#425682

**Topic:** Area of Bounded Regions

Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x-axis in the first quadrant.

**Solution**

The area of the region bounded by the curve, $y^2 = x$, the lines, $x = 1$ and $x = 4$, and the $x$-axis is the area $ABCD$.

Thus area of $ABCD = \int_1^4 y dx$

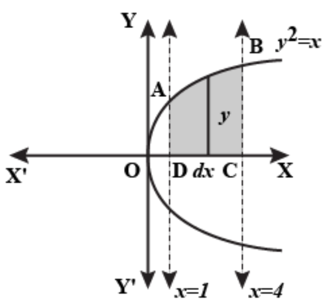
$= \int_1^4 \sqrt{x} dx$

$= \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4$

$= \frac{2}{3}\left[ (4)^{\frac{3}{2}} - (1)^{\frac{3}{2}} \right]$

$= \frac{2}{3}[8 - 1]$

$= \frac{14}{3}$ units




#425683

**Topic:** Area of Bounded Regions

Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the $x$-axis in the first quadrant.

**Solution**

The area of the region bounded by the curve, $y^2 = 9x$, $x = 2$ and $x = 4$, and the $x$-axis is the area $ABCD$.

Area of $ABCD = \int_2^4 y dx$

$= \int_2^4 3\sqrt{x} dx$

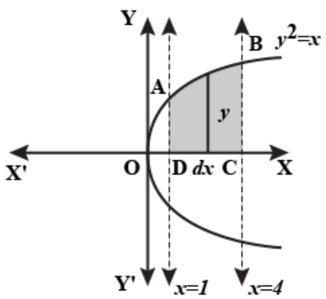
$= 3\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4$

$= 2[x^{\frac{3}{2}}]_2^4$

$= 2[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}]$

$= 2[8 - 2\sqrt{2}]$

$= (16 - 4\sqrt{2}) units$




#425684

**Topic:** Area of Bounded Regions

Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the $y$-axis in the first quadrant.

**Solution**

The area of the region bounded by the curve $x^2 = 4y$, $y = 2$ and $y = 4$, and the $y$-axis is the area $ABCD$.

Area of $ABCD = \int_2^4 x dy$

$= \int_2^4 2\sqrt{y} dy$

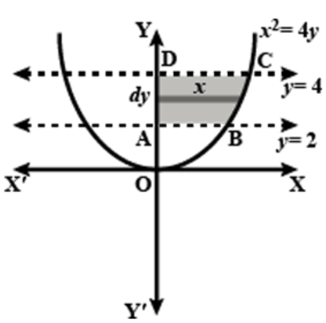
$= 2\int_2^4 \sqrt{y} dy$

$= 2\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4$

$= \frac{4}{3}[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}]$

$= \frac{4}{3}[8 - 2\sqrt{2}]$

$= \left(\frac{32 - 8\sqrt{2}}{3}\right) units$



**#425685**

**Topic:** Area of Bounded Regions

Find the area of the region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$.

**Solution**

The given equation of the ellipse, $\frac{x^2}{16}+\frac{y^2}{9}=1$, can be represented as

It can be observed that the ellipse is symmetrical about $x$-axis and $y$-axis.

$\therefore$ Area bounded by ellipse $=4\times$ Area of $OAB$

Area of $OAB$ $=\int_0^4 ydx$

$=\int_0^4 3\sqrt{1-\frac{x^2}{16}}dx$

$=\frac{3}{4}\int_0^4\sqrt{16-x^2}dx$

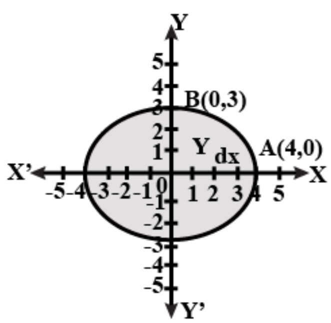
$=\frac{3}{4}\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$

$=\frac{3}{4}[2\sqrt{16-16}+8\sin^{-1}(1)-0-8\sin^{-1}(0)]$

$=\frac{3}{4}\left[\frac{8\pi}{2}\right]$

$=\frac{3}{4}[4\pi]=3\pi$

Therefore, are bounded by the ellipse $=4\times 3\pi=12\pi\ units$




**#425686**

**Topic:** Area of Bounded Regions

Find the area of the region bounded by the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$.

**Solution**

The given equation of the ellipse can be represented as

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$\Rightarrow y = 3\sqrt{1 - \frac{x^2}{4}} \text{ ............. (1)}$$

It can be observed that the ellipse is symmetrical about $x$-axis and $y$-axis.

$\therefore$ Area bounded by ellipse $= 4 \times$ Area $OAB$

$\therefore$ Area of $OAB = \int_0^2 y dx$

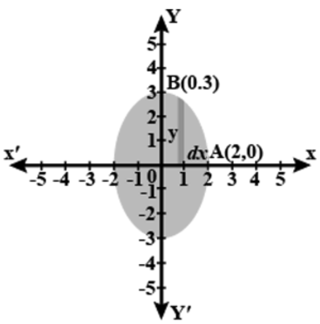
$= \int_0^2 3\sqrt{1 - \frac{x^2}{4}} dx$ [Using (1)]

$= \frac{3}{2}\int_0^2 \sqrt{4 - x^2} dx$

$= \frac{3}{2}\left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$

$= \frac{3}{2}\left[\frac{2\pi}{2}\right] = \frac{3\pi}{2}$

Therefore, area bounded by the ellipse $= 4 \times \frac{3\pi}{2} = 6\pi$ units




**#425687**

**Topic:** Area of Bounded Regions

Find the area of the region in the first quadrant enclosed by $x$-axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$.

**Solution**

The area of the region bounded by the circle, $x^2 + y^2 = 4$, $x = \sqrt{3}y$, and the $x$-axis is the area $OAB$.

The point of intersection of the line and the circle in the first quadrant is $(\sqrt{3}, 1)$.

Area $OAB =$ Area $\Delta OCA +$ Area $ACB$

Area of $OAC = \dfrac{1}{2} \times OC \times AC = \dfrac{1}{2} \times \sqrt{3} \times 1 = \dfrac{\sqrt{3}}{2}$ ............ (1)

Area of $ABC = \int_{\sqrt{3}}^{2} y dx$

$= \int_{\sqrt{3}}^{2} \sqrt{4 - x^2} dx$

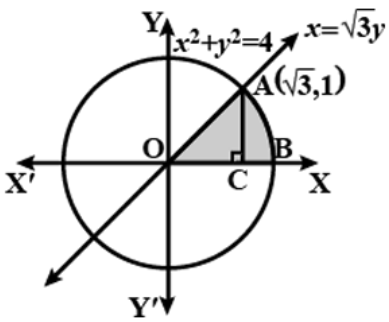
$= \left[\dfrac{x}{2}\sqrt{4 - x^2} + \dfrac{4}{2}\sin^{-1}\dfrac{x}{2}\right]_{\sqrt{3}}^{2}$

$= \left[2 \times \dfrac{\pi}{2} - \dfrac{\sqrt{3}}{2}\sqrt{4 - 3} - 2\sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)\right]$

$= \left[\pi - \dfrac{\sqrt{3}}{2} - \dfrac{2\pi}{3}\right]$

$= \left[\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{2}\right]$ ............. (2)

Therefore, area enclosed by $x$-axis, the line $x = \sqrt{3}y$, and the circle $x^2 + y^2 = 4$ in the first quadrant $= \dfrac{\sqrt{3}}{2} + \dfrac{\pi}{3} - \dfrac{\sqrt{3}}{2} = \dfrac{\pi}{3}$ units

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**#425688**

**Topic:** Area of Bounded Regions

Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \dfrac{a}{\sqrt{2}}$.

