A QUICK REFERENCE...

Here's your guide to the much needed list of such formulae in mathematics and related disciplines that are important owing to their frequent usage in problem solving. Send your comments to managingeditor@careers360.com on the booklet and tell us which other topics you would like to include in the series.

1. ALGEBRA

Make use of symbolic forms to represent, model, and analyse mathematical situations. Using Algebra one can analyse and use linear relations among two or more variables; one can also analyse and use quadratic relations between two variables. Further, algebra is used to represent and solve situations involving change, rates of change, linear equations, and inequalities etc

if $\frac{a}{b} = \frac{c}{d}$ *then:*

$ab = bc$ — *Diagonal product*

$\frac{a}{b} = \frac{c}{d}$ — *Diagonal exchange*

$\frac{b}{a} = \frac{d}{c}$ — *Inverse*

$\frac{a+c}{b} = \frac{c+d}{d}$ — *Addent*

Factors of $x^n - a^n$

$x^n - a^n = (x - a)(x^{n-1} + x^{n-2}a + x^{n-3}a^2 + ... + xa^{n-2} + a^{n-1})$

$a^2 - b^2 = (a + b)(a - b)$

Sum and difference of two cubes

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Expansions of various squares and cubes

$(a + b + c + \cdots + n)^2 = a^2 + b^2 + c^2 + \cdots + n^2 + 2\sum ab$

(where $\sum ab$ represents all possible pairs of a,b,c,...,n)

$(a + b + c + d)^2 = a^2 + b^2 + c^2 + d^2 + 2ab + 2ac + 2ad + 2bc + 2bd + 2cd$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Quadratic equations

The solution of $ax^2 + bx + c = 0$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. COMPLEX NUMBERS

Complex numbers are numbers that are constructed to solve equations where square roots of negative numbers occur. These numbers look like $1 + i$, $2i$, $1 - i$. They are added, subtracted, multiplied and divided with the normal rules of algebra with the additional condition that $i^2 = -1$. The symbol i is treated just like any other algebraic variable.

Definition of i

$$\sqrt{-1} = \pm i$$

Equality

If $a + ib = c + id$, then $a = c$ and $b = d$.

Mod – arg theorems

THEOREM 1

$|Z_1Z_2| = |Z_1| \cdot |Z_2|$

$arg(z_1z_2) = arg\, z1 + arg z_2$

THEOREM 2

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

$$arg \frac{z_1}{z_2} = arg\ z1 - arg\ z_2$$

Euler's formula

$$\cos\theta + i \sin\theta = e^{i\theta}$$

de Moivre's theorem

$$(\cos\theta + i \sin\theta)^n = \cos(n\theta) + i \sin(n\theta)$$

Absolute value

The definition of the absolute value of a is

$$|a| = \begin{cases} a & if\ a > o \\ o & if\ a = o \\ -a & if\ a < o \end{cases}$$

Some properties

$if\ |x| = a$	*then*	$x = a$
$if\ |x| < a$	*then*	$-a < x < a$
$if\ |x| > a$	*then*	$x < a$ or $x > a$

3. INEQUALITIES

Think of it and realize that two real numbers or two algebraic expressions that are related by the symbol < (less than) or > (greater than) and also by signs > or < form an inequality. The system of inequalities (with one unknown or if it contains a modulus etc) usually results in solutions that are in the form of either/or. One has to be clear in the notation of ranges

If a > b, c > d, then:

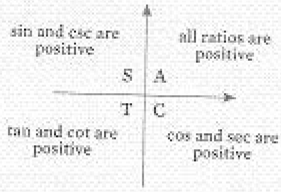
$a \pm c > b \pm c$	
$ac > bc$	$(c > 0)$
$ac < bc$	$(c < 0)$
$ac > bd$	$(a,b,c,d$ all $>0)$
$a^2 > b^2$	$(a, b$ both $>0)$
$\frac{1}{a} < \frac{1}{b}$	$(a,b$ both $>0)$

4. TRIGONOMETRY

Originating from two words in Greek, trigon – 'triangle' and metron - 'a measure', Trigonometry literally means measuring (of angles and sides) of triangles. It is a methodology for getting to know the unknown elements of a triangle (or other geometric shapes) provided the data includes a sufficient amount of linear and angular measurements to define a shape uniquely. Though geometry also depends on treating angles as quantities, but in geometry, angles are not measured, rather they are simply compared or added or subtracted. Trigonometry depends on angle measurement and quantities determined by measure of the angle.

