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Math Chapter 9
Differential Equations

Chapter 9: Differential Equations

Exercise: 9.1

Q.1: Find the degree and order of the differential equation d4ydx4**+sin (y''') = 0.**

Solution:

d4ydx4 + sin (y''') = 0
y'''' + sin (y''') = 0

y'''' is the highest order derivative present in the differential equation.

Therefore, the order is four.

The given differential equation is not a polynomial equation in its derivatives. Hence, its degree is not defined.

Q.2: Find the degree and order of differential equation y' + 5y = 0

Solution:

Given: y' + 5y = 0

y' is the highest order derivative present in the differential equation.

Therefore, the order is one.

The given differential equation is a polynomial equation in y'. The highest degree derivative present in the differential equation is y'.
Therefore, the degree is one.

Q.3: Find the degree and order of differential equation (dsdt)4+3sd2sdt=0.

Solution:

$\left(\frac{ds}{dt}\right)^4+3s\frac{d^2s}{dt^2}=0$

$\frac{d^2s}{dt^2}$ is the highest order derivative present in the differential equation.
Therefore, the order is two.

The given differential equation is a polynomial equation in $\frac{d^2s}{dt^2}$ and $\frac{ds}{dt}$.The power raised to $\frac{d^2s}{dt^2}$ is 1.
Hence, its degree is one.

Q.4: Find the degree and order of differential equation $\left(\frac{d^2y}{dx^2}\right)^2+\cos\left(\frac{dy}{dx}\right)=0$**.**
Solution:

$\left(\frac{d^2y}{dx^2}\right)^2+\cos\left(\frac{dy}{dx}\right)=0$

$\frac{d^2y}{dx^2}$ **is the highest order derivative present in the differential equation.**
Therefore, the order is two.

The given differential equation is not a polynomial equation in its derivatives. Hence, its degree is not defined.

Q.5: Find the degree and order of differential equation $\frac{d^2y}{dx^2}=\cos3x+\sin3x$

Solution:

$\frac{d^2y}{dx^2}=\cos3x+\sin3x\Rightarrow\frac{d^2y}{dx^2}-\cos3x-\sin3x=0$

$\frac{d^2y}{dx^2}$ **is the highest order derivative present in the differential equation.**
Therefore, the order is two.

It is a polynomial equation in $\frac{d^2y}{dx^2}$ and the power raised to $\frac{d^2y}{dx^2}$ is 1.
Hence, its degree is one.

Q.6: Find the degree and order of differential equation $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$

Solution: $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$

y''' is the highest order derivative present in the differential equation.

Therefore, the order is three.

It is a polynomial equation in y''', y'' and y'.

The power of y''' is 2.

Hence, its degree is 2.

Q.7: Find the degree and order of differential equation $y''' + 2y'' + y' = 0$.

Solution:

Given: $y''' + 2y'' + y' = 0$.

y''' is the highest order derivative present in the differential equation.

Therefore, the order is three.

It is a polynomial equation in y''', y'', and y'.

The power of y''' is 1.

Hence, its degree is 1.

Q.8: Find the degree and order of differential equation $y' + y = e^x$

Solution: $y' + y = e^x$

$\Rightarrow y' + y - e^x = 0$

y' is the highest order derivative present in the differential equation.

Therefore, the order is one.

It is a polynomial equation in y'.

The power raised to y''' is 1.

Hence, its degree is 1.

Q.9: Find the degree and order of differential equation y'' + $(y')^2$ +2y = 0.

Solution:

y'' + $(y')^2$ +2y = 0

y'' is the highest order derivative present in the differential equation.

Therefore, the order is two.

It is a polynomial equation in y'' + y'.

The power raised to y'' is 1.

Hence, its degree is 1.

Q.10: The degree of differential equation $\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0$ **is:**

(i) 3

(ii) 2

(iii) 1

(iv) not defined

Solution:

$\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0$

The given differential equation is not a polynomial equation in its derivatives. Hence, its degree is not defined.

Hence, the answer is (iv).

Q.11: The degree of differential equation: $2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$ **is:**

(i) 2

(ii) 1

(iii) 0

(iv) not defined

Solution:

$\frac{d^2y}{dx^2}$ is the highest order derivative present in the differential equation.

Therefore, the order is two.

Hence, the correct answer is (i).

Exercise-9.2

Q.1: y = e^x + 1; y" – y' = 0

Solution:

y = e^x + 1

Differentiate both the sides with respect to x, we get:

$\frac{dy}{dx}=\frac{d}{dx}(e^x+1)$

⇒ y' = e^x . . . . **(1)**

Now, differentiate equation (1) with respect to x, we get:

$\frac{d}{dx}(y')=\frac{d}{dx}(e^x)$

⟶ y" = e^x

Substituting the values of y' and y" in the given differential equation, we get the L.H.S. as:

$y'' - y' = e^x - e^x = 0$ = R.H.S

Thus, the given function is the solution of the corresponding differential equation.

Q.2: $y = x^2 + 2x + C$; $y' - 2x - 2 = 0$

Solution:

$y = x^2 + 2x + C$

Differentiate both the sides with respect to x, we get:

$y' = \frac{d}{dx}(x^2 + 2x + C)$

$\longrightarrow y' = 2x + 2$

Substituting the values of y' in the given differential equation, we get the L.H.S. as:

$y' - 2x - 2 = 2x + 2 - 2x - 2 = 0$ **= R.H.S**

Hence, the given function is the solution of the corresponding differential equation.

Q.3: $y = \cos x + C$; $y' + \sin x = 0$

Solution:

$y = \cos x + C$

Differentiate both the sides with respect to x, we get:

$y' = \frac{d}{dx}(\cos x + C)$

$\longrightarrow y' = -\sin x$

Substituting the values of y' in the given differential equation, we get the L.H.S. as:

$y' + \sin x = -\sin x + \sin x = 0$ = R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Q.4: $\mathbf{y = \sqrt{1+x^2}}$**;** $\mathbf{y' = \frac{xy}{1+x^2}}$

Solution:

$y=\sqrt{1+x^2}$

Differentiate both the sides with respect to x, we get:

$y' = \frac{d}{dx}\left(\sqrt{1+x^2}\right)$

$y' = \frac{1}{2\sqrt{1+x^2}}.\frac{d}{dx}\left(1+x^2\right)$

$y' = \frac{2x}{2\sqrt{1+x^2}}$

$y' = \frac{x}{\sqrt{1+x^2}}$

$y' = \frac{x}{1+x^2}\times\sqrt{1+x^2}$

$y' = \frac{x}{1+x^2}.y$

$y' = \frac{xy}{1+x^2}$

Therefore, L.H.S = R.H.S

Hence, the given function is the solution of the corresponding differential equation.

Q.5: $\mathbf{y = Ax}$**;** $\mathbf{xy' = y \ (x \neq 0)}$

Solution:

Differentiate both the sides with respect to x, we get:

$y'=\frac{d}{dx}(Ax)\Rightarrow y'=A$

Substituting the values of y' in the given differential equation, we get the L.H.S. as:

$xy'=x.A=Ax=y=R.H.S.$

Hence, the given function is the solution of the corresponding differential equation.

Q.6. $y=x\sin x : xy'=y+x\sqrt{x^2-y^2}\ (x\neq 0 \text{ and } x>y \text{ or } x<-y)$

Solution:

y=x sin x

Differentiate both the sides with respect to x, we get:

$$y'=\frac{d}{dx}(x\sin x) \Rightarrow y'=\sin x.\frac{d}{dx}(x)+x.\frac{d}{dx}(\sin x) \Rightarrow y'=\sin x+x\cos x$$

Substitute the value of y' **in the given differential equation, we get:**
L.H.S
$$= xy'=x(\sin x+x\cos x)=x\sin x+x^2\cos x=y+x^2.\sqrt{1-\sin^2 x}=y+x^2\sqrt{1-\left(\frac{y}{x}\right)^2}=y+x\sqrt{y^2-x^2}=\text{R.H.S.}$$
Hence, the given function is the solution of the corresponding differential equation.

Q.7. $xy=\log y+C : y'=\frac{y^2}{1-xy}\ (xy\neq 1)$

Solution:

$xy=\log y+C$

Differentiate both the sides with respect to x, we get:

$$\frac{d}{dx}(xy)=\frac{d}{dx}(\log y) \Rightarrow y.\frac{d}{dx}(x)+x.\frac{dy}{dx}=\frac{1}{y}\frac{dy}{dx} \Rightarrow y+xy'=\frac{1}{y}y' \Rightarrow y^2+xyy'=y' \Rightarrow (xy-1)y'=-y^2 \Rightarrow y'=\frac{y^2}{1-xy}$$

Therefore L.H.S = R.H.S

Hence, the given function is the solution of the corresponding differential equation.

Q.8. $y-\cos y=x : (y\sin y+\cos y+x)y'=y$

Solution:

$y-\cos y=x$ **……….(1)**

Differentiate both the sides with respect to x, we get:

$$\frac{dy}{dx}-\frac{d}{dx}(\cos y)=\frac{d}{dx}(x) \Rightarrow y'+\sin y . y'=1 \Rightarrow y'(1+\sin y)=1 \Rightarrow y'=\frac{1}{1+\sin y}$$

Substitute the value of y' **in the given differential equation, we get:**

L.H.S

= $(y\sin y+\cos y+x)y'=(y\sin y+\cos y+y-\cos y)\times\frac{1}{1+\sin y}=y(1+\sin y).\frac{1}{1+\sin y}=y=R.H.S.$

Hence, the given function is the solution of the corresponding differential equation.

Q.9: $x+y=\tan^{-1}y : y^2y'+y^2+1=0$

Solution: $x+y=\tan^{-1}y$

Differentiate both the sides with respect to x, we get:

$$\frac{d}{dx}(x+y)=\frac{d}{dx}(\tan^{-1}y) \Rightarrow 1+y'=\left[\frac{1}{1+y^2}\right]y' \Rightarrow y'\left[\frac{1}{1+y^2}-1\right]=1 \Rightarrow y'\left[\frac{1-(1+y^2)}{1+y^2}\right]=1 \Rightarrow y'\left[\frac{-y^2}{1+y^2}\right]=1 \Rightarrow y'=\frac{-(1+y^2)}{y^2}$$

Substitute the value of y' **in the given differential equation, we get:**

L.H.S = $y^2y'+y^2+1=y^2\left[\frac{-(1+y^2)}{y^2}\right]+y^2+1=-1-y^2+y^2+1=0=R.H.S.$

Hence, the given function is the solution of the corresponding differential equation.

Q.10: $y=\sqrt{a^2-x^2}\; x\in(-a,a) : x+y\frac{dy}{dx}=0 (y\neq0)$

Solution:

$y=\sqrt{a^2-x^2}$

Differentiate both the sides with respect to x, we get:

$$\frac{dy}{dx}=\frac{d}{dx}\left(\sqrt{a^2-x^2}\right) \Rightarrow \frac{dy}{dx}=\frac{1}{2\sqrt{a^2-x^2}}.\frac{d}{dx}(a^2-x^2)=\frac{1}{2\sqrt{a^2-x^2}}(-2x)=\frac{-x}{\sqrt{a^2-x^2}}$$

Substitute the value of $\frac{dy}{dx}$ **in the given differential equation, we get:**

L.H.S = $x+y\frac{dy}{dx}=x+\sqrt{a^2-x^2}\times\frac{-x}{\sqrt{a^2-x^2}}=x-x=0 R.H.S.$

Hence, the given function is the solution of the corresponding differential equation.

Q.11: The numbers of arbitrary constants in the general solution of a differential equation of fourth order are:

(i) 0

(ii) 2

(iii) 3

(iv) 4

Solution:

We know that, number of constants in the general solution of a differential equation of order n is equal to its order.