**Solution**

The area of the smaller part of the circle, $x^2 + y^2 = a^2$, cut off by the line, $x = \frac{a}{\sqrt{2}}$, is the are ABCDA.

It can be observed that the area ABCD is symmetrical about x-axis.

$\therefore$ Area $ABCD = 2 \times$ Area $ABC$

Area of $ABC = \int_{\frac{a}{\sqrt{2}}}^{a} y dx$

$= \int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{a^2 - x^2} dx$

$= \left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{\frac{a}{\sqrt{2}}}^{a}$

$= \left[\frac{a^2}{2}\left(\frac{\pi}{2}\right) - \frac{a}{2\sqrt{2}}\sqrt{a^2 - \frac{a^2}{2}} - \frac{a^2}{2}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$

$= \frac{a^2\pi}{4} - \frac{a}{2\sqrt{2}} \cdot \frac{a}{\sqrt{2}} - \frac{a^2}{2}\left(\frac{\pi}{4}\right)$

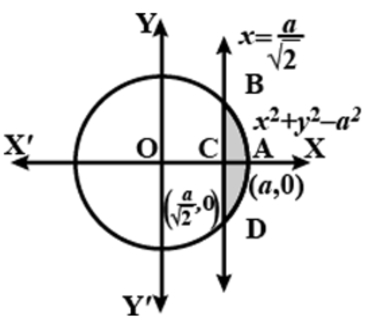
$= \frac{a^2\pi}{4} - \frac{a^2}{4} - \frac{a^2\pi}{8}$

$= \frac{a^2}{4}\left[\pi - 1 - \frac{\pi}{2}\right]$

$= \frac{a^2}{4}\left[\frac{\pi}{2} - 1\right]$

$\Rightarrow$ Area $ABCD = 2\left[\frac{a^2}{4}\left(\frac{\pi}{2} - 1\right)\right] = \frac{a^2}{2}\left(\frac{\pi}{2} - 1\right)$

Therefore, the area of smaller part of the circle, $x^2 + y^2 = a^2$, cut off by the line, $x = \frac{a}{\sqrt{2}}$, is $\frac{a^2}{2}\left(\frac{\pi}{2} - 1\right)$ sq.units.




**#425689**

**Topic:** Area of Bounded Regions

The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, find the value of $a$.

**Solution**

The line, $x = a$, divides the area bounded by the parabola and $x = 4$ into two equal parts.

$\therefore$ Area $OAD$ = Area $ABCD$

It can be observed that the given area is symmetrical about x-axis.

$\Rightarrow$ Area $OED$ = Area $EFCD$

Area $OED = \int_0^a y dx$

$= \int_0^a \sqrt{x} dx$

$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^a$

$= \frac{2}{3}(a)^{\frac{3}{2}}$ ............. (1)

Area of $EFCD = \int_a^4 \sqrt{x} dx$

$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_a^4$

$= \frac{2}{3}[8 - a^{\frac{3}{2}}]$ ........... (2)

From (1) and (2), we obtain

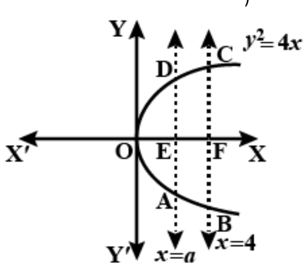
$\frac{2}{3}(a)^{\frac{3}{2}} = \frac{2}{3}[8 - (a)^{\frac{3}{2}}]$

$\Rightarrow 2 \cdot (a)^{\frac{3}{2}} = 8$

$\Rightarrow (a)^{\frac{3}{2}} = 4$

$\Rightarrow a = (4)^{\frac{2}{3}}$

Therefore, the value of a is $(4)^{\frac{2}{3}}$.

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**#425690**

**Topic:** Area of Bounded Regions

Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$.

**Solution**

The area bounded by the parabola, $x^2 = y$, and the line, $y = |x|$, can be represented as

The given area is symmetrical about $y$-axis

$\therefore$ Area $OACO$ = Area $ODBO$

The point of intersection of parabola, $x^2 = y$, and line, $y = x$, is $A(1, 1)$.

Area of $OACO$ = Area $\Delta OAB$ − Area $OBACO$

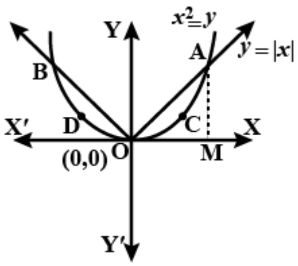
$\therefore$ Area of $\Delta OAB = \frac{1}{2} \times OB \times AB = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

Area of $OBACO = \int_0^1 y dx = \int_0^1 x^2 dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$

$\Rightarrow$ Area of $OACO$ = Area of $\Delta OAB$ − Area of $OBACO$

$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

Therefore, required area $= 2\left[\frac{1}{6}\right] = \frac{1}{3}$ sq.units.

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**#425691**

**Topic:** Area of Bounded Regions

Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

**Solution**

The area bounded by the curve, $x^2 = 4y$, and line, $x = 4y - 2$, is represented by the shaded area $OBAO$.

Let $A$ and $B$ be the points of intersection of the line and parabola.

Coordinates of point $A$ are $\left(-1, \frac{1}{4}\right)$

Coordinates of point $B$ are $(2, 1)$.

We draw $AL$ and $BM$ perpendicular to x-axis.

It can be observed that,

Area $OBAO$ = Area $OBCO$ + Area $OACO$ ........... (1)

Then, Area $OBCO$ = Area $OMBC$ − Area $OMBO$

$$= \int_0^2 \frac{x+2}{4} dx - \int_0^2 \frac{x^2}{4} dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_0^2 - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^2$$

$$= \frac{1}{4}[2 + 4] - \frac{1}{4}\left[\frac{8}{3}\right]$$

$$= \frac{3}{2} - \frac{2}{3} = \frac{5}{6}$$

Similarly, Area $OACO$ = Area $OLAC$ - Area $OLAO$

$$= \int_{-1}^{0} \frac{x+2}{4} dx - \int_{-1}^{0} \frac{x^2}{4} dx$$

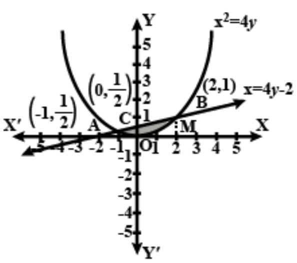
$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_{-1}^{0} - \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{0}$$

$$= -\frac{1}{4}\left[\frac{(-1)^2}{2} + 2(-1)\right] - \left[-\frac{1}{4}\left(\frac{(-1)^3}{3}\right)\right]$$

$$= -\frac{1}{4}\left[\frac{1}{2} - 2\right] - \frac{1}{12}$$

$$= \frac{1}{2} - \frac{1}{8} - \frac{1}{12} = \frac{7}{24}$$

Therefore, required area $= \left(\frac{5}{6} + \frac{7}{24}\right) = \frac{9}{8}$ sq. units.

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**#427458**

**Topic:** Area of Bounded Regions

Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$.