All Silver Tea Cups

Reciprocal ratios



$$csc\ \theta = \frac{1}{sin\ \theta} \qquad \text{(cosec } \theta \text{ in wrtten as csc } \theta\text{)}$$

$$sin\ \theta = \frac{1}{cos\ \theta}$$

$$cot\ \theta = \frac{1}{tan\ \theta} = \frac{cos\theta}{sin\theta}$$

$$tan\theta = \frac{sin\theta}{cos\theta}$$

Pythagorean identities

$$sin^2\theta + cos^2\theta = 1$$

$$tan^2\theta + 1 = sec^2\theta$$

$$1 + cot^2\theta = csc^2\theta$$

Inverse formulas

$$\sin^{-1} x = \tan^{-1} \frac{x}{\sqrt{1 - x^2}} \qquad x \neq 1$$

$$\cos^{-1} x = \tan^{-1} \frac{\sqrt{1 - x^2}}{x} \qquad x \neq 0$$

Co – ratios|(complementary angles)

$\sin(90^o - \theta) = \cos\theta$

$\tan(90^o - \theta) = \cot\theta$

$\sec(90^o - \theta) = \csc\theta$

$\cos(90^o - \theta) = \sin\theta$

$\cot(90^o - \theta) = \tan\theta$

$\csc(90^o - \theta) = \sec\theta$

$\sin(180^o - \theta) = \sin\theta$

$\cos(180^o - \theta) = -\cos\theta$

$\tan(180^o - \theta) = -\tan\theta$

$\sin(180^o + \theta) = -\sin\theta$

$\cos(180^o + \theta) = -\cos\theta$

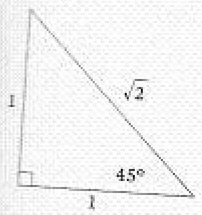
$\tan(180^o + \theta) = \tan\theta$

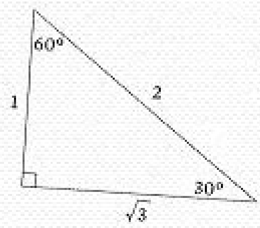
$\sin(360^o - \theta) = \sin -\theta = -\sin\theta$ (odd)

$\cos(360^o - \theta) = \cos -\theta = \cos\theta$ (even)

$\tan(360^o - \theta) = \tan -\theta = -\tan\theta$ (odd)

Exact values





$\sin 45^o = \cos 45^o = \frac{1}{\sqrt{2}}$

$\sin 30^o = \cos 60^o = \frac{1}{2}$

$\sin 60^o = \cos 30^o = \frac{\sqrt{3}}{2}$

$\tan 45^o = \cot 45^o = 1$

$tan\ 30^\circ = cot\ 60^\circ = \frac{1}{\sqrt{3}}$

$tan\ 60^\circ = cot\ 30^\circ = \sqrt{3}$

$sin\ 15^\circ = cos\ 75^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$

$sin\ 75^\circ = cos\ 15^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$

$tan\ 15^\circ = cot\ 75^\circ = 2 - \sqrt{3}$

$tan\ 75^\circ = cot\ 15^\circ = 2 + \sqrt{3}$

Addition formulas

$sin(\theta \pm \phi) = sin\theta\ cos\phi \pm cos\theta\ sin\phi$

$cos(\theta \pm \phi) = cos\theta\ cos\phi \mp sin\theta\ sin\phi$

$tan(\theta \pm \phi) = \frac{tan\theta \pm tan\phi}{1 \mp tan\theta\ tan\phi}$

$cot(\theta \pm \phi) = \frac{cot\theta\ cot\phi \mp 1}{cot\theta \pm cot\ \phi}$ $sin\ 2\theta$ $2sin\theta\ cos\theta$

$cos\ 2\theta = cos^2\theta - sin^2\theta$

$= 1 - 2\ sin^2\theta$

$= 2cos^2\theta - 1$

$tan\ 2\theta = \frac{2tan\theta}{1 - tan^2\theta}$

$sin^2\theta = \frac{1 - cos2\theta}{2}$

$cos^2\theta = \frac{1 + cos2\theta}{2}$

Sine rule

$$\frac{a}{sin\ A} = \frac{b}{sin\ B} = \frac{c}{sin\ C}$$

Area of a triangle

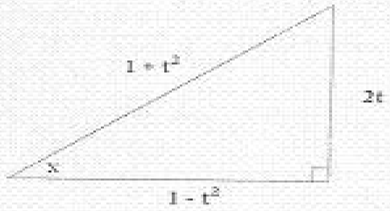
$$Area\ of\ any\ triangle = \frac{ab\ sin\ C}{2}$$

Cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$ (side)

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ (angle)

"Little t" formula (or the substitution formula)



$t = \tan\frac{x}{2}$

$\tan x = \frac{2\tan\frac{x}{2}}{1 - \tan^2\frac{x}{2}}$

$= \frac{2t}{1 - t^2}$

$\sin x = \frac{2t}{1 + t^2}$

$\cos x = \frac{1 - t^2}{1 + t^2}$

if $t = \tan\frac{x}{2}$ then $dx = \frac{2dt}{1 + t^2}$

$\sin 3x = 3\sin x - 4\sin^3 x$

$\cos 3x = 4\cos^3 x - 3\cos x$

$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

5. GEOMETRY AND SPATIAL SENSE

It is often said that if you wish to test someone's knowledge in mathematics, assess him/her in geometry. Using this branch one can analyse properties of objects and relationships among the properties. One can also solve problems involving two- and three-dimensional figures. The transformations and symmetry is used to analyse mathematical situations. More importantly, visualization and spatial reasoning is used to solve problems both within and outside of mathematics. With a good spatial sense, one can select and use different representational systems, including coordinate geometry.

Arc: a curved line that is part of the circumference of a circle

Chord: a line segment within a circle that touches 2 points on the circle.

Circumference: the distance around the circle.

Diameter: the longest distance from one end of a circle to the other.

