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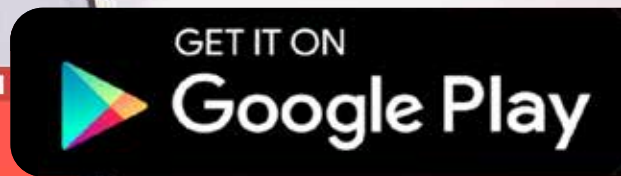
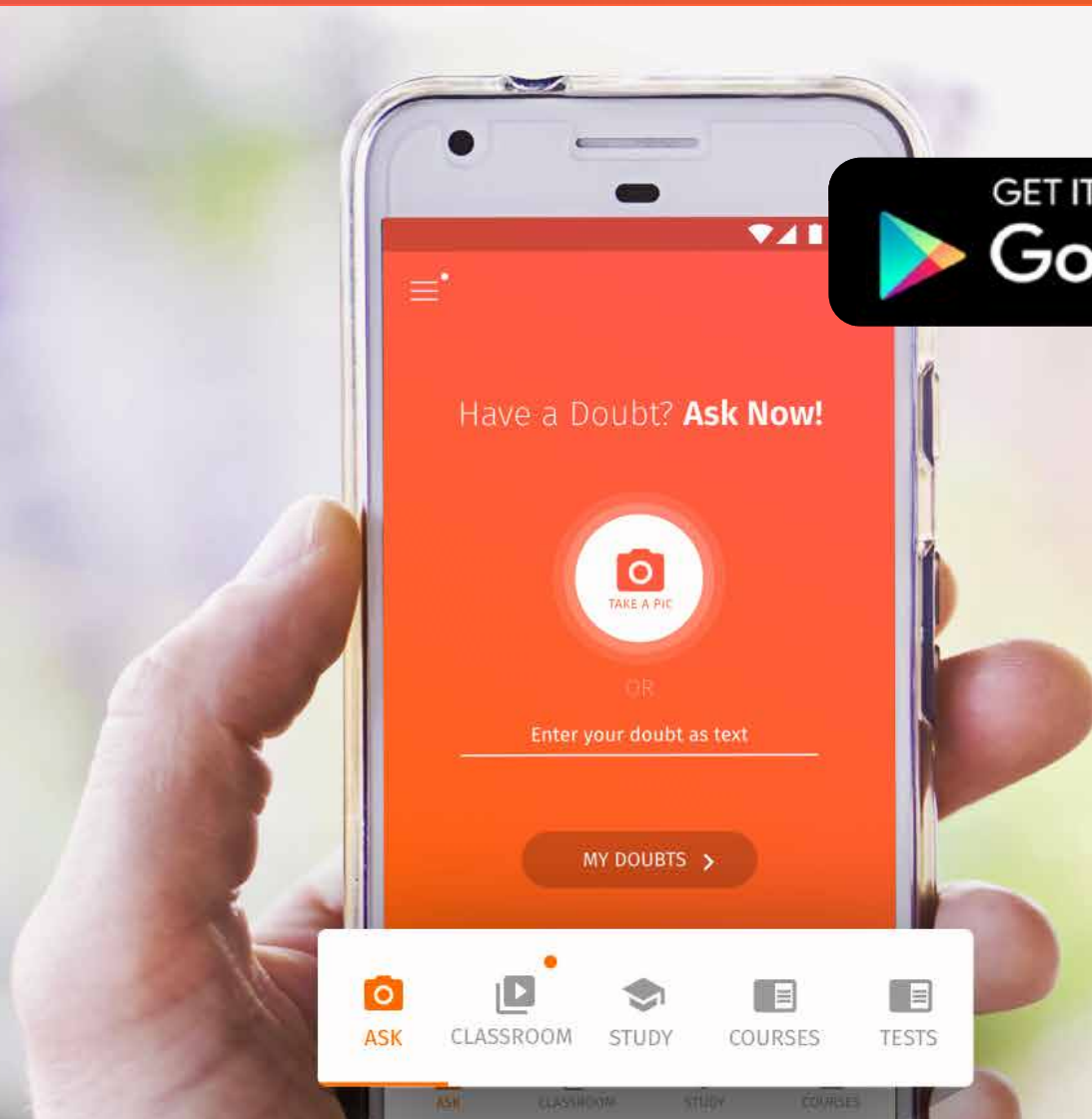
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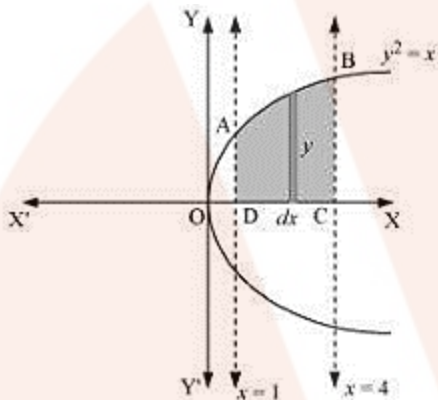


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EXERCISE- 8.1

Question 1:

Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x-axis in the first quadrant.

Solution 1:

The area of the region bounded by the curve, $y^2 = x$, the lines, $x = 1$ and $x = 4$, and the x-axis is the area ABCDA.

$$\text{Area ABCDA} = \int_1^4 \sqrt{x} \, dx$$

Area of ABCDA =

$$= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4$$

$$= \frac{2}{3} \left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}} \right]$$

$$= \frac{2}{3} [8 - 1]$$

$$= \frac{14}{3} \text{ sq. units}$$

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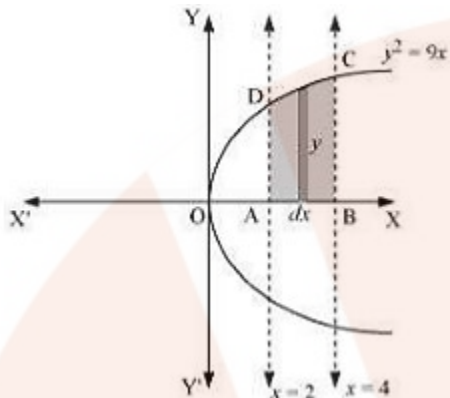
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Question 2:

Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x-axis in the first quadrant.

Solution 2:

The area of the region bounded by the curve, $y^2 = 9x$, $x = 2$, and $x = 4$, and the x-axis is the area ABCDA.

$$\text{Area ABCDA} = \int_2^4 3\sqrt{x} \, dx$$

$$= 3 \left[\frac{x^{3/2}}{3/2} \right]_2^4$$

$$= 2 \left[x^{3/2} \right]_2^4$$

$$= 2 \left[4^{3/2} - 2^{3/2} \right]$$

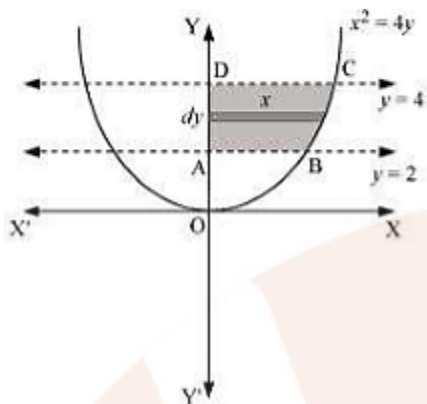
$$= 2 \left[2^3 - 8^{1/2} \right] = 2 \left[8 - 2\sqrt{2} \right]$$

$$= 16 - 4\sqrt{2} \text{ sq. units}$$

Question 3:

Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

Solution 3:



The area of the region bounded by the curve, $x^2 = 4y$, $y = 2$, and $y = 4$, and the y -axis is the area ABCDA.

$$\text{Area of ABCDA} = \int_2^4 x dy$$

$$x^2 = 4y$$

$$x = 2\sqrt{y}$$

$$\int_2^4 x dy = \int_2^4 2\sqrt{y} dy = 2 \int_2^4 \sqrt{y} dy$$

$$= 2 \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_2^4$$

$$= \frac{4}{3} \left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}} \right]$$

$$= \frac{4}{3} [8 - 2\sqrt{2}]$$

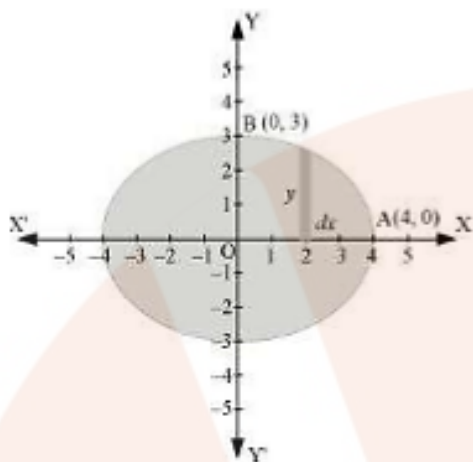
$$= \left(\frac{32 - 8\sqrt{2}}{3} \right) \text{ sq. units}$$

Question 4:

Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Solution 4:

The given equation of the ellipse, $\frac{x^2}{16} + \frac{y^2}{9} = 1$, can be represented as



It can be observed that the ellipse is symmetrical about x-axis and y-axis.

∴ Area bounded by ellipse = 4 × Area of OABO

$$\text{Area OABO} = \int_0^4 y dx$$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$\Rightarrow \frac{y^2}{9} = 1 - \frac{x^2}{16} \Rightarrow y^2 = 9 \left(1 - \frac{x^2}{16} \right)$$

$$y = 3 \sqrt{1 - \frac{x^2}{16}}$$

$$\text{Area OABO} = \int_0^4 3 \sqrt{1 - \frac{x^2}{16}} dx$$

$$= \frac{3}{4} \int_0^4 \sqrt{16 - x^2} dx$$

$$\text{Substitute } x = 4 \sin \theta, \theta = \sin^{-1} \frac{x}{4}$$

$$dx = 4 \cos \theta d\theta$$

$$\text{when, } x = 0 \quad \theta = 0 \quad \& \quad x = 4 \quad \theta = \frac{\pi}{2}$$

$$= \frac{3}{4} \int_0^{\pi/2} \sqrt{16 - 16 \sin^2 \theta} \cdot 4 \cos \theta d\theta$$

$$= 12 \int_0^{\pi/2} \sqrt{1 - \sin^2 \theta} \cdot \cos \theta d\theta$$

$$\begin{aligned}
 &= 12 \int_0^{\pi/2} \cos^2 \theta d\theta = 12 \int_0^{\pi/2} \frac{1 + \cos 2\theta}{2} d\theta \\
 &= 6 \int_0^{\pi/2} (1 + \cos 2\theta) d\theta = 6 \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{\pi/2} \\
 &= 6 \left[\frac{\pi}{2} + \frac{\sin \pi}{2} - 0 - \frac{\sin 0}{2} \right] = 6 \left[\frac{\pi}{2} \right] = 3\pi
 \end{aligned}$$

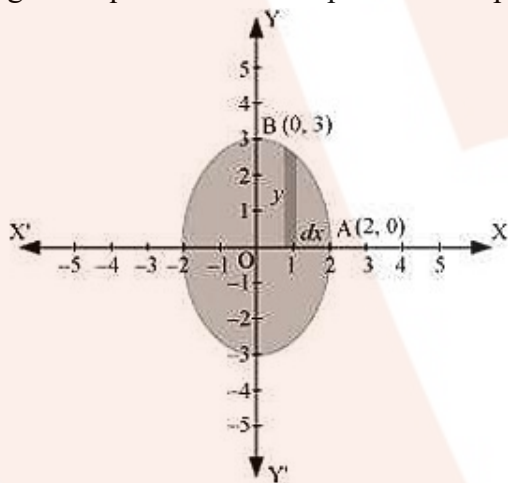
Therefore, area bounded by the ellipse = $4 \times 3\pi = 12\pi$ sq. units

Question 5:

Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Solution 5:

The given equation of the ellipse can be represented as



$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$\frac{y^2}{9} = 1 - \frac{x^2}{4}$$

$$y^2 = 9 \left(1 - \frac{x^2}{4} \right)$$

$$\Rightarrow y = 3\sqrt{1 - \frac{x^2}{4}} \quad \dots(1)$$

It can be observed that the ellipse is symmetrical about x-axis and y-axis.