Therefore, the number of constants in the general equation of fourth order differential equation is four.

Therefore, (iv) is the correct answer.

Q.12: The numbers of arbitrary constants in the particular solution of a differential equation of third order are:

(i) 3

(ii) 2

(iii) 1

(iv) 0

Solution:

In a particular solution of a differential equation, there are no arbitrary constants. Hence, the correct answer is (iv).

Exercise-9.3

Q.1: $\frac{x}{a}+\frac{y}{b}=1$

Solution:

$\frac{x}{a}+\frac{y}{b}=1$

Differentiate both the sides w.r.t x, we get:

$\frac{1}{a}+\frac{1}{b}\frac{dy}{dx}=0 \Rightarrow \frac{1}{a}+\frac{1}{b}y'=0$

Again, differentiate both the sides w.r.t x, we get:

$0+\frac{1}{b}y''=0 \Rightarrow \frac{1}{b}y''=0 \Rightarrow y''=0$

Hence, the required differential equation of the given curve is $y''=0$**.**

Q.2: $y^2=a(b^2-x^2)$

Solution:

$y^2=a(b^2-x^2)$

Differentiate both the sides w.r.t x, we get:

$2y\frac{dy}{dx}=a(-2x) \Rightarrow 2yy'=-2ax \Rightarrow yy'=-ax$ . . . . **(1)**

Again, differentiate both the sides w.r.t x, we get:

$y'.y'+yy''=-a \Rightarrow (y')^2+yy''=-a$**....(2)**

Divide equation (2) by equation(1), we get:

$\frac{(y')^2+yy''}{yy'}=\frac{-a}{-ax} \Rightarrow xyy''+x(y')^2-yy''=0$

This is the required differential equation of the given curve.

Q.3: $y=ae^{3x}+be^{-2x}$

Solution:

$y=ae^{3x}+be^{-2x}$ **…………(1)**

Differentiate both the sides w.r.t x, we get:

$y'=3ae^{3x}-2be^{-2x}\ldots\ldots(2)$

Again, differentiate both the sides w.r.t x, we get:

$y''=9ae^{3x}+4be^{-2x}\ldots\ldots(3)$

Multiply equation(1) with 2 and then add it to equation (2), we get:

$(2ae^{3x}+2be^{-2x})+(3ae^{3x}-2be^{-2x})=2y+y' \Rightarrow 5ae^{3x}=2y+y' \Rightarrow ae^{3x}=\frac{2y+y'}{5}$

Now, multiplying equation (1) with 3 and subtracting equation (2) from it, we get:

$(3ae^{3x}+3be^{-2x})-(3ae^{3x}-2be^{-2x})=3y-y' \Rightarrow 5be^{-2x}=3y-y' \Rightarrow be^{-2x}=\frac{3y-y'}{5}$

Substituting the values of ae^{3x} and be^{-2x} **in equation(3), we get:**

$y''=9\cdot\frac{(2y+y')}{5}+4\cdot\frac{(3y-y')}{5} \Rightarrow y''=\frac{18y+9y'}{5}+\frac{12y-4y'}{5} \Rightarrow y''=\frac{30y+5y'}{5} \Rightarrow y''=6y+y' \Rightarrow y''-y'-6y=0$

This is the required differential equation of the given curve.

Q.4: $y=e^{2x}(a+bx)$

Solution:

$y=e^{2x}(a+bx)$ **…..(1)**

Differentiate both the sides w.r.t x, we get:

$y'=2e^{2x}(a+bx)+e^{2x}.b \Rightarrow y'=e^{2x}(2a+2bx+b)$………**(2)**

Multiply equation (1) with 2 and then add it to equation (2), we get:

$y' - 2y = e^{2x}(2a + 2bx + b) - e^{2x}(2a + 2bx) \Rightarrow y' - 2 = be^{2x}$ ……(3)

Differentiate both the sides w.r.t x, we get:

$y''k - 2y' = 2be^{2x}$…….(4)

Dividing equation (4) by equation (3), we get:

$\frac{y'' - 2y'}{y' - 2y} = 2 \Rightarrow y'' - 2y' = 2y' - 4y \Rightarrow y'' - 4y' + 4y = 0$

This is the required differential equation of the given curve.

Q.5: $y = e^x(a\cos x + b\sin x)$

Solution:

$y = e^x(a\cos x + b\sin x)$ ………… **(1)**

Differentiate both the sides w.r.t x, we get:

$y' = e^x(a\cos x + b\sin x) + e^x(-a\sin x + b\cos x) \Rightarrow y' = e^x[(a+b)\cos x - (a-b)\sin x]$…………(2)

Again, Differentiate both the sides w.r.t x, we get:

$\Rightarrow y'' = e^x[(a+b)\cos x - (a-b)\sin x] + e^x[-(a+b)\sin x - (a-b)\cos x] y'' = e^x[2b\cos x - 2a\sin x]$

$y'' = 2e^x[b\cos x - a\sin x] \Rightarrow \frac{y''}{2} = e^x[b\cos x - a\sin x]$………..(3)

Adding equations (1) and (3), we get:

$y + \frac{y''}{2} = e^x[(a+b)\cos x - (a-b)\sin x] \Rightarrow y + \frac{y''}{2} = y' \Rightarrow 2y + y'' = 2y' \Rightarrow y'' - 2y' + 2y = 0$

This is the required differential equation of the given curve.

Q.6: Form the differential equation of the family of circles touching the y-axis at the origin.

Solution:

The centre of the circle touching the y-axis at origin lies on the x-axis.

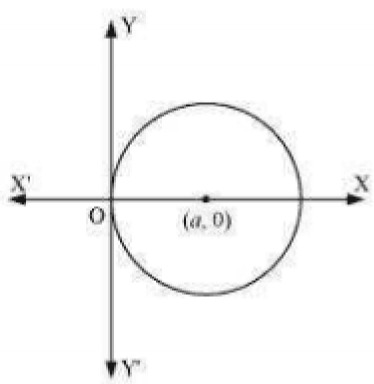
Let (a, 0) be the centre of the circle.

Since it touches the y-axis at origin, its radius is a.

Now, the equation of the circle with centre (a, 0) and radius (a) is

$(x - a)^2 + y^2 = a^2$

i.e. $x^2 + y^2 = 2ax$ . . . . (1)



Differentiating equation (1) with respect to x, we get:

$2x + 2yy' = 2a$

i.e. $x + yy' = a$

Now, on substituting the value of a in equation (1), we get:

$x^2+y^2=2(x+yy')x \Rightarrow x^2+y^2=2x^2+2xyy' \Rightarrow 2xyy'+x^2=y^2$

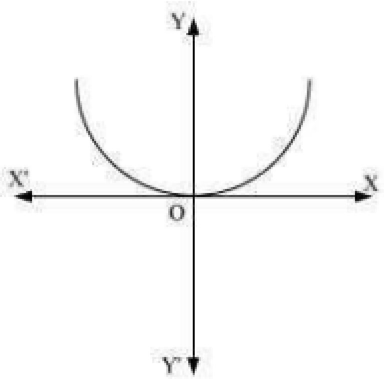
This is the required differential equation.

Q.7. Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis.

Solution:

The equation of the parabola having the vertex at origin and the axis along the positive y-axis is:

$x^2 = 4ay$ . . . .(1)



Differentiate both the sides w.r.t x, we get:

$2x = 4ay'$ . . . . (2)

Dividing equation (2) by equation (1), we get:

$\frac{2x}{x^2}=\frac{4ay'}{4ay} \Rightarrow \frac{2}{x}=\frac{y'}{y} \Rightarrow xy'=2y \Rightarrow xy'-2y=0$

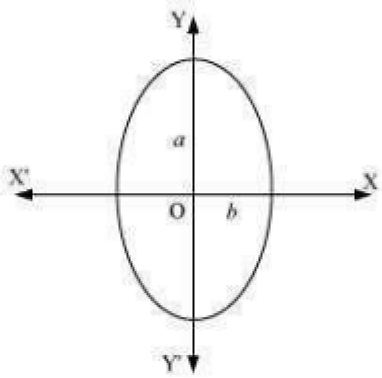
This is the required differential equation.

Q.8: Form the differential equation of the family of ellipses having foci on y-axis and centre at origin.

Solution:

The equation of the family of ellipses having foci on the y-axis and the centre at origin is as follows:

$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$…..(1)



Differentiate both the sides w.r.t x, we get:

$$\frac{2x}{b^2}+\frac{2yy'}{b^2}=0\Rightarrow\frac{x}{b^2}+\frac{yy'}{a^2}=0 \quad (2)$$

Again, differentiate both the sides w.r.t x, we get:

$$\frac{1}{b^2}+\frac{y'\cdot y'+y.y''}{a^2}=0\Rightarrow\frac{1}{b^2}+\frac{1}{a^2}(y'^2+yy'')=0\Rightarrow\frac{1}{b^2}=-\frac{1}{a^2}(y'^2+yy'')$$

Substituting this value in equation (2), we get:

$$x\left[-\frac{1}{a^2}((y'^2)+yy'')\right]+\frac{yy'}{a^2}=0\Rightarrow -x(y')^2-xyy''+yy'=0\Rightarrow xyy''+x(y')^2-yy'=0$$

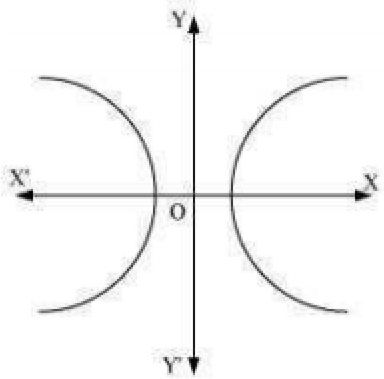
This is the required differential equation.

Q.9: Form the differential equation of the family of hyperbolas having foci on x-axis and centre at origin.

Solution:

The equation of the family of hyperbolas with the centre at origin and foci along the x-axis is:

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ **…………(1)**



Differentiate both the sides w.r.t x, we get:

$\frac{2x}{a^2}-\frac{2yy'}{b^2}=0\Rightarrow\frac{x}{a^2}-\frac{yy'}{b^2}=0$ **……….. (2)**

Again, differentiate both the sides w.r.t x, we get:

$\frac{1}{a^2}-\frac{y'\cdot y'+yy''}{b^2}=0\Rightarrow\frac{1}{a^2}=\frac{1}{b^2}((y')^2+yy'')$

Substituting the value of $\frac{1}{a^2}$ **in equation (2):**

$\frac{x}{b^2}((y')^2+yy'')-\frac{yy'}{b^2}=0$ $x(y')^2+xyy''-yy'=0\Rightarrow xyy''+x(y')^2-yy'=0$

This is the required differential equation.

Q.10: Form the differential equation of the family of circles having centre on y-axis and radius 3 units.

Solution:

Let the center of the circle on y-axis be (0, b).

The differential equation of the family of circles with centre at (0, b) and radius 3 is as follows:

$x^2+(y-b)^2=3^2\Rightarrow x^2+(y-b)^2=9\ldots(1)$

Differentiate equation (1) with respect to x, we get:

$2x+2(y-b)\cdot y'=0\Rightarrow(y-b)\cdot y'=-x\Rightarrow y-b=\frac{-x}{y'}$

Substitute the value of (y – b) in equation (1), we get:

$$x^2+\left(\frac{-x}{y'}\right)^2=9 \Rightarrow x^2\left[1+\frac{1}{(y')^2}\right]=9 \Rightarrow x^2((y')^2+1)=9(y')^2 \Rightarrow (x^2-9)(y')^2+x^2=0$$

This is the required differential equation.

