frac {1}{n}\right)+\frac {6n-6}{2}\right]

=3\displaystyle \lim_{n\rightarrow \infty}\left [1+\frac {9}{6}\left (1-\frac {1}{n}\right)\left (2-\frac {1}{n}\right)+3-\frac {3}{n}\right]

=3[1+3+3]=3[7]

I_1=21 ....(2)

For I_2=\int_1^4xdx,

a=1, b=4, and f(x)=x

\Rightarrow h=\dfrac {4-1}{n}=\dfrac {3}{n}

\therefore I_2=(4-1)\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}[f(1)+f(1+h)+....f(a+(n-1)h)]

=3\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}[1+(1+h)+....+(1+(n-1)h]

=3\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [1+\left (1+\frac {3}{n}\right)+....+\left \{1+(n-1)\frac {3}{n}\right \}\right]

=3\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [(1+\underset {n times}{1+....}+1)+\frac {3}{n}(1+2+....+(n-1))\right]

=3\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [n+\frac {3}{n}\left \{\frac {(n-1)n}{2}\right \}\right]

=3\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [1+\frac {3}{2}\left (1-\frac {1}{n}\right)\right]

=3\left [1+\dfrac {3}{2}\right]=3\left [\dfrac {5}{2}\right]

I_2=\dfrac {15}{2} .... (3)

From equations (2) and (3), we obtain

\displaystyle I=I_1-I_2=21-\dfrac {15}{2}=\dfrac {27}{2}

#424500

Topic: Definite Integral as a Limit of Sum

Evaluate the given definite integrals as limit of sums:

\displaystyle \int_{-1}^1e^xdx

Solution

Let I=\int_{-1}^1e^xdx

We know that

\displaystyle \int_a^b f(x)dx=(b-a)\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}[f(a)+f(a+h) ....f(a+(n-1)h)], where h=\cfrac {b-a}{n}

Here, a=-1, b=1, and f(x)=e^x

\therefore \displaystyle h=\frac {1+1}{n}=\frac {2}{n}

\therefore I=(1+1)\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [f(-1)+f\left (-1+\frac {2}{n}\right)+f\left (-1+2\cdot \frac {2}{n}\right)+ ....+f\left (-1+\frac {(n-1)2}{n}\right)\right]

=2\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [e^{-1}+e^{\left (-1+\cfrac {2}{n}\right)}+e^{\left (-1+2\cdot \cfrac {2}{n} \right)}+....e^{\left (-1+(n-1)\cfrac {2}{n} \right) }\right]

=2\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [e^{-1}\left \{1+e^{\frac {2}{n}}+e^{\frac {4}{n}}+e^{\frac {6}{n}}+e^{(n-1)\frac {2}{n}}\right \}\right]

which forms a G.P.

\displaystyle =2\lim _{ n\rightarrow \infty } \frac { e^{ -1 } }{ n } \left[\frac { { \left(e^{ \cfrac { 2 }{ n } } \right) }^{ n }-1 }{ e^{ \frac { 2 }{ n } }-1 } \right]   (Sum of n terms of G.P.)

=e^{-1}\times 2\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [\frac {e^2-1}{e^{\frac {2}{n}}-1}\right]

\displaystyle =(e^{ 2 }-1)2e^{ -1 }\lim _{ n\rightarrow \infty } \frac { 1 }{ n(\cfrac { 2 }{ n }) } \left[\frac { 1 }{ \frac { e^{ \frac { 2 }{ n } }-1 }{ \cfrac { 2 }{ n } } } \right]

=e^{-1}\left [\dfrac {2(e^2-1)}{2}\right]   \left[\lim _{ h\rightarrow 0 } \left(\cfrac { e^{ h }-1 }{ h } \right) =1 \right]

=\cfrac {e^2-1}{e}

=e-\cfrac {1}{e}

#424636

Topic: Definite Integral as a Limit of Sum

Evaluate the given definite integrals as limit of sums:

\displaystyle \int_0^4(x+e^{2x})dx

Solution

\int_a^bf(x)dx=(b-a)\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}[f(a)+f(a+h)+...+f(a+(n-1)h)], where h=\dfrac {b-a}{n}

Here, a=0, b=4, and f(x)=x+e^{2x}

\therefore h=\dfrac {4-0}{n}=\dfrac {4}{n}

\Rightarrow \int_0^4(x+e^{2x})dx=(4-0)\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}[f(0)+f(h)+f(2h)+...+f((n-1)h)]

=4\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}[(0+e^0)+(h+e^{2h})+(2h+e^{2\cdot 2h})+....+\left \{(m-1)h+e^{2(n-1)h}\right \}]

=4\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}[1+(h+e^{2h})+(2h+e^{4h})+...+\left \{(m-1)h+e^{2(n-1)h}\right \}]

=4\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}[\left \{h+2h+3h+....+(n-1)h\right \}+(1+e^{2h}+e^{4h}+...+e^{2(n-1)h})]

=4\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [h\left \{1+2+...(n-1)\right \}+\left (\frac {2^{2hn}-1}{e^{2h}-1}\right)\right]

=4\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [\frac {h(n-1)n)}{2}+\left (\frac {e^{2hn}-1}{e^{2h}-1}\right)\right]

=\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [\frac {4}{n}\cdot \frac {(n-1)n}{2}+\left (\frac {e^8-1}{e^{\frac {8}{n}}-1}\right)\right]

=4(2)+4\displaystyle \lim_{n\rightarrow \infty}\frac {(e^8-1}{\left (\frac {e^{\frac {8}{n}}-1}{\frac {8}{n}}\right)8}

=8+\dfrac {4\cdot (e^8-1)}{8}, \left (\because \displaystyle \lim_{n\rightarrow \infty}\dfrac {e^x-1}{x}=1\right)

\displaystyle =8+\frac {e^8-1}{2}=\frac {15+e^8}{2}

#424645

Topic: Changing Variable

Evaluate the definite integral \displaystyle \int_{\tfrac {\pi}{6}}^{\tfrac {\pi}{4}}co\sec x dx

Solution

Let \displaystyle I=\int_{\tfrac {\pi}{6}}^{\tfrac {\pi}{4}}co\sec x dx

\Rightarrow\displaystyle \int co\sec x dx=\log |co\sec x-\cot x|=F(x)

By second fundamental theorem of calculus, we obtain

\displaystyle I=F\left (\cfrac {\pi}{4}\right)-F\left (\cfrac {\pi}{6}\right)

\displaystyle =\log |co\sec \cfrac {\pi}{4}-\cot \cfrac {\pi}{4}|-\log |co\sec \cfrac {\pi}{6}-\cot \cfrac {\pi}{6}|

\displaystyle =\log|\sqrt 2-1|-\log|2-\sqrt 3|

\displaystyle =\log \left (\cfrac {\sqrt 2-1}{2-\sqrt 3}\right)

#424646

Topic: Changing Variable

Evaluate the definite integral \displaystyle \int_0^1\frac {dx}{\sqrt {1-x^2}}

Solution

Let I=\displaystyle \int_0^1\frac {dx}{\sqrt {1-x^2}}

\Rightarrow \displaystyle \int \frac {dx}{\sqrt {1-x^2}}=\sin^{-1}x=F(x)

By second fundamental theorem of calculus, we obtain

I=F(1)-F(0)

=\sin^{-1}(1)-\sin^{-1}(0)