\therefore Area bounded by ellipse = $4 \times$ Area OABO

$$\therefore \text{Area of OABO} = \int_0^2 y dx$$

$$= \int_0^2 3\sqrt{1 - \frac{x^2}{4}} dx \quad [\text{Using (1)}]$$

$$= \frac{3}{2} \int_0^2 \sqrt{4 - x^2} dx$$

$$\text{Substitute } x = 2 \sin \theta \Rightarrow \theta = \sin^{-1}\left(\frac{x}{2}\right)$$

$$dx = 2 \cos \theta d\theta$$

$$\text{when, } x = 0 \quad \theta = 0 \quad \& \quad x = 2 \quad \theta = \frac{\pi}{2}$$

$$\therefore \frac{3}{2} \int_0^2 \sqrt{4 - x^2} dx = \frac{3}{2} \int_0^{\pi/2} \sqrt{4 - 4 \sin^2 \theta} \cdot 2 \cos \theta d\theta$$

$$= 3 \int_0^{\pi/2} \sqrt{4 - 4 \sin^2 \theta} \cdot \cos \theta d\theta = 6 \int_0^{\pi/2} \sqrt{1 - \sin^2 \theta} \cos \theta d\theta$$

$$= 6 \int_0^{\pi/2} \cos 2\theta d\theta = 6 \int_0^{\pi/2} \frac{1 + \cos 2\theta}{2} d\theta$$

$$= \frac{6}{2} \int_0^{\pi/2} (1 + \cos 2\theta) d\theta = 3 \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{\pi/2}$$

$$= 3 \left[\frac{\pi}{2} + \frac{\sin \pi}{2} - 0 \right] = 3 \times \frac{\pi}{2} = \frac{3\pi}{2}$$

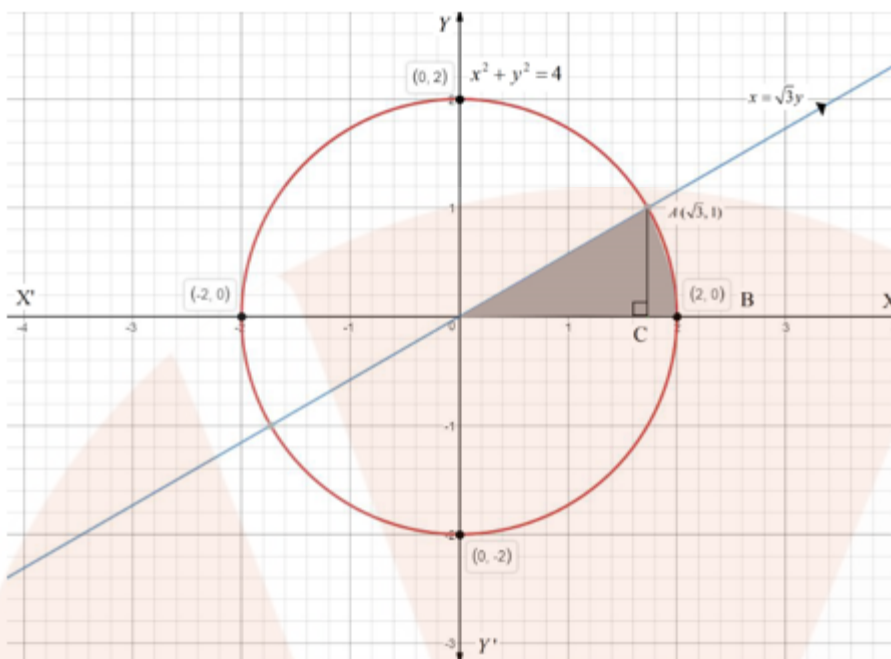
Therefore, area bounded by the ellipse = $4 \times \frac{3\pi}{2} = 6\pi$ sq. units

Question 6:

Find the area of the region in the first quadrant enclosed by x-axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$

Solution 6:

The area of the region bounded by the circle, $x^2 + y^2 = 4$, $x = \sqrt{3}y$, and the x-axis is the area OAB.



Substituting $x = \sqrt{3}y$ in $x^2 + y^2 = 4$, for finding the point of intersection.

$$\therefore (\sqrt{3}y) + y^2 = 4 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1, x = \pm \sqrt{3}$$

The point of intersection of the line and the circle in the first quadrant is $(\sqrt{3}, 1)$.

$$\text{Area OABO} = \text{Area } \triangle OCA + \text{Area ACBA}$$

$$\text{Area of OAC} = \frac{1}{2} \times OC \times AC = \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2} \quad \dots(1)$$

$$\text{Area of ABCA} = \int_{\sqrt{3}}^2 y dx$$

$$\int_{\sqrt{3}}^2 y dx = \int_{\sqrt{3}}^2 \sqrt{4 - x^2} dx$$

$$\int_{\sqrt{3}}^2 x = 2 \sin \theta \quad \theta = \sin^{-1} \left(\frac{x}{2} \right)$$

$$\text{when } x = 2 \quad \theta = \frac{\pi}{2}$$

$$x = \sqrt{3} \quad \theta = \frac{\pi}{3}$$

$$\therefore \int_{\sqrt{3}}^2 \sqrt{4-x} \, dx = \int_{\pi/3}^{\pi/2} \sqrt{4-4\sin^2\theta} (2\cos\theta) d\theta$$

$$= 4 \int_{\pi/3}^{\pi/2} \cos^2\theta d\theta = 4 \int_{\pi/3}^{\pi/2} 1 + \frac{\cos 2\theta}{2} d\theta$$

$$= 2 \int_{\pi/3}^{\pi/2} (1 + \cos 2\theta) d\theta = 2 \left[\theta + \frac{\sin 2\theta}{2} \right]_{\pi/3}^{\pi/2}$$

$$= 2 \left[\frac{\pi}{2} + \frac{1}{2} \sin \pi - \frac{\pi}{3} - \frac{1}{2} \sin \frac{2\pi}{3} \right]$$

$$= 2 \left[\frac{\pi}{2} - \frac{\pi}{3} - \frac{1}{2} \times \frac{\sqrt{3}}{2} \right] = 2 \left[\frac{\pi}{6} - \frac{\sqrt{3}}{4} \right] \quad \dots(2)$$

From (1) & (2)

$$\text{Area of OAB} = \frac{\sqrt{3}}{2} + 2 \left[\frac{\pi}{6} - \frac{\sqrt{3}}{4} \right] = \frac{\pi}{3}$$

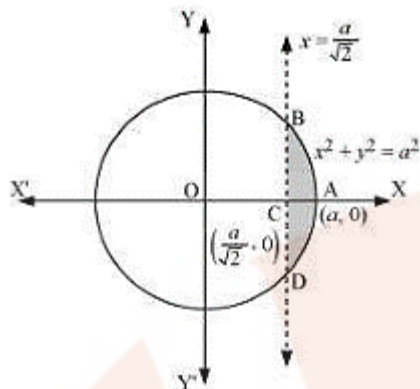
Therefore, area enclosed by x-axis, the line $x = \sqrt{3}y$, and the circle $x^2 + y^2 = 4$ in the first quadrant = $\frac{\pi}{3}$ sq. units

Question 7:

Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$.

Solution 7:

The area of the smaller part of the circle, $x^2 + y^2 = a^2$, cut off by the line $x = \frac{a}{\sqrt{2}}$, is the area ABCDA.



It can be observed that the area ABCDA is symmetrical about x-axis.

\therefore Area ABCDA = $2 \times$ Area ABCA

$$\text{Area of ABCA} = \int_{\frac{a}{\sqrt{2}}}^a y dx$$

$$\int_{\frac{a}{\sqrt{2}}}^a \sqrt{a^2 - x^2} dx$$

$$x = a \sin \theta \quad dx = a \cos \theta d\theta$$

$$x = \frac{a}{\sqrt{2}} \quad \theta = \sin^{-1}\left(\frac{x}{a}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$x = a, \quad \theta = \sin^{-1}\left(\frac{a}{a}\right) = \frac{\pi}{2}$$

$$\therefore \int_{\frac{a}{\sqrt{2}}}^a \sqrt{a^2 - x^2} dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{a^2 - a^2 \sin^2 \theta} \cdot (a \cos \theta) d\theta$$

$$\Rightarrow a^2 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2 \theta = a^2 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1 + \cos 2\theta}{2} d\theta$$

$$= \frac{a^2}{2} \left[\theta + \frac{\sin 2\theta}{2} \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \frac{a^2}{2} \left[\frac{\pi}{2} + \frac{\sin \pi}{2} - \frac{\pi}{4} - \frac{\sin \frac{\pi}{2}}{2} \right]$$

$$= \frac{a^2}{2} \left[\frac{\pi}{4} - \frac{1}{2} \right]$$

$$= \frac{a^2}{4} \left[\frac{\pi}{2} - 1 \right]$$

$$\Rightarrow \text{Area } ABCD = 2 \left[\frac{a^2}{4} \left(\frac{\pi}{2} - 1 \right) \right] = \frac{a^2}{2} \left(\frac{\pi}{2} - 1 \right)$$

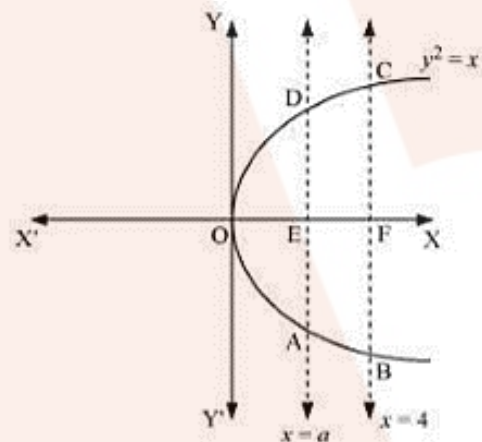
Therefore, the area of smaller part of the circle, $x^2 + y^2 = a^2$, cut off by the line $x = \frac{a}{\sqrt{2}}$, is $\frac{a^2}{2} \left(\frac{\pi}{2} - 1 \right)$ sq. units.

Question 8:

The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, find the value of a.

Solution 8:

The line $x = a$, divides the area bounded by the parabola $x = y^2$ and $x = 4$ into two equal parts.
 \therefore Area OADO = Area ABCDA



It can be observed that the given area is symmetrical about x-axis.

$$\text{Area of OEDO} = \frac{1}{2} \text{Area of OADO}$$

$$\text{Area of EFCDE} = \frac{1}{2} \text{Area of ABCDA}$$

Therefore, Area OEDO = Area EFCDE

$$\text{Area OEDO} = \int_0^a y dx$$

$$= \int_0^a \sqrt{x} dx$$

$$\begin{aligned}
 &= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^a \\
 &= \frac{2}{3} (a)^{\frac{3}{2}} \quad \dots(1)
 \end{aligned}$$

$$\text{Area of EFCDE} = \int_a^4 y \, dx = \int_a^4 \sqrt{x} \, dx$$

$$\begin{aligned}
 &= \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_a^4 \\
 &= \frac{2}{3} \left[4^{\frac{3}{2}} - a^{\frac{3}{2}} \right] \\
 &= \frac{2}{3} \left[8 - a^{\frac{3}{2}} \right] \quad \dots(2)
 \end{aligned}$$

From (1) and (2), we obtain

$$\frac{2}{3} (a)^{\frac{3}{2}} = \frac{2}{3} \left[8 - (a)^{\frac{3}{2}} \right]$$

$$\Rightarrow 2 \cdot (a)^{\frac{3}{2}} = 8$$

$$\Rightarrow (a)^{\frac{3}{2}} = 4$$

$$\Rightarrow a = (4)^{\frac{2}{3}}$$

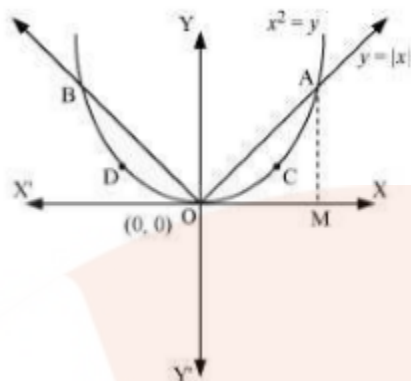
Therefore, the value of a is $(4)^{\frac{2}{3}}$.