Q.11.Which of the following differential equations has $y=c_1e^x+c_2e^{-x}$ as the general solution?
(i) $\frac{d^2y}{dx^2}+y=0$
(ii) $\frac{d^2y}{dx^2}-y=0$
(iii) $\frac{d^2y}{dx^2}+1=0$
(iv) $\frac{d^2y}{dx^2}-1=0$

Solution:

The given equation is:

$y=c_1e^x+c_2e^{-x}$ **. . . . . (1)**
Differentiate equation (1) with respect to x, we get:

$$\frac{dy}{dx}=c_1e^x-c_2e^{-x}$$

Again, differentiate with respect to x, we get:

$$\frac{d^2y}{dx^2}=c_1e^x-c_2e^{-x} \Rightarrow \frac{d^2y}{dx^2}=y \Rightarrow \frac{d^2y}{dx^2}-y=0$$

This is the required differential equation of the given equation of curve.

Hence, the correct answer is (ii).

Q.12: Which of the following differential equation has y = x as one of its particular solution?

(i) $\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=x$
(ii) $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+xy=x$
(iii) $\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=0$
(iv) $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+xy=0$

Solution:

The given equation of curve is **y = x.**

Differentiate with respect to x, we get:

$\frac{dy}{dx}=1 \ldots.(1)$

Again, differentiate with respect to x, we get:

$\frac{d^2y}{dx^2}=0 \ldots.(2)$

Now, on substituting the values of y:

$\frac{d^2y}{dx^2}$, and $\frac{dy}{dx}$ from equation (1) and (2) in each of the given alternatives, we find that only the differential equation given in alternative **C is correct.**

$\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=0-x^2\cdot1+x\cdot x=-x^2+x^2=0$

Hence, the correct answer is (iii).

Exercise-9.4

Q.1: $\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$

Solution:

The given differential equation is:

$\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$ $\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}\Rightarrow\frac{dy}{dx}=\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}=\tan^2\frac{x}{2}\Rightarrow\frac{dy}{dx}=\left(\sec^2\frac{x}{2}-1\right)$

Separate the variables, we get:

$dy=\left(\sec^2\frac{x}{2}-1\right)dx$

Now, integrating both sides of this equation, we get:

$\int dy=\int\left(\sec^2\frac{x}{2}-1\right)dx=\int\sec^2\frac{x}{2}dx-\int dx\Rightarrow y=2\tan\frac{x}{2}-x+C$

This is the required general solution of the given differential equation.

Q.2: $\frac{dy}{dx}=\sqrt{4-y^2}\ (-2<y<2)$

Solution:

The given differential equation is:

$\frac{dy}{dx}=\sqrt{4-y^2}$

Separate the variables, we get:

$\Rightarrow \frac{dy}{\sqrt{4-y^2}}=dx$

Now, integrating both sides of this equation, we get:

$\int \frac{dy}{\sqrt{4-y^2}}=\int dx \Rightarrow \sin^{-1}\frac{y}{2}=x+C \Rightarrow \frac{y}{2}=\sin(x+C) \Rightarrow y=2\sin(x+C)$

This is the required general solution of the given differential equation.

Q.3: $\frac{dy}{dx}+y=1\ (y\neq 1)$

Solution:

The given differential equation is:

$\frac{dy}{dx}+y=1 \Rightarrow dy+y\,dx=dx \Rightarrow dy=(1-y)dx$

Separate the variables, we get:

$\Rightarrow \frac{dy}{1-y}=dx$

Now, integrating both sides, we get:

$\int \frac{dy}{1-y}=\int dx \Rightarrow \log(1-y)=x+\log C \Rightarrow -\log C-\log(1-y)=x \Rightarrow \log C(1-y)=e^{-x} \Rightarrow 1-y=\frac{1}{C}e^{-x} \Rightarrow y=1-\frac{1}{C}e^{-x} \Rightarrow y=1+Ae^{-x} \left(\text{where } A=-\frac{1}{C}\right)$

This is the required general solution of the given differential equation.

Q.4: $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

Solution:

The given differential equation is:

$$\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0 \Rightarrow \frac{\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy}{\tan x \tan y} = 0 \Rightarrow \frac{\sec^2 x}{\tan x} dx + \frac{\sec^2 y}{\tan y} dy = 0 \Rightarrow \frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 y}{\tan y} dy$$

Integrating both sides of this equation, we get:

$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy$ **. . . . . (1)**

Let, tan x = t

Therefore, $\frac{d}{dx}(\tan x) = \frac{dt}{dx} \Rightarrow \sec^2 x = \frac{dt}{dx} \Rightarrow \sec^2 x\, dx = dt$

Now,

$\int \frac{\sec^2 x}{\tan x} dx = \int \frac{1}{t} dt = \log t = \log(\tan x)$

Similarly, $\int \frac{\sec^2 x}{\tan x} dx = \log(\tan y)$

Substituting these values in equation (1), we get:

$$\log(\tan x) = -\log(\tan y) + \log C \Rightarrow \log(\tan x) = \log\left(\frac{C}{\tan y}\right) \Rightarrow \tan x = \frac{C}{\tan y} \Rightarrow \tan x \tan y = C$$

This is the required general solution of the given differential equation.

Q.5: $(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0$

Solution:

The given differential equation is:

$$(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0 \Rightarrow (e^x + e^{-x})dy = (e^x - e^{-x})dx \Rightarrow dy = \left[\frac{e^x - e^{-x}}{e^x + e^{-x}}\right]dx$$

Integrating both sides of this equation, we get:

$$\int dy = \int \left[\frac{e^x - e^{-x}}{e^x + e^{-x}}\right]dx + C \Rightarrow y = \int \left[\frac{e^x - e^{-x}}{e^x + e^{-x}}\right]dx + C \ldots.(1)$$

Let (e^x+e^{-x})

$\frac{d}{dx}(e^x+e^{-x})=\frac{dt}{dx}\Rightarrow e^x-e^{-x}=\frac{dt}{dx}\Rightarrow(e^x-e^{-x})dx=dt$

Substituting this value in equation (1), we get:

$y=\int\frac{1}{t}dt+C\Rightarrow y=\log(t)+C\Rightarrow y=\log(e^x+e^{-x})+C$

This is the required general solution of the given differential equation.

Q.6: $\frac{dy}{dx}=(1+x^2)(1+y^2)$

Solution:

The given differential equation is:

$\frac{dy}{dx}=(1+x^2)(1+y^2)\Rightarrow\frac{dy}{1+y^2}=(1+x^2)dx$

Integrating both sides of this equation, we get:

$\int\frac{dy}{1+y^2}=\int(1+x^2)dx\Rightarrow\tan^{-1}y=\int dx+\int x^2dx\Rightarrow\tan^{-1}y=x+\frac{x^3}{3}+C$

This is the required general solution of the given differential equation.

Q.7: $y\log y dx-xdy=0$

Solution:

The given differential equation is:

$y\log y dx-xdy=0\Rightarrow y\log y dx=xdy\Rightarrow\frac{dy}{y\log y}=\frac{dx}{x}$

Integrating both sides, we get:

$\int\frac{dy}{y\log y}=\int\frac{dx}{x}$ **. . . . . (1)**

Let, log y = t

Therefore, $\frac{d}{dy}(\log y)=\frac{dt}{dy}\Rightarrow\frac{1}{y}=\frac{dt}{dy}\Rightarrow\frac{1}{y}dy=dt$

Substituting this value in equation (1), we get:

$$\int \frac{dt}{t}=\int \frac{dx}{x} \Rightarrow \log t=\log x+\log C \Rightarrow \log(\log y)=\log Cx \Rightarrow \log y=Cx \Rightarrow y=e^{Cx}$$

This is the required general solution of the given differential equation.

Q.8: $x^5\frac{dy}{dx}=-y^5$

Solution:

The given differential equation is:

$$x^5\frac{dy}{dx}=-y^5 \Rightarrow \frac{dy}{y^5}=-\frac{dx}{x^5} \Rightarrow \frac{dx}{x^5}+\frac{dy}{y^5}=0$$

Integrating both sides, we get:

$\int \frac{dx}{x^5}+\int \frac{dy}{y^5}=k$ (where k is any constant)

$\int x^{-5}dx+\int y^{-5}dy=k \Rightarrow \frac{x^{-4}}{-4}+\frac{y^{-4}}{-4}=k \Rightarrow x^{-4}+y^{-4}=-4k \Rightarrow x^{-4}+y^{-4}=C$ (C=-4k)

This is the required general solution of the given differential equation.

Q.9. $\frac{dy}{dx}=\sin^{-1}x$

Solution:

The given differential equation is:

$$\frac{dy}{dx}=\sin^{-1}x \Rightarrow dy=\sin^{-1}x\,dx$$

Integrating both sides, we get:

$$\int dy=\int \sin^{-1}x\,dx \Rightarrow y=\int(\sin^{-1}x \cdot 1)dx \Rightarrow y=\sin^{-1}x \cdot \int(1)dx-\int\left[\left(\frac{d}{dx}(\sin^{-1}x)\cdot\int(1)dx\right)\right]dx \Rightarrow y=\sin^{-1}x \cdot x-\int\left(\frac{1}{\sqrt{1-x^2}}\cdot x\right)dx \Rightarrow y=x\sin^{-1}x+\int\frac{-x}{\sqrt{1-x^2}}dx \ldots.(1)$$

Let, $1-x^2=t \Rightarrow \frac{d}{dx}(1-x^2)=\frac{dt}{dx} \Rightarrow -2x=\frac{dt}{dx} \Rightarrow x\,dx=-\frac{1}{2}dt$

Substituting this value in equation (1), we get:

$$y=x\sin^{-1}x+\int\frac{1}{2\sqrt{t}}dt \Rightarrow y=x\sin^{-1}x+\frac{1}{2}\cdot\int(t)^{-\frac{1}{2}}dt \Rightarrow y=x\sin^{-1}x+\frac{1}{2}\cdot\frac{t^{\frac{1}{2}}}{\frac{1}{2}}+C \Rightarrow y=x\sin^{-1}x+\sqrt{t}+C \Rightarrow y=x\sin^{-1}x+\sqrt{1-x^2}+C$$

This is the required general solution of the given differential equation.

Q.10. $e^x \tan y\,dx + (1 - e^x)\sec^2 y\,dy = 0$

Solution:

$$e^x \tan y\,dx + (1 - e^x)\sec^2 y\,dy = 0 \quad (1 - e^x)\sec^2 y\,dy = -e^x \tan y\,dx$$

Separating the variables, we get:

$$\frac{\sec^2 y}{\tan y}dy = \frac{-e^x}{1 - e^x}dx$$

Integrating both sides, we get:

$$\int \frac{\sec^2 y}{\tan y}dy = \int \frac{-e^x}{1 - e^x}dx \ldots (1)$$

Let, tan y = u

$$\frac{d}{dy}\tan y = \frac{du}{dy} \Rightarrow \sec^2 y = \frac{du}{dy} \Rightarrow \sec^2 y\,dy = du$$

Therefore, $\int \frac{\sec^2 y}{\tan y}dy = \int \frac{du}{u} = \log u = \log(\tan y)$

Now, **let 1 – e^x = t**

Therefore, $\frac{d}{dx}(1 - e^x) = \frac{dt}{dx} \Rightarrow -e^x = \frac{dt}{dx} \Rightarrow -e^x dx = dt \Rightarrow \int \frac{-e^x}{1 - e^x}dx = \int \frac{dt}{t} = \log t = \log(1 - e^x)$

Substituting the values of $\int \frac{\sec^2 y}{\tan y}dy$ and $\int \frac{-e^x}{1 - e^x}dx$

$$\Rightarrow \log(\tan y) = \log(1 - e^x) + \log C \Rightarrow \log(\tan y) = \log[C(1 - e^x)] \Rightarrow \tan y = C(1 - e^x)$$

This is the required general solution of the given differential equation.