Origin: the center of the circle

pi (π): A number, 3.141592..., equal to (the circumference) / (the diameter) of any circle.

Radius: distance from center of circle to any point on it.

Sector: is like a slice of pie (a circle wedge).

Tangent of circle: a line perpendicular to the radius that touches only one point on the circle.

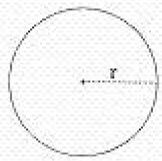
Diameter = 2 x radius of circle

Circumference of Circle

= $\pi \times diameter = 2\pi \times radius$

Area of Circle:

area = πr^2



Length of a Circular Arc:

(with central angle θ)

if the angle θ *is in degrees, then length* = $\theta \, x(\pi /180) \, x \, r$

if the angle θ *is in radians, then length* = $r \, x \, \theta$

Area of Circle Sector:

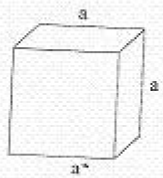
(with central angle θ)

if the angle θ *is in degrees, then area* = $(\theta /360) x \pi r^2$

if the angle θ *is in radians, then area* = $((\theta /(2)) x \pi r^2$

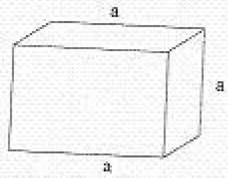
CUBE

Surface Area of a Cube = $6a^2$



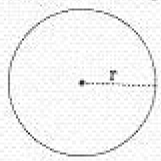
(a is the length of the side of each edge of the cube)

Surface Area of a Rectangular Prism = $2ab + 2bc + 2ac$



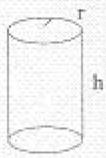
(a, b, and c are the lengths of the 3 sides)

» **Surface Area of a Sphere** = $4 \pi r^2$



(r is radius of circle)

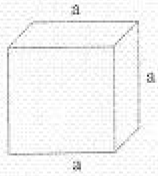
Surface Area of a Cylinder = $2\pi r^2 + 2\pi rh$



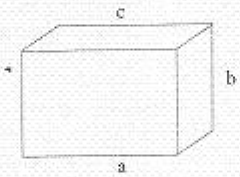
(h is the height of the cylinder, r is the radius of the top)
Surface Area = Areas of top and bottom + Area of the side
Surface Area = 2(Area of top) + (perimeter of top) height*
Surface Area = $2(\pi r^2) + (2\pi r) * h$

Volume Formulas

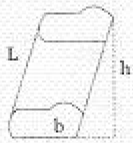
cube = a^3



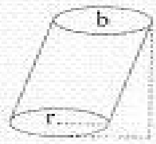
rectangular prism = $a\,b\,c$



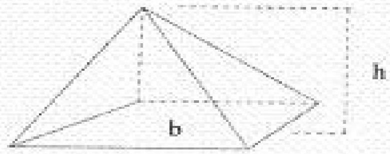
irregular prism = $b\,h$



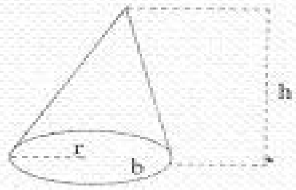
cylinder = $b\,h = \pi r^2 h$



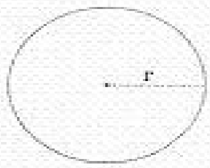
pyramid = $(1/3)\,b\,h$



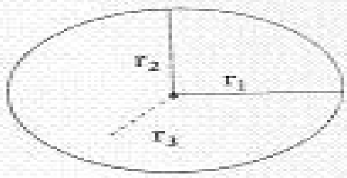
cone = $(1/3)\,b\,h = 1/3\;\pi r^2 h$



sphere = $(4/3)\,\pi r^3$

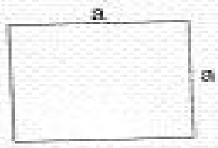


ellipsoid = $(4/3)\pi\, r1\, r2\, r3$

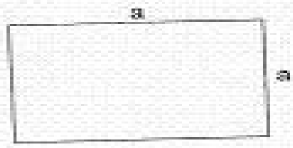


Area Formulas

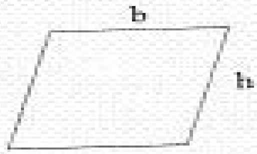
square = a^2



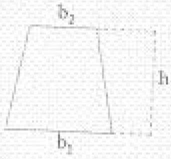
rectangle = ab



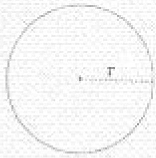
parallelogram = bh



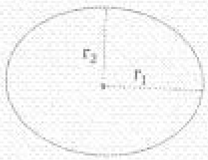
trapezoid = $h/2\,(b_1 + b_2)$

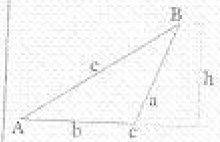


circle = πr^2



ellipse = $\pi r_1 r_2$



triangle	$\frac{1}{2}$ (bh)		*one half times the base length time the height of the triangle*
equilateral triangle =	$\frac{\sqrt{3}}{4}$ (a^2)		

triangle given SAS (two sides and the opposite angle)
= (1/2) a b sin C
triangle given a,b,c = $\sqrt{\ }$ [s(s − a)(s − b)(s − c)] when s = (a + b + c)/2
(Hero's formula)