Question 9:

Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$.

Solution 9:

The area bounded by the parabola, $x^2 = y$, and the line, $y = |x|$, can be represented as



The given area is symmetrical about y-axis.

\therefore Area OACO = Area ODBO

The point of intersection of parabola $x^2 = y$ and line $y = x$ is A (1, 1).

Area of OACO = Area Δ OAM – Area OMACO

$$\therefore \text{Area of } \Delta\text{OAM} = \frac{1}{2} \times OM \times AM = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$$

$$\text{Area of OMACO} = \int_0^1 y dx = \int_0^1 x^2 dx = \left[\frac{x^3}{3} \right]_0^1 = \frac{1}{3}$$

\Rightarrow Area of OACO = Area of Δ OAM – Area of OMACO

$$= \frac{1}{2} - \frac{1}{3}$$

$$= \frac{1}{6}$$

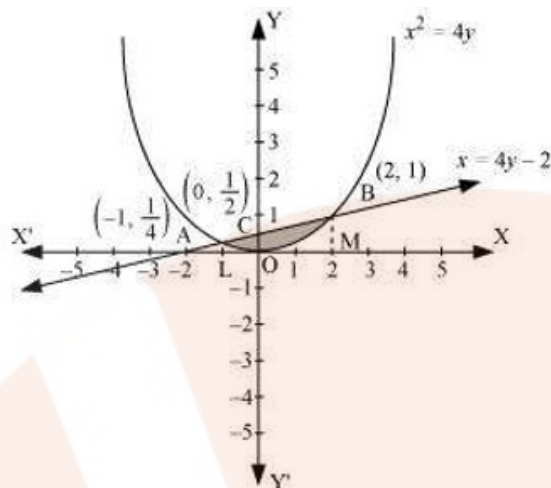
Therefore, required area = $2 \left[\frac{1}{6} \right] = \frac{1}{3}$ sq. units.

Question 10:

Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

Solution 10:

The area bounded by the curve, $x^2 = 4y$, and line, $x = 4y - 2$, is represented by the shaded area OBAO.



Let A and B be the points of intersection of the line and parabola.

Substituting $x = 4y - 2$ in $x^2 = 4y$

$$(4y - 2)^2 = 4y$$

$$16y^2 - 16y + 4 = 4y$$

$$16y^2 - 20y + 4 = 0$$

$$4y^2 - 5y + 1 = 0$$

$$(4y - 1)(y - 1) = 0$$

$$y = \frac{1}{4}, x = -1$$

$$y = 1, x = 2$$

Coordinates of point A are $\left(-1, \frac{1}{4}\right)$.

Coordinates of point B are (2, 1).

We draw AL and BM perpendicular to x-axis.

It can be observed that,

$$\text{Area OBAO} = \text{Area OBCO} + \text{Area OACO} \dots (1)$$

Area OMBCO = Area under the line $x = 4y - 2$ between $x = 0$ and $x = 2$

$$\text{Area OMBCO} = \int_0^2 \frac{x+2}{4} dx$$

Area OMBO = Area under the curve $x^2 = 4y$ between $x = 0$ and $x = 2$

$$\text{Area OMBO} = \int_0^2 \frac{x^2}{4} dx$$

Then, Area OBCO = Area OMBCO – Area OMBO

$$\begin{aligned}
 &= \int_0^2 \frac{x+2}{4} dx - \int_0^2 \frac{x^2}{4} dx \\
 &= \frac{1}{4} \left[\frac{x^2}{2} + 2x \right]_0^2 - \frac{1}{4} \left[\frac{x^3}{3} \right]_0^2 \\
 &= \frac{1}{4} [2+4] - \frac{1}{4} \left[\frac{8}{3} \right] \\
 &= \frac{3}{2} - \frac{2}{3} \\
 &= \frac{5}{6}
 \end{aligned}$$

Area OLACO = Area under the line $x = 4y - 2$ between $x = -1$ and $x = 0$

$$\text{Area OLACO} = \int_{-1}^0 \frac{x+2}{4} dx$$

Area OLAO = Area under the curve $x^2 = 4y$ between $x = -1$ and $x =$

$$\text{Area OMBO} = \int_0^2 \frac{x^2}{4} dx$$

Area OACO = Area OLACO – Area OLAO

$$\begin{aligned}
 &= \int_{-1}^0 \frac{x+2}{4} dx - \int_{-1}^0 \frac{x^2}{4} dx \\
 &= \frac{1}{4} \left[\frac{x^2}{2} + 2x \right]_{-1}^0 - \frac{1}{4} \left[\frac{x^3}{3} \right]_{-1}^0 \\
 &= \frac{1}{4} \left[\frac{0}{2} + 0 - \frac{(-1)^2}{2} - 2(-1) \right] - \frac{1}{4} \left[\frac{0^3}{3} - \frac{(-1)^3}{3} \right]
 \end{aligned}$$

$$\begin{aligned}
 &= -\frac{1}{4} \left[\frac{(-1)}{2} + 2(-1) \right] - \left[-\frac{1}{4} \left(\frac{(-1)^3}{3} \right) \right] \\
 &= -\frac{1}{4} \left[\frac{1}{2} - 2 \right] - \frac{1}{12} \\
 &= \frac{1}{2} - \frac{1}{8} - \frac{1}{12} \\
 &= \frac{7}{24}
 \end{aligned}$$

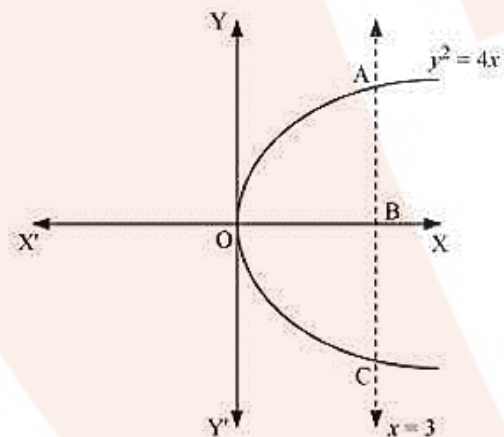
Therefore, required area = $\left(\frac{5}{6} + \frac{7}{24} \right) = \frac{9}{8}$ sq. units

Question 11:

Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$.

Solution 11:

The region bounded by the parabola, $y^2 = 4x$, and the line, $x = 3$, is the area OACO



The area OACO is symmetrical about x-axis.

∴ Area of OACO = 2 (Area of OABO)

$$\text{Area OACO} = 2 \left[\int_0^3 y dx \right]$$

$$= 2 \left[\int_0^3 2\sqrt{x} dx \right]$$

$$= 4 \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^3$$

$$= \frac{8}{3} \left[(3)^{\frac{3}{2}} \right]$$

$$= 8\sqrt{3}$$

Therefore, the required area is $8\sqrt{3}$ sq. units.

Question 12:

Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

A. π

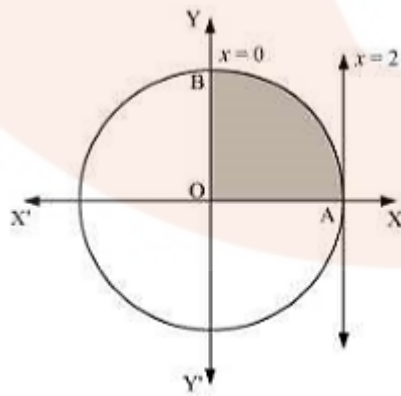
B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Solution 12:

The area bounded by the circle and the lines, $x = 0$ and $x = 2$, in the first quadrant is represented as



$$\begin{aligned}\therefore \text{Area OAB} &= \int_0^2 y dx \\ &= \int_0^2 \sqrt{4-x^2} dx \\ &= \left[\frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 \\ &= 2 \left(\frac{\pi}{2} \right) \\ &= \pi \text{ units}\end{aligned}$$

Alternate Solution:

Area OABO = $\frac{1}{2}$ Area of circle

Radius = 2

Area OABO = $\frac{1}{4} \times \pi \times 2^2 = \pi$ sq. units

Thus, the correct answer is A.

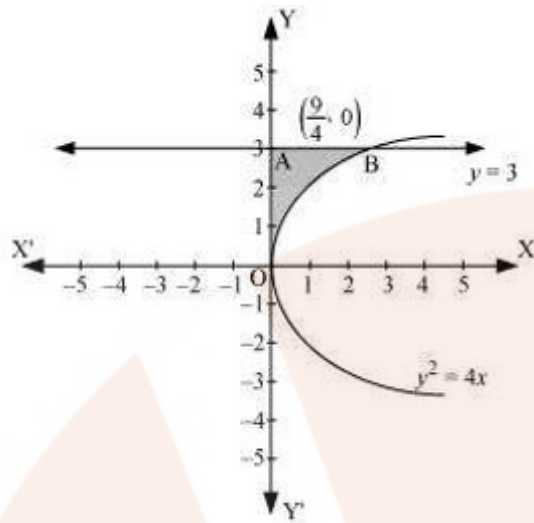
Question 13:

Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

- A. 2
- B. $\frac{9}{4}$
- C. $\frac{9}{3}$
- D. $\frac{9}{2}$

Solution 13:

The area bounded by the curve $y^2 = 4x$, y-axis, and $y = 3$ is represented as



$$\therefore \text{Area OABO} = \int_0^3 x dy$$

$$= \int_0^3 \frac{y^2}{4} dy$$

$$= \frac{1}{4} \left[\frac{y^3}{3} \right]_0^3$$

$$= \frac{1}{12} (27)$$

$$= \frac{9}{4} \text{ sq. units}$$

Thus, the correct answer is B.

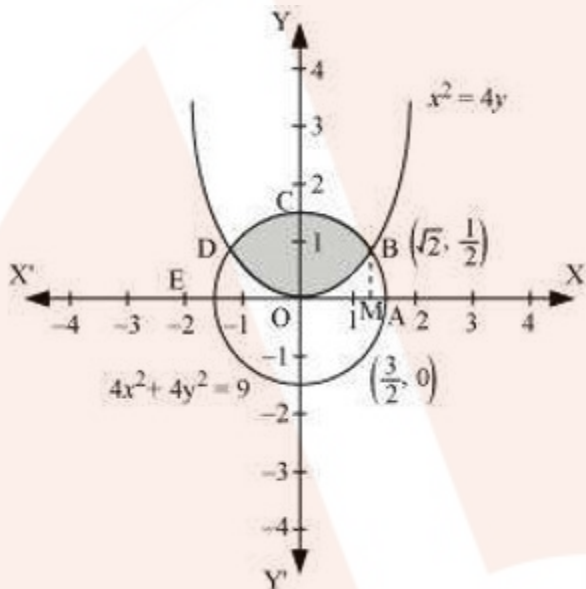
EXERCISE- 8.2

Question 1:

Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $x^2 = 4y$

Solution 1:

The required area is represented by the shaded area OBCDO.



Solving the given equation of circle, $4x^2 + 4y^2 = 9$, and parabola, $x^2 = 4y$,

$$4(4y) + 4y^2 = 9$$

$$4y^2 + 16y - 9 = 0$$

$$y = \frac{-16 \pm \sqrt{16^2 - 4 * 4 * (-9)}}{2 * 4}$$

$$y = \frac{1}{2} \text{ or } -4.5$$

The value of y cannot be negative as $x^2 = 4y$.

We obtain the point of intersection as $B\left(\sqrt{2}, \frac{1}{2}\right)$ and $D\left(-\sqrt{2}, \frac{1}{2}\right)$.