Exercise-9.5

Q.1: $(x^2 + xy)dy = (x^2 + y^2)dx$

Ans:

Given:

$(x^2 + xy)dy = (x^2 + y^2)dx$

$\frac{dy}{dx}=\frac{x^2+y^2}{x^2+xy}$ . . . . **(1)**

Let, F(x, y) = $\frac{x^2+y^2}{x^2+xy}$

Now,

F(λx, λy) = $\frac{(\lambda x)^2+(\lambda y)^2}{(\lambda x)^2+(\lambda x)(\lambda y)}=\frac{x^2+y^2}{x^2+xy}=\lambda^0\cdot F(x,y)$

Here we have observed that equation (1) is a homogeneous equation.

Let, y = vx

Differentiate both the sides w.r.t. x, we get:

$\frac{dy}{dx}=v+x\frac{dv}{dx}$

Substitute the values of v and $\frac{dy}{dx}$ in equation(1), we get:

$\Rightarrow\ v+x\frac{dv}{dx}=\frac{x^2+(vx)^2}{x^2+x(vx)}\Rightarrow v+x\frac{dv}{dx}=\frac{1+v^2}{1+v}\Rightarrow x\frac{dv}{dx}=\frac{1+v^2}{1+v}-v=\frac{(1+v^2)-v(1+v)}{1+v}\Rightarrow x\frac{dv}{dx}=\frac{1-v}{1+v}\Rightarrow\left(\frac{1+v}{1-v}\right)dv=\frac{dx}{x}\Rightarrow\left(\frac{2-1+v}{1-v}\right)dv=\frac{dx}{x}\Rightarrow\left(\frac{2}{1-v}-1\right)dv=\frac{dx}{x}$

Integrate on both the sides, we get:

⇒ -2log(1-v)-v=log x – log k

$\Rightarrow v=-2\log(1-v)-\log x+\log k\Rightarrow v=\log\left[\frac{k}{x(1-v)^2}\right]\Rightarrow\frac{y}{x}=\log\left[\frac{k}{x(1-\frac{y}{x})^2}\right]\Rightarrow\frac{y}{x}=\log\left[\frac{kx}{(x-y)^2}\right]\Rightarrow\frac{kx}{(x-y)^2}=e^{\frac{y}{x}}\Rightarrow(x-y)^2=kxe^{-\frac{y}{x}}$

This is the required solution of the given differential equation.

Q.2: y = $\frac{x+y}{x}$

Ans:

Given:

y = $\frac{x+y}{x}$

$\Rightarrow\frac{dy}{dx}=\frac{x+y}{x}$ . . . . **(1)**

Let F(x,y)= $\frac{x+y}{x}$

Now, F(λx,λy)= $\frac{\lambda x+\lambda y}{\lambda x}=\frac{x+y}{x}=\lambda^0F(x,y)$

Here we have observed that equation (1) is a homogeneous equation.

Let, y = vx

Differentiate both the sides w.r.t. x, we get

$\frac{dy}{dx}=v+x\frac{dv}{dx}$

Substitute the values of v and $\frac{dy}{dx}$ **in equation(1), we get:**

$\Rightarrow\ v+x\frac{dv}{dx}=\frac{x+vx}{x}\Rightarrow v+x\frac{dv}{dx}=1+v\Rightarrow x\frac{dv}{dx}=1\Rightarrow dv=\frac{dx}{x}$

Integrate on both the sides, we get:

V = log x + C

$\Rightarrow\frac{y}{x}=\log x+c\Rightarrow y=x\log x+Cx$

This is the required solution of the given differential equation.

Q.3: (x-y)dy-(x+y)dx=0

Ans:

Given:

(x – y)dy – (x + y)dx = 0

$\Rightarrow\frac{dy}{dx}=\frac{x+y}{x-y}$ **. . . . . (1)**

Let, F(x, y) = $\frac{x+y}{x-y}$

Therefore, $F(\lambda x,\lambda y)=\frac{\lambda x+\lambda y}{\lambda x-\lambda y}=\frac{x+y}{x-y}=\lambda^0\cdot F(x,y)$

Here we have observed that equation (1) is a homogeneous equation.

Let, y = vx

Differentiate both the sides w.r.t. x, we get:

$\frac{dy}{dx}=v+x\frac{dv}{dx}$

Substitute the values of v and $\frac{dy}{dx}$ **in equation(1), we get:**

$\Rightarrow\ v+x\frac{dv}{dx}=\frac{x+vx}{x-vx}=\frac{1+v}{1-v}\ x\frac{dv}{dx}=\frac{1+v}{1-v}-v=\frac{1+v-v(1-v)}{1-v}\Rightarrow x\frac{dv}{dx}=\frac{1+v^2}{1-v}\Rightarrow\frac{1-v}{(1+v^2)}dv=\frac{dx}{x}\Rightarrow\left(\frac{1}{1+v^2}-\frac{v}{1-v^2}\right)dv=\frac{dx}{x}$

Integrate on both the sides, we get:

$\Rightarrow \tan^{-1}v - \frac{1}{2}\log(1+v^2) = \log x + C \Rightarrow \tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log\left[1+\left(\frac{y}{x}\right)^2\right] = \log x + C \Rightarrow \tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log\left(\frac{x^2+y^2}{x^2}\right) = \log x + C \Rightarrow \tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\left[\log(x^2+y^2) - \log x^2\right] = \log x + C \Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\log(x^2+y^2) + C$

This is the required solution of the given differential equation.

Q.4: $(x^2 – y^2)\, dx + 2xy\, dy = 0$

Ans:

Given,

$(x^2 – y^2)dx + 2xy\, dy = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{(x^2-y^2)}{2xy}$ **. . . . . (1)**

Let, F(x, y) = $-\frac{(x^2-y^2)}{2xy}$

Therefore, $F(\lambda x, \lambda y) = \left[\frac{(\lambda x)^2-(\lambda y)^2}{2xy}\right] = -\frac{(x^2-y^2)}{2(\lambda x)(\lambda y)} = -\frac{(x^2-y^2)}{2xy} = \lambda^{\circ}\cdot F(x,y)$

Here we have observed that equation (1) is a homogeneous equation.

Let, y = vx

Differentiate both the sides w.r.t. x, we get:

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Substitute the values of v and $\frac{dy}{dx}$ **in equation(1), we get:**

$\Rightarrow v + x\frac{dv}{dx} = -\left[\frac{x^2-(vx)^2}{2x\cdot(vx)}\right] = \frac{v^2-1}{2v} \Rightarrow x\frac{dv}{dx} = \frac{v^2-1}{2v} - v = \frac{v^2-1-2v^2}{2v} \Rightarrow x\frac{dv}{dx} = -\frac{(1+v^2)}{2v} \Rightarrow \frac{2v}{1+v^2}dv = -\frac{dx}{x}$

Integrate on both the sides, we get:

$\Rightarrow \text{Log}(1 + v^2) = -\log x + \log C = \log\frac{C}{x}$

$\Rightarrow 1+v^2 = \frac{C}{x} \Rightarrow \left[1+\frac{y^2}{x^2}\right] = \frac{C}{x} \Rightarrow x^2+y^2 = Cx$

This is the required solution of the given differential equation.

Q.5: $x^2\frac{dy}{dx} - x^2 - 2y^2 + xy$

Ans:

Given:

$x2dydx−x2−2y2+xy$

$dydx=x2−2y2+xyx2$ **. . . . . (1)**

Let $F(x,y)= x2−2y2+xyx2$

$F(λx,λy)=(λx)2−2(λy)2+(λx)(λx)(λx)2=x2–2y2+xyx2=λ∘⋅F(x,y)$

Here we have observed that equation (1) is a homogeneous equation.

Let, y = vx

Differentiate both the sides w.r.t. x, we get:

$dydx=v+xdvdx$

Substitute the values of v and $dydx$ **in equation(1), we get:**

$⇒ v+xdvdx=x2−2(vx)2+x⋅(vx)x2⇒v+xdvdx=1−2v2+v⇒xdvdx=1−2v2⇒dv1−2v2=dxx⇒12⋅dv12−v2=dxx⇒12[dv(12√)2−v2]=dxx$

Integrate on both the sides, we get:

$⇒ 12⋅12×12√log|||12√+v1 sqrt2−v|||=log|x|+C⇒122√log|||12√+yx12√−yx|||=log|x|+C⇒12×12√log||x+22√yx+22√y||=log|x|+C$

This is the required solution of the given differential equation.

Q.6: xdy – ydx = $x2+y2−−−−−−√dx$

Ans:

xdy – ydx = $x2+y2−−−−−−√dx$

$⇒xdy=[y+x2+y2−−−−−−√]dx⇒dydx=y+x2+y2√x2$ **. . . . . (1)**

Let, F(x,y) = $y+x2+y2√x2$

Therefore, $F(λx,λy)=λx+(λx)2(λy)2√x=λ0⋅F(x,y)$

Here we have observed that equation (1) is a homogeneous equation.

Let, y = vx

Differentiate both the sides w.r.t. x, we get:

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Substitute the values of v and $\frac{dy}{dx}$ **in equation(1), we get:**
$\Rightarrow v + x\frac{dv}{dx} = \frac{vx+\sqrt{x^2+(vx)^2}}{x} \Rightarrow v + x\frac{dv}{dx} = v+\sqrt{1+v^2} \Rightarrow \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$
Integrate on both the sides, we get:

$\Rightarrow \log\left|v+\sqrt{1+v^2}\right| = \log|x| + \log C \Rightarrow \log\left|\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right| = \log|Cx| \Rightarrow \log\left|\frac{y+\sqrt{x^2+y^2}}{x}\right| = \log|Cx| \Rightarrow y+\sqrt{x^2+y^2} = Cx^2$
This is the required solution of the given differential equation.

Q.7: $\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}y\,dx = \left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}x\,dy$

Ans:

Given:

$\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}y\,dx = \left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}x\,dy$

$\frac{dy}{dx} = \frac{\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}y}{\left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}x}$ **. . . . . (1)**
Let, F(x, y) = $\frac{dy}{dx} = \frac{\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}y}{\left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}x}$
Therefore, $F(\lambda x,\lambda y) = \frac{\left\{\lambda x\cos\left(\frac{\lambda y}{\lambda x}\right)+\lambda y\sin\left(\frac{\lambda y}{\lambda x}\right)\right\}\lambda y}{\left\{\lambda y\sin\left(\frac{\lambda y}{\lambda x}\right)-\lambda x\cos\left(\frac{\lambda y}{\lambda x}\right)\right\}\lambda x} = \frac{\left\{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)\right\}y}{\left\{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)\right\}x} = \lambda^0\cdot F(x,y)$
Here we have observed that equation (1) is a homogeneous equation.

