NOTES –INEQUALITIES (One Variable, Linear, and Systems)

ONE VARIABLE INEQUALITIES

Inequality Symbols:

$<$ less than

$>$ greater than

$\leq$ less than or equal to

$\geq$ greater than or equal to

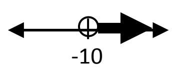
Open circles do NOT include that number.

Closed circles DO include that number.

Solving One Variable Inequalities:
Solve inequalities the same way you would solve an equation.

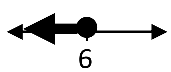
- If you **multiply** or **divide** by a negative number, FLIP the inequality symbol over.

  EX:
  $$3(x - 2) - 8x < 44$$
  $$3x - 6 - 8x < 44$$
  $$-5x - 6 < 44$$
  $$+6 \quad +6$$
  $$\frac{-5x}{-5} < \frac{50}{-5}$$
  $$x > -10$$

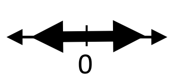
- If the variable is on the right side of the inequality after you have solved the problem, FLIP the entire problem over.

  EX:
  $$-4x + 9 \geq x - 21$$
  $$+4x \quad +4x$$
  $$9 \geq 5x - 21$$
  $$+21 \quad +21$$
  $$\frac{30}{5} \geq \frac{5x}{5}$$
  $$6 \geq x$$
  becomes $x \leq 6$

- If the variable cancels out while solving and you are left with a true statement, the answer is ALL REAL NUMBERS.

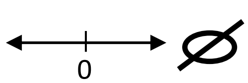
  EX:
  $$2x + 3 > 2(-3 + x)$$
  $$2x + 3 > -6 + 2x$$
  $$-2x \quad -2x$$
  $$3 > -6$$

  The answer is <u>all real numbers</u> since 3 is greater than or equal to -6.

- If the variable cancels out while solving and you are left with a false statement, the answer is NO SOLUTION.

  EX:
  $$4x - 6 > -2(3 - 2x)$$
  $$4x - 6 > -6 + 4x$$
  $$-4x \quad -4x$$
  $$-6 > -6$$

  The answer is <u>no solution</u> since -6 is NOT less than -6.

ONE VARIABLE INEQUALITIES – cont.

Determining if a Number is a Solution to an Inequality:
There are two options when checking to see if a number is a solution to an inequality.

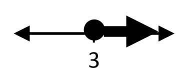
Option 1: Substitute the number for the variable in the inequality. If it makes a true statement, it is a solution. If it makes a false statement, it is NOT a solution.

EX: Is 1 a solution to the inequality $3(x + 2) < 4x + 5$?
$$3(1 + 2) < 4(1) + 5$$
$$3(3) < 4 + 5$$
$$9 < 9 \ ✗$$
1 is **NOT** a solution because when substituted into the inequality, it makes a false statement (9 is not less than 9).

EX: Is -5 a solution to the inequality $4x + 3 < -13$?
$$4(-5) + 3 < -13$$
$$-20 + 3 < -13$$
$$-17 < -13 \ ✓$$
-5 **IS** a solution because when substituted into the inequality, it makes a true statement (-17 is less than -13.)

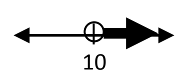
Option 2: See if the number is in the shaded part of the graph of the inequality. If it is in the shaded area or on the closed circle, it is a solution. If it is on an open circle, it is NOT a solution.

EX: Is 4 a solution to the inequality $9 - 2x \leq 6x - 15$?
$$9 - 2x \leq 6x - 15$$
$$+2x \quad +2x$$
$$9 \leq 8x - 15$$
$$+15 \quad +15$$
$$\frac{24}{8} \leq \frac{8x}{8}$$
$$3 \leq x$$
becomes $x \geq 3$



4 **IS** a solution to the inequality because it is in the shaded area.

EX: Is 10 a solution to the inequality $10+6x-2x > 50$?
$$10 + 6x - 2x > 50$$
$$10 + 4x > 50$$
$$-10 \quad -10$$
$$\frac{4x}{4} > \frac{40}{4}$$
$$x > 10$$



10 is **NOT** a solution to the inequality because it is on an open circle which means it is NOT included as part of the solution.

TWO VARIABLE (LINEAR) INEQUALITIES

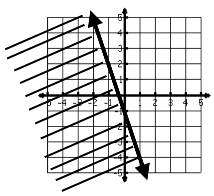
$y < mx + b$	dotted and shaded below the line
$y > mx + b$	dotted and shaded above the line
$y \le mx + b$	solid and shaded below the line
$y \ge mx + b$	solid and shaded above the line

<u>Solving and Graphing Linear Inequalities</u>:
When solving for *y*, don't forget to FLIP the inequality symbol if you multiply or divide by a negative!

- Points on a dotted line do NOT count as part of the solution.
- Points on a solid line DO count as part of the solution.
- ALL points in the shaded area are part of the solution.
- Points in the non-shaded area are NOT part of the solution.