It can be observed that the required area is symmetrical about y -axis.

$$\therefore \text{Area OBCDO} = 2 \times \text{Area OBCO}$$

We draw BM perpendicular to OA .

Therefore, the coordinates of M are $\left(\sqrt{2}, 0\right)$.

Therefore, $\text{Area OBCO} = \text{Area OMBCO} - \text{Area OMBO}$

Area OMBCO = Area under the circle $4x^2 + 4y^2 = 9$ between $x = 0$ & $x = \sqrt{2}$

$$\text{Area OMBCO} = \int_0^{\sqrt{2}} \sqrt{\frac{9-4x^2}{4}} dx$$

$$= \int_0^{\sqrt{2}} \sqrt{\frac{9}{4} - x^2} dx$$

substitute $x = \frac{3}{2} \sin \theta$, $dx = \frac{3}{2} \cos \theta d\theta$

$$\int \sqrt{\frac{9}{4} - \frac{9}{4} \sin^2 \theta} \cdot \frac{3}{2} \cos \theta d\theta$$

$$= \frac{9}{4} \int \sqrt{1 - \sin^2 \theta} \cos \theta d\theta = \frac{9}{4} \int \cos^2 \theta d\theta$$

$$= \frac{9}{4} \int \frac{1 + \cos 2\theta}{2} d\theta$$

$$= \frac{9}{4} \int \frac{1}{2} + \frac{\cos 2\theta}{2} d\theta$$

$$= \frac{9}{8} \left[\theta + \frac{\sin 2\theta}{2} \right]$$

$$= \frac{9}{8} \left[\sin^{-1} \frac{2x}{3} + \frac{\cancel{2} \cdot \sin \theta \cos \theta}{\cancel{2}} \right]$$

$$= \frac{9}{8} \left[\sin^{-1} \frac{2x}{3} + \frac{2x}{3} \sqrt{1 - \frac{4x^2}{9}} \right]$$

$$= \frac{9}{8} \left[\sin^{-1} \frac{2x}{3} + \frac{2x}{9} \sqrt{9 - 4x^2} \right]$$

Applying the limits

$$= \frac{9}{8} \left[\sin^{-1} \frac{2x}{3} + \frac{2x}{9} \sqrt{9 - 4x^2} \right]_0^{\sqrt{2}}$$

$$= \frac{9}{8} \left[\sin^{-1} \left(\frac{2\sqrt{2}}{3} \right) + \frac{2\sqrt{2}}{9} \right]$$

$$\text{Area OMBCO} = \frac{1}{4} \left[\sqrt{2} + \frac{9}{2} \sin^{-1} \frac{2\sqrt{2}}{3} \right] \quad \dots(1)$$

Area OMBCO = Area under $x^2 = 4y$ between $x = 0$ and $x = \sqrt{2}$

$$\text{Area OMBO} = \int_0^{\sqrt{2}} \frac{x^2}{4} dx$$

$$\text{Area OMBO} = \frac{1}{4} \left[\frac{x^3}{3} \right]_0^{\sqrt{2}}$$

$$\text{Area OMBO} = \frac{1}{12} [2\sqrt{2}] = \frac{\sqrt{2}}{6} \quad \dots(2)$$

From (1) and (2)

Area OBCO =

$$= \frac{\sqrt{2}}{4} + \frac{9}{8} \sin^{-1} \frac{2\sqrt{2}}{3} - \frac{\sqrt{2}}{6}$$

$$= \frac{\sqrt{2}}{12} + \frac{9}{8} \sin^{-1} \frac{2\sqrt{2}}{3}$$

$$= \frac{1}{2} \left(\frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} \right)$$

Therefore, the required area OBCDO is

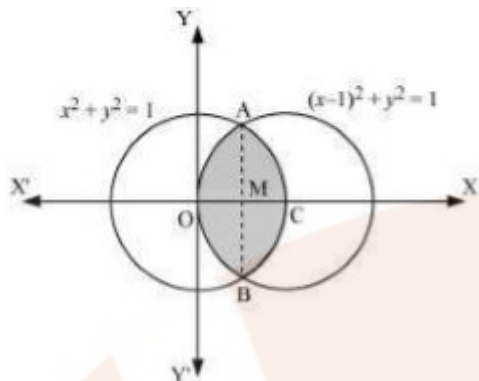
$$\left(2 \times \frac{1}{2} \left[\frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} \right] \right) = \left[\frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3} \right] \text{sq. units}$$

Question 2:

Find the area bounded by curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$

Solution 2:

The area bounded by the curves, $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$, is represented by the shaded area as



On solving the equations, $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$,

$$(x-1)^2 + 1 - x^2 = 1$$

$$-2x + 1 = 0$$

$$x = 1/2$$

$$y^2 = 1 - x^2 = 1 - 1/4 = 3/4$$

we obtain the point of intersection as A $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and B $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

It can be observed that the required area is symmetrical about x-axis.

\therefore Area OBCAO = 2 \times Area OCAO

We join AB, which intersects OC at M, such that AM is perpendicular to OC.

The coordinates of M are $\left(\frac{1}{2}, 0\right)$

\Rightarrow Area OCAO = Area OMAO + Area MCAM

$$= \left[\int_0^{1/2} \sqrt{1-(x-1)^2} dx + \int_{1/2}^1 \sqrt{1-x^2} dx \right]$$

$$= \left[\frac{x-1}{2} \sqrt{1-(x-1)^2} + \frac{1}{2} \sin^{-1}(x-1) \right]_0^{1/2} + \left[\frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x \right]_{1/2}^1$$

$$= \left[-\frac{1}{4} \sqrt{1-\left(-\frac{1}{2}\right)^2} + \frac{1}{2} \sin^{-1}\left(\frac{1}{2}-1\right) - \frac{1}{2} \sin^{-1}(-1) \right] + \left[\frac{1}{2} \sin^{-1}(+1) - \frac{1}{4} \sqrt{1-\left(+\frac{1}{2}\right)^2} - \frac{1}{2} \sin^{-1}\left(\frac{1}{2}\right) \right]$$

$$= \left[-\frac{\sqrt{3}}{8} + \frac{1}{2} \left(-\frac{\pi}{6}\right) - \frac{1}{2} \left(-\frac{\pi}{2}\right) \right] + \left[\frac{1}{2} \left(\frac{\pi}{2}\right) - \frac{\sqrt{3}}{8} - \frac{1}{2} \left(\frac{\pi}{6}\right) \right]$$

$$= \left[-\frac{\sqrt{3}}{4} - \frac{\pi}{12} + \frac{\pi}{4} + \frac{\pi}{4} - \frac{\pi}{12} \right]$$

$$= \left[-\frac{\sqrt{3}}{4} - \frac{\pi}{6} + \frac{\pi}{2} \right]$$

$$= \left[\frac{2\pi}{6} - \frac{\sqrt{3}}{4} \right]$$

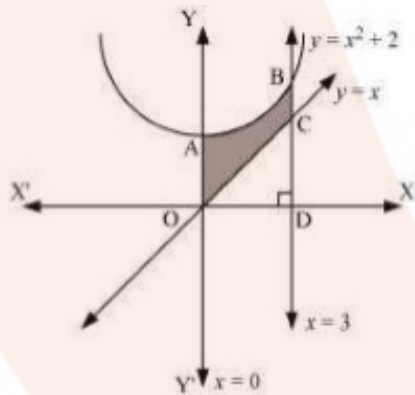
Therefore, required area OBCAO = $2 \times \left(\frac{2\pi}{6} - \frac{\sqrt{3}}{4} \right) = \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right)$ sq. units

Question 3:

Find the area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$.

Solution 3:

The area bounded by the curves, $y = x^2 + 2$, $y = x$, $x = 0$, and $x = 3$, is represented by the shaded area OCBAO as



Then, Area OCBAO = Area ODBAO – Area ODCO

$$= \int_0^3 (x^2 + 2) dx - \int_0^3 x dx$$

$$= \left[\frac{x^3}{3} + 2x \right]_0^3 - \left[\frac{x^2}{2} \right]_0^3$$

$$\begin{aligned}
 &= [9 + 6] - \left[\frac{9}{2} \right] \\
 &= 15 - \frac{9}{2} \\
 &= \frac{21}{2} \text{ sq. units}
 \end{aligned}$$

Question 4:

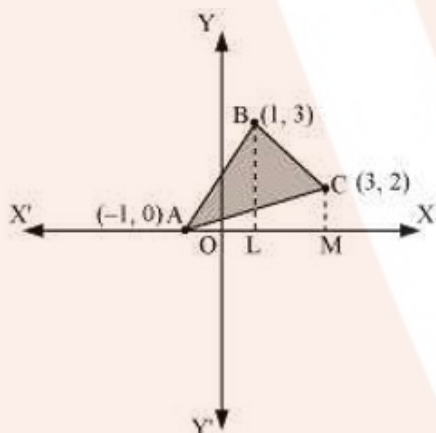
Using integration find the area of the region bounded by the triangle whose vertices are $(-1, 0)$, $(1, 3)$ and $(3, 2)$.

Solution 4:

BL and CM are drawn perpendicular to x-axis.

It can be observed in the following figure that,

$$\text{Area } (\triangle ACB) = \text{Area } (ALBA) + \text{Area } (BLMCB) - \text{Area } (AMCA) \dots(1)$$



Equation of line segment AB is

$$y - 0 = \frac{3 - 0}{1 + 1}(x + 1)$$

$$y = \frac{3}{2}(x + 1)$$

$$\therefore \text{Area } (ALBA) = \int_{-1}^1 \frac{3}{2}(x + 1) dx = \frac{3}{2} \left[\frac{x^2}{2} + x \right]_{-1}^1 = \frac{3}{2} \left[\frac{1}{2} + 1 - \frac{1}{2} + 1 \right] = 3 \text{ sq. units}$$

Equation of line segment BC is

$$y - 3 = \frac{2-3}{3-1}(x-1)$$

$$y = \frac{1}{2}(-x+7)$$

$$\therefore \text{Area (BLMCB)} = \int_1^3 \frac{1}{2}(-x+7) dx = \frac{1}{2} \left[-\frac{x^2}{2} + 7x \right]_1^3 = \frac{1}{2} \left[-\frac{9}{2} + 21 + \frac{1}{2} - 7 \right] = 5 \text{ sq. units}$$

Equation of line segment AC is

$$y - 0 = \frac{2-0}{3+1}(x+1)$$

$$y = \frac{1}{2}(x+1)$$

$$\therefore \text{Area (AMCA)} = \frac{1}{2} \int_{-1}^3 (x+1) dx = \frac{1}{2} \left[\frac{x^2}{2} + x \right]_{-1}^3 = \frac{1}{2} \left[\frac{9}{2} + 3 - \frac{1}{2} + 1 \right] = 4 \text{ sq. units}$$

Therefore, from equation (1), we obtain

$$\text{Area } (\Delta ABC) = (3 + 5 - 4) = 4 \text{ sq. units}$$

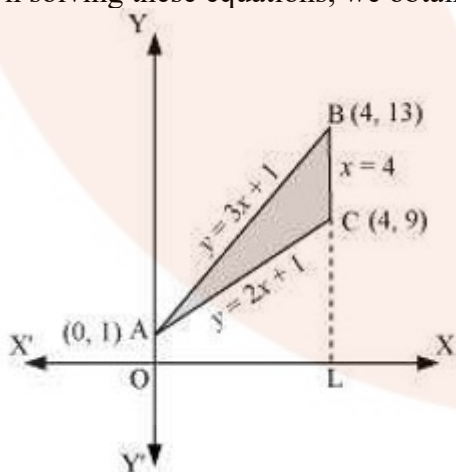
Question 5:

Using integration find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

Solution 5:

The equations of sides of the triangle are $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

On solving these equations, we obtain the vertices of triangle as A(0, 1), B(4, 13), and C(4, 9).