Let, y = vx

Differentiate both the sides w.r.t. x, we get:

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow$ Substitute the values of v and $\frac{dy}{dx}$ in equation(1), we get:
$v+x\frac{dv}{dx} = \frac{(x\cos v+vx\sin v)\cdot vx}{(vx\sin v-x\cos v)\cdot x} \Rightarrow x\frac{dv}{dx} = \frac{v\cos v+v^2\sin v}{v\sin v-\cos v} - v \Rightarrow x\frac{dv}{dx} = \frac{v\cos v+v^2\sin v-v^2\sin v+v\cos v}{v\sin v-\cos v} \Rightarrow x\frac{dv}{dx} = \frac{2v\cos v}{v\sin v-\cos v} \Rightarrow \left[\frac{v\sin v-\cos v}{v\cos v}\right]dv = \frac{2dx}{x} \Rightarrow \left(\tan v-\frac{1}{v}\right)dv = \frac{2dx}{x}$

Integrate on both the sides, we get:

$\Rightarrow$ Log(sec v) – log v = 2 log x + log C

$\Rightarrow \log\left(\frac{\sec v}{v}\right)=\log(Cx^2) \Rightarrow \log\left(\frac{\sec v}{v}\right)=Cx^2 \Rightarrow \sec v = Cx^2 v \Rightarrow \sec\left(\frac{y}{x}\right)=C-x^2\cdot\frac{y}{x} \Rightarrow \sec\left(\frac{y}{x}\right)=Cxy \Rightarrow \sec\left(\frac{y}{x}\right)=\frac{1}{Cxy}=\frac{1}{C}\cdot\frac{1}{xy} \Rightarrow xy\cos\left(\frac{y}{x}\right)=k \left(k=\right)\frac{1}{C}$

This is the required solution of the given differential equation.

Q.8: $x\frac{dy}{dx}-y+x\sin\left(\frac{y}{x}\right)=0$

Ans:

$x\frac{dy}{dx}-y+x\sin\left(\frac{y}{x}\right)=0$

$\Rightarrow x\frac{dy}{dx}=y-x\sin\left(\frac{y}{x}\right) \Rightarrow \frac{dy}{dx}=\frac{y-x\sin\left(\frac{y}{x}\right)}{x}$ **. . . . . (1)**

Let, F(x , y) = $\frac{y-x\sin\left(\frac{y}{x}\right)}{x}$

Therefore, $F(\lambda x,\lambda y)=\frac{\lambda y-\lambda x\sin\left(\frac{\lambda y}{\lambda x}\right)}{\lambda x}=\frac{y-x\sin\left(\frac{y}{x}\right)}{x}=\lambda^0\cdot F(x,y)$

Here we have observed that equation (1) is a homogeneous equation.

Let, y = vx

Differentiate both the sides w.r.t. x, we get:

$\frac{dy}{dx}=v+x\frac{dv}{dx}$

Substitute the values of v and $\frac{dy}{dx}$ **in equation(1), we get:**

$\Rightarrow v+x\frac{dv}{dx}=\frac{vx-x\sin v}{x} \Rightarrow v+x\frac{dv}{dx}=v-\sin v \Rightarrow -\frac{dv}{\sin v}=-\frac{dx}{x} \Rightarrow \operatorname{cosec} v\, dv=-\frac{dx}{x}$

Integrate on both the sides:

$\Rightarrow \log|\operatorname{cosec} v-\cot v|=-\log x+\log C=\log\frac{C}{x} \Rightarrow \operatorname{cosec}\left(\frac{y}{x}\right)-\cot\left(\frac{y}{x}\right)=\frac{C}{x} \Rightarrow \frac{1}{\sin\left(\frac{y}{x}\right)}-\frac{\cos\left(\frac{y}{x}\right)}{\sin\left(\frac{y}{x}\right)}=\frac{C}{x} \Rightarrow x\left[1-\cos\left(\frac{y}{x}\right)\right]=C\sin\left(\frac{y}{x}\right)$

This is the required solution of the given differential equation.

Q.9: y dx + x log ($\frac{y}{x}$) dy – 2x dy = 0

Ans:

$y\,dx + x\log\left(\frac{y}{x}\right)dy - 2x\,dy = 0 \Rightarrow y\,dx = \left[2x - x\log\left(\frac{y}{x}\right)\right]dy \Rightarrow \frac{dy}{dx} = \frac{y}{2x - x\log\left(\frac{y}{x}\right)}$ . . . . **(1)**

Let, F(x, y) = $\frac{y}{2x - x\log\left(\frac{y}{x}\right)}$

Therefore, $F(\lambda x, \lambda y) = \frac{\lambda y}{2\lambda x - \lambda x\log\left(\frac{\lambda y}{\lambda x}\right)} = \lambda^0 \cdot F(x,y)$

Here we have observed that equation (1) is a homogeneous equation.

Let y = vx

Differentiate both the sides w.r.t. x, we get:

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Substitute the values of v and $\frac{dy}{dx}$ **in equation(1), we get:**

$\Rightarrow v + x\frac{dv}{dx} = \frac{vx}{2x - x\log v} \Rightarrow v + x\frac{dv}{dx} = \frac{v}{2 - \log v} \Rightarrow x\frac{dv}{dx} = \frac{v}{2 - \log v} - v \Rightarrow x\frac{dv}{dx} = \frac{v - 2v + v\log v}{2 - \log v} \Rightarrow x\frac{dv}{dx} = \frac{v\log v - v}{2 - \log v} \Rightarrow \frac{2 - \log v}{v\log v - v}dv = \frac{dx}{x} \Rightarrow \left[\frac{1 + (1 - \log v)}{v(\log v - 1)}\right]dv = \frac{dx}{x} \Rightarrow \left[\frac{1}{v(\log v - 1)} - \frac{1}{v}\right]dv = \frac{dx}{x}$

Integrate on both the sides:

$\int \frac{1}{v(\log v - 1)}dv - \int \frac{1}{v}dv = \int \frac{1}{x}dx \Rightarrow \int \frac{1}{v(\log v - 1)}dv - \log v = \log x + \log C$ . . . . **(2)**

Let, log v – 1 =t

$\Rightarrow \frac{d}{dv}(\log v - 1) = \frac{dt}{dv} \Rightarrow \frac{1}{v} = \frac{dt}{dv} \Rightarrow \frac{dv}{v} = dt$

So, equation (1) will become:

$\int \frac{dt}{t} - \log v = \log x + \log C \Rightarrow \log t - \log\left(\frac{y}{x}\right) = \log(Cx) \Rightarrow \log\left[\log\left(\frac{y}{x}\right) - 1\right] - \log\left(\frac{y}{x}\right) = \log(Cx) \Rightarrow \log\left[\frac{\log\left(\frac{y}{x}\right) - 1}{\frac{y}{x}}\right] = \log(Cx) \Rightarrow \frac{x}{y}\left[\log\left(\frac{y}{x}\right) - 1\right] = Cx \Rightarrow \log\left(\frac{y}{x}\right) - 1 = Cy$

This is the required solution of the given differential equation.

Q.10: $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$

Ans:

$\Rightarrow \left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0 \Rightarrow \left(1 + e^{\frac{x}{y}}\right)dx = -e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy \Rightarrow \frac{dx}{dy} = \frac{-e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)}{1 + e^{\frac{x}{y}}}$ . . . . **(1)**

Let F(x,y) = $\frac{-e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)}{1 + e^{\frac{x}{y}}}$

Therefore, $F(\lambda x, \lambda y) = \frac{-e^{\frac{\lambda x}{\lambda y}}\left(1 - \frac{\lambda x}{\lambda y}\right)}{1 + e^{\frac{\lambda x}{\lambda y}}} = \frac{-e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)}{1 + e^{\frac{x}{y}}} = \lambda^0 \cdot F(x,y)$

Here we have observed that equation (1) is a homogeneous equation.

Let, x = vy

$\frac{d}{dy}(x)=\frac{d}{dy}(vy)\Rightarrow\frac{dx}{dy}=v+y\frac{dv}{dy}$

Differentiate both the sides w.r.t. x, we get

Substitute the values of v and $\frac{dx}{dy}$ **in equation(1), we get:**

$\Rightarrow\ v+y\frac{dv}{dx}=-\frac{e^v(1-v)}{1+e}\Rightarrow y\frac{dv}{dy}=\frac{-e^v+ve^v}{1+e^v}-v\Rightarrow y\frac{dv}{dy}=\frac{-e^v+ve^v-v-ve^v}{1+e^v}\Rightarrow y\frac{dv}{dy}=-\left[\frac{v+e^v}{1+e^v}\right]\Rightarrow\left[\frac{v+e^v}{1+e^v}\right]dv=-\frac{dy}{y}$

Integrate on both the sides, we get:

$\log(v+e^v)=-\log y+\log C=\log\left(\frac{C}{y}\right)\Rightarrow\left[\frac{x}{y}+e^{\frac{x}{y}}\right]=\frac{C}{y}\Rightarrow x+ye^{\frac{x}{y}}=C$

This is the required solution of the given differential equation.

Q.11: (x + y)dy + (x – y)dx = 0 ; y = 1 when x = 1

Ans:

(x + y)dy + (x – y)dx = 0

⇒ (x + y)dy = -(x – y)dx

⇒ $\frac{dy}{dx}=-\frac{(x-y)}{x+y}$ **. . . . . (1)**

Let, F(x , y) = $-\frac{(x-y)}{x+y}$

Therefore, $F(\lambda x,\lambda y)=-\frac{(\lambda x-\lambda y)}{\lambda x+\lambda y}=-\frac{(x-y)}{x+y}=\lambda^0\cdot F(x,y)$

Here we have observed that equation (1) is a homogeneous equation.

Let, y=vx

Differentiate both the sides w.r.t. x, we get:

$\Rightarrow\frac{d}{dx}(y)=\frac{d}{dx}(vx)\ \frac{dy}{dx}=v+x\frac{dv}{dx}$

Substitute the values of v and $\frac{dy}{dx}$ **in equation(1), we get:**

$\Rightarrow\ v+x\frac{dv}{dx}=-\frac{(x-vx)}{x+vx}\Rightarrow v+x\frac{dv}{dx}=\frac{v-1}{v+1}X\frac{dv}{dx}=\frac{v-1}{v+1}-vX\frac{dv}{dx}=\frac{v-1-v(v+1)}{v+1}X\frac{dv}{dx}=\frac{v-1-v^2-v}{v+1}=-\frac{(1+v^2)}{v+1}\Rightarrow\frac{(v+1)}{1+v^2}dv=-\frac{dx}{x}\Rightarrow\left[\frac{v}{1+v^2}+\frac{1}{1+v^2}\right]dv=-\frac{dx}{x}$

Integrate on bothe the sides, we get:

$$\Rightarrow \frac{1}{2}\log(1+v^2)+\tan^{-1}v=-\log x+k \Rightarrow \log(1+v^2)+2\tan^{-1}v=-2\log x+2k \Rightarrow \log[(1+v^2)\cdot x^2]+2\tan^{-1}v=2k \Rightarrow \log\left[\left(1+\frac{y^2}{x^2}\right)\cdot x^2\right]+2\tan^{-1}\frac{y}{x}=2k \Rightarrow \log(x^2+y^2)+2\tan^{-1}\frac{y}{x}=2k$$

…..(2)

Now y = 1 at x = 1:

$$\Rightarrow \log 2+2\tan^{-1}1=2k \Rightarrow \log 2+2\times\frac{\pi}{4}=2k \Rightarrow \frac{\pi}{2}+\log 2=2k$$

Substitute value of 2k in equⁿ(2), we get:

$$\log(x^2+y^2)+2\tan^{-1}\left(\frac{y}{x}\right)=\frac{\pi}{2}+\log 2$$

This is the required solution of the given differential equation.

Q.12: $x^2\,dy + (xy + y^2)\,dx = 0$, y = 1 when x =1

Ans:

$x^2\,dy + (xy + y^2)\,dx = 0$

$$\Rightarrow x^2dy=-(xy+y^2)dx \Rightarrow \frac{dy}{dx}=-\frac{(xy+y^2)}{x^2} \quad \ldots\ldots\ldots\ldots \textbf{(1)}$$

Let $F(x, y) = -\frac{(xy+y^2)}{x^2}$

Therefore, $F(\lambda x,\lambda y)=-\frac{(\lambda x\cdot\lambda y+(\lambda y)^2)}{(\lambda x)^2}=-\frac{(xy+y^2)}{x^2}=\lambda^0\cdot F(x,y)$

Here we have observed that equation (1) is a homogeneous equation.