=\dfrac {\pi}{2}-0=\dfrac {\pi}{2}

#424647

Topic: Changing Variable

Evaluate the definite integral \displaystyle \int_0^1\frac {dx}{1+x^2}

Solution

Let I=\displaystyle \int_0^1\frac {dx}{1+x^2}

\Rightarrow \displaystyle \int \frac {dx}{1+x^2}=\tan^{-1}x=F(x)

By second fundamental theorem of calculus, we obtain

I=F(1)-F(0)

=\tan^{-1}(1)-\tan^{-1}(0)=\dfrac {\pi}{4}

#424648

Topic: Changing Variable

Evaluate the definite integral \displaystyle \int_2^3\frac {dx}{x^2-1}

Solution

Let \displaystyle I=\int_2^3\frac {dx}{x^2-1}

\Rightarrow \displaystyle \int \frac {dx}{x^2-1}=\frac {1}{2}\log \left |\frac {x-1}{x+1}\right |=F(x)

By second fundamental theorem of calculus, we obtain

I=F(3)-F(2)

\displaystyle =\frac {1}{2}\left [\log\left |\frac {3-1}{3+1}\right |-\log \left |\frac {2-1}{2+1}\right |\right]

\displaystyle =\frac {1}{2}\left [\log \left |\frac {2}{4}\right |-\log\left |\frac {1}{3}\right |\right]

\displaystyle =\frac {1}{2} \left [\log \frac {1}{2}-\log \frac {1}{3}\right]

\displaystyle=\frac {1}{2}\left [\log \frac {3}{2}\right]

#424650

Topic: Changing Variable

Evaluate the definite integral \displaystyle \int_2^3\frac {xdx}{x^2+1}

Solution

Let \displaystyle I=\int_2^3\frac {xdx}{x^2+1}

\displaystyle \int \frac {x}{x^2+1}dx=\frac {1}{2}\int \frac {2x}{x^2+1}dx=\frac {1}{2}\log (1+x^2)=F(x)

By second fundamental theorem of calculus, we obtain

I=F(3)-F(2)

\displaystyle =\frac {1}{2}[\log (1+(3)^2)-\log (1+(2)^2)]

\displaystyle =\frac {1}{2}[\log (10)-\log (5)]

\displaystyle =\frac {1}{2}\log \left (\frac {10}{5}\right)=\frac {1}{2}\log 2

#424652

Topic: Changing Variable

Evaluate the definite integral \displaystyle \int_0^1\frac {2x+3}{5x^2+1}dx

Solution

Let \displaystyle I=\int_0^1\frac {2x+3}{5x^2+1}dx

\Rightarrow \displaystyle \int \frac {2x+3}{5x^2+1}dx=\frac {1}{5}\int \frac {5(2x+3)}{5x^2+1}dx

\displaystyle =\frac {1}{5}\int \frac {10x+15}{5x^2+1}dx

\displaystyle =\frac {1}{5}\int \frac {10x}{5x^2+1}dx+\int \frac {1}{5x^2+1}dx

\displaystyle =\frac {1}{5}\int \frac {10x}{5x^2+1}dx+3\int \frac {1}{5\left (x^2+\frac {1}{5}\right)}dx

\displaystyle =\frac {1}{5}\log (5x^2+1)+\frac {3}{5}\cdot \frac {1}{\frac {1}{\sqrt 5}}\tan^{-1}\frac {x}{\frac {1}{\sqrt 5}}

\displaystyle =\frac {1}{5}\log (5x^2+1)+\frac {3}{\sqrt 5}\tan^{-1}(\sqrt 5x)

By second fundamental theorem of calculus, we obtain

I=F(1)-F(0)

=\displaystyle \left \{\frac {1}{5}\log (5+1)+\frac {3}{\sqrt 5}\tan^{-1}(\sqrt 5)\right \}-\left \{\frac {1}{5}(1)+\frac {3}{\sqrt 5}\tan^{-1}(0)\right \}

\displaystyle =\frac {1}{5}\log 6+\frac {3}{\sqrt 5}\tan^{-1}\sqrt 5

#424653

Topic: Changing Variable

Evaluate the definite integral \displaystyle \int_0^1xe^{x^2}dx

Solution

Let I=\int_0^1xe^{x^2}dx

Put x^2=t\Rightarrow 2x dx =dt

As x\rightarrow 0, t\rightarrow 0 and as x\rightarrow 1, t\rightarrow 1,

\therefore I=\dfrac {1}{2}\int_0^1e^tdt

\Rightarrow \dfrac {1}{2}\int e^tdt=\dfrac {1}{2}e^t=F(t)

By second fundamental theorem of calculus, we obtain

I=F(1)-F(0)

=\dfrac {1}{2}e-\frac {1}{2}e^0

=\dfrac {1}{2}(e-1)

#424654

Topic: Changing Variable

Evaluate the definite integral \displaystyle \int_1^2\frac {5x^2}{x^2+4x+3}

Solution

Let \displaystyle I=\int_1^2\frac {5x^2}{x^2+4x+3}

Since, degree of numerator is equal to degree of denominator, so dividing 5x^2 by x^2+4x+3

\displaystyle I=\int_1^2\left \{5-\frac {20x+15}{x^2+4x+3}\right \}dx

\displaystyle =\int_1^25 dx-\int_1^2\frac {20x+15}{x^2+4x+3}dx

\displaystyle =[5x]_1^2-\int_1^2\frac {20x+15}{x^2+4x+3}dx

I=5-I_1 .... (1)

where I_1=\displaystyle \int_1^2\frac {20x+15}{x^2+4x+3}dx

Consider I_1=\displaystyle \int_1^2\frac {20x+15}{x^2+4x+3}dx

Let 20x+15=A\cfrac {d}{dx}(x^2+4x+3)+B

\Rightarrow 20x+15=A(2x+4)+B ....(2)

=2Ax+(4A+B)

Equating the coefficients of x and constant term, we obtain

A=10 and B=-25

Substituting these values in (2),

20x+15=10(2x+4)-25

\Rightarrow \displaystyle I_1=10\int_1^2\cfrac {2x+4}{x^2+4x+3}dx-25\int_1^2\cfrac {dx}{x^2+4x+3}

Let x^2+4x+3=t

\Rightarrow x^2+4x+3=t

\Rightarrow (2x+4)dx=dt

\Rightarrow\displaystyle I_1=10\int_{8}^{15} \frac {dt}{t}-25\int_{1}^2 \frac {dx}{(x+2)^2-1^2}

\displaystyle =10 [\log t]_{8}^{15}-25\left [\frac {1}{2}\log \left (\frac {x+2-1}{x+2+1}\right)\right]_{1}^2

\displaystyle =10 [\log 15 - \log 8]-25\left [\frac {1}{2}\log \left (\frac {x+1}{x+3}\right)\right]_1^2

\displaystyle =[10 \log 15-10\log 8]-25\left [\frac {1}{2}\log \frac {3}{5}-\frac {1}{2}\log \frac {2}{4}\right]

\displaystyle =[10\log (5\times 3)-10 \log (4\times 2)]-\frac {25}{2}[\log 3-\log 5-\log 2+\log 4]

\displaystyle =[10 \log 5+10 \log 3-10 \log 4-10\log 2-\frac {25}{2}[\log 3-\log 5-\log 2 +\log 4]

\displaystyle =\left [10+\frac {25}{2}\right]\log 5+\left [-10-\frac {25}{2}\right]\log 4+\left [10-\frac {25}{2}\right]\log 3+\left [-10+\frac {25}{2}\right]\log 2