It can be observed that,

$$\text{Area } (\Delta ACB) = \text{Area (OLBAO)} - \text{Area (OLCAO)}$$

$$\begin{aligned}
 &= \int_0^4 (3x+1) dx - \int_0^4 (2x+1) dx \\
 &= \left[\frac{3x^2}{2} + x \right]_0^4 - \left[\frac{2x^2}{2} + x \right]_0^4 \\
 &= (24+4) - (16+4) \\
 &= 28 - 20 \\
 &= 8 \text{ sq. units}
 \end{aligned}$$

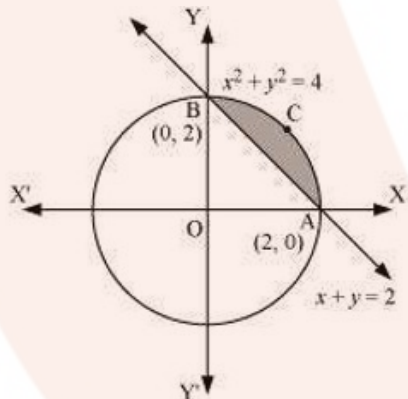
Question 6:

Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$ is

- A. $2(\pi - 2)$
- B. $\pi - 2$
- C. $2\pi - 1$
- D. $2(\pi + 2)$

Solution 6:

The smaller area enclosed by the circle, $x^2 + y^2 = 4$, and the line, $x + y = 2$, is represented by the shaded area ACBA as



It can be observed that,

$$\text{Area ACBA} = \text{Area OACBO} - \text{Area } (\Delta OAB)$$

$$\begin{aligned}
 &= \int_0^2 \sqrt{4-x^2} dx - \int_0^2 (2-x) dx \\
 &= \left[\frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 - \left[2x - \frac{x^2}{2} \right]_0^2 \\
 &= \left[2 \cdot \frac{\pi}{2} \right] - [4 - 2] \\
 &= (\pi - 2) \text{ sq. units}
 \end{aligned}$$

Thus, the correct answer is B.

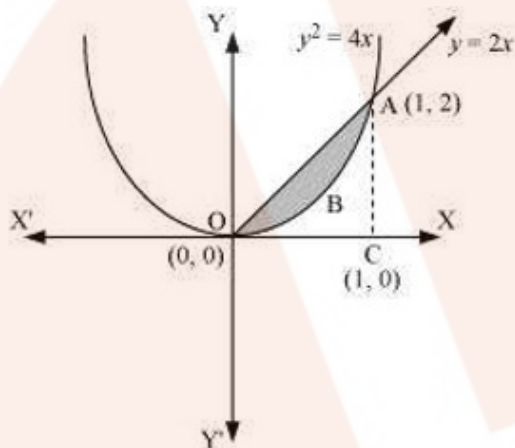
Question 7:

Area lying between the curves $y^2 = 4x$ and $y = 2x$ is

- A. $\frac{2}{3}$ C. $\frac{1}{4}$
 B. $\frac{1}{3}$ D. $\frac{3}{4}$

Solution 7:

The area lying between the curves, $y^2 = 4x$ and $y = 2x$, is represented by the shaded area OBAO as



The points of intersection of these curves are O (0, 0) and A (1, 2).

We draw AC perpendicular to x-axis such that the coordinates of C are (1, 0).

∴ Area OBAO = Area (ΔOCA) – Area (OCABO)

$$= \int_0^1 2x dx - \int_0^1 2\sqrt{x} dx$$

$$= 2 \left[\frac{x^2}{2} \right]_0^1 - 2 \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1$$

$$= \left| 1 - \frac{4}{3} \right|$$

$$= \left| -\frac{1}{3} \right|$$

$$= \frac{1}{3} \text{ sq. units}$$

Thus, the correct answer is B.

Miscellaneous Exercise

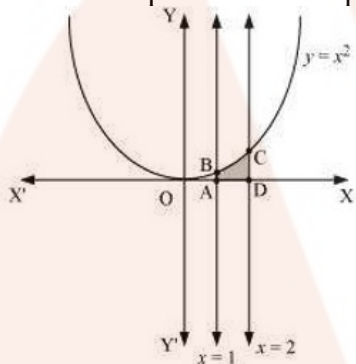
Question 1:

Find the area under the given curves and given lines:

- (i) $y = x^2$, $x = 1$, $x = 2$ and x-axis
 (ii) $y = x^4$, $x = 1$, $x = 5$ and x-axis

Solution 1:

i. The required area is represented by the shaded area ADCBA as



$$\text{Area ADCBA} = \int_1^2 y dx$$

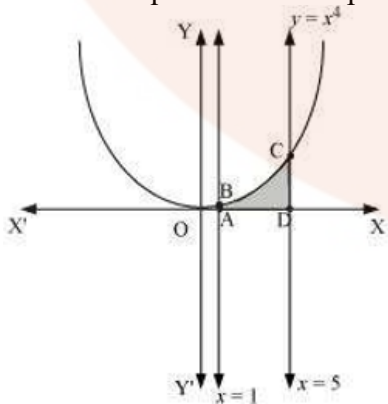
$$= \int_1^2 x^2 dx$$

$$= \left[\frac{x^3}{3} \right]_1^2$$

$$= \frac{8}{3} - \frac{1}{3}$$

$$= \frac{7}{3} \text{ sq. units}$$

ii. The required area is represented by the shaded area ADCBA as



$$\text{Area of ADCBA} = \int_1^5 x^4 dx$$

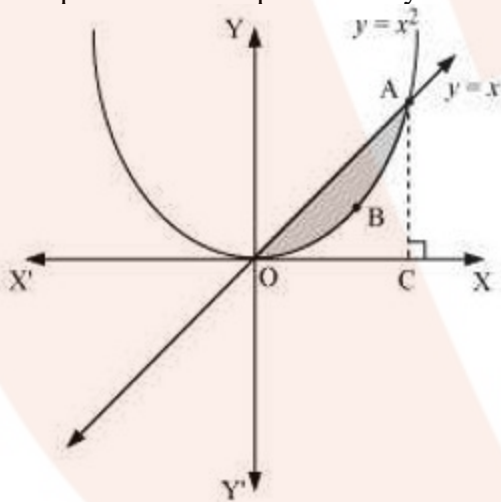
$$\begin{aligned}
 &= \left[\frac{x^5}{5} \right]_1^5 \\
 &= \frac{(5)^5}{5} - \frac{1}{5} \\
 &= (5)^4 - \frac{1}{5} \\
 &= 625 - \frac{1}{5} \\
 &= 624.8 \text{ sq. units}
 \end{aligned}$$

Question 2:

Find the area between the curves $y = x$ and $y = x^2$

Solution 2:

The required area is represented by the shaded area OBAO as



The points of intersection of the curves, $y = x$ and $y = x^2$, is A (1, 1).

We draw AC perpendicular to x-axis.

$$\therefore \text{Area (OBAO)} = \text{Area } (\Delta OCA) - \text{Area (OCABO)} \dots (1)$$

$$= \int_0^1 x dx - \int_0^1 x^2 dx$$

$$= \left[\frac{x^2}{2} \right]_0^1 - \left[\frac{x^3}{3} \right]_0^1$$

$$= \frac{1}{2} - \frac{1}{3}$$

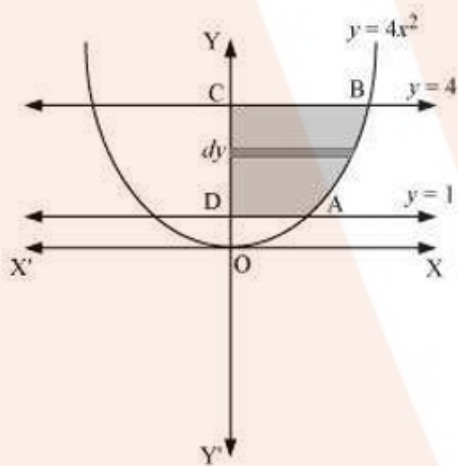
$$= \frac{1}{6} \text{ sq. units}$$

Question 3:

Find the area of the region lying in the first quadrant and bounded by $y = 4x^2$, $x = 0$, $y = 1$ and $y = 4$

Solution 3:

The area in the first quadrant bounded by $y = 4x^2$, $x = 0$, $y = 1$, and $y = 4$ is represented by the shaded area ABCDA as



$$\text{Area ABCDA} = \int_1^4 x dy$$

$$= \int_1^4 \frac{1}{2} \sqrt{y} dy$$

$$= \frac{1}{2} \left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4$$

$$= \frac{1}{3} \left[(4)^{\frac{3}{2}} - 1 \right]$$

$$= \frac{1}{3}[8-1]$$

$$= \frac{7}{3} \text{ sq. units}$$

Question 4:

Sketch the graph of $y = |x+3|$ and evaluate $\int_{-6}^0 |x+3| dx$

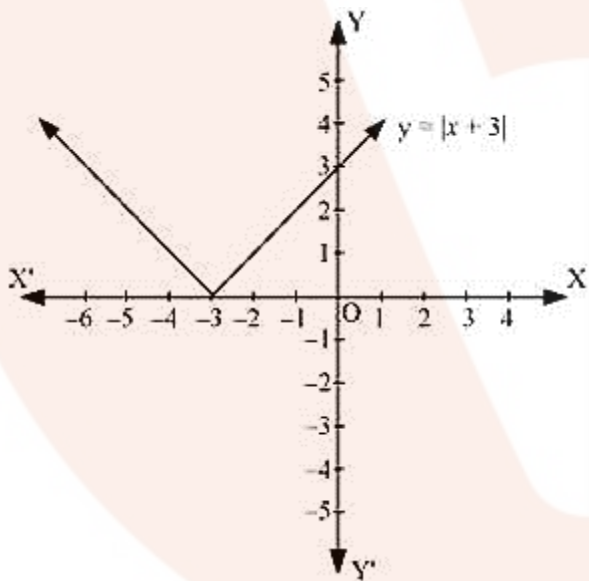
Solution 4:

The given equation is $y = |x+3|$

The corresponding values of x and y are given in the following table.

x	-6	-5	-4	-3	-2	-1	0
y	3	2	1	0	1	2	3

On plotting these points, we obtain the graph of $y = |x+3|$ as follows.



It is known that, $(x+3) \leq 0$ for $-6 \leq x \leq -3$ and $(x+3) \geq 0$ for $-3 \leq x \leq 0$

$$\therefore \int_{-6}^0 |(x+3)| dx = -\int_{-6}^{-3} (x+3) dx + \int_{-3}^0 (x+3) dx$$

$$= -\left[\frac{x^2}{2} + 3x \right]_{-6}^{-3} + \left[\frac{x^2}{2} + 3x \right]_{-3}^0$$

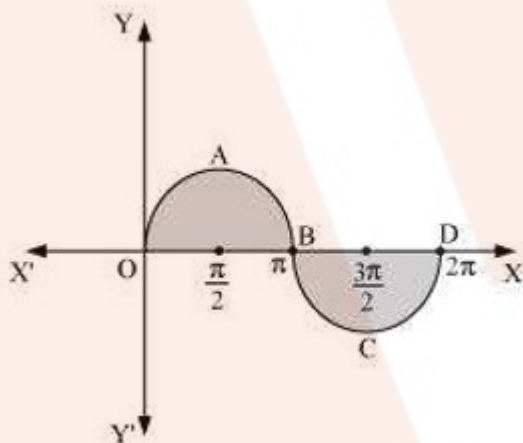
$$\begin{aligned}
 &= -\left[\left(\frac{(-3)^2}{2} + 3(-3)\right) - \left(\frac{(-6)^2}{2} + 3(-6)\right)\right] + \left[0 - \left(\frac{(-3)^2}{2} + 3(-3)\right)\right] \\
 &= -\left[-\frac{9}{2}\right] - \left[-\frac{9}{2}\right] \\
 &= 9
 \end{aligned}$$

Question 5:

Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$

Solution 5:

The graph of $y = \sin x$ can be drawn as



\therefore Required area = Area OABO + Area BCDB

$$= \int_0^{\pi} \sin x dx + \left| \int_{\pi}^{2\pi} \sin x dx \right|$$

$$= [-\cos x]_0^{\pi} + \left| [-\cos x]_{\pi}^{2\pi} \right|$$

$$= [-\cos \pi + \cos 0] + |-\cos 2\pi + \cos \pi|$$

$$= 1 + 1 + |(-1 - 1)|$$

$$= 2 + |-2|$$

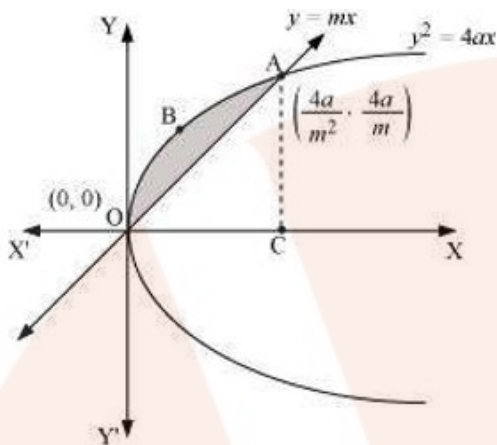
$$= 2 + 2 = 4 \text{ sq. units}$$

Question 6:

Find the area enclosed between the parabola $y^2 = 4ax$ and the line $y = mx$.