Let, y = vx

Differentiate both the sides w.r.t. x, we get:

$$\Rightarrow \frac{d}{dx}(y)=\frac{d}{dx}(vx) \quad \frac{dy}{dx}=v+x\frac{dv}{dx}$$

Substitute the values of v and $\frac{dy}{dx}$ in equation(1), we get:

$$\Rightarrow v+x\frac{dv}{dx}=-\frac{[x\cdot vx+(vx)^2]}{x^2}=-v-v^2 \Rightarrow x\frac{dv}{dx}=-v^2-2v=-v(v+2) \Rightarrow \frac{dv}{v(v+2)}=-\frac{dx}{x} \Rightarrow \frac{1}{2}\left[\frac{(v+2)-v}{v(v+2)}\right]dv=-\frac{dx}{x} \Rightarrow \frac{1}{2}\left[\frac{1}{v}-\frac{1}{v+2}\right]dv=-\frac{dx}{x}$$

Integrate on both the sides, we get:

$$\Rightarrow \frac{1}{2}[\log v-\log(v+2)]=-\log x+\log C \Rightarrow \frac{1}{2}\log\left(\frac{v}{v+2}\right)=\log\frac{C}{x} \Rightarrow \frac{v}{v+2}=\left(\frac{C}{x}\right)^2 \Rightarrow \frac{\frac{y}{x}}{\frac{y}{x}+2}=\left(\frac{C}{x}\right)^2 \Rightarrow \frac{y}{y+2x}=\frac{C^2}{x^2}$$

$$x^2\frac{y}{y+2x}=C^2 \quad \ldots\ldots\ldots\ldots \textbf{(2)}$$

Now, y = 1 at x = 1:

$\Rightarrow \frac{1}{1+2} = C^2 \Rightarrow C^2 = \frac{1}{3}$

Substituting $C^2 = \frac{1}{3}$

$\frac{x^2 y}{y+2x} = \frac{1}{3} \Rightarrow y + 2x = 3x^2 y$

This is the required solution for the given differential equation.

Q.13: $\left[x\sin^2\left(\frac{x}{y}\right) - y\right]dx + x\,dy = 0; y = \frac{\pi}{4}$ when $x = 1$

Ans:

$\left[x\sin^2\left(\frac{x}{y}\right) - y\right]dx + x\,dy = 0 \Rightarrow \frac{dy}{dx} = \frac{-\left[x\sin^2\left(\frac{y}{x}\right) - y\right]}{x}$ . . . . **(1)**

Let, F(x , y) = $\frac{-\left[x\sin^2\left(\frac{y}{x}\right) - y\right]}{x}$

Therefore, $F(\lambda x, \lambda y) = \frac{-\left[\lambda x \cdot \sin^2\left(\frac{\lambda x}{\lambda y}\right) - \lambda y\right]}{\lambda x} = \frac{-\left[x\sin^2\left(\frac{y}{x}\right) - y\right]}{x} = \lambda^0 \cdot F(x,y)$

So, the given differential equation is a homogeneous equation.

Let y = vx

Differentiate both the sides w.r.t. x, we get

$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Substitute the values of v and $\frac{dy}{dx}$ **in equation(1), we get:**

$\Rightarrow v + x\frac{dv}{dx} = \frac{-[x\sin^2 v - vx]}{x} \Rightarrow v + x\frac{dv}{dx} = -[\sin^2 v - v] = v - \sin^2 v \Rightarrow x\frac{dv}{dx} = -\sin^2 v \Rightarrow \frac{dv}{\sin^2 v} = -\frac{dx}{x} \Rightarrow \mathrm{cosec}^2 v\,dv = -\frac{dx}{x}$

Integrate on both the sides, we get:

$\Rightarrow$ **-cot v = -log|x|- C**

$\Rightarrow \cot v = \log|x| + C \Rightarrow \cot\left(\frac{y}{x}\right) = \log|x| + \log C \Rightarrow \cot\left(\frac{y}{x}\right) = \log|Cx|$ . . . . **(2)**

Now, y = $\frac{\pi}{4}$ **at x = 1**

$\Rightarrow \cot\frac{\pi}{4} = \log|C|$

$\Rightarrow$ 1 = log C

$\Rightarrow$ $C = e^1 = e$

Substituting C = e in equation (2), we get:

$\cot\left(\frac{y}{x}\right) = \log|ex|$

This is the required solution for the given differential equation.

Q.14: $\frac{dy}{dx}-\frac{y}{x}+cosec\left(\frac{y}{x}\right)=0; y=0 \text{ when } x=1$

Ans:

$\frac{dy}{dx}-\frac{y}{x}+cosec\left(\frac{y}{x}\right)=0 \Rightarrow \frac{dy}{dx}=\frac{y}{x}-cosec\left(\frac{y}{x}\right)$ . . . . **(1)**
Let, F(x , y) = $\frac{y}{x}-cosec\left(\frac{y}{x}\right)$
Therefore, $F(\lambda x,\lambda y)=\frac{\lambda y}{\lambda x}-cosec\left(\frac{\lambda y}{\lambda x}\right)$
$\Rightarrow$ $F(\lambda x,\lambda y)=\frac{y}{x}-cosec\left(\frac{y}{x}\right)=F(x,y)=\lambda^0\cdot F(x,y)$
So, the given differential equation is a homogeneous equation.

Let, y = vx

Differentiate both the sides w.r.t. x, we get

$\Rightarrow \frac{d}{dx}(y)=\frac{d}{dx}(vx)$ $\frac{dy}{dx}=v+x\frac{dv}{dx}$

Substitute the values of v and $\frac{dy}{dx}$ **in equation(1), we get:**
$\Rightarrow$ $v+x\frac{dv}{dx}=v-cosec v \Rightarrow -\frac{dv}{cosec v}=-\frac{dx}{x} \Rightarrow -\sin v\, dv=\frac{dx}{x}$
Integrate on both the sides, we get:

$\Rightarrow$ $\cos v=\log x+\log C=\log|Cx| \Rightarrow \cos\left(\frac{y}{x}\right)=\log|Cx|$ . . . . **(2)**
This is the required solution for the given differential equation.

Now, y = 0 at x = 1

$\Rightarrow$ $\cos(0)=\log C$
$\Rightarrow$ 1 = log C
$\Rightarrow$ C = e^1 = e
This is the required solution for the given differential equation.

Q.15: $2xy+y^2-2x^2\frac{dy}{dx}=0; y=2 \text{ when } x=1$

Ans:

$2xy+y^2-2x^2\frac{dy}{dx}=0$

$\Rightarrow 2x^2\frac{dy}{dx}=2xy+y^2\Rightarrow\frac{dy}{dx}=\frac{2xy+y^2}{2x^2}$ . . . . **(1)**

Let, F(x , y) = $\frac{2xy+y^2}{2x^2}$

Therefore, $F(\lambda x,\lambda y)=\frac{2(\lambda x)(\lambda y)+(\lambda y)^2}{2(\lambda x)^2}=\frac{2xy+y^2}{2x^2}=\lambda^0\cdot F(x,y)$

So, the given differential equation is a homogeneous equation.

Let, y=vx

Differentiate both the sides w.r.t. x, we get:

$\Rightarrow\ \frac{d}{dx}(y)=\frac{d}{dx}(vx)$

$\frac{dy}{dx}=v+x\frac{dv}{dx}$

Substitute the values of v and $\frac{dy}{dx}$ **in equation(1), we get:**

$\Rightarrow v+x\frac{dv}{dx}=\frac{2x(vx)+(vx)^2}{2x^2}\Rightarrow v+x\frac{dv}{dx}=\frac{2v+v^2}{2}\Rightarrow v+x\frac{dv}{dx}=v+\frac{v^2}{2}\Rightarrow\frac{2}{v^2}dv=\frac{dx}{x}$

Integrate on both the sides, we get:

$\Rightarrow 2\cdot\frac{v^{-2+1}}{-2+1}=\log|x|+C\Rightarrow-\frac{2}{v}=\log|x|+C\Rightarrow-\frac{2}{\frac{y}{x}}=\log|x|+C\Rightarrow-\frac{2x}{y}=\log|x|+C$ . . . . **(2)**

Now, y = 2 at x = 1

$\Rightarrow$ -1 = log(1)+C

$\Rightarrow$ C = -1

Substutute C = -1 in equation (2), we get:

$\Rightarrow-\frac{2x}{y}=\log|x|-1\Rightarrow\frac{2x}{y}=1-\log|x|\Rightarrow y=\frac{2x}{1-\log|x|},(x\neq0,x\neq e)$

This is the required solution of the given differential equation.

Q.16: A homogeneous differential equation of the form $\frac{dx}{dy}=h\left(\frac{x}{y}\right)$ **can be solved by making the substitution**

(i) y = vx

(ii) v = yx

(iii) x = vy

(iv) x = v

Ans:

For solving the homogeneous equation of the form $\frac{dx}{dy}=h\left(\frac{x}{y}\right)$, we need to make the substitution as x = vy. **Hence, the correct answer is (iii).**

Q.17: Which of the following is a homogeneous differential equation?

(i) (4x + 6y + 5)dy – (3y + 2x + 4)dx = 0

(ii) (xy)dx – (x^3 + y^3)dy = 0

(iii) (x^3 + $2y^2$)dx + 2xy dy = 0

(iv) y^2 dx + (x^2 –xy^2 – y^2)dy = 0

Ans:

Function F(x, y) is said to be the homogenous function of degree n, if

F(λx , λy) = λ^n F(x, y) for any non-zero constant (λ).

Consider the equation given in alternative IV:

Y^2 dx + (x^2 – xy –y^2)dy = 0

$\Rightarrow \quad \frac{dy}{dx}=-\frac{y^2}{x^2-xy-y^2}=\frac{y^2}{y^2+xy-x^2}$

Let F(x , y) = $\frac{y^2}{y^2+xy-x^2}$

$\Rightarrow \quad F(\lambda x,\lambda y)=\frac{(\lambda y)^2}{(\lambda y)^2+(\lambda x)(\lambda y)-(\lambda x)^2}=\frac{\lambda^2 y^2}{\lambda^2\left(y^2+xy-x^2\right)}$

$\Rightarrow \quad \lambda^0\left(\frac{y^2}{y^2+xy-x^2}\right)=\lambda^0\cdot F(x,y)$

Hence, the differential equation given in alternative (iv) is a homogenous equation.