**Solution**

The region bounded by the parabola, $y^2 = 4x$, and the line $x = 3$ is the area $OACO$.

The area $OACO$ is symmetrical about $x$-axis.

$\therefore$ Area of $OACO$ = 2 (Area of $OAB$)

Area $OACO = 2\left[\int_0^3 y dx\right]$

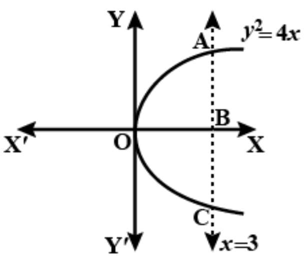
$= 2\int_0^3 2\sqrt{x} dx$

$= 4\left[\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^3$

$= \dfrac{8}{3}[(3)^{\frac{3}{2}}]$

$= 8\sqrt{3}$

Therefore, the required area is $8\sqrt{3}$ units.




**#427461**
**Topic:** Area of Bounded Regions

Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

A $\pi$

B $\dfrac{\pi}{2}$

C $\dfrac{\pi}{3}$

D $\dfrac{\pi}{4}$

**Solution**

The area bounded by the circle and the lines, $x = 0$ and $x = 2$, in the first quadrant is represented as shaded region in the plot.

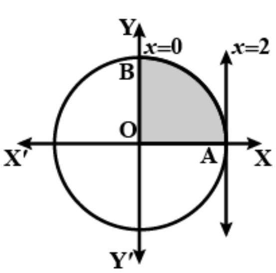
$\therefore$ Area $OAB = \int_0^2 y dx$

$= \int_0^2 \sqrt{4 - x^2} dx$

$= \left[\dfrac{x}{2}\sqrt{4 - x^2} + \dfrac{4}{2}\sin^{-1}\dfrac{x}{2}\right]_0^2$

$= 2\left(\dfrac{\pi}{2}\right) = \pi$sq. units

Thus, the correct answer is A.




**#427464**
**Topic:** Area of Bounded Regions

Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

A $2$

B $\frac{9}{4}$

C $\frac{9}{3}$

D $\frac{9}{2}$

**Solution**

The area bounded by the curve, $y^2 = 4x$, $y$-axis, and $y = 3$ is represented as shown in the diagram.

$\therefore$ Area $OAB = \int_0^3 x dy$

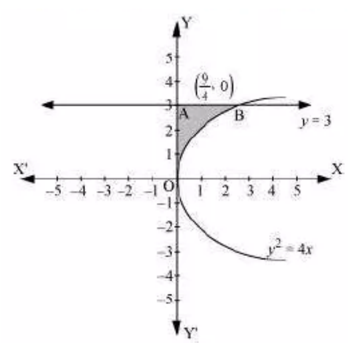
$= \int_0^3 \frac{y^2}{4} dy$

$= \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3$

$= \frac{1}{12}(27)$

$= \frac{9}{4}$ sq. units

Thus, the correct answer is B.




**#427467**

**Topic:** Area of Bounded Regions

Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$.

**Solution**

The required area is represented by the shaded area $OBCDO$.

Solving the given equation of circle, $4x^2 + 4y^2 = 9$, and parabola, $x^2 = 4y$, we obtain the point of intersection as $B\left(\sqrt{2}, \frac{1}{2}\right)$ and $D\left(-\sqrt{2}, \frac{1}{2}\right)$.

It can be observed that the required area is symmetrical about $y$-axis.

$\therefore$ Area $OBCDO = 2 \times Area\ OBCO$

We draw $BM$ perpendicular to $OA$.

Therefore, the coordinates of M are $(\sqrt{2}, 0)$.

Therefore, Area $OBCO =$ Area $OMBCO$ - Area $OMBO$

$$= \int_0^{\sqrt{2}} \sqrt{\frac{(9 - 4x^2)}{4}}\, dx - \int_0^{\sqrt{2}} \frac{x^2}{4}\, dx$$

$$= \frac{1}{2}\int_0^{\sqrt{2}} \sqrt{9 - 4x^2}\, dx - \frac{1}{4}\int_0^{\sqrt{2}} x^2\, dx$$

$$= \frac{1}{4}\left[x\sqrt{9 - 4x^2} + \frac{9}{2}\sin^{-1}\frac{2x}{3}\right]_0^{\sqrt{2}} - \frac{1}{4}\left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$$

$$= \frac{1}{4}\left[\sqrt{2}\sqrt{9 - 8} + \frac{9}{2}\sin^{-1}\frac{2\sqrt{2}}{3}\right] - \frac{1}{12}(\sqrt{2})^3$$

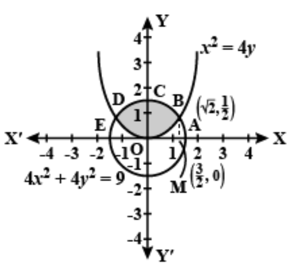
$$= \frac{\sqrt{2}}{4} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{6}$$

$$= \frac{\sqrt{2}}{12} + \frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3}$$

$$= \frac{1}{2}\left(\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right)$$

Therefore, the required area $OBCDO$ is

$$\left(2 \times \frac{1}{2}\left[\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right]\right) = \left[\frac{\sqrt{2}}{6} + \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right] \text{ sq. units}$$




#427472

**Topic:** Area of Bounded Regions

Find the area bounded by curves $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$.

**Solution**

The area bounded by the curves, $(x-1)^2+y^2=1$ and $x^2+y^2=1$, is represented by the shaded area as

On solving the equations, $(x-1)^2+y^2=1$ and $x^2+y^2=1$, we obtain the point of intersection as $A\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ and $B\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$.

It can be observed that the required area is symmetrical about $x$-axis.

$\therefore$ Area $OBCAO = 2\times Area\ OCAO$

We join $AB$, which intersects $OC$ at $M$, such that $AM$ is perpendicular to $OC$.

The coordinates of $M$ are $\left(\frac{1}{2},0\right)$.

$\Rightarrow Area\ OCAO = Area\ OMAO + Area\ MCAM$

$$=\left[\int_0^{\frac{1}{2}}\sqrt{1-(x-1)^2}dx+\int_{\frac{1}{2}}^{1}\sqrt{1-x^2}dx\right]$$

$$=\left[\frac{x-1}{2}\sqrt{1-(x-1)^2}+\frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}}+\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}$$

$$=\left[-\frac{1}{4}\sqrt{1-\left(-\frac{1}{2}\right)^2}+\frac{1}{2}\sin^{-1}\left(\frac{1}{2}-1\right)-\frac{1}{2}\sin^{-1}(-1)\right]+\left[\frac{1}{2}\sin^{-1}(1)-\frac{1}{4}\sqrt{1-\left(\frac{1}{2}\right)^2}-\frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)\right]$$

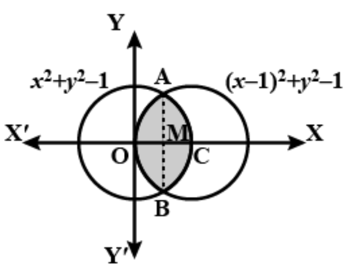
$$=\left[-\frac{\sqrt{3}}{8}+\frac{1}{2}\left(-\frac{\pi}{6}\right)-\frac{1}{2}\left(-\frac{\pi}{2}\right)\right]+\left[\frac{1}{2}\left(\frac{\pi}{2}\right)-\frac{\sqrt{3}}{8}-\frac{1}{2}\left(\frac{\pi}{6}\right)\right]$$

$$=\left[-\frac{\sqrt{3}}{4}-\frac{\pi}{12}+\frac{\pi}{4}+\frac{\pi}{4}-\frac{\pi}{12}\right]$$

$$=\left[-\frac{\sqrt{3}}{4}-\frac{\pi}{6}+\frac{\pi}{2}\right]$$

$$=\left[\frac{2\pi}{6}-\frac{\sqrt{3}}{4}\right]$$

Therefore, required area $OBCAO = 2\times\left(\frac{2\pi}{6}-\frac{\sqrt{3}}{4}\right)=\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$ units




**#427496**

**Topic:** Area of Bounded Regions

Find the area of the region bounded by the curves $y=x^2+2$, $y=x$, $x=0$ and $x=3$.