\displaystyle =\frac {45}{2}\log 5-\frac {45}{2}\log 4-\frac {5}{2}\log 3+\frac {5}{2}\log 2

\displaystyle =\frac {45}{2}\log \frac {5}{4}-\frac {5}{2}\log \frac {3}{2}

Substituting the value of I_1 in (1), we get

\displaystyle I=5-\left [\frac {45}{2}\log \frac {5}{4}-\frac {5}{2}\log \frac {3}{2}\right]

\displaystyle =5-\frac {5}{2}\left [9 \log \frac {5}{4}-\log \frac {3}{2}\right]

#424657

Topic: Changing Variable

Evaluate the definite integral $\displaystyle \int_0^2\frac {6x+3}{x^2+4}dx$

Solution

Let $\displaystyle I=\int_0^2\frac {6x+3}{x^2+4}dx$

$\Rightarrow \displaystyle \int \frac {6x+3}{x^2+4}dx=3\int \frac {2x+1}{x^2+4}dx$

$\displaystyle =3\int \frac {2x}{x^2+4}dx+3\int \frac {1}{x^2+4}dx$

$\displaystyle =3 \log (x^2+4)+\frac {3}{2}\tan^{-1}\frac {x}{2}=F(x)$

By second fundamental theorem of calculus, we obtain

$I=F(2)-F(0)$

$\displaystyle =\left \{3 \log (2^2+4)+\frac {3}{2}\tan^{-1}\left (\frac {2}{2}\right)\right \}-\left \{3 \log (0+4)+\frac {3}{2}\tan^{-1}\left (\frac {0}{2}\right)\right \}$

$\displaystyle =3 \log 8+\frac {3}{2}\tan^{-1}1-3 \log 4-\frac {3}{2}\tan^{-1}0$

$\displaystyle =3 \log 8+\frac {3}{4}\left (\frac {\pi}{4}\right)-3 \log 4-0$

$\displaystyle =3\log \left (\frac {8}{4}\right)+\frac {3\pi}{8}$

$\displaystyle =3 \log 2+\frac {3\pi}{8}$

#424658

Topic: Changing Variable

Evaluate the definite integral $\displaystyle \int_0^1\left (xe^x+\sin \frac {\pi x}{4}\right)dx$

Solution

Let $\displaystyle I=\int_0^1\left (xe^x+\sin \frac {\pi x}{4}\right)dx$

$\Rightarrow \displaystyle \int \left (xe^x+\sin \frac {\pi x}{4}\right)dx=x\int e^xdx-\int \left \{\left (\frac {d}{dx}x\right)\int e^xdx\right \}dx+\left \{\frac {-\cos \frac {\pi x}{4}}{\frac {\pi}{4}}\right \}$

$\displaystyle =xe^x-\int e^xdx-\frac {4}{\pi}\cos \frac {\pi x}{4}$

$\displaystyle =xe^x-e^x-\frac {4}{\pi}\cos \frac {\pi x}{4}=F(x)$

By second fundamental theorem of calculus, we obtain

$I=F(1)-F(0)$

$\displaystyle =\left (1.e^1-e^1-\frac {4}{\pi}\cos \frac {\pi}{4}\right)-\left (0.e^0-e^0-\frac {4}{\pi}\cos 0\right)$

$\displaystyle =e-e-\frac {4}{\pi}\left (\frac {1}{\sqrt 2}\right)+1+\frac {4}{\pi}$

$\displaystyle =1+\frac {4}{\pi}-\frac {2\sqrt 2}{\pi}$

#424660

Topic: Changing Variable

$\displaystyle \int_1^{\sqrt 3}\frac {dx}{1+x^2}$ equals

A $\dfrac {\pi}{3}$

B $\dfrac {2\pi}{3}$

C $\dfrac {\pi}{6}$

[✓ D] $\dfrac {\pi}{12}$

Solution

$\displaystyle \int \frac {d}{1+x^2}=\tan^{-1}x=F(x)$

By second fundamental theorem of calculus, we obtain

$\displaystyle \int_1^{\sqrt 3}\frac {dx}{1+x^2}=F(\sqrt 3)-F(1)$

$=\tan^{-1}\sqrt 3-\tan^{-1}1$

$\displaystyle =\frac {\pi}{3}-\frac {\pi}{4}=\frac {\pi}{12}$

#424661

Topic: Changing Variable

\displaystyle \int_0^{\frac {2}{3}}\frac {dx}{4+9x^2} equals

A \dfrac {\pi}{6}

B \dfrac {\pi}{12}

[✓ **C**] \dfrac {\pi}{24}

D \dfrac {\pi}{4}

Solution

\displaystyle \int \frac {dx}{4+9x^2}=\int \frac {dx}{(2)^2+(3x)^2}

Put 3x=t\Rightarrow 3dx=dt

\therefore\displaystyle \int \frac {dx}{(2)^2+(3x)^2}=\frac {1}{3}\int \frac {dt}{(2)^2+t^2}

\displaystyle =\frac {1}{3}\left [\frac {1}{2}\tan^{-1}\frac {t}{2}\right]

\displaystyle =\frac {1}{6}\tan^{-1}\left (\frac {3x}{2}\right)

=F(x)

By second fundamental theorem of calculus, we obtain

\displaystyle \int_0^{\frac {2}{3}}\frac {dx}{4+9x^2}=F\left (\frac {2}{3}\right)-F(0)

=\dfrac {1}{6}\tan^{-1}\left (\dfrac {3}{2}\cdot \dfrac {2}{3}\right)-\dfrac {1}{6}\tan^{-1}0

=\dfrac {1}{6}\tan^{-1}1-0

\displaystyle =\frac {1}{6}\times \frac {\pi}{4}=\frac {\pi}{24}

#424663

Topic: Changing Variable

Evaluate the integral \displaystyle \int_0^1\frac {x}{x^2+1}dx using substitution.

Solution

\displaystyle \int_0^1\frac {x}{x^2+1}dx

Let x^2+1=t\Rightarrow 2x dx=dt

When x=0, t=1 and when x=1, t=2

\therefore\displaystyle \int_0^1\frac {x}{x^2+1}dx=\frac {1}{2}\int_1^2\frac {dt}{t}

\displaystyle =\frac {1}{2}[\log|t|]_1^2

\displaystyle =\frac {1}{2}[\log 2-\log 1]=\frac {1}{2}\log 2

#424664

Topic: Changing Variable

Evaluate the integral \displaystyle \int_0^{\frac {\pi}{2}}\sqrt {\sin \phi}\cos^5\phi d\phi using substitution.

Solution

Let \displaystyle I=\int_0^{\tfrac {\pi}{2}}\sqrt {\sin \phi}cos^5\phi d\phi =\int_0^{\tfrac {\pi}{2}}\sqrt {\sin \phi}\cos^4\phi d\phi

Also, let \sin\phi =t\Rightarrow \cos \phi d\phi =dt

When \phi=0, t=0 and when \phi =\cfrac {\pi}{2}, t=1

\therefore I=\int_0^1 \sqrt t(1-t^2)^2dt

=\int_0^1 t^{\tfrac {1}{2}}(1+t^4-2t^2)dt

=\int_0^1\left [t^{\tfrac {1}{2}}+t^{\tfrac {9}{2}}-2t^{\tfrac {5}{2}}\right]dt

\displaystyle =\left [\frac {t^{\tfrac {3}{2}}}{\frac {3}{2}}+\frac {t^{\tfrac {11}{2}}}{\frac {11}{2}}-\frac {2t^{\tfrac {7}{2}}}{\frac {7}{2}}\right]_0^1

\displaystyle =\cfrac {2}{3}+\frac {2}{11}-\frac {4}{7}

\displaystyle =\cfrac {154+42-132}{231}=\frac {64}{231}

#424665

Topic: Changing Variable

Evaluate the integral $\displaystyle \int_0^1\sin^{-1}\left (\frac {2x}{1+x^2}\right)dx$ using substitution.