Exercise-9.6

Q.1: $\frac{dy}{dx}+2y=\sin x$

Ans:

Given:

$\frac{dy}{dx}+2y=\sin x$

We know that:

$\frac{dy}{dx}+py=Q$ **[where, p = 2 and Q = sin x]**

Now, I.F = $e^{\int p dx}=e^{\int 2 dx}=e^{2x}$

The solution of the given differential equation is given by the relation:

Y(I.F.) = $\int(Q\times I.F.)dx+C$

$\Rightarrow\ ye^{2x}=\int \sin x\cdot e^{2x}dx+C$ . . . . **(1)**

Let, I = $\int \sin x\cdot e^{2x}$

$\Rightarrow\ I=\sin x.\int e^{2x}dx-\int\left(\frac{d}{dx}(\sin x).\int e^{2x}dx\right)dx \Rightarrow I=\sin x.\frac{e^{2x}}{2}-\int\left(\cos x.\frac{e^{x}}{2}\right)dx \Rightarrow I=\frac{e^{2x}.\sin x}{2}-\frac{1}{2}\left[\cos x.\int e^{2x}-\int\left(\frac{d}{dx}(\cos x).\int e^{2x}dx\right)dx\right] \Rightarrow I=\frac{e^{2x}.\sin x}{2}-\frac{1}{2}\left[\cos x.\frac{e^{2x}}{2}-\int\left[(-\sin x).\frac{e^{2x}}{2}dx\right]\right] \Rightarrow I=\frac{e^{2x}.\sin x}{2}-\frac{e^{2x}.\cos x}{4}-\frac{1}{4}\int(\sin.x.e^{2x})dx \Rightarrow I=\frac{e^{2x}}{4}(2\sin x-\cos x)-\frac{1}{4}I \Rightarrow \frac{5}{4}I=\frac{e^{2x}}{4}(2\sin x-\cos x) \Rightarrow I=\frac{e^{2x}}{5}(2\sin x-\cos x)$

So, equation (1) becomes:

$ye^{2x}=\frac{e^{2x}}{5}(2\sin x-\cos x)+C$

$\Rightarrow\ y=\frac{1}{5}(2\sin x-\cos x)+Ce^{-2x}$

This is the required general solution of the given differential equation.

Q.2: $\frac{dy}{dx}+3y=e^{-2x}$

Ans:

The given differential equation is:

$\frac{dy}{dx}+py=Q$ (where p=3 and Q=e^{-2x})

Now, I.F = $e^{\int p dx}=e^{\int 3 dx}=e^{3x}$

The solution of the given differential equation is given by the relation:

$y(I.F.)=\int(Q\times I.F.)dx+C \Rightarrow ye^{3x}=\int(e^{-2x}\times e^{3x})+C \Rightarrow ye^{3x}=\int e^{x}dx+C \Rightarrow ye^{3x}=e^{x}+C \Rightarrow y=e^{-2x}+Ce^{-3x}$

This is the required general solution of the given differential equation.

Q.3: $\frac{dy}{dx}+\frac{y}{x}=x^2$

Ans:

The given differential equation is:

$\frac{dy}{dx}+py=Q$ (where $p=\frac{1}{x}$ and Q = x^2)
Now, I.F. = $e^{\int p\,dx}=e^{\int \frac{1}{x}dx}=e^{\log x}=x$
The solution of the given differential equation is given by the relation:

$$y(I.F.)=\int(Q\times I.F.)dx+C$$

$$\Rightarrow \quad y(x)=\int(x^2.x)dx+C \Rightarrow xy=\int x^3dx+C \Rightarrow xy=\frac{x^4}{4}+C$$
This is the required general solution of the given differential equation.

Q.4: $\frac{dy}{dx}+\sec x\,y=\tan x \left(0\le x<\frac{\pi}{2}\right)$
Ans:

The given differential equation is:

$\frac{dy}{dx}+py=Q$ (where p = sec x and Q = tan x)
Now,

Now, I.F. = $e^{\int p\,dx}=e^{\int \sec x\,dx}=e^{\log(\sec x+\tan x)}=\sec x+\tan x$
The general solution of the given differential equation is given by the relation:

$$y(I.F.)=\int(Q\times I.F.)dx+C$$

$$\Rightarrow \quad y(\sec x+\tan x)=\int \tan x(\sec x+\tan x)dx+C \Rightarrow y(\sec x+\tan x)=\int \sec x\tan x\,dx+\int \tan^2 x\,dx+C \Rightarrow y(\sec x+\tan x)=\sec x+\int(\sec^2 x-1)dx+C \Rightarrow y(\sec x+\tan x)=\sec x+\tan x-x+C$$

Q.5: $\int_0^{\frac{\pi}{2}}\cos 2x\,dx$

Ans:

Let, I = $\int_0^{\frac{\pi}{2}} \cos 2x dx$

$\int \cos 2x dx = \left(\frac{\sin 2x}{2}\right) = F(x)$

By second fundamental theorem of calculus, we get:

$I = F\left(\frac{\pi}{2}\right) - F(0) = \frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right) - \sin 0\right] = \frac{1}{2}[\sin\pi - \sin 0] = \frac{1}{2}[0-0] = 0$

Q.6: $x\frac{dy}{dx} + 2y = x^2 \log x$

Ans:

The given differential equation is:

$x\frac{dy}{dx} + 2y = x^2 \log x$

$\Rightarrow \quad \frac{dy}{dx} + \frac{2}{x}y = x\log x$

This equation is in the form of a linear differential equation as:

$\frac{dy}{dx} + py = Q$ (where p = $\frac{2}{x}$ and Q = x log x)

Now, I.F. = $e^{\int p dx} = e^{\int \frac{2}{x} dx} = e^{2\log x} = e^{\log x^2} = x^2$

The general solution of the given differential equation is given by the relation:

$y(I.F.) = \int (Q \times I.F.) dx + C$

$\Rightarrow \quad y.x^2 = \int (x\log x . x^2) dx + C \Rightarrow x^2 y = \int (x^3 \log x) dx + C \Rightarrow x^2 y = \log x . \int x^3 - \int \left[\frac{d}{dx}(\log x) . \int x^3 dx\right] dx + C \Rightarrow x^2 y = \log x . \frac{x^4}{4} - \int \left(\frac{1}{x} . \frac{x^4}{4}\right) dx + C \Rightarrow x^2 y = \frac{x^4 \log x}{4} - \frac{1}{4}\int x^3 dx + C \Rightarrow x^2 y = \frac{x^4 \log x}{4} - \frac{1}{4} . \frac{x^4}{4} + C \Rightarrow x^2 y = \frac{1}{16} x^4 (4\log x - 1) + C \Rightarrow y = \frac{1}{16} x^2 (4\log x - 1) + Cx^{-2}$

Q.7: $x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$

Ans:

The given differential equation is:

$x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x \Rightarrow \frac{dy}{dx}+\frac{y}{x\log x}=\frac{2}{x^2}$

This equation is the form of a linear differential equation as:

$\frac{dy}{dx}+py=Q$ (where p = $\frac{1}{x\log x}$ and Q = $\frac{2}{x^2}$)
Now, I.F. = $e^{\int pdx}=e^{\int \frac{1}{x\log x}dx}=e^{\log(\log x)}=\log x$
The general solution of the given differential equation is given by the relation:

$$y(I.F.)=\int(Q\times I.F.)dx+C$$

$\Rightarrow\quad y\log x=\int\left(\frac{2}{x^2}\log x\right)dx+C$ . . . . **(1)**
Now,

$$\int\left(\frac{2}{x^2}\log x\right)dx=2\int\left(\log x.\frac{1}{x^2}\right)dx:$$

$\Rightarrow\quad =2\left[\log x.\int\frac{1}{x^2}dx-\int\left\{\frac{d}{dx}(\log x).\int\frac{1}{x^2}dx\right\}dx\right]=2\left[\log x\left(-\frac{1}{x}\right)-\int\left(\frac{1}{x}.\left(-\frac{1}{x}\right)\right)dx\right]=2\left[-\frac{\log x}{x}+\int\frac{1}{x^2}dx\right]=2\left[-\frac{\log x}{x}-\frac{1}{x}\right]=-\frac{2}{x}(1+\log x)$
Substituting the value of $\int\left(\frac{2}{x^2}\log x\right)dx$ **in equation (1), we get:**
y log x = $-\frac{2}{x}(1+\log x)+C$
This is the required general solution of the given differential equation.

Q.8.(1 + x²)dy+ 2xy dx = cot xdx

Ans:

$\Rightarrow\frac{dy}{dx}+\frac{2xy}{1+x^2}=\frac{\cot x}{1+x^2}$

This equation is a linear differential equation of the form:

$\frac{dy}{dx}+py=Q$ (where p = $\frac{2x}{1+x^2}$ and Q = $\frac{\cot x}{1+x^2}$)
Now, I.F. = $e^{\int pdx}=e^{\int\frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=1+x^2$
The general solution of the given differential equation is given by the relation:

$$y(I.F.)=\int(Q\times I.F.)dx+C$$

$$\Rightarrow\quad y(1+x^2)=\int\left[\frac{\cot x}{1+x^2}\times(1+x^2)\right]dx+C\Rightarrow y(1+x^2)=\int\cot x dx+C\Rightarrow y(1+x^2)=\log|\sin x|+C$$

Q.9: $x\frac{dy}{dx}+y-x+xy\cot x=0\ (x\neq 0)$

Ans:

$x\frac{dy}{dx}+y-x+xy\cot x=0$

$\Rightarrow\ x\frac{dy}{dx}+y(1+x\cot x)=x$

$\Rightarrow\ \frac{dy}{dx}+\left(\frac{1}{x}+\cot x\right)y=1$

This equation is a linear differential equation of the form:

$\frac{dy}{dx}+py=Q$ (where p = $\frac{1}{x}+\cot x$ and Q = 1)

Now, I.F. = $e^{\int p\,dx}=e^{\int\left(\frac{1}{x}+\cot x\right)dx}=e^{\log x+\log(\sin x)}=e^{\log(x\sin x)}=x\sin x$

The general solution of the given differential equation is given by the relation,

$y(I.F.)=\int(Q\times I.F.)dx+C$

$\Rightarrow\ y(x\sin x)=\int(1\times x\sin x)dx+C\Rightarrow y(x\sin x)=\int(x\sin x)dx+C\Rightarrow y(x\sin x)=x\int\sin x\,dx-\int\left[\frac{d}{dx}(x).\int\sin x\,dx\right]+C\Rightarrow y(x\sin x)=x(-\cos x)-\int 1.(-\cos x)dx+C\Rightarrow y(x\sin x)=-x\cos x+\sin x+C\Rightarrow y=\frac{-x\cos x}{x\sin x}+\frac{\sin x}{x\sin x}+\frac{C}{x\sin x}\Rightarrow y=-\cot x+\frac{1}{x}+\frac{C}{x\sin x}$

Q.10: $(x+y)\frac{dy}{dx}=1$

Ans:

$(x+y)\frac{dy}{dx}=1$

$\Rightarrow\ \frac{dy}{dx}=\frac{1}{x+y}\Rightarrow\frac{dx}{dy}=x+y\Rightarrow\frac{dx}{dy}-x=y$

This is a linear differential equation of the form:

$\frac{dy}{dx}+py=Q$ (where p = -1 and Q = y)

Now, I.F. = $e^{\int p\,dx}=e^{\int -dy}=e^{-y}$

The general solution of the given differential equation is given by the relation:

$y(I.F.)=\int(Q\times I.F.)dx+C\ \Rightarrow xe^{-y}=\int(y.e^{-y})dy+C\Rightarrow xe^{-y}=y.\int e^{-y}dy-\int\left[\frac{d}{dy}(y)\int e^{-y}dy\right]dy+C\Rightarrow xe^{-y}=y(-e^{-y})-\int(-e^{-y})dy+C\Rightarrow xe^{-y}=-ye^{-y}+\int e^{-y}dy+C\Rightarrow xe^{-y}=-ye^{-y}-e^{-y}+C\Rightarrow x=-y-1+Ce^{y}\Rightarrow x+y+1=Ce^{y}$

Q.11: y dx + (x – y^2)dy = 0

Ans:

y dx + (x – y^2)dy = 0

$\Rightarrow y\,dx=(y^2-x)dy \Rightarrow \frac{dx}{dy}=\frac{y^2-x}{y}=y-\frac{x}{y} \Rightarrow \frac{dx}{dy}+\frac{x}{y}=y$

This is a linear differential equation of the form:

$\frac{dy}{dx}+py=Q$ (where p = $\frac{1}{y}$ and Q = y)