**Solution**

The area bounded by the curves, $y = x^2 + 2$, $y = x$, $x = 0$, and $x = 3$, is represented by the shaded area $OCBAO$ as shown in the diagram.

Then, Area $OCBAO$ = Area ODBAO - Area ODCO

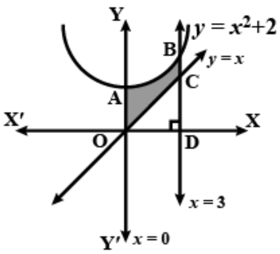
$= \int_0^3 (x^2 + 2)dx - \int_0^3 x dx$

$= \left[\frac{x^3}{3} + 2x\right]_0^3 - \left[\frac{x^2}{2}\right]_0^3$

$= [9 + 6] - \left[\frac{9}{2}\right]$

$= 15 - \frac{9}{2}$

$= \frac{21}{2}$ sq. units




#427505

**Topic:** Area of Bounded Regions

Using integration find the area of the region bounded by the triangle whose vertices are $(-1, 0)$, $(1, 3)$ and $(3, 2)$.

**Solution**

$BL$ and $CM$ are drawn perpendicular to x-axis.

It can be observed in the following figure that,

$Area(\Delta ACB) = Area(ALBA) + Area(BLMCB) - Area(AMCA)$ ......... (1)

Equating of line segment $AB$ is

$y - 0 = \frac{3-0}{1+1}(x+1)$

$y = \frac{3}{2}(x+1)$

$\therefore Area(ALBA) = \int_{-1}^{1}\frac{3}{2}(x+1)dx = \frac{3}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{1} = \frac{3}{2}\left[\frac{1}{2}+1-\frac{1}{2}+1\right] = 3$sq. units

Equating of line segment $BC$ is

$y - 3 = \frac{2-3}{3-1}(x-1)$

$y = \frac{1}{2}(-x+7)$

$\therefore Area(BLMCB) = \int_{1}^{3}\frac{1}{2}(-x+7)dx = \frac{1}{2}\left[-\frac{x^2}{2}+7x\right]_{1}^{3} = \frac{1}{2}\left[-\frac{9}{2}+21+\frac{1}{2}-7\right] = 5$ sq. units

Equation of line segment $AC$ is

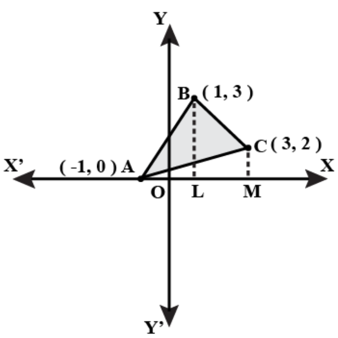
$y - 0 = \frac{2-0}{3+1}(x+1)$

$y = \frac{1}{2}(x+1)$

$\therefore Area(AMCA) = \frac{1}{2}\int_{-1}^{3}(x+1)dx = \frac{1}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{3} = \frac{1}{2}\left[\frac{9}{2}+3-\frac{1}{2}+1\right] = 4$ sq.units

Therefore, from equation (1), we obtain

$Area(\Delta ABC) = (3+5-4) = 4$ sq. units




**#427507**

**Topic:** Area of Bounded Regions

Using integration find the area of the triangular region whose sides have the equations y $= 2x+1$, $y = 3x+1$ and $x = 4$

**Solution**

The equations of sides of the triangles are $y = 2x + 1$, $y = 3x + 1$, and $x = 4$.

On solving these question, we obtain the vertices of triangle as $A(0, 1)$, $B(4, 13)$, and $C(4, 9)$.

It can be observed that,

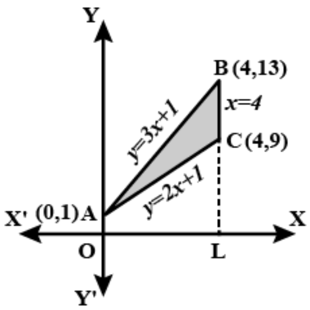
$Area(\Delta ACB) = Area(OLBAO) - Area(OLCAO)$

$= \int_0^4 (3x + 1)dx - \int_0^4 (2x + 1)dx$

$= \left[\frac{3x^2}{2} + x\right]_0^4 - \left[\frac{2x^2}{2} + x\right]_0^4$

$= (24 + 4) - (16 + 4)$

$= 28 - 20 = 8$ sq. units.




**#427516**
**Topic:** Area of Bounded Regions

Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is

A $2(\pi - 2)$

B $\pi - 2$

C $2\pi - 1$

D $2(\pi + 2)$

**Solution**

The smaller area enclosed by the circle, $x^2 + y^2 = 4$ and the line, $x + y = 2$ is represented by the shaded area $ACBA$ as shown in the diagram.

It can be observed that,

$Area\ ACBA = Area\ OACBO - Area\ (\Delta OAB)$

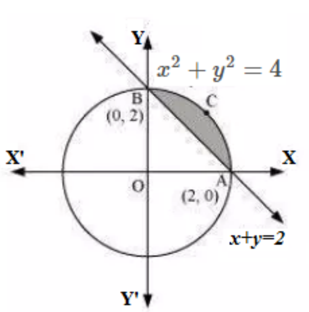
$= \int_0^2 \sqrt{4 - x^2}dx - \int_0^2 (2 - x)dx$

$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$

$= \left[2 \cdot \frac{\pi}{2}\right] - [4 - 2]$

$= (\pi - 2)$ sq. units

Thus, the correct answer is $B$.



**#427522**

**Topic:** Area of Bounded Regions

Area lying between the curve $y^2 = 4x$ and $y = 2x$ is

A $\frac{2}{3}$

B $\frac{1}{3}$

C $\frac{1}{4}$

D $\frac{3}{4}$

**Solution**

The area lying between the curve, $y^2 = 4x$ and $y = 2x$ is represented by the shaded area $OBAO$ as shaded in the diagram.

The points of intersection of these curves are $O(0, 0)$ and $A(1, 2)$.

We draw $AC$ perpendicular to $x$-axis such that the coordinates of $C$ are $(1, 0)$.

$\therefore$ Area $OBAO = Area(\Delta OCA) - Area(OCABO)$

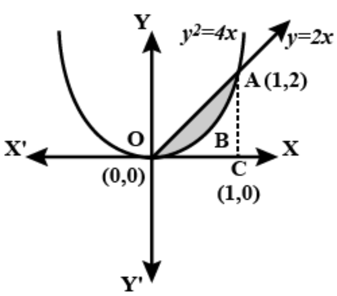
$= \int_0^1 2x dx - \int_0^1 2\sqrt{x} dx$

$= 2\left[\frac{x^2}{2}\right]_0^1 - 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$

$= \left|1 - \frac{4}{3}\right|$

$= \left|-\frac{1}{3}\right| = \frac{1}{3}$ sq. units

Thus, the correct answer is B.