Areas and volumes of geometric shapes

length of an arc with angle θ and radius r is $\ell = r\theta$
Area of a sector angle θ and radius r is

$$A = \frac{1}{2}r^2\theta$$

Area of a segment of a circle with angle θ and radius r is

$$A = \frac{1}{2}r^2(\theta - \sin\theta)$$

$$\text{Area of triangle} = \frac{1}{2}r^2 \sin\theta$$

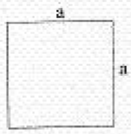
$$\text{area of segment} = \frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin\theta$$

Perimeter Formulas

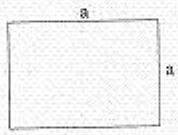
The perimeter of any polygon is the sum of the lengths of all the sides.

Examples

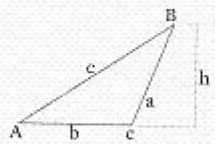
square = *4a*



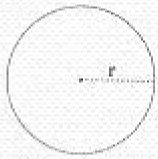
rectangle = *2a + 2b*



triangle = $a + b + c$



circle = $2\pi r$



circle = πd *(where d is the diameter)*

The perimeter of a circle is more commonly known as the circumference.

Polygon Formulas

(N = # of sides and S = length from center to a corner)

Area of a regular polygon = (1/2) N sin(360°/N) S2

Sum of the interior angles of a polygon = (N – 2) x 180°

The number of diagonals in a polygon = 1/2 N(N – 3)

The number of triangles (when you draw all the diagonals from one vertex) in a polygon = (N – 2)

Coordinate Geometry

Equations of straight lines

The equation of a line || y axis through (a, b) is

$y = b$

the equation of a line || x axis through (a, b) is

$x = a$

the equation of a line with slope m and y intercept b is

$y = mx + b$

The equation of a line with slope m through (x_1, y_1) is

$y - y_1 = m(x - x_1)$

The equation of a line through (x_1, y_1) *and* (x_2, y_2) *is*

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

The equation with x intercept a and y intercept b is

$$\frac{x}{a} + \frac{y}{b} = 1$$

This "general form" of equation for a line has slope $= \frac{-A}{B}$,

x *intercept* $= \frac{-C}{A}$, *y intercept* $= \frac{-C}{B}$

$$Ax + By + C = 0$$

Distance and point formulas

The distance between (x_1, x_2) *and* (x_2, y_2) *is*

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance of (x_1, y_1) *from* $Ax + Bx + C = 0$ *is*

$$d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$$

The coordinates of the point which divides the join of $Q = (x_1, y_1)$ *and* $R = (x_2, y_2)$ *in the ration* $m_1 : m_2$ *is*

$$P = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

From Q to P to R. If P is outside of the line QR, then QP, RP are measured in opposite senses, and $\frac{m_1}{m_2}$ *is negative.*

The coordinates of the midpoint of the join of (x_1, y_1) *and* (x_2, y_2) *is*

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

The angle α *between two lines with slopes* m_1, m_2 *is given by*

$$\tan\alpha = \left|\frac{m_2 - m_1}{1 + m_2 m_1}\right| \qquad \textit{for } \alpha \textit{ acute}$$

$$\tan\alpha = -\left|\frac{m_2 - m_1}{1 + m_2 m_1}\right| \qquad \textit{for } \alpha \textit{ obtuse}$$

6. LOGARITHMS AND INDEXES

If one has to quickly compute the quantum of money growth in real terms or if an investor wishes to know how much he/she can expect to receive after a period of inflation, logarithm will be able to easily solve such problem in a jiffy. Thus the powers and indices are significantly important to economic calculations such as growth, compounding and rates of change.

Basic index laws

$$a^m X a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^0 = 1$$

$$a^{-1} = \frac{1}{a^n}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \ (\text{if } a^m > 0)$$

$$(ab)^n = a^n b^n$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Definition of a logarithm

if $\log_a x = y$ *then* $x = a^y$

The number a is called the base of the logarithm.

Change of base of logarithms

$$log_a x = \frac{log_b x}{log_b a}$$

Basic laws of logarithms

$$log_a mn = log_a m + log_a n$$

$$log_a \frac{m}{n} = log_a m - log_a n$$

$$log_a m^x = x log_a m$$

$$a^x = e^{x \ln a}$$

$$x^e = e^{e \log x}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... \forall x.$$

7. SERIES AND SEQUENCE

Series and sequence individually are a set of numbers that are in order, which in mathematics are normally used for extrapolating. If the sequence goes on forever it is called an infinite sequence, else we call it a finite sequence. In an Arithmetic Sequence, the difference between one term and the next is a constant, while in a Geometric Sequence each term is found by multiplying the previous term by a constant.