Now, I.F. = $e^{\int p\,dx}=e^{\int \frac{1}{y}dy}=e^{\log y}=y$

The general solution of the given differential equation is given by the relation:

$x(I.F.)=\int(Q\times I.F.)dy+C$

$\Rightarrow \quad xy=\int(y.y)dy+C$

$\Rightarrow \quad xy=\int y^2dy+C=\frac{y^3}{3}+C$

$\Rightarrow \quad x=\frac{y^3}{3}+\frac{C}{y}$

Q.12: $(x+3y^2)\frac{dy}{dx}=y\,(y>0)$

Ans:

$(x+3y^2)\frac{dy}{dx}=y$

$\Rightarrow \quad \frac{dy}{dx}=\frac{y}{x+3y^2}$

$\Rightarrow \quad \frac{dx}{dy}=\frac{x+3y^2}{y}=\frac{x}{y}+3y$

$\Rightarrow \quad \frac{dx}{dy}-\frac{x}{y}=3y$

This is a linear differential equation of the form:

$\frac{dx}{dy}+px=Q$ **[where, p =** $-\frac{1}{y}$ **and Q = 3y]**

Now, I.F. = $e^{\int p\,dx}=e^{-\int\frac{dy}{y}}=e^{-\log y}=e^{\frac{1}{y}}=\frac{1}{y}$

The general solution of the given differential equation is given by the relation:

$x(I.F.)=\int(Q\times I.F.)dy+C$

$\Rightarrow \quad x\times\frac{1}{y}=\int(3y\times\frac{1}{y})dy+C$

$\Rightarrow \quad \frac{x}{y}=3y+C$

$\Rightarrow$ **x = 3y^2 + Cy**

Q.13: $\frac{dy}{dx}+2y\tan x=\sin x; y=0 \text{ when } x=\frac{\pi}{3}$

Ans:

Given:

$\frac{dy}{dx}+2y\tan x=\sin x$

This is a linear equation of the form:

$\frac{dy}{dx}+py=Q$ (where p = 2 tan x and Q = sin x)
Now, I.F.= $e^{\int p dx}=e^{\int 2\tan x dx}=e^{2\log|\sec x|}=e^{\log(\sec^2 x)}=\sec^2 x$
The general solution of the given differential equation is given by the relation,

$y(\text{I.F.})=\int(Q\times \text{I.F.})dx+C$

$\Rightarrow y(\sec^2 x)=\int(\sin x.\sec^2 x)dx+C \Rightarrow y\sec^2 x=\int(\sec x.\tan x)dx+C \Rightarrow y\sec^2 x=\sec x+C$ …
….(1)
Now, y = 0 at $x=\frac{\pi}{3}$
Therefore,

$0\times\sec^2\frac{\pi}{3}=\sec\frac{\pi}{3}+C$

0 = 2 + C i.e C= – 2

Substituting C = –2 in equation (1), we get:

y $\sec^2$ x = sec x – 2

$\Rightarrow$ y = cos x – 2$\cos^2$ x
Hence, the required solution of the given differential equation is y = cos x – 2$\cos^2$ x

Q.14. $(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2}; y=0 \text{ when } x=1$

Ans:

$(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{1+x^2} \Rightarrow \frac{dy}{dx}+\frac{2xy}{1+x^2}=\frac{1}{(1+x^2)^2}$

This is a linear differential equation of the form:

$\frac{dy}{dx}$ + py = Q   [where, p = $\frac{2x}{1+x^2}$ and Q = $\frac{1}{(1+x^2)^2}$)]
Now, I.F. = $e^{\int p dx}=e^{\int \frac{2x dx}{1+x^2}}=e^{\log(1+x^2)}=1+x^2$
The general solution of the given differential equation is given by the relation:

$y(I.F.)=\int(Q\times I.F.)dx+C$

$\Rightarrow \quad y(1+x^2)=\int\left[\frac{1}{1+x^2}.(1+x^2)\right]dx+C \Rightarrow y(1+x^2)=\int\frac{1}{1+x^2}dx+C \Rightarrow y(1+x^2)=\tan^{-1}x+C$ . . . . .
. . . . .**(1)**
Now, y = 0 at x = 1

Therefore,

$0 = \tan^{-1}1 + C$

$\Rightarrow C = -\frac{\pi}{4}$
Substitute C = $-\frac{\pi}{4}$ in equation(1), we get:
$y(1 + x^2) = \tan^{-1} x - \frac{\pi}{4}$
This is the required general solution of the given differential equation.

Q.15: $\frac{dy}{dx}-3y\cot x=\sin 2x; y=2 \text{ when } x=\frac{\pi}{2}$

Ans:

Given:

$\frac{dy}{dx}-3y\cot x=\sin 2x$

This is a linear differential equation of the form:

$\frac{dy}{dx}$ + py = Q   **[where, p = -3 cot x and Q = sin 2x]**
Now, I.F. = $e^{\int p dx}=e^{-3\int \cot x dx}=e^{-3\log|\sin x|}=e^{\log\left|\frac{1}{\sin^3 x}\right|}=\frac{1}{\sin^3 x}$
The general solution of the given differential equation is given by the relation:

$y(I.F.)=\int(Q\times I.F.)dx+C$

$\Rightarrow \quad y\cdot\frac{1}{\sin^3 x}=\int\left[\sin 2x\cdot\frac{1}{\sin^3 x}\right]dx+C$

$\Rightarrow$ $y \operatorname{cosec}^3 x = 2\int(\cot x \operatorname{cosec} x)\,dx + C$
$\Rightarrow$ $y\operatorname{cosec}^3 x = 2\operatorname{cosec} x + C = -\frac{2}{\operatorname{cosec}^2 x} + \frac{3}{\operatorname{cosec}^3 x}$
$\Rightarrow$ $y = -2\sin^2 x + C\sin^3 x$ . . . . . . **(1)**
Now,

$y = 2$ at $x = \frac{\pi}{2}$
Therefore, we get:

2 = -2 + C

$\Rightarrow C = 4$

Substitute C = 4 in equation (1), we get:

$$y = -2\sin^2 x + 4\sin^3 x \Rightarrow y = 4\sin^3 x - 2\sin^2 x$$

This is the required particular solution of the given differential equation.

Q.16: Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinates of the point.

Ans:

Let, F (x , y) be the curve passing through the origin.

At point (x, y), the slope of the curve will be $\frac{dy}{dx}$.
According to the given information:

$\frac{dy}{dx} = x + y \Rightarrow \frac{dy}{dx} - y = x$

This is a linear differential equation of the form:

$\frac{dy}{dx} + py = Q$ **[where, p =-1 and Q =x]**
Now, I.F. = $e^{\int p\,dx} = e^{\int(-1)\,dx} = e^{-1}$
The general solution of the given differential equation is given by the relation,

$$y(I.F.) = \int(Q \times I.F.)\,dx + C$$

$\Rightarrow$ $ye^{-1} = \int xe^{-1}\,dx + C$ . . . . **(1)**

Now ,

$$\int xe^{-1}dx = x\int e^{-1}dx - \int\left[\frac{d}{dx}(x).\int e^{-x}dx\right]dx = -xe^{-x} - \int -e^{-1}dx = -xe^{-x}(-e^{-x}) = -e^{-x}(x+1)$$

Substituting in equation (1), we get:

$Ye^{-1} = -e^{-x}(x + 1) + C$

⇒ $y = -(x + 1) + Ce^{x}$

⇒ $x + y + 1 = Ce^{x}$ **. . . . . (2)**

The curve passes through the origin.

Therefore, equation (2) becomes:

C = 1

Substituting **C = 1** in equation (2), we get:

$x + y + 1 = e^{x}$

Hence, the required equation of curve passing through the origin is $x + y + 1 = e^{x}$

Q.17. Find the equation of a curve passing through the point (0, 2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

Ans:

Let, F (x, y) be the curve and let (x, y) be a point on the curve.

The slope of the tangent to the curve at (x, y) is $\frac{dy}{dx}$.

According to the given information:

$$\frac{dy}{dx} + 5 = x + y \Rightarrow \frac{dy}{dx} - y = x - 5$$

This is a linear differential equation of the form:

$\frac{dy}{dx} + py = Q$ **[where, p = -1 and Q = x – 5]**

Now, I.F. = $e^{\int p dx} = e^{\int (-1)dx} = e^{-x}$

The general equation of the curve is given by the relation:

$y(I.F.)=\int(Q\times I.F.)dx+C$

$\Rightarrow \; y.e^{-x}=\int(x-5)e^{-x}dx+C$ . . . . **(1)**

Now,

$\Rightarrow \; \int(x-5)e^{-x}dx=(x-5)\int e^{-x}dx-\int[\frac{d}{dx}(x-5).\int e^{-x}dx]$

$\Rightarrow \; (x-5)(-e^{-x})-\int(-e^{-x})dx$

$\Rightarrow \; (5-x)e^{-x}+(-e^{-x})$

$\Rightarrow \; (4-x)e^{-x}$

Therefore, equation (1) becomes:

$\mathbf{ye^{-x} = (4 - x) e^{-x} + C}$

$\Rightarrow \; y = 4 - x + Ce^{x}$

$\Rightarrow \; x + y - 4 = Ce^{x}$ . . . . **(2)**

The curve passes through point (0, 2).

Therefore, equation (2) becomes:

$0 + 2 - 4 = Ce^{0}$

$\Rightarrow \; -2 = C$

or, $C = -2$

Substituting $C = -2$ in equation (2), we get:

$x + y - 4 = -2e^{x}$

$\Rightarrow \; \mathbf{y = 4 - x - 2e^{x}}$

This is the required equation of the curve.

Q.18: The integrating factor of the differential equation $x\frac{dy}{dx}-y=2x^2$ **is**

(i) $\mathbf{e^{-x}}$

(ii) $\mathbf{e^{-y}}$

(iii) $\frac{1}{x}$

(iv) $\mathbf{x}$

Ans:

The given differential equation is:

$x\frac{dy}{dx} - y = 2x^2$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x} = 2x$

This is a linear differential equation of the form:

$\frac{dy}{dx} + py = Q$ **[where, p =-$\frac{1}{x}$ and Q =2x]**

The integrating factor (I.F) is given by the relation:

$\Rightarrow \text{I.F.} = e^{\int \frac{1}{x} dx} = e^{-\log x} = e^{\log(x^{-1})} = x^{-1} = \frac{1}{x}$

Hence, the correct answer is (iii)

Q.19: The integrating factor of the differential equation.

$(1 - y^2)\frac{dx}{dy} + yx = ay \quad (-1 < y < 1)$

(i) $\frac{1}{y^2 - 1}$

(ii) $\frac{1}{\sqrt{y^2 - 1}}$

(iii) $\frac{1}{1 - y^2}$

(iv) $\frac{1}{\sqrt{1 - y^2}}$

Ans:

The given differential equation is:

$\frac{dx}{dy} + yx = ay$

$\Rightarrow \frac{dx}{dy} + \frac{yx}{1 - y^2} = \frac{ay}{1 - y^2}$

This is a linear differential equation of the form:

$\frac{dx}{dy} + py = Q \left(\text{where } p = \frac{y}{1 - y^2} \text{ and } Q = \frac{ay}{1 - y^2}\right)$

The integrating factor (I.F) is given by the relation:

$\Rightarrow \text{I.F.} = e^{\int p dy} = e^{\int \frac{y}{1 - y^2} dy} = e^{-\frac{1}{2}\log(1 - y^2)} = e^{\log\left[\frac{1}{\sqrt{1 - y^2}}\right]} = \frac{1}{\sqrt{1 - y^2}}$

Hence, the correct answer is (iv)