Solution 6:

The area enclosed between the parabola, $y^2 = 4ax$ and the line $y = mx$, is represented by the shaded area OABO as



$$y^2 = 4ax, y = mx$$

$$m^2 x^2 = 4ax$$

$$m^2 x^2 - 4ax = 0$$

$$x(m^2 x - 4a) = 0$$

$$x = 0 \text{ or } x = 4/m^2$$

The points of intersection of both the curves are $(0, 0)$ and $\left(\frac{4a}{m^2}, \frac{4a}{m}\right)$.

We draw AC perpendicular to x-axis.

\therefore Area OABO = Area OCABO – Area (Δ OCA)

$$= \int_0^{\frac{4a}{m^2}} 2\sqrt{ax} dx - \int_0^{\frac{4a}{m^2}} mx dx$$

$$= 2\sqrt{a} \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{4a}{m^2}} - m \left[\frac{x^2}{2} \right]_0^{\frac{4a}{m^2}}$$

$$= \frac{4}{3} \sqrt{a} \left(\frac{4a}{m^2} \right)^{\frac{3}{2}} - \frac{m}{2} \left[\left(\frac{4a}{m^2} \right)^2 \right]$$

$$= \frac{32a^2}{3m^3} - \frac{m}{2} \left(\frac{16a^2}{m^3} \right)$$

$$= \frac{32a^2}{3m^3} - \frac{8a^2}{m^3}$$

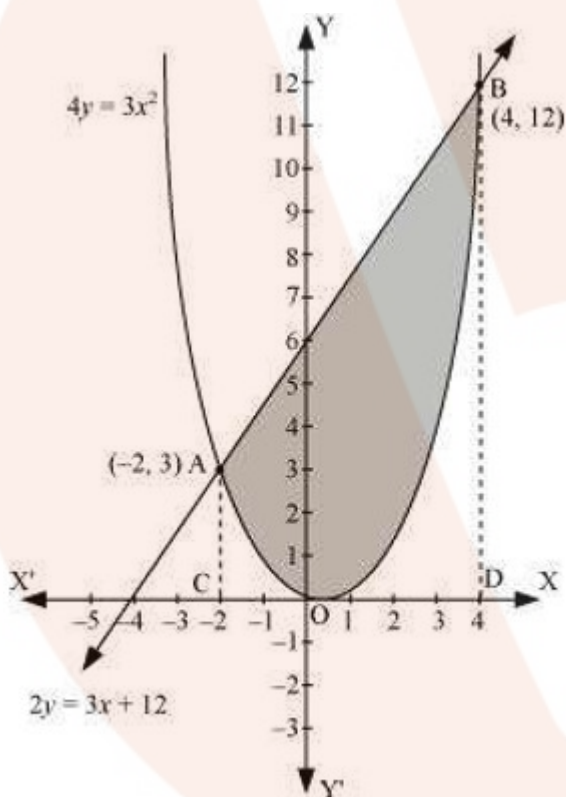
$$= \frac{8a^2}{3m^3} \text{ sq. units}$$

Question 7:

Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

Solution 7:

The area enclosed between the parabola $4y = 3x^2$ and the line $2y = 3x + 12$, is represented by the shaded area OBAO as



From the given equation of line, we have

$$2y = 3x + 12$$

$$y = \frac{3x + 12}{2} \quad \dots(1)$$

From the given equation of parabola, we have

$$4y = 3x^2$$

$$4\left(\frac{3x+12}{2}\right) = 3x^2 \quad [From (1)]$$

$$6x + 24 = 3x^2$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x = 4, x = -2$$

The points of intersection of the given curves are A (-2, 3) and (4, 12).

We draw AC and BD perpendicular to x-axis.

∴ Area OBAO = Area CDBAC – (Area ODBO + Area OACO)

$$= \int_{-2}^4 \frac{1}{2}(3x+12)dx - \int_{-2}^4 \frac{3x^2}{4}dx$$

$$= \frac{1}{2} \left[\frac{3x^2}{2} + 12x \right]_{-2}^4 - \frac{3}{4} \left[\frac{x^3}{3} \right]_{-2}^4$$

$$= \frac{1}{2} [24 + 48 - 6 + 24] - \frac{1}{4} [64 + 8]$$

$$= \frac{1}{2} [90] - \frac{1}{4} [72]$$

$$= 45 - 18$$

$$= 27 \text{ sq. units}$$

Question 8:

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$

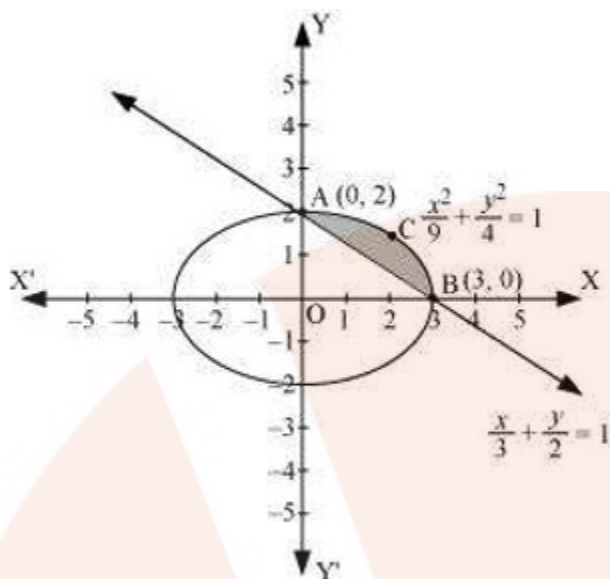
Solution 8:

The area of the smaller region bounded by the ellipse, $\frac{x^2}{9} + \frac{y^2}{4} = 1$, and the line, $\frac{x}{3} + \frac{y}{2} = 1$, is represented by the shaded region BCAB as

∴ Area BCAB = Area (OBCAO) – Area (OBAO)

$$= \int_0^3 2\sqrt{1 - \frac{x^2}{9}}dx - \int_0^3 2\left(1 - \frac{x}{3}\right)dx$$

$$= \frac{2}{3} \left[\int_0^3 \sqrt{9 - x^2}dx \right] - \frac{2}{3} \int_0^3 (3 - x)dx$$



$$\begin{aligned}
 &= \frac{2}{3} \left[\frac{x}{2} \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} \right]_0^3 - \frac{2}{3} \left[3x - \frac{x^2}{2} \right]_0^3 \\
 &= \frac{2}{3} \left[\frac{9}{2} \left(\frac{\pi}{2} \right) \right] - \frac{2}{3} \left[9 - \frac{9}{2} \right] \\
 &= \frac{2}{3} \left[\frac{9\pi}{4} - \frac{9}{2} \right] \\
 &= \frac{2}{3} \times \frac{9}{4} (\pi - 2) \\
 &= \frac{3}{2} (\pi - 2) \text{ sq. units}
 \end{aligned}$$

Question 9:

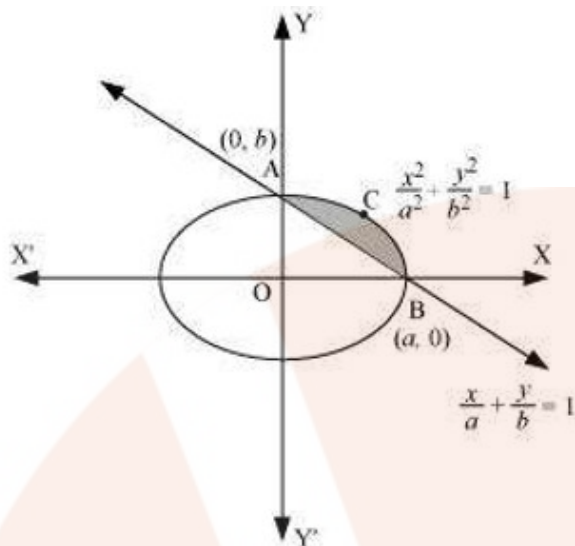
Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$

Solution 9:

The area of the smaller region bounded by the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, and the line, $\frac{x}{a} + \frac{y}{b} = 1$, is represented by the shaded region BCAB as

\therefore Area BCAB = Area (OBCAO) – Area (OBAO)

$$= \int_0^a b \sqrt{1 - \frac{x^2}{a^2}} dx - \int_0^a b \left(1 - \frac{x}{a} \right) dx$$



$$\begin{aligned}
 &= \frac{b}{a} \int_0^a \sqrt{a^2 - x^2} dx - \frac{b}{a} \int_0^a (a - x) dx \\
 &= \frac{b}{a} \left[\left\{ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right\}_0^a - \left\{ ax - \frac{x^2}{2} \right\}_0^a \right] \\
 &= \frac{b}{a} \left[\left\{ \frac{a^2}{2} \left(\frac{\pi}{2} \right) \right\} - \left\{ a^2 - \frac{a^2}{2} \right\} \right] \\
 &= \frac{b}{a} \left[\frac{a^2 \pi}{4} - \frac{a^2}{2} \right] \\
 &= \frac{ba^2}{2a} \left[\frac{\pi}{2} - 1 \right] \\
 &= \frac{ab}{2} \left[\frac{\pi}{2} - 1 \right] \\
 &= \frac{ab}{4} (\pi - 2)
 \end{aligned}$$

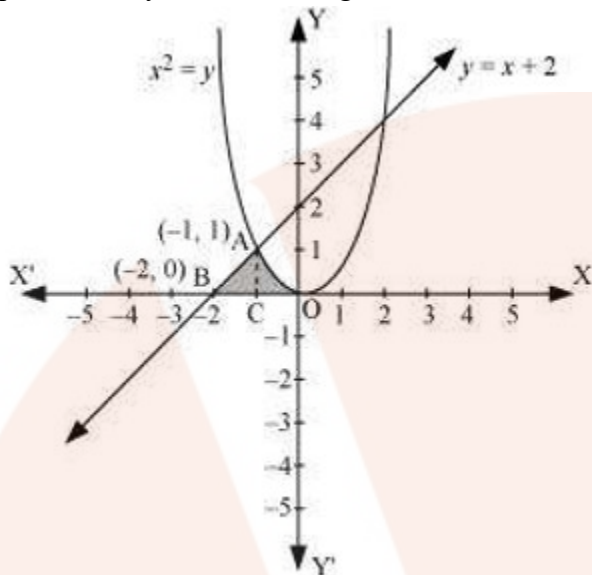
\therefore The required area is $\frac{ab}{4}(\pi - 2)$ sq. units

Question 10:

Find the area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and x axis

Solution 10:

The area of the region enclosed by the parabola, $x^2 = y$, the line, $y = x + 2$, and x-axis is represented by the shaded region OABCO as



$$y = x + 2, x^2 = y$$

$$\therefore x^2 = x + 2$$

$$\Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x - 2)(x + 1) = 0$$

$$\Rightarrow x = 2 \text{ or } x = -1$$

The point of intersection of the parabola, $x^2 = y$, and the line, $y = x + 2$, is A $(-1, 1)$.