**#428060**

**Topic:** Area of Bounded Regions

Find the area under the given curves and given lines:

(i) $y = x^2$, $x = 1$, $x = 2$ and $x$-axis

(ii) $y = x^4$, $x = 1$, $x = 5$ and $x$-axis

**Solution**

(i)

The required area is represented by the shaded area $ADCBA$ shown in the diagram.

Area $ADCBA = \int_1^2 y dx$

$= \int_1^2 x^2\ dx$

$= \left[\frac{x^3}{3}\right]_1^2$

$= \frac{8}{3} - \frac{1}{3}$

$= \frac{7}{3}$ sq.units

(ii)

The required area is represented by the shaded area $ADCBA$ in the diagram.

Area $ADCBA = \int_1^5\ x^4\ dx$

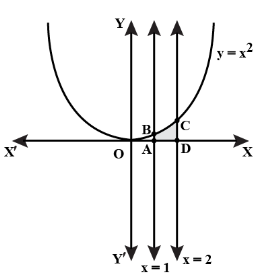
$= \left[\frac{x^5}{5}\right]_1^5$

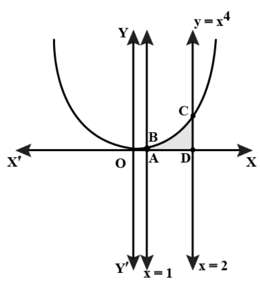
$= \frac{(5)^5}{5} - \frac{1}{5}$

$= (5)^4 - \frac{1}{5}$

$= 625 - \frac{1}{5}$

$= 624.8$ sq. units




**#428075**

**Topic:** Area of Bounded Regions

Find the area between the curves $y = x$ and $y = x^2$.

**Solution**

The required area is represented by the shaded area $OBAO$ in the diagram.

The points of intersection of the curves, $y = x$ and $y = x^2$, is $A\,(1, 1)$.

We draw $AC$ perpendicular to x-axis

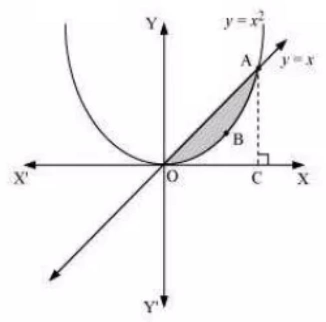
$\therefore\ Area\,(OBAO) = Area\,(\Delta OCA) - Area\,(OCABO)\ldots\ldots\ldots(1)$

$= \int_0^1 x\,dx - \int_0^1 x^2\,dx$

$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1$

$= \frac{1}{2} - \frac{1}{3}$

$= \frac{1}{6}$ sq. units




**#428082**

**Topic:** Area of Bounded Regions

Find the area of the region lying in the first quadrant and bounded by

$y = 4x^2$, $x = 0$, $y = 1$ and $y = 4$.

**Solution**

The area in the first quadrant bounded by $y = 4x^2$, $x = 0$, $y = 1$, and $y = 4$ is represented by the shaded area $ABCDA$

$\therefore\ Area\ ABCD = \int_1^4 x\,dx$

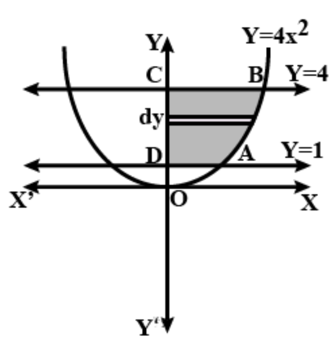
$= \int_1^4 \frac{\sqrt{y}}{2}\,dx$

$= \frac{1}{2}\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^4$

$= \frac{1}{3}\left[(4)^{\frac{3}{2}} - 1\right]$

$= \frac{1}{3}[8 - 1]$

$= \frac{7}{3}$ sq. units




**#428438**

**Topic:** Area of Bounded Regions

Sketch the graph of $y = |x+3|$ and evaluate $\int_{-6}^{0}|x+3|dx$

**Solution**

The given equation is $y = |x+3|$

Graph is plotted in the diagram.

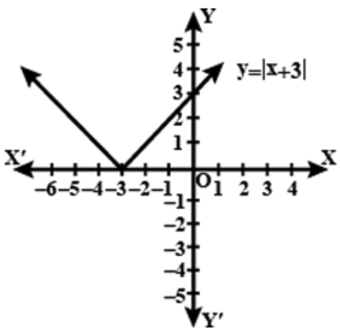
It is known that, $(x+3) \le 0$ for $-6 \le x \le -3$ and $(x+3) \ge 0$ for $-3 \le x \le 0$

$\therefore \int_{-6}^{0}|(x+3)|dx = -\int_{-6}^{-3}(x+3)dx + \int_{-3}^{0}(x+3)dx$

$= -\left[\frac{x^2}{2}+3x\right]_{-6}^{-3} + \left[\frac{x^2}{2}+3x\right]_{-3}^{0}$

$= -\left[\left(\frac{(-3)^2}{2}+3(-3)\right) - \left(\frac{(-6)^2}{2}+3(-6)\right)\right] + \left[0 - \left(\frac{(-3)^2}{2}+3(-3)\right)\right]$

$= -\left[-\frac{9}{2}\right] - \left[-\frac{9}{2}\right] = 9$




#428443
**Topic:** Area of Bounded Regions

Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$

**Solution**

The graph of $y = \sin x$ can be drawn as shown in the diagram.

$\therefore$ Required area $= Area\ OABO + Area\ BCDB$

$= \int_{0}^{\pi}\sin x\, dx + \left|\int_{\pi}^{2\pi}\sin x\, dx\right|$

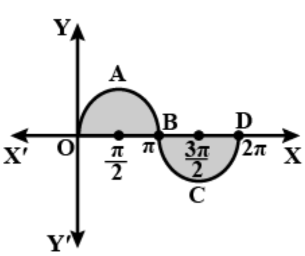
$= [-\cos x]_{0}^{\pi} + \left|[-\cos x]_{\pi}^{2\pi}\right|$

$= [-\cos\pi + \cos 0] + |-\cos 2\pi + \cos \pi|$

$= 1 + 1 + |(-1-1)|$

$= 2 + |-2|$

$= 2 + 2 = 4$ sq. units




#428446
**Topic:** Area of Bounded Regions

Find the area enclosed between the parabola $y^2 = 4ax$ and the line $y = mx$.

**Solution**

The area enclosed between the parabola, $y^2 = 4ax$ and the line, $y = mx$ is represented by the shaded area $OABO$ as

The points of intersection of both the curves are $(0, 0)$ and $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$.

We draw $AC$ perpendicular to $x$-axis.