- **Arithmetic progression (AP): is of the form:**

  $a, a + d, a + 2d, ..., u_n$

  $u_n = a + (a - 1)d$

Tests for arithmetic progression

$u_2 - u_1 = u_3 - u_2 = ... = u_n - u_{n-1}$

if a,b,c are sequential terms in AP, then $b = \frac{a + c}{2}$

Sum of an AP

$$S_n = \sum_{k=1}^{n} (a + (k - 1)d)$$

$$u_n = s_n - s_{n-1}$$

$$s_n = \frac{n(a + u_n)}{2}$$ *if given last term u_n*

$$s_n = \frac{n(2a + (n-1)d)}{2}$$ *if last term not given*

Geometric progression(GP):

is of the form:

$a, ar, ar^2, \ldots, ar^{n-1}$

$u_n = ar^{n-1}$

Test for GP

$$\frac{u_2}{u_1} = \frac{u_3}{u_2} = \ldots = \frac{u_n}{u_{n-1}}$$

$$u_2^2 = u_1 u_3, \ldots, u_{n-1}^2 = u_n - 2^{un}$$

Sum of GP

$$S_n = a \sum_{k=1}^{n} r^{k-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$ *use if $|r| > 1$*

$$S_n = \frac{a(1 - r^n)}{1 - r}$$ *use if $|r| < 1$*

Sum to infinity

$$S_\infty = \frac{a}{1 - r}$$ *where $|r| < 1$*

$$\sum_{k=1}^{n} r^k = r \sum_{k=1}^{n} r^{n-1}$$

$$= \frac{r(r^n - 1)}{r - 1}$$

$$\frac{r^{n+1} - r}{r - 1}$$

some other useful sums

$$\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \sum_{k=0}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$a^x = e^{x \ln a} = 1 + x \ln a + \frac{(x \ln a)^2}{2!} + \frac{(x \ln a)^3}{3!} + \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, -1 < x \leq 1.$$

8. CALCULUS

Simply put, Calculus is all about studying how things change. Using Calculus one can construct relatively simple quantitative models of change, and thus deduce their consequences. One can find instantaneous change that is changes over tiny intervals of time (called the 'derivative') of various functions, which is commonly called as 'differentiation'. One can also go back from the derivative of a function to the function itself (Integration). Using Calculus we can solve various geometric problems, such as computation of areas and volumes of certain regions. Calculus was invented by Isaac Newton, who discovered that acceleration, which means change of speed of objects could be modelled by his relatively simple laws of motion. In other words, by studying calculus it is possible to understand the interrelations between the concepts exemplified by speed and acceleration and that represented by position.

Differentiation (from first principles:)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\text{or } f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}c = 0$$

$$\frac{d}{dx}cf(x) = cf'(x)$$

$$\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$$

Product rule

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}(uvw) = uv\frac{dw}{dx} + uw\frac{dv}{dx} + vw\frac{du}{dx}$$

Quotient rule

$$\frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Chain rule (function of a function rule, composite function rule, substitution rule)

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

Derivatives of trigonometric functions

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \quad -\frac{\pi}{2} < \sin^{-1} x < \frac{\pi}{2}$$

$$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}} \quad -\frac{\pi}{2} < \cos^{-1} x < \frac{\pi}{2}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2} \quad -\frac{\pi}{2} < \tan^{-1}x < \frac{\pi}{2}$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sinh x = \cosh x$$

$$\frac{d}{dx}\cosh x = \sinh x$$ (here also cosec is written as csc)

Derivatives of exponential and log functions

$$\frac{d}{dx}\ln f(x) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}e^{f(x)} = f'(x)\,e^{f(x)}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}a^x = \ln a . a^x \text{ since } a^k = e^{k \ln a}$$

$$\frac{d}{dx}\ln y = \frac{d}{dx}\ln y \frac{dy}{dx}$$

$$\frac{d}{dx}\ln y = \frac{1}{y}\frac{dy}{dx}$$

Integration

Integration by parts

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

Numerical integration

Trapezoidal rule

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}(y_0 + y_n + 2(y_1 + y_2 + y_3 + \ldots + y_{n-1}))$$

Simpson's rule

$$\int_a^b f(x)\,dx \approx \frac{b-a}{3n}\,(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \dots + 4y_{n-1} + y_n)$$

n is even

Indefinite integrals

$$\int k\,dx = kx + c$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

$$\int \sin ax\,dx = \frac{-1}{a}\cos ax + c$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax + c$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + c$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{dx}{x} = \ln x + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln f(x) + c$$

$$\int \frac{dx}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + c$$

$$\int \frac{-1}{\sqrt{1-x^2}}\,dx = \cos^{-1} x + c$$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x + c$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$$

$$= \cos^{-1}\frac{x}{a} + c2, \quad -a < x < a$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$$

$$\int e^u \, du = e^u + c, \quad i,e \int e^{f(x)} f'(x) \, dx = e^{f(x)} + c$$

$$\int e^{ax} \, dx = \frac{e^{ax}}{a} + c$$

$$\int e^{ax} \, dx = \frac{e^{ax}}{a} + c$$

$$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \frac{x - a}{a + a} + c$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \frac{a + x}{a - x} + c$$

$$\int \ln x \, dx = x \ln x - x + c$$

$$\int \frac{dx}{\sqrt{a^2 + x^2}} = \log(x + \sqrt{a^2 + x^2}) + c$$

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log(x + \sqrt{x^2 - a^2}) + c \quad \forall x: |x| > a$$

$$\int \tan x \, dx = \log \sec x + c$$

$$\int \sec x \, dx = \ln (\sec x + \tan x) + c$$

$$= - \ln (\sec x - \tan x) + c$$

$$= \ln \tan\left(\frac{\pi}{4} + \frac{x}{2}\right) + c$$

$$\int \cot x \, dx = \ln \sin x + c$$

$$\int \csc^2 (ax) \, dx = - \frac{1}{a} \cot (ax) + c$$

$$\int a^x \, dx = \int e^{x \ln a} \, dx = \frac{e^{x \ln a}}{\ln a} + c = \frac{a^x}{\ln a} + c2$$