Solution

Let $I\displaystyle =\int_0^1\sin^{-1}\left (\frac {2x}{1+x^2}\right)dx$

Also, let $x=\tan\theta \Rightarrow dx=\sec^2\theta d\theta$

When $x=0, \theta =0$ and when $x=1, \theta =\dfrac {\pi}{4}$

$\therefore\displaystyle I=\int_0^{\frac {\pi}{4}}\sin^{-1}\left (\frac {2 \tan\theta}{1+\tan^2\theta}\right)\sec^2\theta d\theta$

$=\int_0^{\frac {\pi}{4}}\sin^{-1} (\sin 2\theta)\sec^2 \theta d\theta$

$=\int_0^{\frac {\pi}{4}} 2\theta \cdot \sec^2\theta d\theta$

$=2\int_0^{\frac {\pi}{4}}\theta \cdot \sec^2\theta d\theta$

Taking θ as first function and $\sec^2\theta$ as second function and integrating by parts, we obtain

$I=2\left [\theta \int \sec^2\theta d\theta -\int \left \{\left (\frac {d}{dx}\theta \right)\int \sec^2 \theta d\theta \right \}d\theta \right]_0^{\frac {\pi}{4}}$

$=2[\theta \tan \theta -\int \tan \theta d\theta]_0^{\frac {\pi}{4}}$

$=2[\theta \tan \theta +\log l\cos \theta l]_0^{\frac {\pi}{4}}$

$=2\left [\dfrac {\pi}{4}\tan \dfrac {\pi}{4}+\log \left l\cos \dfrac {\pi}{4}\right l-\log l\cos 0l\right]$

$=2\left [\dfrac {\pi}{4}+\log \left (\dfrac {1}{\sqrt 2}\right)-\log 1\right]$

$=2\left [\dfrac {\pi}{4}-\dfrac {1}{2}\log 2\right]$

$=\dfrac {\pi}{2}-\log 2$

#424666

Topic: Changing Variable

Evaluate the integral $\displaystyle \int_0^2x\sqrt {x+2}$ using substitution.

Solution

$\int_0^2x\sqrt {x+2}dx$

Let $x+2=t^2\Rightarrow dx=2tdt$

When $x=0, t=\sqrt 2$ and when $x=2, t=2$

$\therefore \int_0^2x\sqrt {x+2}dx=\int_{\sqrt 2}^2(t^2-2)\sqrt {t^2}2tdt$

$=2\int_{\sqrt 2}^2(t^2-2)t^2dt$

$=2\int_{\sqrt 2}^2(t^4-2t^2)dt$

$\displaystyle =2\left [\frac {t^5}{5}-\frac {2t^3}{3}\right]_{\sqrt 5}^2$

$\displaystyle=2\left [\frac {32}{5}-\frac {16}{3}-\frac {4\sqrt 2}{5}+\frac {4\sqrt 2}{3}\right]$

$\displaystyle=2\left [\frac {96-80-12\sqrt 2+20\sqrt 2}{15}\right]$

$\displaystyle=2\left [\frac {16+8\sqrt 2}{15}\right]$

$\displaystyle=\frac {16(2+\sqrt 2)}{15}$

$\displaystyle=\frac {16\sqrt 2(\sqrt 2+1)}{15}$

#424667

Topic: Changing Variable

Evaluate the integral $\displaystyle \int_0^{\tfrac {\pi}{2}}\frac {\sin x}{1+\cos^2x}dx$ using substitution.

Solution

$\displaystyle \int_0^{\tfrac {\pi}{2}}\frac {\sin x}{1+\cos^2x}dx$

Let $\cos x=t\Rightarrow - \sin x dx =dt$

When $x=0, t=1$ and when $x=\cfrac {\pi}{2}, t=0$

$\displaystyle \Rightarrow \int_0^{\tfrac {\pi}{2}}\frac {\sin x}{1+\cos^2x}dx=\int_1^0\frac {dt}{1+t^2}$

$=-[\tan^{-1}t]_1^0$

$=-[\tan^{-1}0-\tan^{-1}1]$

$=-\left [-\cfrac {\pi}{4}\right]=\cfrac {\pi}{4}$

#424668

Topic: Changing Variable

Evaluate the integral \displaystyle \int_0^2\frac {dx}{x+4-x^2} using substitution.

Solution

\displaystyle\int_0^2\frac {dx}{x+4-x^2}=\int_0^2\frac {dx}{-(x^2-x-4)}

\displaystyle=\int_0^2\frac {dx}{-(x^2-x+\frac {1}{4}-\frac {1}{4}-4)}

\displaystyle=\int_0^2\frac {dx}{-\left [\left (x-\frac {1}{2}\right)^2-\frac {17}{4}\right]}

\displaystyle=\int_0^2\frac {dx}{\left (\frac {\sqrt {17}}{2}\right)^2-\left (x-\frac {1}{2}\right)^2}

Let \displaystyle x-\frac {1}{2}=t\Rightarrow dx=dt

When x=0, t=-\frac {1}{2} and when x=2, t=\frac {3}{2}

\displaystyle\therefore \int_0^2\frac {dx}{\left (\frac {\sqrt {17}}{2}\right)^2-\left (x-\frac {1}{2}\right)^2}=\int_{-\frac {1}{2}}^{\frac {3}{2}}\frac {dt}{\left (\frac {\sqrt {17}}{2}\right)^2-t^2}

\displaystyle=\left [\frac {1}{2\left (\frac {\sqrt {17}}{2}\right)}\log \frac {\frac {\sqrt {17}}{2}+t}{\frac {\sqrt {17}}{2}-t}\right]_{-\frac {1}{2}}^{\frac {3}{2}}

\displaystyle=\frac {1}{\sqrt {17}}\left [\log \frac {\frac {\sqrt {17}}{2}+\frac {3}{2}}{\frac {\sqrt {17}}{2}-\frac {3}{2}}-\log\frac {\frac {\sqrt {17}}{2}-\frac {1}{2}}{\frac {\sqrt {17}}{2}+\frac {1}{2}}\right]

\displaystyle=\frac {1}{\sqrt {17}}\left [\log \frac {\sqrt {17}+3}{\sqrt {17}-3}-\log \frac {\sqrt {17}-1}{\sqrt {17}+1}\right]

\displaystyle=\frac {1}{\sqrt {17}}\left [\log \frac {\sqrt {17}+3}{\sqrt {17}-3}\times \frac {\sqrt {17}+1}{\sqrt {17}-1}\right]

\displaystyle=\frac {1}{\sqrt {17}}\log\left [\frac {17+3+4\sqrt {17}}{17+3-4\sqrt {17}}\right]

\displaystyle=\frac {1}{\sqrt {17}}\log \left [\frac {20+4\sqrt {17}}{20-4\sqrt {17}}\right]

\displaystyle=\frac {1}{\sqrt {17}}\log \left (\frac {5+\sqrt {17}}{5-\sqrt {17}}\right)

\displaystyle=\frac {1}{\sqrt {17}}\log \left [\frac {(5+\sqrt {17})(5+\sqrt {17})}{25-17}\right]

\displaystyle=\frac {1}{\sqrt {17}}\log \left [\frac {25+17+10\sqrt {17}}{8}\right]

\displaystyle=\frac {1}{\sqrt {17}}\log\left (\frac {42+10\sqrt {17}}{8}\right)

\displaystyle=\frac {1}{\sqrt {17}}\log \left (\frac {21+5\sqrt {17}}{4}\right)

#424680

Topic: Changing Variable

Evaluate the integral \displaystyle \int_{-1}^1\frac {dx}{x^2+2x+5} using substitution.