$\therefore Area\ OABO = Area\ OCABO - Area\ (\Delta OCA)$

$= \int_0^{\frac{4a}{m^2}} 2\sqrt{ax}dx - \int_0^{\frac{4a}{m^2}} mxdx$

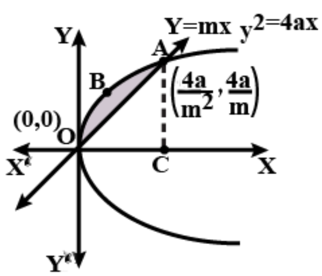
$= 2\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{\frac{4a}{m^2}} - m\left[\frac{x^2}{2}\right]_0^{\frac{4a}{m^2}}$

$= \frac{4}{3}\sqrt{a}\left(\frac{4a}{m^2}\right)^{\frac{3}{2}} - \frac{m}{2}\left[\left(\frac{4a}{m^2}\right)^2\right]$

$= \frac{32a^2}{3m^3} - \frac{m}{2}\left(\frac{16a^2}{m^4}\right)$

$= \frac{32a^2}{3m^3} - \frac{8a^2}{m^3}$

$= \frac{8a^2}{3m^3}$ sq. units




#428540

**Topic:** Area of Bounded Regions

Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$

**Solution**

The area enclosed between the parabola, $4y = 3x^2$ and the line, $2y = 3x + 12$ is represented by the shaded are $OBAO$

The points of intersection of the given curves are $A(-2, 3)$ and $(4, 12)$.

We draw $AC$ and $BD$ perpendicular to $x$-axis.

$\therefore Area\ OBAO = Area\ CDBA - (Area\ ODBO + Area\ OACO)$

$= \int_{-2}^{4} \frac{1}{2}(3x + 12)dx - \int_{-2}^{4} \frac{3x^2}{4} dx$

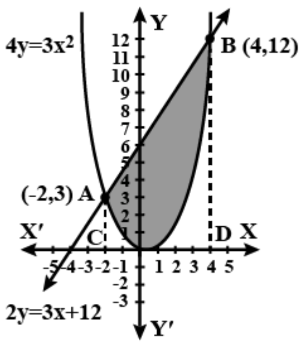
$= \frac{1}{2}\left[\frac{3x^2}{2} + 12x\right]_{-2}^{4} - \frac{3}{4}\left[\frac{x^3}{3}\right]_{-2}^{4}$

$= \frac{1}{2}[24 + 48 - 6 + 24] - \frac{1}{4}[64 + 8]$

$= \frac{1}{2}[90] - \frac{1}{4}[72]$

$= 45 - 18$

$= 27$ sq. units




#428542

**Topic:** Area of Bounded Regions

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$

**Solution**

The area of the smaller region by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$ represented by the shaded region $BCAB$

$\therefore Area\ BCAB = Area\ (OBCAO) - Area\ (OBAO)$

$= \int_0^3 2\sqrt{1 - \frac{x^2}{9}}\,dx - \int_0^3 2\left(1 - \frac{x}{3}\right)dx$

$= \frac{2}{3}\left[\int_0^3 \sqrt{9 - x^2}\,dx\right] - \frac{2}{3}\int_0^3 (3 - x)dx$

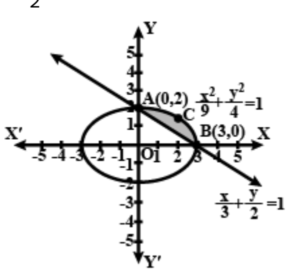
$= \frac{2}{3}\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3 - \frac{2}{3}\left[3x - \frac{x^2}{2}\right]_0^3$

$= \frac{2}{3}\left[\frac{9}{2}\left(\frac{\pi}{2}\right)\right] - \frac{2}{3}\left[9 - \frac{9}{2}\right]$

$= \frac{2}{3}\left[\frac{9\pi}{4} - \frac{9}{2}\right]$

$= \frac{2}{3} \times \frac{9}{4}(\pi - 2)$

$= \frac{3}{2}(\pi - 2)$ sq. units



**#428546**

**Topic:** Area of Bounded Regions

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$

**Solution**

The area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$ is represented by the shaded region $BCAB$

$\therefore$ $Area\ BACB = Area\ (OBCAO) - Area(OBAO)$

$= \int_0^a b\sqrt{1-\frac{x^2}{a^2}}dx - \int_0^a b\left(1-\frac{x}{a}\right)dx$

$= \frac{b}{a}\int_0^a \sqrt{a^2-x^2}dx - \frac{b}{a}\int_0^a (a-x)dx$

$= \frac{b}{a}\left[\left\{\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right\}_0^a - \left\{ax-\frac{x^2}{2}\right\}_0^a\right]$

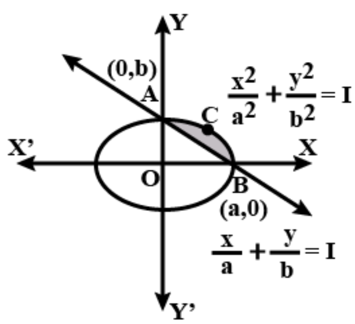
$= \frac{b}{a}\left[\left\{\frac{a^2}{2}\left(\frac{\pi}{2}\right)\right\} - \left\{a^2-\frac{a^2}{2}\right\}\right]$

$= \frac{b}{a}\left[\frac{a^2\pi}{4}-\frac{a^2}{2}\right]$

$= \frac{ba^2}{2a}\left[\frac{\pi}{2}-1\right]$

$= \frac{ab}{2}\left[\frac{\pi}{2}-1\right]$

$= \frac{ab}{4}(\pi-2)$




**#428548**

**Topic:** Area of Bounded Regions

Find the area of the region enclosed by the parabola $x^2=y$, the line $y=x+2$ and $x$-axis.

**Solution**

The region enclosed by the parabola $x^2 = y$, the line $y = x + 2$, and $x$-axis is represented by the shaded region $OABCO$ as

The point of intersection of the parabola $x^2 = y$ and the line $y = x + 2$ is $A(-1, 1)$.

$\therefore Area\ OABCO = Area\ (BCA) + Area\ COAC$

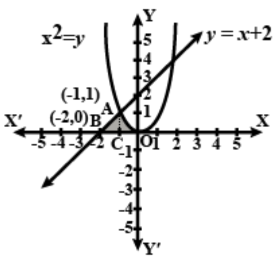
$= \int_{-2}^{-1}(x+2)dx + \int_{-1}^{0}x^2dx$

$= \left[\frac{x^2}{2} + 2x\right]_{-2}^{-1} + \left[\frac{x^3}{3}\right]_{-1}^{0}$

$= \left[\frac{(-1)^2}{2} + 2(-1) - \frac{(-2)^2}{2} - 2(-2)\right] + \left[0 - \frac{(-1)^3}{3}\right]$

$= \left[\frac{1}{2} - 2 - 2 + 4 + \frac{1}{3}\right]$

$= \frac{5}{6}$ sq. units




#428550

**Topic:** Area of Bounded Regions

Using the method of integration find the area bounded by the curve $|x| + |y| = 1$

**Solution**

The required region is bounded by lines $x + y = 1$, $x - y = 1$, $-x + y = 1$ and $-x - y = 11$

The area bounded by the curve $|x| + |y| = 1$ is represented by the shaded region $ADCB$

The curve intersects the axes at points $A(0, 1)$, $B(1, 0)$, $C(0, -1)$ and $D(-1, 0)$.

It can be observed that the given curve is symmetrical about $x$-axis and $y$-axis.