(let $y = a^x$, then $\ln y = x \ln a$, so $y = e^{x \ln a}$)

$$\int \cosh ax \, dx = \frac{1}{a} \sinh ax + c$$

$$\int \sinh ax \, dx = \frac{1}{a} \cosh ax + c$$

9. PROBABILITY, STATISTICS AND DATA ANALYSIS

Interpreting data can be fun using methods of exploratory data analysis. You will be able to develop and evaluate inferences, predictions, and arguments that are based on a given set of data. Here one can understand and apply basic notions of chance and probability and be able to explain concepts of certainty and fairness in real world situations. Predicting the likelihood of random events and testing predictions by experiment is possible using probability and statistics. One can also recognize random variables in real situations (e.g., insurance, life expectancy) estimate and compute expectations. In this branch, one can pose questions and collect, organize, and represent data to answer such questions.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) . P(B|A)$$

$$P(A|B) = \frac{P(B/A)\,P(A)}{P(B/A)\,P(A) + P(B/A')\,P(A')}$$

Bayes' Theorem: $P(A_j|B) = \frac{P(A_j)P(B/A_j)}{\Sigma P(A_i)\,P(B/A_i)}$

Populations

Discrete distributions

X is a random variable taking values x_i in a discrete distribution with

$P(X = x_i)^* = p_i$

Expectation: $\mu = E(X) = \Sigma\ x_i p_i$

Variance: $\sigma^2 = Var(X) = \Sigma\ (x_i - \mu)^2 p_i = \Sigma\ x_i^2 p_i - \mu^2$

For a function g(X): $E[g(X)] = \sum\ g(x_i)p_i$

Continuous distributions

X is a continuous variable with probability density function (p.d.f.) f(x)

Expectation: $\mu = E(X) = \int x\,f(x)dx$

Variance: $\sigma^2 = Var(X)$

$= \int (x - \mu)^2 f(x)dx = \int x^2 f(x)dx - \mu^2$

For a function g(X): $E[g(X)] = \int g(x)f(x)dx$
Cumulative
Distribution function $F(x) = P(X \leq x) = \int^{x}_{-\infty} f(t)dt$

Correlation and regression

for a sample of n pairs of observations (x_i, y_i)

$$S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n}, S_{yy} = \sum (y_i - \bar{y})^2 = \sum y_i^2 - \frac{(\sum y_i)^2}{n}$$

$$S_{xx} = \sum (x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}$$

Covariance $\frac{S_{xy}}{n} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n} = \frac{\sum x_i y_i}{n} - \bar{x}\bar{y}$

Product – moment correlation: Pearson's coefficient

$$r = \frac{Sxy}{\sqrt{SxxSyy}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{[\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2]}} = \frac{\frac{\sum xiyi}{n} - \bar{x}\bar{y}}{\sqrt{\left[\left(\frac{\sum xi2}{n} - \bar{x}^2\right)\left(\frac{\sum yi^2}{n} - \bar{y}^2\right)\right]}}$$

Rank correlation: Spearman's coefficient

$$r_s = 1 - \frac{6\sum d_i^2}{n(n^2 - 1)}$$

Regression

Least squares regression line of y on x: $y - \bar{y} = b(x - \bar{x})$

Estimates

Unbiased estimates from a single sample

$\bar{X}$ *for population mean* μ*; Var* $= \bar{X} = \frac{\sigma^2}{n}$

S^2 *for population variance* σ^2 *where* $S^2 = \frac{1}{n-1}\sum (x_i - \bar{x})^2 f_i$

Probability generating functions

For a discrete distribution

$G(t) = E(t^X)$

$E(X) = G'(1); Var(X) = G''(1) + \mu - \mu^2$

$G_{X+Y}(t) = G_X(t)\,G_Y(t)$ *for independent X, Y*

Moment generating functions:

$M_X(\theta) = E(e^{\theta x})$

$E(X) = M'(0) = \mu ; E(X^n) = M^{(n)}(0)$

$Var(X) = M''(0) - \{M'(0)\}^2$

$M_{X+Y}(\theta) = M_X(\theta)\,M_Y(\theta)$ *for independent X, Y*

Markov Chains

$\mathbf{P}_{n+1} = \mathbf{p}_n\mathbf{P}$

long run proportion $\mathbf{P} = \mathbf{P}^{\mathbf{P}}$

Bivariate distributions

Covariance $Cov(X, Y) = E[(X - \mu_X)(Y - \mu_Y)] = E(XY) - \mu_x\mu_Y$

Product – moment correlation coefficient $P = \dfrac{cov(X,Y)}{\sigma_X\sigma_Y}$