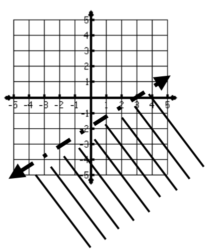
<u>EXAMPLES</u>:

* $3x + y \le -1$
$\quad -3x \qquad -3x$
$y \le -3x - 1$



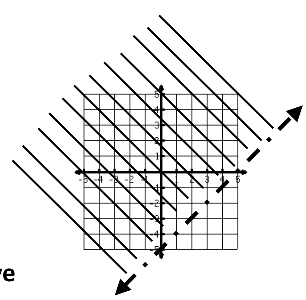
Solid & shaded below

* $2x - 3y > 6$
$\quad -2x \qquad -2x$
$\frac{-3y}{-3} > \frac{-2x}{-3} + \frac{6}{-3}$
$y < \frac{2}{3}x - 2$



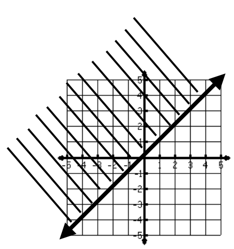
Dotted & shaded below

* $x - y < 5$
$\quad -x \qquad -x$
$\frac{-y}{-1} < \frac{-x}{-1} + \frac{5}{-1}$
$y > x - 5$



dotted & shaded above

$4x \le 2(x + y)$
$4x \le 2x + 2y$
$-2x \quad -2x$
$\frac{2x}{2} \le \frac{2y}{2}$
$x \le y$
becomes **$y \ge x$**



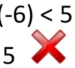
solid and shaded above

TWO VARIABLE (LINEAR) INEQUALITIES – cont.

<u>Determining if a Point is a Solution to a Linear Inequality</u>:
There are two options when checking to see if a point is a solution to a linear inequality.

<u>Option 1</u>: Substitute the point for the variables in the inequality. If it makes a true statement, it is a solution. If it makes a false statement, it is NOT a solution.

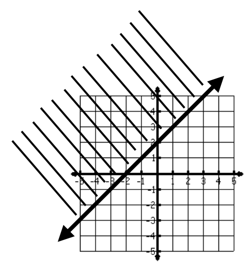
<u>EX</u>: Is (5, -2) a solution to the inequality $2x - 3y < 5$?

$2(5) - 3(-2) < 5$
$10 - (-6) < 5$
$16 < 5$ 

No, (5, -2) is **NOT** a solution because when you substitute the point into the inequality, 16 is NOT less than 5.

<u>Option 2</u>: See if the point is in the shaded part of the graph of the inequality. If it is in the shaded area, it is a solution. If it is on a dotted line, it is NOT a solution.

<u>EX</u>: Is (-3, 0) a solution to the inequality $y \ge x + 2$?



Yes, (-3, 0) **IS** a solution to $y \ge x + 2$ because it is in the shaded area.

CALCULATOR TIPS

When graphing linear inequalities, the calculator will NOT show you if it should be a solid or dotted line. It will help you determine which side of the line to shade. It can also help you find the "double" shaded area for systems of linear inequalities.

1. Solve for *y*. (Don't forget to flip the inequality symbol if you multiply or divide by a negative number.)
2. Put the inequality into [Y=].
3. Arrow in front of the \\$y_1$= (or y_2, y_3, etc). Push [ENTER] until you change the \\$y_1$= to ◣$y_1$=for < and ≤ or ◥$y_1$= for > and ≥.
4. [GRAPH] to see the shading.

SYSTEMS OF LINEAR INEQUALITIES

System of Linear Inequalities: two or more linear inequalities graphed on the same coordinate plane. The solution is the area where all the shading overlaps.

Graphing Systems of Linear Inequalities:

1. Solve for y. (Don't forget to flip the symbol if you multiply or divide by a negative number.)
2. Graph ALL the inequalities on the same coordinate plane.

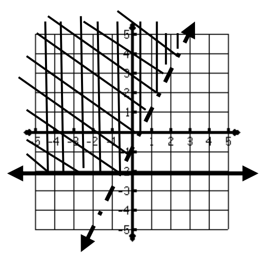
   Remember:

$y < mx + b$	dotted and shaded below
$y > mx + b$	dotted and shaded above
$y \le mx + b$	solid and shaded below
$y \ge mx + b$	solid and shaded above

3. The solution is the area where all the shading overlaps. If the areas do never overlap, it has NO SOLUTION.

EXAMPLES:

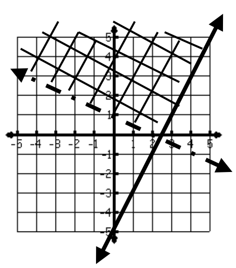
* Graph the system $\begin{cases} y > 2x - 1 \\ y \ge -2 \end{cases}$