\therefore Area OABCO = Area (BCAB) + Area COAC

$$= \int_{-2}^{-1} (x+2) dx + \int_{-1}^0 x^2 dx$$

$$= \left[\frac{x^2}{2} + 2x \right]_{-2}^{-1} + \left[\frac{x^3}{3} \right]_{-1}^0$$

$$= \left[\frac{(-1)^2}{2} + 2(-1) - \frac{(-2)^2}{2} - 2(-2) \right] + \left[-\frac{(-1)^3}{3} \right]$$

$$= \left[\frac{1}{2} - 2 - 2 + 4 + \frac{1}{3} \right]$$

$$= \frac{5}{6} \text{ sq. units}$$

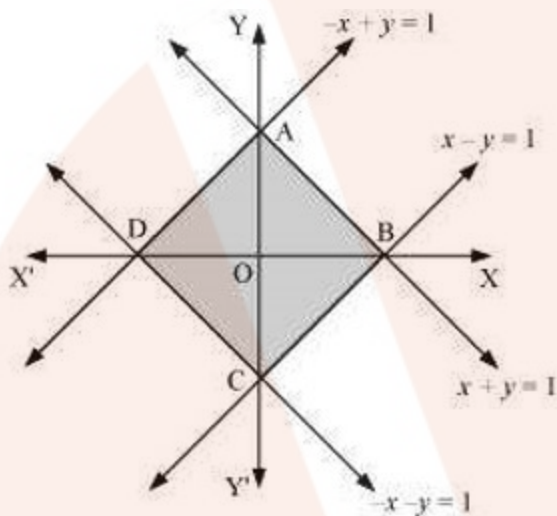
Question 11:

Using the method of integration find the area bounded by the curve $|x|+|y|=1$

[Hint: the required region is bounded by lines $x+y=1$, $x-y=1$, $-x+y=1$ and $-x-y=1$]

Solution 11:

The area bounded by the curve, $|x|+|y|=1$, is represented by the shaded region ADCBA as



The curve intersects the axes at points A (0, 1), B (1, 0), C (0, -1), and D (-1, 0).

It can be observed that the given curve is symmetrical about x-axis and y-axis.

\therefore Area ADCBA = $4 \times$ Area OBAO

$$= 4 \int_0^1 (1-x) dx$$

$$= 4 \left(x - \frac{x^2}{2} \right)_0^1$$

$$= 4 \left[1 - \frac{1}{2} \right]$$

$$= 4 \left(\frac{1}{2} \right)$$

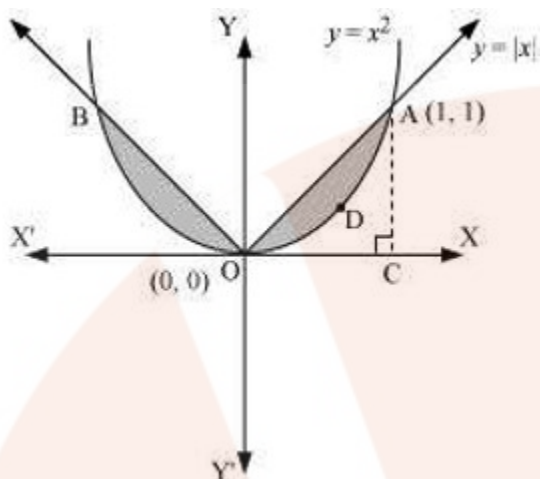
$$= 2 \text{ sq. units}$$

Question 12:

Find the area bounded by curves $\{(x, y) : y \geq x^2 \text{ and } y = |x|\}$

Solution 12:

The area bounded by the curves, $\{(x, y) : y \geq x^2 \text{ and } y = |x|\}$, is represented by the shaded region as



It can be observed that the required area is symmetrical about y-axis.

Required area = 2 [Area (OCAO) – Area (OCADO)]

$$= 2 \left[\int_0^1 x dx - \int_0^1 x^2 dx \right]$$

$$= 2 \left[\left[\frac{x^2}{2} \right]_0^1 - \left[\frac{x^3}{3} \right]_0^1 \right]$$

$$= 2 \left[\frac{1}{2} - \frac{1}{3} \right]$$

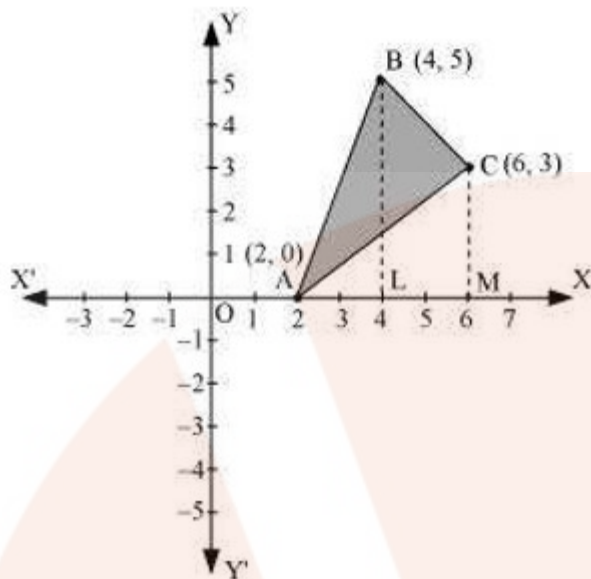
$$= 2 \left[\frac{1}{6} \right] = \frac{1}{3} \text{ sq. units}$$

Question 13:

Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are A (2, 0), B (4, 5) and C (6, 3)

Solution 13:

The vertices of $\triangle ABC$ are A (2, 0), B (4, 5), and C (6, 3).



Equation of line segment AB is

$$y - 0 = \frac{5 - 0}{4 - 2}(x - 2)$$

$$y = \frac{5}{2}(x - 2) \quad \dots(1)$$

Equation of line segment BC is

$$y - 5 = \frac{3 - 5}{6 - 4}(x - 4)$$

$$2y - 10 = -2x + 8$$

$$2y = -2x + 18$$

$$y = -x + 9 \quad \dots(2)$$

Equation of line segment CA is

$$y - 3 = \frac{0 - 3}{2 - 6}(x - 6)$$

$$-4y + 12 = -3x + 18$$

$$4y = 3x - 6$$

$$y = \frac{3}{4}(x - 2) \quad \dots(3)$$

Area (ΔABC) = Area (ABLA) + Area (BLMCB) – Area (ACMA)

$$= \int_2^4 \frac{5}{2}(x - 2) dx + \int_4^6 (-x + 9) dx - \int_2^6 \frac{3}{4}(x - 2) dx$$

$$= \frac{5}{2} \left[\frac{x^2}{2} - 2x \right]_2^4 + \left[\frac{-x^2}{2} + 9x \right]_4^6 - \frac{3}{4} \left[\frac{x^2}{2} - 2x \right]_2^6$$

$$\begin{aligned}
 &= \frac{5}{2}[8-8-2+4] + [-18+54+8-36] - \frac{3}{4}[18-12-2+4] \\
 &= 5+8 - \frac{3}{4}(8) \\
 &= 13-6 \\
 &= 7 \text{ sq. units}
 \end{aligned}$$

Question 14:

Using the method of integration find the area of the region bounded by lines:

$$2x + y = 4, 3x - 2y = 6 \text{ and } x - 3y + 5 = 0$$

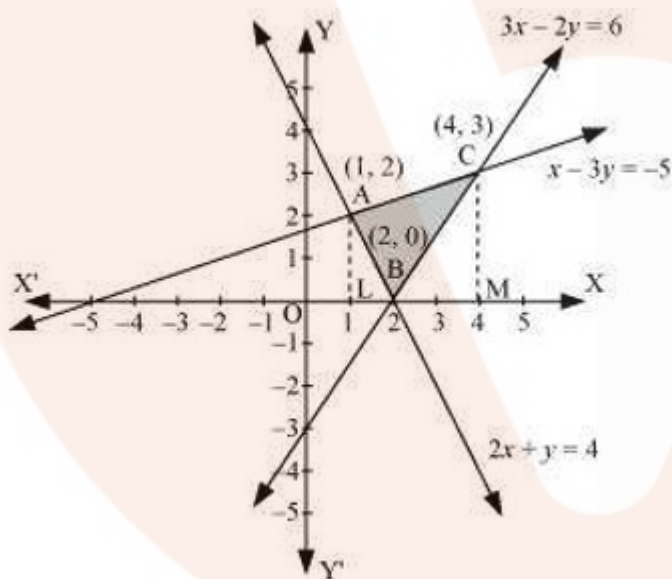
Solution 14:

The given equations of lines are

$$2x + y = 4 \dots(1)$$

$$3x - 2y = 6 \dots(2)$$

$$\text{And, } x - 3y + 5 = 0 \dots(3)$$



The area of the region bounded by the lines is the area of ΔABC . AL and CM are the perpendiculars on x -axis.

$$\text{Area}(\Delta ABC) = \text{Area}(\text{ALMCA}) - \text{Area}(\text{ALBA}) - \text{Area}(\text{CMBC})$$

$$= \int_1^4 \left(\frac{x+5}{3} dx \right) - \int_1^2 (4-2x) dx - \int_2^4 \left(\frac{3x-6}{2} \right) dx$$

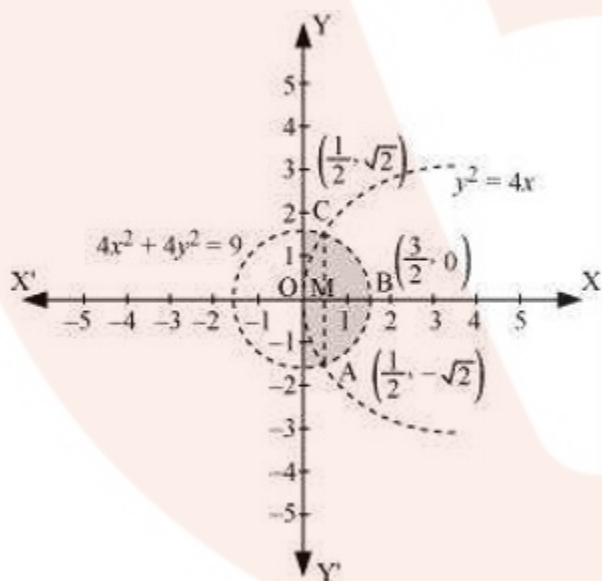
$$\begin{aligned}
 &= \frac{1}{3} \left[\frac{x^2}{2} + 5x \right]_1^4 - \left[4x - x^2 \right]_1^2 - \frac{1}{2} \left[\frac{3x^2}{2} - 6x \right]_2^4 \\
 &= \frac{1}{3} \left[8 + 20 - \frac{1}{2} - 5 \right] - [8 - 4 - 4 + 1] - \frac{1}{2} [24 - 24 - 6 + 12] \\
 &= \left(\frac{1}{3} \times \frac{45}{2} \right) - (1) - \frac{1}{2} (6) \\
 &= \frac{15}{2} - 1 - 3 \\
 &= \frac{15}{2} - 4 = \frac{15 - 8}{2} = \frac{7}{2} \text{ sq. units}
 \end{aligned}$$

Question 15:

Find the area of the region $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

Solution 15:

The area bounded by the curves, $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$ is represented as



$$y^2 = 4x$$

$$\Rightarrow 4x^2 + 4(4x) = 9$$

$$\Rightarrow 4x^2 + 16x - 9 = 0$$

$$\Rightarrow 4x^2 + 18x - 2x - 9 = 0$$

$$\Rightarrow 2x(2x + 9) - (2x + 9) = 0$$

$$\Rightarrow (2x - 1)(2x + 9) = 0$$

$$\therefore x = \frac{1}{2} \text{ \& } y = \pm\sqrt{4x} = \pm\sqrt{2}$$

The points of intersection of both the curves are $\left(\frac{1}{2}, \sqrt{2}\right)$ and $\left(\frac{1}{2}, -\sqrt{2}\right)$.