Sum and difference

$Var(aX \pm bY) = a^2Var(X) + b^2Var(Y) \pm 2ab\,Cov(X,Y)$

If X, Y are independent: $Var(aX \pm bY) = a^2Var(X) + b^2Var(Y)$

$E(XY) = E(X)\,E(Y)$

Coding

$$\left.\begin{matrix} X = aX' + b \\ Y = cY' + d \end{matrix}\right\} \Rightarrow Cov(X, Y) = ac\,Cov(X', Y')$$

Analysis of variance

One – factor model: $x_{ij} = \mu + \alpha_1 + \varepsilon_{ij}$, *where* $\varepsilon_{ij} \sim N(0, \sigma^2)$

$$SS_B = \sum_i (\bar{x}_i - \bar{x})^2 \frac{T_i^2}{n_i} - \frac{T^2}{n}$$

$$SS_T = \sum_i \sum_j (x_{ij} - \bar{x})^2 = \sum_i \sum_j x_{ij}^2 - \frac{T^2}{n}$$

Regression

y_i	RSS	no. of Parameters, p
$\alpha + \beta x_i + \varepsilon_i$	$\sum (y_i - a - bx_i)^2$	2
$\alpha + \beta f(x_i) + \varepsilon_i$	$\sum (y_i - a - bf(x_i))^2$	2
$\alpha + \beta x_i + \gamma z_i + \varepsilon_i$	$\sum (y_i - a - bx_i - cz_i)^2$	3

$\varepsilon_i \sim N(0, \sigma^2)$ *a,b, c are estimates for* $\alpha\ \beta\ \gamma$ $\hat{\sigma}^2 = \frac{RSS}{n-p}$

For the model $Y_i = \alpha + \beta x_i + \varepsilon_i$,

$$a = \bar{y} - b\bar{x}, a \sim N\left(\alpha, \frac{\sigma^2 \sum x_i^2}{nS_{xx}}\right)$$

$$a = \bar{y} - b\bar{x}, a \sim N\left(\alpha, \frac{\sigma^2 \sum x_i^2}{nS_{xx}}\right)$$

$$a = bx_0 \sim N(\alpha + \beta x_0 \cdot \sigma^2 \left\{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right\}$$

$$RSS = S_{yy} - \frac{(S_{xy})^2}{S_{xx}} = S_{yy}(1 - r^2)$$

Binomial

The coefficients in the expansion $(a + b)^n$ where $n = 0,1,2,3,...$ are given by Pascal's triangle.

Binomial theorem

$$(a+b)^n = \sum_{r=0}^{n} T_{r+1}$$

$$= \sum_{r=0}^{n} \frac{n!}{r!(n-r)!} a^{n-r} b^r$$

$$T_{r+1} = {}^nC_r\, a^{n-r} b^r$$

Where ${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

Pascal's triangle relationship

$$^{n+1}C_r = {}^nC_{r-1} + {}^nC_{r-n}$$

Sum of Coefficients

$$\sum_{r=0}^{n} {}^nC_r = 2^n$$

Symmetrical relationship

$$^nC_r = {}^nC_{n-r}$$

10. OTHERS

While there are several other mathematics disciplines that find application in our day-to-day activities, one can make use of the formulae given below given the nature of problem in different areas. These may be in interest calculation, profit and loss or in systems of units in measurement. One can develop and use one's own techniques and tools,and also combine other formulae using multiplication or ratios to produce measures such as force, work, velocity, acceleration, density, pressure, or trigonometric ratios.

Simple interest

$$I = prt$$

Where P = Principal, t = time, r = rate $(i.e., 8\% \rightarrow \frac{8}{100})$

Compound interest

$$A = P(1 + r)^n$$

Where P = Principal, n = time, r = rate as above,

A = amount you get.

Reducible interest:

This uses $S_n = \frac{a(r^n - 1)}{r - 1}$ *Where a is M,*

$$r = 1 + \frac{R}{100}$$

$$\frac{M\left(\left(1+\frac{R}{100}\right)^{n}-1\right)}{\frac{R}{100}} = P\left(1+\frac{R}{100}\right)^{n}$$

$$i.e., M = \frac{P\left(1+\frac{R}{100}\right)^{n} \cdot \frac{R}{100}}{\left(1+\frac{R}{100}\right)^{n}-1}$$

Where M = equal instalment paid per unit time
R = % rate of interest per unit time (Note: this is the same unit of time as for M)
n = number of instalments.

Superannuation

$$S_n = \frac{A\left(1+\frac{R}{100}\right)\left(\left(1+\frac{R}{100}\right)^{n}-1\right)}{\frac{R}{100}}$$

Where A is an equal instalment per unit time. n, R are as above

Simple Harmonic Motion

$\ddot{x} = -n^2x$ *n is any constant*

Other properties

$v^2 = n^2(a^2-x^2)$ *a = amplitude*

$x = a\cos nt$

$T = \frac{2\pi}{n}$ *T is the period*

$v_{max} = |na|$

$\ddot{x}_{max} = -n^2a$

Typical Problems

Distance = Speed X Time

If one has to convert one kilometre per hour (1km/hr) into metre per second (m/s), multiply it by 5/18

Crossing Trains:

When two trains are moving in the same direction, they cross each other at a speed equal to the difference between their speeds