* Graph the system $\begin{cases} 2x - y \le 5 \\ x + 2y > 2 \end{cases}$

$2x - y \le 5$
$-2x \quad -2x$
$\frac{-y}{-1} \le \frac{5}{-1} - \frac{2x}{-1}$
$y \ge -5 + 2x$

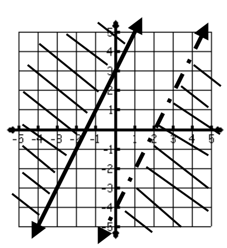
$x + 2y > 2$
$-x \quad -x$
$\frac{2y}{2} > \frac{-x}{2} + \frac{2}{2}$
$y > -\frac{1}{2}x + 1$



* Graph the system $\begin{cases} -y - 4 > -2x \\ -8x + 4y \ge 12 \end{cases}$

$-y - 4 > -2x$
$+4 \quad +4$
$\frac{-y}{-1} > \frac{-2x}{-1} + \frac{4}{-1}$
$y < 2x - 4$

$-8x + 4y \ge 12$
$+8x \quad +8x$
$\frac{4y}{4} \ge \frac{8x}{4} + \frac{12}{4}$
$y \ge 2x + 3$



$\varnothing$

This system has **NO SOLUTION** because there will never be any overlapping shaded areas.

Determining if a Point is a Solution to a Linear Inequality System:

There are two options when checking to see if a point is a solution to a linear inequality system.

Option 1: Substitute the point for the variables in ALL of the inequalities. If it makes a true statement in ALL of the inequalities, it is a solution. If it makes a false statement in even one inequality, it is NOT a solution.

EX: Is (-3, -3) a solution to the system $\begin{cases} 3y \ge -9 \\ x - 3y < 6 \end{cases}$?

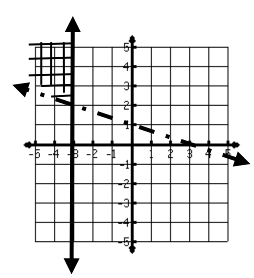
$3y \ge -9$	$x - 3y < 6$
$3(-3) \ge -9$	$-3 - 3(-3) < 6$
$-9 \ge -9$ ✓	$6 < 6$ ✗

(-3, -3) is **NOT** a solution to the system because when you substitute it into the inequality it does NOT make a true statement for both inequalities.

Option 2: See if the point is in the overlapped area of the graphs of the inequalities. If it is in the overlapped area, it is a solution. If it is on any dotted line, it is NOT a solution.

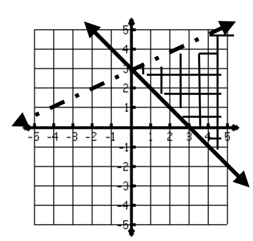
* Is (-3, 2) a solution to the system $\begin{cases} 3y + x > 3 \\ x \le -3 \end{cases}$

$3y + x > 3$
$-x \quad -x$
$\frac{3y}{3} > \frac{-x}{3} + \frac{3}{3}$
$y > -\frac{1}{3}x + 1$



(-3, 2) is **NOT** a solution to the inequality because it is on the intersection of a solid and **dotted** line.

* Is (2, 3) a solution to the system $\begin{cases} y \ge -x + 3 \\ y < \frac{1}{2}x + 3 \end{cases}$



(2, 3) **IS** a solution to the system because it is in the double shaded area.

APPLICATIONS OF INEQUALITIES

When solving problems with inequalities, make sure to read the question carefully. You may have to round up or round down depending on the wording of the problem. Check your answer to make sure it makes sense! When you check your answer, you should be able to tell if you rounded correctly.

Some key terms:

$<$ is less than, fewer than, below

$\leq$ at most, no more than, at or below

$>$ is greater than, more than, above

$\geq$ at least, no less than, at or above

<u>EXAMPLES</u>:

- Fred the Farmer has started a savings account for a tractor. He saved \$600 last month and plans to add \$100 each month until he has saved more than \$2000. Write and solve the inequality. Then, interpret the answer.

  more than \$2000 means greater than \$2000

  $$600 + 100m > 2000$$
  $$-600 \qquad -600$$
  $$\frac{100m}{100} > \frac{1400}{100}$$
  $$\mathbf{m > 14}$$

  Since the number of month is greater than 14, it will take him a <u>minimum of 15 months</u> to save enough money.

- Gabby the gabber likes to text message her friends using her cell phone. She is charged \$0.10 each time she types a message plus \$50 for the phone plan. She is only allowed to have a bill that is at most \$60. Write and solve the inequality. Then, interpret the answer.

  at most means less than or equal to

  $$0.10m + 50 \leq 60$$
  $$-50 \quad -50$$
  $$\frac{0.10m}{0.10} \leq \frac{10}{0.10}$$
  $$m \leq 100$$

  Since the number of messages is less than or equal to 100, she can send a maximum of 100 text messages.