The required area is given by OABCO.

It can be observed that area OABCO is symmetrical about x-axis.

$$\therefore \text{Area OABCO} = 2 \times \text{Area OBCO}$$

$$\text{Area OBCO} = \text{Area OMCO} + \text{Area MBCM}$$

$$= \int_0^{\frac{1}{2}} 2\sqrt{x} dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2} \sqrt{9 - 4x^2} dx$$

$$= \int_0^{\frac{1}{2}} 2\sqrt{x} dx + \int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{2} \sqrt{(3)^2 - (2x)^2} dx$$

$$\text{Let } 2x = t, dx = \frac{dt}{2}$$

$$\text{When } x = \frac{1}{2}, t = 1 \text{ and when } x = \frac{3}{2}, t = 3$$

$$\Rightarrow \int_0^{\frac{1}{2}} 2\sqrt{x} dx + \frac{1}{4} \int_1^3 \sqrt{9 - t^2} dt$$

$$\Rightarrow 2 \left[\frac{2}{3} x^{3/2} \right]_0^{\frac{1}{2}} + \frac{1}{4} \left[\frac{t}{2} \sqrt{9 - t^2} + \frac{9}{2} \sin^{-1} \left(\frac{t}{3} \right) \right]_1^3$$

$$\Rightarrow \frac{4}{3} \left[\frac{1}{2\sqrt{2}} \right] + \frac{1}{4} \left[\frac{9}{2} \cdot \frac{\pi}{2} - \frac{1}{2} \sqrt{8} - \frac{9}{2} \sin^{-1} \left(\frac{1}{3} \right) \right]$$

$$\Rightarrow \frac{2}{3\sqrt{2}} + \frac{9\pi}{16} - \frac{1}{2\sqrt{2}} - \frac{9}{8} \sin^{-1} \left(\frac{1}{3} \right)$$

$$\Rightarrow \frac{1}{6\sqrt{2}} + \frac{9\pi}{16} - \frac{9}{8} \sin^{-1} \left(\frac{1}{3} \right)$$

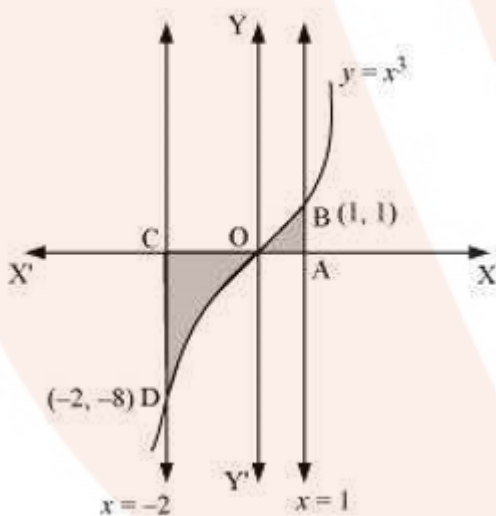
$$\text{Area of OABCD} = 2 \text{ Area of OBCO}$$

$$\begin{aligned} \therefore \text{Required area} &= 2 \left[\frac{1}{6\sqrt{2}} + \frac{9\pi}{16} - \frac{9}{8} \sin^{-1} \left(\frac{1}{3} \right) \right] \\ &= \frac{9\pi}{8} + \frac{1}{3\sqrt{2}} - \frac{9}{4} \sin^{-1} \left(\frac{1}{3} \right) \text{ sq. units} \end{aligned}$$

Question 16:

Area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is

- A. -9
- B. $-\frac{15}{4}$
- C. $\frac{15}{4}$
- D. $\frac{17}{4}$

Solution 16:

$$\begin{aligned} \text{Required area} &= \int_{-2}^1 y dx \\ &= \int_{-2}^1 x^3 dx \\ &= \left[\frac{x^4}{4} \right]_{-2}^1 \end{aligned}$$

$$\left[\frac{1}{4} - \frac{(-2)^4}{4} \right]$$

$$\left(\frac{1}{4} - 4 \right) = -\frac{15}{4}$$

$$\therefore \text{Area} = \left| -\frac{15}{4} \right| = \frac{15}{4} \text{ sq. units}$$

Thus, the correct answer is C.

Question 17:

The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by

[Hint: $y = x^2$ if $x > 0$ and $y = -x^2$ if $x < 0$]

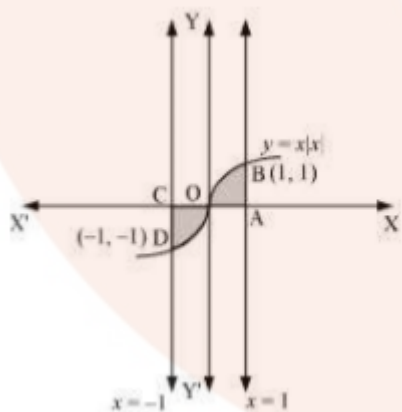
A. 0

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. $\frac{4}{3}$

Solution 17:



$$\text{Required area} = \int_{-1}^1 y dx$$

$$\begin{aligned} &= \int_{-1}^1 x|x| dx = \left| \int_{-1}^0 x^2 dx \right| + \left| \int_0^1 x^2 dx \right| \\ &= \left| \int_{-1}^0 -x^2 dx \right| + \left| \int_0^1 x^2 dx \right| \\ &= \left| \left[\frac{-x^3}{3} \right]_{-1}^0 \right| + \left| \left[\frac{x^3}{3} \right]_0^1 \right| \\ &= \left| -\left(-\frac{1}{3} \right) \right| + \frac{1}{3} \\ &= \frac{2}{3} \text{ sq. units} \end{aligned}$$

Thus, the correct answer is C.

Question 18:

The area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$ is

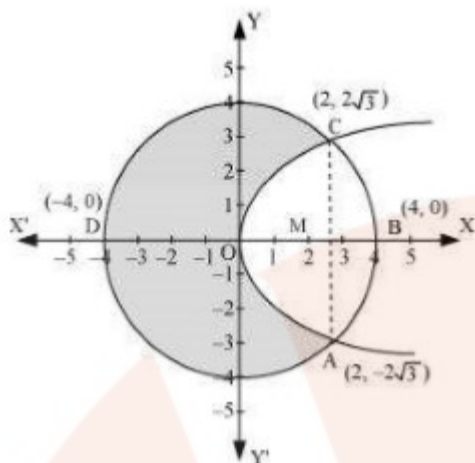
- A. $\frac{4}{3}(4\pi - \sqrt{3})$
- B. $\frac{4}{3}(4\pi + \sqrt{3})$
- C. $\frac{4}{3}(8\pi - \sqrt{3})$
- D. $\frac{4}{3}(8\pi + \sqrt{3})$

Solution 18:

The given equations are

$$x^2 + y^2 = 16 \dots(1)$$

$$y^2 = 6x \dots(2)$$



$$y^2 = 6x$$

$$\Rightarrow x^2 + 6x = 16$$

$$\Rightarrow x^2 + 6x - 16 = 0$$

$$\Rightarrow (x + 8)(x - 2) = 0$$

$$\Rightarrow x = -8 \text{ \& } x = 2$$

$$\text{when } x = 2 \quad y = \pm\sqrt{15} = \pm 2\sqrt{3}$$

Area bounded by the circle and parabola (unshaded portion)

= Area (OCBAC)

= 2[Area (OBCO)]

= 2[Area(OMCO) + Area (BMCB)]

$$= 2 \left[\int_0^2 \sqrt{16x} dx + \int_2^4 \sqrt{16 - x^2} dx \right]$$

$$= 2\sqrt{6} \int_0^2 \sqrt{x} dx + 2 \int_2^4 \sqrt{16 - x^2} dx$$

$$= 2\sqrt{6} \times \frac{2}{3} \left[(x)^{3/2} \right]_0^2 + 2 \left[\frac{x}{2} \sqrt{16 - x^2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right]_2^4$$

$$= 2\sqrt{6} \times \frac{2}{3} \left[x^{3/2} \right]_0^2 + 2 \left[8 \cdot \frac{\pi}{2} - \sqrt{16 - 4} - 8 \sin^{-1} \left(\frac{1}{2} \right) \right]$$

$$= \frac{4\sqrt{6}}{3} (2\sqrt{2}) + 2 \left[4\pi - \sqrt{12} - 8 \frac{\pi}{6} \right]$$

$$= \frac{16\sqrt{3}}{3} + 8\pi - 4\sqrt{3} - \frac{8}{3}\pi$$

$$= \frac{4}{3} [4\sqrt{3} + 6\pi - 3\sqrt{3} - 2\pi]$$

$$= \frac{4}{3} [\sqrt{3} + 4\pi]$$

$$= \frac{4}{3} [4\pi + \sqrt{3}] \text{ sq. units}$$

$$\text{Area of circle} = \pi(r)^2$$

$$= \pi(4)^2$$

$$= 16\pi \text{ sq. units}$$

\therefore Area of the circle $x^2 + y^2 = 16$ exterior to the parabola $y^2 = 6x$

$$= \text{Area of circle} - \frac{4}{3} [4\pi + \sqrt{3}]$$

$$= \pi(4)^2 - \frac{4}{3} [4\pi + \sqrt{3}]$$

$$= \frac{4}{3} [4 \times 3\pi - 4\pi - \sqrt{3}]$$

$$= \frac{4}{3} (8\pi - \sqrt{3}) \text{ sq. units}$$

Thus, the correct answer is C.

Question 19:

The area bounded by the y-axis, $y = \cos x$ and $y = \sin x$ when $0 \leq x \leq \frac{\pi}{2}$

A. $2(\sqrt{2} - 1)$

B. $\sqrt{2} - 1$

C. $\sqrt{2} + 1$

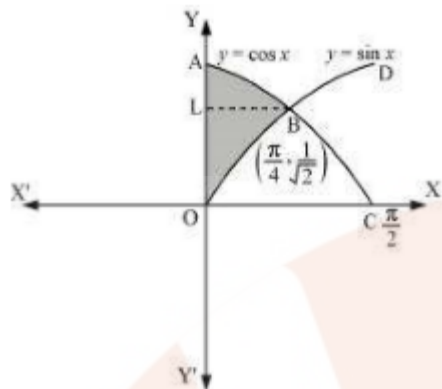
D. $\sqrt{2}$

Solution 19:

The given equations are

$$y = \cos x \dots (1)$$

$$\text{And, } y = \sin x \dots (2)$$



Required area = Area (ABLA) + area (OBLO)

$$= \int_{\frac{1}{\sqrt{2}}}^1 x dy + \int_0^{\frac{1}{\sqrt{2}}} x dy$$

$$= \int_{\frac{1}{\sqrt{2}}}^1 \cos^{-1} y dy + \int_{\frac{1}{\sqrt{2}}}^1 \sin^{-1} y dy$$

Integrating by parts, we obtain

$$= \left[y \cos^{-1} y - \sqrt{1-y^2} \right]_{\frac{1}{\sqrt{2}}}^1 + \left[y \sin^{-1} y + \sqrt{1-y^2} \right]_0^{\frac{1}{\sqrt{2}}}$$

$$= \left[\cos^{-1}(1) - \frac{1}{\sqrt{2}} \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} \right] + \left[\frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + \sqrt{1-\frac{1}{2}} - 1 \right]$$

$$= \frac{-\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}} + \frac{1}{\sqrt{2}} - 1$$

$$= \frac{2}{\sqrt{2}} - 1$$

$$= \sqrt{2} - 1 \text{ sq. units}$$

\therefore The correct answer is option B.

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