When two trains are moving in opposite directions, they cross each other at a speed equal to the sum of their speeds

The distance travelled by a train to clear the platform is equal to the sum of lengths of the train and the platform

If the train is crossing a stationary object then,

$$\text{Crossing time} = \frac{\text{Length of the train} + \text{Length of the object}}{\text{Speed of the train}}$$

Pipes and Cisterns

If one pipe can fill one tank in x hours, then part of the tank filled in *1* hour is equal to $1/x$ hours

If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours then on opening both the pipes, the net filled portion in the tank in 1 hour will be $(1/x - 1/y)$

Boats and Streams

If the speed of a boat in still water is *10* km/hour and the speed of stream is *1* km/hour, then the boat will travel with the stream (downstream) at the rate of *(10+1)* km/hr and if it travels against the stream (upstream), then it will travel at the rate of *(10-1)* km/hr.

If a man also joins and if his rate in free water is x km/hr and y km/hr is the rate of the water current then,

$x + y$ = *man's rate with the current* and

$x - y$ = *man's rate against the current*

Man's rate in still water is half the sum of his rates with and against the water current

The rate of water current is half the difference between the rates of the man with and against the current

Profit and Loss

Cost Price: The price, at which an article is purchased, is called its cost price or CP.

Selling Price: The price, at which an article is sold, is called its selling price or SP

Profit or Loss:

If SP is greater than CP, the seller is said to have a profit or gain.

Loss: If SP is less than CP, the seller is said to have incurred a loss.

Profit = (SP - CP)

Loss = (CP - SP)

Profit or Loss is always considered on CP

Profit Percentage:

$$Profit\ \% = \frac{Profit \times 100}{CP}$$

$$Loss\ \% = \frac{Loss \times 100}{CP}$$

Selling Price: (SP)

$$SP = \left[\frac{100 + Profit\%}{100}\right] \times CP$$

$$SP = \left[\frac{100 - Loss\%}{100}\right] \times CP$$

Cost Price: (CP)

$$CP = \left[\frac{100}{100 + Profit\%}\right] \times SP$$

$CP = \left[\frac{100}{100 - Loss\%}\right] \times SP$

If an article is sold at a gain of say 40%, then $SP = 140\ \%\ of\ CP$
If an article is sold at a loss of say, 40% then $SP = 60\%\ of\ CP$
When a person sells two similar items, one at a profit of say x%, and the other at a loss of x%, then the seller always incurs a loss given by:

$$Loss\ \% = \left[\frac{Common\ Loss\ and\ Profit\%}{10}\right]^2 = \left[\frac{x}{10}\right]^2$$

If a trader professes to sell his goods at cost price, but uses false weights, then

$$Profit\ \% = \left[\frac{Error}{(True\ Value) - (Error)} \times 100\right]\%$$

Discount

If Marked Price is denoted as MP, then

Actual Discount = MP – SP

Discount % = (Actual Discount ÷ MP) × 100%

SP = MP – Actual Discount

SP = MP – Discount % of MP

Time and Work

Work from Days:

If A can do a piece of work in n days, then A*'s 1 day's work* $= 1/n$

Days from Work:

If A's 1 day's work $= 1/n$, then A can finish the work in n days.

Mathematical Constants

$\pi = 3.14159$

$\pi^2 = 9.8696$

$\sqrt{\pi} = 1.7724$

$\frac{1}{\pi} = 0.3183$

$e = 2.7182$

$log_{10}\pi = 0.4971$

$log_{10}e = 0.4343$

$log_e 10 = 2.3026$

Plank's Constant = 6.624×10^{-27} *erg.sec*
Avogadro's number = 6.023×10^{23} *mole*
Constant of Gravitation = 6.67×10^{-8} *dyne.cm*^{2}
$g = 32.16\ ft/sec^2 = 980 cm/sec^2$

Fraction to Decimal

1/1 = 1	2/3 = 0.6	3/5 = 0.6	4/5 = 0.8
1/2 = 0.5	3/4 = 0.75	3/7 = 0.428	4/7 = 0.572
1/3 = 0.3	2/5 = 0.4	6/7 = 0.857	7/8 = 0.875
1/4 = 0.25	5/6 = 0.83	5/8 = 0.625	5/9 = 0.5
1/5 = 0.2	2/7 = 0.286	4/9 = 0.4	9/10 = 0.9
1/6 = 0.16	3/8 = 0.375	8/9 = 0.8	4/11 = 0.36
1/7 = 0.143	2/9 = 0.2	7/10 = 0.7	7/11 = 0.63
1/8 = 0.125	7/9 = 0.7	3/11 = 0.27	10/11 = 0.90
1/9 = 0.1	2/11 = 0.18	6/11 = 0.54	11/12 = 0.916
1/10 = 0.1	5/11 = 0.45	9/11 = 0.81	7/16 = 0.4375
1/11 = 0.09	8/11 = 0.72	7/12 = 0.583	15/16 = 0.9375
1/12 = 0.083	5/12 = 0.416	5/16 = 0.3125	7/32 = 0.21875
1/16 = 0.0625	3/16 = 0.1875	13/16 = 0.8125	
1/32 = 0.03125	3/32 = 0.09375	5/32 = 0.15625	