Solution

\displaystyle \int_{-1}^1\frac {dx}{x^2+2x+5}=\int_{-1}^1\frac {dx}{(x^2+2x+1)+4}=\int_{-1}^1\frac {dx}{(x+1)^2+(2)^2}

Let x+1=t\Rightarrow dx=dt

When x=-1, t=0 and When x=1, t=2

\therefore\displaystyle \int_{-1}^1\frac {dx}{(x+1)^2+(2)^2}=\int_0^2\frac {dt}{t^2+2^2}

=\left [\dfrac {1}{2}\tan^{-1}\dfrac {t}{2}\right]_0^2

=\dfrac {1}{2}\tan^{-1}1-\dfrac {1}{2}\tan^{-1}0

\displaystyle =\frac {1}{2}\left (\frac {\pi}{4}\right)=\frac {\pi}{8}

#424689

Topic: Changing Variable

Evaluate the integral \displaystyle \int_1^2\left (\frac {1}{x}-\frac {1}{2x^2}\right)e^{2x}dx using substitution.

Solution

\int_1^2\left (\dfrac {1}{x}-\dfrac {1}{2x^2}\right)e^{2x}dx

Let 2x=t\Rightarrow 2dx=dt

When x=1, t=2 and when x=2, t=4

\therefore \displaystyle \int_1^2\left (\frac {1}{x}-\frac {1}{2x^2}\right)e^{2x}dx=\int_2^4\frac {1}{2}\left (\frac {2}{t}-\frac {2}{t^2}\right)e^tdt

=\displaystyle \int_2^4\left (\frac {1}{t}-\frac {1}{t^2}\right)e^tdt

Let \dfrac {1}{t}=f(t)

Then, f'(t)=-\dfrac {1}{t^2}

\Rightarrow \displaystyle \int_2^4\left (\frac {1}{t}-\frac {1}{t^2}\right)e^tdt=\int_2^4e^t[f(t)+f'(t)]dt

=[e^tf(t)]_2^4

=\left [e^t\cdot \dfrac {1}{t}\right]_2^4

=\left [\dfrac {e^t}{t}\right]_2^4

=\dfrac {e^4}{4}-\dfrac {e^2}{2}

=\dfrac {e^2(e^2-2)}{4}

#424707
Topic: Changing Variable

The value of the integral \displaystyle \int_{\frac {1}{3}}^1\frac {(x-x^3)^{\frac {1}{3}}}{x^4}dx

$\boxed{\text{A}}$ 6

B 0

C 3

D 4

Solution

Let \displaystyle I=\int_{ {1}/{3}}^1\frac {(x-x^3)^{\frac {1}{3}}}{x^4}dx

Put x=\sin\theta

\Rightarrow dx=\cos\theta \, d\theta

When x=\cfrac {1}{3},\theta =\sin^{-1}\left (\cfrac {1}{3}\right) and when x=1, \theta =\cfrac {\pi}{2}

\Rightarrow \displaystyle I=\int_{\sin^{-1}\left (\frac {1}{3}\right)}^{\frac {\pi}{2}}\frac {(\sin\theta-\sin^3\theta)^{\frac {1}{3}}}{\sin^4\theta}cos \theta \,d\theta

\displaystyle =\int_{\sin^{-1}\left (\frac {1}{3}\right)}^{\frac {\pi}{2}}\frac {(\sin\theta)^{\frac {1}{3}}(1-\sin^2\theta)^{\frac {1}{3}}}{\sin^4\theta}\cos \theta \, d\theta

\displaystyle =\int_{\sin^{-1}\left (\frac {1}{3}\right)}^{\frac {\pi}{2}}\frac {(\sin\theta)^{\frac {1}{3}}(\cos\theta)^{\frac {2}{3}}}{\sin^4\theta}\cos \theta \, d\theta

\displaystyle =\int_{\sin^{-1}\left (\frac {1}{3}\right)}^{\frac {\pi}{2}}\frac {(\sin\theta)^{\frac {1}{3}}(\cos\theta)^{\frac {2}{3}}}{\sin^2\theta \sin^2\theta}\cos \theta d\theta

\displaystyle =\int_{\sin^{-1}\left (\frac {1}{3}\right)}^{\frac {\pi}{2}}\frac {(\cos\theta)^{\frac {5}{3}}}{(\sin\theta)^{\frac {5}{3}}}cosec^2\theta d\theta

\displaystyle=\int_{sin^{-1}\left (\frac {1}{3}\right)}^{\frac {\pi}{2}}(\cot\theta)^{\frac {5}{3}}cosec^2\theta d\theta

Put \cot \theta=t

\Rightarrow -cosec^2\theta \,d\theta=dt

When \theta=\sin^{-1}\left (\dfrac {1}{3}\right),t=2\sqrt 2 and when \theta=\dfrac {\pi}{2}, t=0

\therefore I=-\int_{2\sqrt 2}^0(t)^{\frac {5}{3}}dt

\Rightarrow \displaystyle I=\int_0^{2\sqrt 2} (t)^{\frac {5}{3}}dt

\displaystyle =\left [\frac {3}{8}(t)^{\frac {8}{3}}\right]_0^{2\sqrt 2}

\displaystyle =\frac {3}{8}[(2\sqrt 2)^{\frac {8}{3}}]

=\dfrac {3}{8}[(\sqrt 8)^{\frac {8}{3}}]

=\cfrac {3}{8}[(8)^{\frac {4}{3}}]

=\cfrac {3}{8}[16]

=3\times 2

=6

Hence, the correct Answer is A.

#425533

Topic: Integration using Partial Fractions

Integrate the function \cfrac {1}{x-x^3}

Solution

\displaystyle \frac {1}{x-x^3}=\frac {1}{x(1-x^2)}=\frac {1}{x(1-x)(1+x)}

Let \displaystyle \frac {1}{x(1-x)(1+x)}=\frac {A}{x}+\frac {B}{(1-x)}+\frac {C}{1+x} .... (1)

\Rightarrow 1=A(1-x^2)+Bx(1+x)+Cx(1-x)

\Rightarrow 1=A-Ax^2+Bx+Bx^2+Cx-Cx^2

Equating the coefficients of x^2, x, and constant term, we obtain

A + B C = 0

B + C = 0

A = 1

On solving these equations, we obtain

A=1, B=\dfrac {1}{2}, and C=-\dfrac {1}{2}

From equation (1), we obtain

\displaystyle\frac {1}{x(1-x)(1+x)}=\frac {1}{x}+\frac {1}{2(1-x)}-\frac {1}{2(1+x)}

\Rightarrow\displaystyle \int \frac {1}{x(1-x)(1+x)}dx=\int \frac {1}{x}dx+\frac {1}{2}\int \frac {1}{1-x}dx-\frac {1}{2}\int \frac {1}{1+x}dx