$\therefore Area\ ADCB = 4 \times Area\ OBAO$

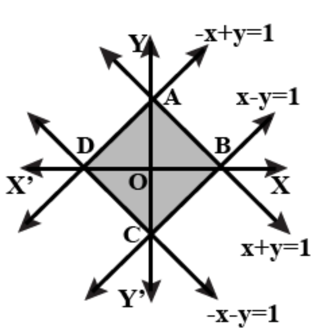
$= 4\int_{0}^{1}(1 - x)dx$

$= 4\left(x - \frac{x^2}{2}\right)_0^1$

$= 4\left[1 - \frac{1}{2}\right]$

$= 4\left(\frac{1}{2}\right)$

$= 2$ sq. units




#428553

**Topic:** Area of Bounded Regions

Find the area bounded by curves $\left\{(x, y): y \geq x^2 \text{ and } y = |x|\right\}$

**Solution**

The area bounded by curves $\left\{(x, y): y \geq x^2 \text{ and } y = |x|\right\}$, is represented by the shaded region as

It can be observed that the required area is symmetrical about $y$-axis.

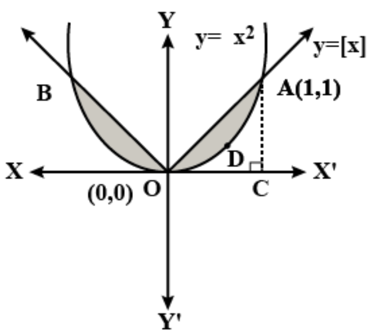
Required area $= 2[Area(OCAO) - Area(OCADO)]$

$= 2\left[\int_0^1 x dx - \int_0^1 x^2 dx\right]$

$= 2\left[\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1\right]$

$= 2\left[\frac{1}{2} - \frac{1}{3}\right]$

$= 2\left[\frac{1}{6}\right] = \frac{1}{3}$ sq. units




**#428554**

**Topic:** Area of Bounded Regions

Using the method of integration find the area of the triangle $ABC$, coordinates of whose vertices are $A(2, 0)$, $B(4, 5)$ and $C(6, 3)$

**Solution**

The vertices of $\Delta ABC$ are $A(2, 0)$, $B(4, 5)$, and $C(6, 3)$.

Equation of line segment $AB$ is $y - 0 = \dfrac{5 - 0}{4 - 2}(x - 2)$

$2y = 5x - 10$

$y = \dfrac{5}{2}(x - 2)..........(1)$

Equation of line segment $BC$ is $y - 5 = \dfrac{3 - 5}{6 - 4}(x - 4)$

$2y - 10 = -2x + 8$

$2y = -2x + 18$

$y = -x + 9.........(2)$

Equation of line segment $CA$ is $y - 3 = \dfrac{0 - 3}{2 - 6}(x - 6)$

$-4y + 12 = -3x + 18$

$4y = 3x - 6$

$y = \dfrac{3}{4}(x - 2)...........(3)$

$\therefore Area(\Delta ABC) = Area(ABLA) + Area(BLMCB) - Area(ACMA)$

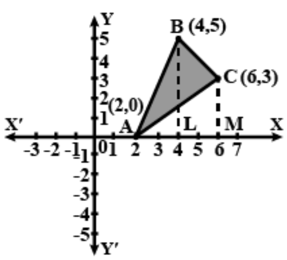
$= \int_2^4 \frac{5}{2}(x - 2)dx + \int_4^6 (-x + 9)dx - \int_2^6 \frac{3}{4}(x - 2)dx$

$= \frac{5}{2}\left[\frac{x^2}{2} - 2x\right]_2^4 + \left[\frac{-x^2}{2} + 9x\right]_4^6 - \frac{3}{4}\left[\frac{x^2}{2} - 2x\right]_2^6$

$= \frac{5}{2}[8 - 8 - 2 + 4] + [-18 + 54 + 8 - 36] - \frac{3}{4}[18 - 12 - 2 + 4]$

$= 5 + 8 - \frac{3}{4}(8)$

$= 13 - 6 = 7$ sq. units

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**#428556**

**Topic:** Area of Bounded Regions

Using the method of integraton find the area of the region bounded by lines: $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$

**Solution**

The given equations of lines are

$2x + y = 4 .........(1)$

$3x - 2y = 6 .........(2)$

And, $x - 3y + 5 = 0 ..........(3)$

The area of the region bounded by the lines is the area of $\Delta ABC$. $AL$ and $CM$ are the perpendicular on $x$-axis.

$Area(\Delta ABC) = Area(ALMCA) - Area(ALB) - Area(CMB)$

$= \int_1^4 \left(\frac{x+5}{3}\right)dx - \int_1^2 (4-2x)dx - \int_2^4 \left(\frac{3x-6}{2}\right)dx$

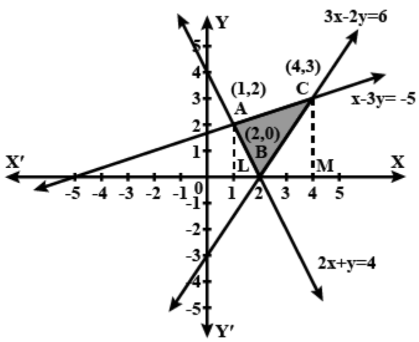
$= \frac{1}{3}\left[\frac{x^2}{2} + 5x\right]_1^4 - [4x - x^2]_1^2 - \frac{1}{2}\left[\frac{3x^2}{2} - 6x\right]_2^4$

$= \frac{1}{3}\left[8 + 20 - \frac{1}{2} - 5\right] - [8 - 4 - 4 + 1] - \frac{1}{2}[24 - 24 - 6 + 12]$

$= \left(\frac{1}{3} \times \frac{45}{2}\right) - (1) - \frac{1}{2}(6)$

$= \frac{15}{2} - 1 - 3$

$= \frac{15}{2} - 4 = \frac{15-8}{2} = \frac{7}{2}$ sq. units




**#428557**

**Topic:** Area of Bounded Regions

Find the area of the region $\left\{(x, y) : y^2 \le 4x, 4x^2 + 4y^2 \le 9\right\}$

**Solution**

Given curves are $y^2 = 4x$ ....(1)

$4x^2 + 4y^2 = 9$ ...(2)

$\Rightarrow x^2 + y^2 = \frac{9}{4}$

Center of circle is (0,0) and radius of circle is $\frac{3}{2}$

Put the value from eqn (1) in eqn (2),

$4x^2 + 16x - 9 = 0$

$\Rightarrow x = \frac{1}{2}, -\frac{9}{2}$

But $x = -\frac{9}{2}$, not possible .

So, $x = \frac{1}{2}$

$\Rightarrow y = \pm\sqrt{2}$

So, the points of intersection of both the curves are $\left(\frac{1}{2}, \sqrt{2}\right)$ and $\left(\frac{1}{2}, -\sqrt{2}\right)$.

The shaded region $OABCO$ represents the area bounded by the curves $\{(x, y): y^2 \le 4x, 4x^2 + 4y^2 \le 9\}$

Since, the area $OABCO$ is symmetrical about x-axis.