=\log |x|-\dfrac {1}{2}\log |(1-x)|-\dfrac {1}{2}\log |(1+x)|

=\log|x|-\log|(1-x)^{\frac {1}{2}}|-\log|(1+x)^{\frac {1}{2}}|

=\log \left |\dfrac {x}{(1-x)^{\frac {1}{2}}(1+x)^{\frac {1}{2}}}\right |+C

=\log \left |\left (\dfrac {x^2}{1-x^2}\right)^{\frac {1}{2}}\right |+C

=\dfrac {1}{2}\log \left |\dfrac {x^2}{1-x^2}\right |+C

#425614

Topic: Changing Variable

Evaluate the definite integral \displaystyle \int_{\frac {\pi}{2}}^{\pi}e^x\left (\frac {1-\sin x}{1-\cos x}\right)dx

Solution

I\displaystyle =\int_{\frac {\pi}{2}}^{\pi}e^x\left (\frac {1-\sin x}{1-\cos x}\right)dx

\displaystyle=\int_{\frac {\pi}{2}}^{\pi}e^x\left (\frac {1-2 \sin\frac {x}{2}\cos \frac {x}{2}}{2 \sin^2\frac {x}{2}}\right)dx

\displaystyle=\int_{\frac {\pi}{2}}^{\pi}e^x\left (\frac {\text{cosec}^2\frac {\pi}{2}}{2}-\cot \frac {x}{2}\right)dx

Let f(x)=-\cot \frac {x}{2}

\Rightarrow f'(x)=-\left (-\frac {1}{2} \text{cosec}^2\frac {x}{2}\right)=\frac {1}{2}\text{cosec}^2\frac {x}{2}

\therefore\displaystyle I=\int_{\frac {\pi}{2}}^{\pi}e^x(f(x)+f'(x)]dx

=[e^x\cdot f(x)dx]_{\frac {\pi}{2}}^{\pi}

=-\left [e^x\cdot \cot \frac {x}{2}\right]_{\frac {\pi}{2}}^{\pi}

=-\left [e^{\pi}\times \cot \frac {\pi}{2}-e^{\frac {\pi}{2}}\times \cot \frac {\pi}{4}\right]

=-\left [e^{\pi}\times 0-e^{\frac {\pi}{2}}\times 1\right]=e^{\frac {\pi}{2}}

#425616

Topic: Changing Variable

Evaluate the definite integral \displaystyle \int_0^{\frac {\pi}{4}}\frac {\sin x \cos x}{\cos^4 x+\sin^4x}dx

Solution

Let \displaystyle I=\int_0^{\frac {\pi}{4}}\frac {\sin x \cos x}{\cos^4 x+\sin^4x}dx

\displaystyle\Rightarrow I=\int_0^{\frac {\pi}{4}}\frac {\frac {(\sin x \cos x)}{\cos^4x}}{\frac {(\cos^4x+\sin^4x)}{\cos^4x}}dx

\displaystyle\Rightarrow I=\int_0^{\frac {\pi}{4}}\frac {\tan x \sec^2x}{1+\tan^4x}dx

Let \tan^2x=t\Rightarrow 2 \tan x \sec^2x dx=dt

When x=0, t=0 and when x=\cfrac {\pi}{4}, t=1

\therefore\displaystyle I=\frac {1}{2}\int_0^1\frac {dt}{1+t^2}

\displaystyle=\frac {1}{2}[\tan^{-1}t]_0^1

\displaystyle=\frac {1}{2}[\tan^{-1}1-\tan^{-1}0]

\displaystyle=\frac {1}{2}\left [\frac {\pi}{4}\right]=\frac {\pi}{8}

#425617

Topic: Changing Variable

Evaluate the definite integral \displaystyle \int_0^{\frac {\pi}{2}}\frac {\cos^2x dx}{\cos^2x+4 \sin^2x}

Solution

Let I\displaystyle=\int_0^{\frac {\pi}{2}}\frac {\cos^2x}{\cos^2x+4 \sin^2x}dx

\displaystyle\Rightarrow I=\int_0^{\frac {\pi}{2}}\frac {\cos^2x}{\cos^2x+4(1-\cos^2x)}dx

\displaystyle\Rightarrow I=\int_0^{\frac {\pi}{2}}\frac {\cos^2x}{\cos^2 x+4-4\cos^2x}dx

\displaystyle\Rightarrow I=\frac {-1}{3}\int_0^{\frac {\pi}{2}}\frac {4-3\cos^2x-4}{4-3\cos^2x}dx

\Rightarrow\displaystyle I=\frac {-1}{3}\int_0^{\frac {\pi}{2}}\frac {4-3 \cos^2x}{4-3\cos^2x}dx+\frac {1}{3}\int_0^{\frac {\pi}{2}}\frac {4}{4-3\cos^2x}dx

\Rightarrow\displaystyle I=\frac {-1}{3}\int_0^{\frac {\pi}{2}}1 dx+\frac {1}{3}\int_0^{\frac {\pi}{2}}\frac {4 \sec^2x}{4 \sec^2x-3}dx

\Rightarrow\displaystyle I =\frac {-1}{3}[x]_0^{\frac {\pi}{2}}+\frac {1}{3}\int_0^{\frac {\pi}{2}}\frac {4\sec^2x}{4(1+\tan^2x)-3}dx

\Rightarrow\displaystyle I=-\frac {\pi}{6}+\frac {2}{3}\int_0^{\frac {\pi}{2}}\frac {2 \sec^2x}{1+4 \tan^2x}dx .... (1)

Consider, \displaystyle\int_0^{\frac {\pi}{2}}\frac {2 \sec^2x}{1+4 \tan^2x}dx

Let 2 \tan x=t\Rightarrow 2 \sec^2x dx=dt

When x=0, t=0 and when x=\frac {\pi}{2}, t=\infty

\Rightarrow\displaystyle \int_0^{\frac {\pi}{2}}\frac {2 \sec^2x}{1+4 \tan^2x}dx=\int_0^{\infty}\frac {dt}{1+t^2}

=[\tan^{-1}t]_0^{\infty}

\displaystyle =[\tan^{-1}(\infty)-\tan^{-1}(0)]=\frac {\pi}{2}

Therefore, from (1), we obtain

I\displaystyle =-\frac {\pi}{6}+\frac {2}{3}\left [\frac {\pi}{2}\right]=\frac {\pi}{3}-\frac {\pi}{6}=\frac {\pi}{6}

#425620

Topic: Changing Variable

Evaluate the definite integral \displaystyle \int_{\frac {\pi}{6}}^{\frac {\pi}{3}}\frac {\sin x+\cos x}{\sqrt {\sin 2x}}dx

Solution

Let I\displaystyle =\int_{\frac {\pi}{6}}^{\frac {\pi}{3}}\frac {\sin x+\cos x}{\sqrt {\sin 2x}}dx

\Rightarrow\displaystyle I=\int_{\frac {\pi}{6}}^{\frac {\pi}{3}}\frac {(\sin x+\cos x)}{\sqrt {-(-\sin 2x)}}dx

\Rightarrow\displaystyle I=\int_{\frac {\pi}{6}}^{\frac {\pi}{3}}\frac {\sin x+\cos x}{\sqrt {-(-1+1-2\sin x \cos x)}}dx

\Rightarrow\displaystyle I=\int_{\frac {\pi}{6}}^{\frac {\pi}{3}}\frac {(\sin x+\cos x)}{\sqrt {1-(\sin^2x+\cos^2x-2 \sin x \cos x)}}dx