$\therefore$ *Area OABCO* = 2 × *Area OBC*

*Area OBCO* = *Area OMC* + *Area MBC*

$= \int_0^{\frac{1}{2}} 2\sqrt{x}dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2}\sqrt{9 - 4x^2}dx$

$= 2\left[\frac{x^{3/2}}{3/2}\right]_0^{1/2} + \int_{1/2}^{3/2} \sqrt{(\frac{3}{2})^2 - (x)^2}dx$

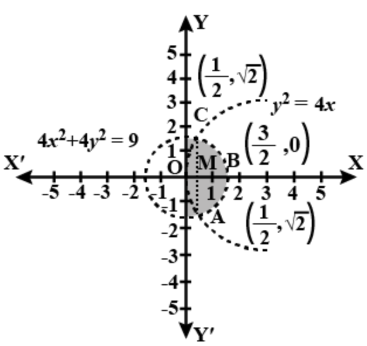
$= \frac{4}{3}\frac{1}{2\sqrt{2}} + \left[\frac{x\sqrt{(\frac{3}{2})^2 - (x)^2}}{2} + \frac{\frac{9}{4}}{2}\sin^{-1}\left(\frac{x}{\frac{3}{2}}\right)\right]_{1/2}^{3/2}$

$= \frac{\sqrt{2}}{3} + [0 + \frac{9}{8}\sin^{-1}(1) - \frac{\frac{1}{2}\sqrt{2}}{2} - \frac{9}{8}\sin^{-1}(\frac{1}{3})]$

$= \frac{\sqrt{2}}{3} + \frac{9\pi}{16} - \frac{\sqrt{2}}{4} - \frac{9}{8}\sin^{-1}(\frac{1}{3})$

Required Area $= 2[\frac{\sqrt{2}}{3} + \frac{9\pi}{16} - \frac{\sqrt{2}}{4} - \frac{9}{8}\sin^{-1}(\frac{1}{3})]$

$= \frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}(\frac{1}{3})$ sq.units




**#428558**

**Topic:** Area of Bounded Regions

Area bounded by the curves $y = x^3$, the $x$-axis and the ordinates $x = -2$ and $x = 1$ is

**A** $-9$

**B** $-\frac{15}{4}$

**C** $\frac{15}{4}$

D $\frac{17}{4}$

**Solution**

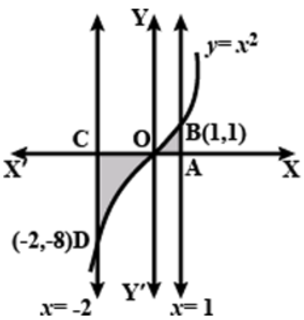
Required area $= -\int_{-2}^{0} x^3 dx + \int_{0}^{1} x^3 dx$

$= -\left[\frac{x^4}{4}\right]_{-2}^{0} + \left[\frac{x^4}{4}\right]_{0}^{1}$

$= -\left[0 - \frac{(-2)^4}{4}\right] + \frac{1}{4}$

$= \left(\frac{1}{4} + 4\right) = \frac{17}{4}$ sq. units

Thus, the correct answer is D.




**#428560**

**Topic:** Area of Bounded Regions

The area bounded by the curve $y = x|x|$, $x$-axis and the ordinates $x = -1$ and $x = 1$ is given by

A 0

B $\frac{1}{3}$

C $\frac{2}{3}$

D $\frac{4}{3}$

**Solution**

$y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$

Required area $= \int_{-1}^{1} y dx$

$= \int_{-1}^{1} x|x| dx$

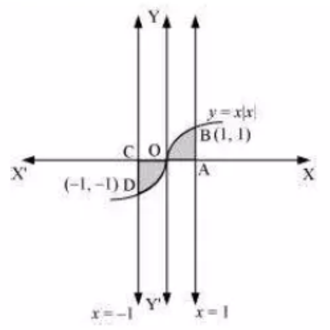
$= \int_{-1}^{0} x^2 dx + \int_{0}^{1} x^2 dx$

$= \left[\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1}$

$= -\left(-\frac{1}{3}\right) + \frac{1}{3}$

$= \frac{2}{3}$ sq. units

Thus, the correct answer is C.




**#428561**

**Topic:** Area of Bounded Regions

The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is

A $\frac{4}{3}(4\pi - \sqrt{3})$

B $\frac{4}{3}(4\pi + \sqrt{3})$

C $\frac{4}{3}(8\pi - \sqrt{3})$

D $\frac{4}{3}(8\pi + \sqrt{3})$

**Solution**

The given equations are

$x^2 + y^2 = 16$........(1)

$y^2 = 6x$. . . . . . . (2)

Area bounded by the circle and parabola

$= 2[Area(OADO) + Area(ADBA)]$

$= 2[\int_0^2 \sqrt{16x}dx + \int_2^4 \sqrt{16 - x^2}dx]$

$= 2\left[\sqrt{6}\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)\right]_0^2 + 2\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_2^4$

$= 2\sqrt{6} \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^2 + 2\left[8 \cdot \frac{\pi}{2} - \sqrt{16 - 4} - 8\sin^{-1}\left(\frac{1}{2}\right)\right]$

$= \frac{4\sqrt{6}}{3}(2\sqrt{2}) + 2\left[4\pi - \sqrt{12} - 8\frac{\pi}{6}\right]$

$= \frac{16\sqrt{3}}{3} + 8\pi - 4\sqrt{3} - \frac{8}{3}\pi$

$= \frac{4}{3}[4\sqrt{3} + 6\pi - 2\sqrt{3} - 2\pi] = \frac{4}{3}[\sqrt{3} + 4\pi]$

$= \frac{4}{3}[4\pi + \sqrt{3}]$ sq. units

Area of circle $= \pi(r)^2$

$= \pi(4)^2$

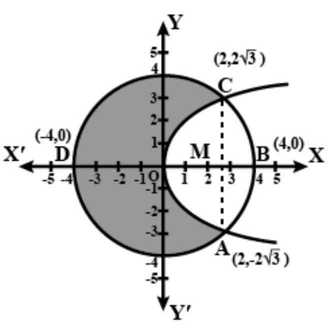
$= 16\pi$ sq. units

$\therefore$ Required area $= 16\pi - \frac{4}{3}[4\pi + \sqrt{3}]$

$= \frac{4}{3}[4 \times 3\pi - 4\pi - \sqrt{3}]$

$= \frac{4}{3}(8\pi - \sqrt{3})$ sq. units

Thus, the correct answer is C.

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**#428566**

**Topic:** Area of Bounded Regions

The area bounded by the $y$-axis, $y = \cos x$ and $y = \sin x$ when $0 \le x \le \frac{\pi}{2}$

A $2(\sqrt{2} - 1)$

**[B]** $\sqrt{2} - 1$

C $\sqrt{2} + 1$

D $\sqrt{2}$

**Solution**

The given equations are

$y = \cos x \ldots\ldots\ldots (1)$ And, $y = \sin x \ldots\ldots\ldots (2)$

Required area $= Area(ABLA) + area(OBLO)$

$= \int_{\frac{1}{\sqrt{2}}}^{1} x dy + \int_{0}^{\frac{1}{\sqrt{2}}} x dy$

$= \int_{\frac{1}{\sqrt{2}}}^{1} \cos^{-1} y dy + \int_{0}^{\frac{1}{\sqrt{2}}} \sin^{-1} y dy$

Integrating by parts, we obtain

$= \left[ y\cos^{-1} y - \sqrt{1-y^2} \right]_{\frac{1}{\sqrt{2}}}^{1} + \left[ x\sin^{-1} x + \sqrt{1-x^2} \right]_{0}^{\frac{1}{\sqrt{2}}}$

$= \left[ \cos^{-1}(1) - \frac{1}{\sqrt{2}}\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} \right] + \left[ \frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} - 1 \right]$

$= \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} - 1$

$= \frac{2}{\sqrt{2}} - 1$

$= \sqrt{2} - 1$ sq. units

Thus the correct answer is B.