\Rightarrow\displaystyle I=\int_{\frac {\pi}{6}}^{\frac {\pi}{3}}\frac {(\sin x+\cos x)dx}{\sqrt {1-(\sin x-\cos x)^2}}

Let (\sin x - \cos x)=t\Rightarrow (\sin x+\cos x)dx=dt

when x=\dfrac {\pi}{6},t=\left (\dfrac {1-\sqrt 3}{2}\right) and when x=\dfrac {\pi}{3}, t=\left (\dfrac {\sqrt 3-1}{2}\right)

I\displaystyle=\int_{\frac {1-\sqrt 3}{3}}^{\frac {\sqrt 3-1}{2}}\frac {dt}{\sqrt {1-t^2}}

\Rightarrow\displaystyle I=\int_{-\left (\frac {\sqrt 3-1}{2}\right)}^{\frac {\sqrt 3-1}{1}}\frac {dt}{\sqrt {1-t^2}}

As \displaystyle\frac {1}{\sqrt {1-(-t)^2}}=\frac {1}{\sqrt {1-t^2}}, therefore, \cfrac {1}{\sqrt {1-t^2}} is an even function.

It is known that if f(x) is an even function, then \displaystyle\int_{-a}^af(x)dx=2\int_0^af(x)dx

\Rightarrow\displaystyle I=2\int_0^{\frac {\sqrt 3-1}{2}}\frac {dt}{\sqrt {1-t^2}}

=[2\sin^{-1}t]_{0}^{\frac {\sqrt 3-1}{2}}

=2 \sin^{-1}\left (\dfrac {\sqrt 3-1}{2}\right)

#425622

Topic: Changing Variable

Evaluate the definite integral \displaystyle \int_0^1\frac {dx}{\sqrt {1+x}-\sqrt x}

Solution

Let \displaystyle I=\int_0^1\frac {dx}{\sqrt {1+x}-\sqrt x}

\displaystyle I=\int_0^1\frac {1}{(\sqrt {1+x}-\sqrt x)}\times \frac {(\sqrt {1+x}+\sqrt x)}{(\sqrt {1+x}+\sqrt x)}dx

\displaystyle =\int_0^1\frac {\sqrt {1+x}+\sqrt x}{1+x-x}dx

\displaystyle =\int_0^1\sqrt {1+x}dx+\int_0^1\sqrt xdx

\displaystyle =\left [\frac {2}{3}(1+x)^{\frac {3}{2}}\right]_0^1+\left [\frac {2}{3}(x)^{\frac {3}{2}}\right]_0^1

\displaystyle =\frac {2}{3}[(2)^{\frac {3}{2}}-1]+\frac {2}{3}[1]

\displaystyle =\frac {2}{3}(2)^{\frac {3}{2}}

\displaystyle =\frac {2\cdot 2\sqrt 2}{3}=\frac {4\sqrt 2}{3}

#425624

Topic: Changing Variable

Evaluate the definite integral \displaystyle \int_0^{\frac {\pi}{4}}\frac {\sin x+\cos x}{9+16 \sin 2x}dx

Solution

Let I\displaystyle =\int_0^{\frac {\pi}{4}}\frac {\sin x+\cos x}{9+16 \sin 2x}dx

Also, let \displaystyle\sin x-\cos x=t\Rightarrow (\cos x+\sin x)dx=dt

When x=0, t=-1 and when x=\frac {\pi}{4}, t=0

\Rightarrow (\sin x-\cos x)^2=t^2

\Rightarrow \sin^2x+\cos^2x-2\sin x \cos x=t^2

\Rightarrow 1-\sin 2x=t^2

\Rightarrow \sin 2x=1-t^2

\therefore\displaystyle I=\int_{-1}^0\frac {dt}{9+16(1-t^2)}

\displaystyle=\int_{-1}^0\frac {dt}{9+16-16t^2}

\displaystyle=\int_{-1}^0\frac {dt}{25-16t^2}=\int_{-1}^0\frac {dt}{(5)^2-(4t)^2}

\displaystyle=\frac {1}{4}\left [\frac {1}{2(5)}\log \left |\frac {5+4t}{5-4t}\right |\right]_{-1}^0

\displaystyle=\frac {1}{40}\left [\log(1)-\log\left |\frac {1}{9}\right |\right]

\displaystyle=\frac {1}{40}\log 9

#425629

Topic: Changing Variable

Evaluate the definite integral \displaystyle \int_0^{\frac {\pi}{2}}\sin 2x \tan^{-1}(\sin x)dx

Solution

Let I\displaystyle =\int_0^{\frac {\pi}{2}}\sin 2x \tan^{-1}(\sin x)dx=\int_0^{\frac {\pi}{2}}2 \sin x \cos x \tan^{-1}(\sin x)dx

Also, let \sin x=t\Rightarrow \cos x dx=dt

When x=0, t=0 and when x=\dfrac {\pi}{2}, t=1

\Rightarrow\displaystyle I=2\int_0^1\tan^{-1}(t)dt .... (1)

Consider \displaystyle\int t\cdot \tan^{-1}t dt=\tan^{-1}t\cdot \int t dt-\int \left \{\frac {d}{dt}(\tan^{-1}t)\int t dt\right \}dt

\displaystyle=\tan^{-1}t\cdot \frac {t^2}{2}-\int \frac {1}{1+t^2}\cdot \frac {t^2}{2}dt

\displaystyle=\frac {t^2\tan^{-1}t}{2}-\frac {1}{2}\int \frac {t^2+1-1}{1+t^2}dt

\displaystyle=\frac {t^2\tan^{-1}t}{2}-\frac {1}{2}\int 1 dt+\frac {1}{2}\int \frac {1}{1+t^2}dt

\displaystyle=\frac {t^2\tan^{-1}t}{2}-\frac {1}{2}\cdot t+\frac {1}{2}\tan^{-1}t

\Rightarrow\displaystyle \int_0^1t\cdot \tan^{-1}t dt=\left [\frac {t^2\cdot\tan^{-1}t}{2}-\frac {t}{2}+\frac {1}{2}\tan^{-1}t\right]_0^1

\displaystyle=\frac {1}{2}\left [\frac {\pi}{4}-1+\frac {\pi}{4}\right]

\displaystyle=\frac {1}{2}\left [\frac {\pi}{2}-1\right]=\frac {\pi}{4}-\frac {1}{2}

From equation (1), we obtain

I\displaystyle=2\left [\frac {\pi}{4}-\frac {1}{2}\right]=\frac {\pi}{2}-1

#425642

Topic: Changing Variable

Evaluate the definite integral \int_1^4[|x-1|+|x-2|+|x-3|]dx

Solution

Let I=\int_1^4[|x-1|+|x-2|+|x-3|]dx$

\Rightarrow I=\int_1^4|x-1|dx+\int_1^4|x-2|dx+\int_1^4|x-3|dx

I=I_1+I_2+I_3 .... (1)

where, I_1=\int_1^4|x-1|dx, I_2=\int_1^4|x-2|dx, and I_3=\int_1^4|x-3|dx

I_1=\int_1^4|x-1|dx

(x-1)\geq 0 for 1\leq x\leq 4

\therefore I_1=\int_1^4(x-1)dx

\Rightarrow I_1=\left [\dfrac {x^2}{2}-x\right]_1^4

\Rightarrow I_1=\left [8-4-\dfrac {1}{2}+1\right]=\dfrac {9}{2} .... (2)

I_2=\int_1^4|x-2|dx

x-2\leq 0 for 2\leq x\leq 4 and x-2\leq 0 for 1\leq x\leq 2

\therefore I_2=\int_1^2(2-x)dx+\int_2^4(x-2)dx

\Rightarrow I_2=\left [2x-\dfrac {x^2}{2}\right]_1^2+\left [\dfrac {x^2}{2}-2x\right]_2^4

\Rightarrow I_2=[4-2-2+\dfrac {1}{2}]+[8-8-2+4]

\Rightarrow I_2=\dfrac {1}{2}+2=\dfrac {5}{2} .... (3)

I_3=\int_1^4|x-3|dx

x-3\geq 0 for 3\leq x\leq 4 and x-3\leq 0 for 1\leq x\leq 3

\therefore I_3=\int_1^3(3-x)dx+\int_3^4(x-3)dx

\Rightarrow I_3=\left [3x-\dfrac {x^2}{2}\right]_1^3+\left [\dfrac {x^2}{2}-3x\right]_3^4

\Rightarrow I_3=\left [9-\dfrac {9}{2}-3+\dfrac {1}{2}\right]+\left [8-12-\dfrac {9}{2}+9\right]

\Rightarrow I_3=[6-4]+\left [\dfrac {1}{2}\right]=\dfrac {5}{2} .... (4)

From equations (1), (2), (3), and (4), we obtain

I=\displaystyle \frac {9}{2}+\frac {5}{2}+\frac {5}{2}=\frac {19}{2}

#425655
Topic: Changing Variable

Prove \displaystyle \int_0^1xe^xdx=1

Solution

Let \displaystyle I=\int_0^1xe^xdx=1

Integrating by parts, we obtain

I=\displaystyle x\int_0^1e^xdx-\int_0^1\left \{\left (\frac {d}{dx}(x)\right)\int e^xdx\right \}dx

\displaystyle=[xe^x]_0^1-\int_0^1e^xdx

\displaystyle=[xe^x]_0^1-[e^x]_0^1

=e-e+1=1

Hence, the given result is proved.

#425659
Topic: Changing Variable

Prove \displaystyle \int_0^{\frac {\pi}{2}}\sin^3xdx=\frac {2}{3}

Solution

Let I=\int_0^{\frac {\pi}{2}}\sin^3x dx

I=\int_0^{\frac {\pi}{2}}\sin^2x\cdot \sin x dx

=\int_0^{\frac {\pi}{2}}(1-\cos^2x)\sin x dx

=\int_0^{\frac {\pi}{2}}\sin x dx-\int_0^{\frac {\pi}{2}}\cos^2x\cdot \sin x dx

Put \cos x = t in second part \Rightarrow -\sin xdx=dt

\displaystyle \therefore I=[-\cos x]_0^{\frac {\pi}{2}}-\left [\frac {t^3}{3}\right]_0^{1}

\displaystyle =1-\frac {1}{3}=1-\frac {1}{3}=\frac {2}{3}

Hence, the given result is proved.

#425663
Topic: Changing Variable

Prove \displaystyle \int_0^{\frac {\pi}{4}}2\tan^3x dx=1-\log 2

Solution

Let \displaystyle I=\int_0^{\frac {\pi}{4}}2 \tan^3x dx

I=\displaystyle2\int_0^{\frac {\pi}{4}}\tan^2 x \tan x dx=2\int_0^{\frac {\pi}{4}}(\sec^2x-1)\tan x dx

\displaystyle=2\int_0^{\frac {\pi}{4}}\sec^2x \tan x dx-2\int_0^{\frac {\pi}{4}}\tan x dx

\displaystyle=2\left [\frac {\tan^2x}{2}\right]_0^{\frac {\pi}{4}}+2[\log \cos x]_0^{\frac {\pi}{4}}

=1+2\left [\log \cos\dfrac {\pi}{4}-\log \cos 0\right]

=1+2\left [\log \dfrac {1}{\sqrt 2}-\log 1\right]

=1-\log 2-\log 1=1-\log 2

Hence, the given result is proved.

#425666
Topic: Changing Variable

Prove \displaystyle \int_0^1\sin^{-1}x dx=\frac {\pi}{2}-1

Solution

Let I\displaystyle =\int_0^1\sin^{-1}x dx

\Rightarrow\displaystyle I=\int_0^1\sin^{-1}x\cdot 1\cdot dx

Integrating by parts, we obtain

I\displaystyle=[\sin^{-1}x\cdot x]_0^1-\int_0^1\frac {1}{\sqrt {1-x^2}}\cdot x dx

\displaystyle=[x\sin^{-1}x]_0^1+\frac {1}{2}\int_0^1\frac {(-2x)}{\sqrt {1-x^2}}dx

Let 1-x^2=t\Rightarrow -2x dx=dt

When x=0, t=1 and when x=1, t=0

\displaystyle I=[x \sin^{-1}x]_0^1+\frac {1}{2}\int_1^0\frac {dt}{\sqrt t}

\displaystyle=[x \sin^{-1}x]_0^1+\frac {1}{2}[2\sqrt t]_1^0

=\sin^{-1}(1)+[-\sqrt 1]

\displaystyle=\frac {\pi}{2}-1

Hence, the given result is proved.

#425673
Topic: Definite Integral as a Limit of Sum

Evaluate \displaystyle \int_0^1e^{2-3x}dx as a limit of a sum.

Solution

Let I=\int_0^1e^{2-3x}dx

It is known that,

\int_a^bf(x)dx=(b-a)\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}[f(a)+f(a+h)+ ... +f(a+(n-1)h]

Where, h=\frac {b-a}{n}

Here, a=0, b=1, and f(x)=e^{2-3x}

\Rightarrow h=\frac {1-0}{n}=\frac {1}{n}

\therefore \int_0^1e^{2-3x}dx=(1-0)\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}[f(0)+f(0+h)+ ... +f(0+(n-1)h)]

=\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}[e^2+e^{2-3h}+ .... e^{2-3(n-1)h}]

=\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}[e^2\left \{1+e^{-3h}+e^{-6h}+e^{-9h}+ ... e^{-3(n-1)h}\right \}]

=\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [e^2\left \{\frac {1-(e^{-3h})^n}{1-(e^{-3h})}\right \}\right]

=\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [e^2\left \{\frac {1-e^{-\frac {3}{n}\times n}}{1-e^{-\frac {3}{n}}}\right \}\right]

=\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [\frac {e^2(1-e^{-3})}{1-e^{-\frac {3}{4}}}\right]

=e^2(e^{-3}-1)\displaystyle \lim_{n\rightarrow \infty}\frac {1}{n}\left [\frac {1}{e^{-\frac {3}{n}}-1}\right]

=e^2(e^{-3}-1)\displaystyle \lim_{n\rightarrow \infty}\left (-\frac {1}{3}\right)\left [\frac {-\frac {3}{n}}{e^{-\frac {3}{n}}-1}\right]

=\dfrac {-e^2(e^{-3}-1)}{3}\displaystyle \lim_{n\rightarrow \infty}\left [\frac {-\frac {3}{n}}{e^{-\frac {3}{n}}-1}\right]

=\dfrac {-e^2(e^{-3}-1)}{3} (1) \left [\displaystyle \lim_{n\rightarrow \infty}\frac {x}{e^x-1}\right]

=\dfrac {-e^{-1}+e^2}{3}

=\dfrac {1}{3}\left (e^2-\dfrac {1}{e}\right)