ALGEBRA

Assignment Problems
Solving Equations and Inequalities

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Preface

Here are a set of assignment problems for the Algebra notes. Please note that these problems do not have any solutions available. These are intended mostly for instructors who might want a set of problems to assign for turning in. Having solutions available (or even just final answers) would defeat the purpose the problems.

If you are looking for some practice problems (with solutions available) please check out the Practice Problems. There you will find a set of problems that should give you quite a bit practice.

Chapter 2 : Solving Equations and Inequalities

Here are a set of assignment problems for the Solving Equations and Inequalities chapter of the Algebra notes. Please note that these problems do not have any solutions available. These are intended mostly for instructors who might want a set of problems to assign for turning in. Having solutions available (or even just final answers) would defeat the purpose the problems.

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Here is a list of all the sections for which assignment problems have been written as well as a brief description of the material covered in the notes for that particular section.

Solutions and Solution Sets – In this section we introduce some of the basic notation and ideas involved in solving equations and inequalities. We define solutions for equations and inequalities and solution sets.
Linear Equations – In this section we give a process for solving linear equations, including equations with rational expressions, and we illustrate the process with several examples. In addition, we discuss a subtlety involved in solving equations that students often overlook.
Applications of Linear Equations – In this section we discuss a process for solving applications in general although we will focus only on linear equations here. We will work applications in pricing, distance/rate problems, work rate problems and mixing problems.
Equations With More Than One Variable – In this section we will look at solving equations with more than one variable in them. These equations will have multiple variables in them and we will be asked to solve the equation for one of the variables. This is something that we will be asked to do on a fairly regular basis.
Quadratic Equations, Part I – In this section we will start looking at solving quadratic equations. Specifically, we will concentrate on solving quadratic equations by factoring and the square root property in this section.
Quadratic Equations, Part II – In this section we will continue solving quadratic equations. We will use completing the square to solve quadratic equations in this section and use that to derive the quadratic formula. The quadratic formula is a quick way that will allow us to quickly solve any quadratic equation.
Quadratic Equations : A Summary – In this section we will summarize the topics from the last two sections. We will give a procedure for determining which method to use in solving quadratic equations and we will define the discriminant which will allow us to quickly determine what kind of solutions we will get from solving a quadratic equation.
Applications of Quadratic Equations – In this section we will revisit some of the applications we saw in the linear application section, only this time they will involve solving a quadratic equation. Included are examples in distance/rate problems and work rate problems.
Equations Reducible to Quadratic Form – Not all equations are in what we generally consider quadratic equations. However, some equations, with a proper substitution can be turned into a quadratic equation. These types of equations are called quadratic in form. In this section we will solve this type of equation.
Equations with Radicals – In this section we will discuss how to solve equations with square roots in them. As we will see we will need to be very careful with the potential solutions we get as the process used in solving these equations can lead to values that are not, in fact, solutions to the equation.

Linear Inequalities – In this section we will start solving inequalities. We will concentrate on solving linear inequalities in this section (both single and double inequalities). We will also introduce interval notation.
Polynomial Inequalities – In this section we will continue solving inequalities. However, in this section we move away from linear inequalities and move on to solving inequalities that involve polynomials of degree at least 2.
Rational Inequalities – We continue solving inequalities in this section. We now will solve inequalities that involve rational expressions, although as we'll see the process here is pretty much identical to the process used when solving inequalities with polynomials.
Absolute Value Equations – In this section we will give a geometric as well as a mathematical definition of absolute value. We will then proceed to solve equations that involve an absolute value. We will also work an example that involved two absolute values.
Absolute Value Inequalities – In this final section of the Solving chapter we will solve inequalities that involve absolute value. As we will see the process for solving inequalities with a < (*i.e.* a less than) is very different from solving an inequality with a > (*i.e.* greater than).

Section 2-1 : Solutions and Solution Sets

For each of the following determine if the given number is a solution to the given equation or inequality.

1. Is $u=-1$ a solution to $4u^2-40=10(2u-1)-6$?

2. Is $t=7$ a solution to $7(t+2)=5(t+4)+2$?

3. Is $z=-\frac{1}{3}$ a solution to $6(z-1)+5=9z$?

4. Is $x=-6$ a solution to $x^2=-10x-24$?

5. Is $t=\frac{1}{4}$ a solution to $3t^2+8t=3(1-t)$?

6. Is $w=-3$ a solution to $2w^2-10=w^2-7w+8$?

7. Is $x=\frac{1}{2}$ a solution to $\frac{3}{x}-\frac{1}{x^2}=2$?

8. Is $v=-2$ a solution to $\frac{v^2+v-2}{v-1}=0$?

9. Is $v=1$ a solution to $\frac{v^2+v-2}{v-1}=0$?

10. Is $x=-1$ a solution to $\frac{3x+1}{x^2}-\frac{6}{x+2}=\frac{x-7}{3x+4}$?

11. Is $y=4$ a solution to $4y^2-y^3\le 5y+2$?

12. Is $w=0$ a solution to $3(w-7)+2(w+1)>10w$?

13. Is $x=7$ a solution to $3+4x<x+24$?

Section 2-2 : Linear Equations

Solve each of the following equations and check your answer.

1. $13+2(1-u)=8u-5(u+7)$

2. $8(2+3z)+1=z-10(z+1)$

3. $8-(4-12t)+2=3t+2(7-3t)$

4. $2x(6x-1)+21=8x-x(3-12x)$

5. $\dfrac{3w-1}{5}+1=\dfrac{7w+2}{15}$

6. $\dfrac{10y}{9}+\dfrac{1}{3}=\dfrac{2y-1}{9}$

7. $2\left(3-\dfrac{x}{4}\right)=\dfrac{2x+5}{3}-\dfrac{1}{3}$

8. $\dfrac{6x+24}{x+4}=5$

9. $\dfrac{3}{v+7}-\dfrac{2-7v}{v^2+5v-14}=\dfrac{4}{v-2}$

10. $\dfrac{6t-1}{t^2+5t+4}=-\dfrac{19}{t+1}$

11. $\dfrac{8-4z}{3z-2}=2-\dfrac{10z}{3z-2}$

12. $\dfrac{4w-1}{w-2}+\dfrac{8w}{w^2-6w+8}=\dfrac{4w+3}{w-4}$

Section 2-3 : Applications of Linear Equations

1. In a clearance bin everything has been reduced by 75%. One item is listed in the bin for $32.40. How much was the price of the item before it was put into the clearance bin?

2. A piece of electronics has been marked up 20% and is selling for $21.50. How much did the store pay for the item?

3. A widget is on sale for $715.80 and has been marked down by 11%. What was the original price of the widget?

4. Two cars start at the same point and move in the same direction. One car travels 5mph faster than the twice the speed of other car. After 10 hours the distance separating the two cars is 60 miles. What was the speed of each car?

5. Two people start out 100 meters apart from each other and start moving towards each other at the same time. One person is moving at half the speed of the other person and they meet after 25 seconds of travel time. What was the speed of each person?

6. Two boats start at the same point. One boat starts traveling to the east at 45 mph and two hours later the second boat starts traveling to the east at 60 mph. At some point in time the faster boat will be 145 miles in front of the slower boat. How long has each boat been traveling when this happens?

7. One machine can complete a production run at a factory in 46 hours. Two machines can complete the production run in 25 hours if they work together. How long would it take the second machine to complete the production run if it had to do the job by itself?

8. One person can mow a field in 52 minutes and a second can mow the same field in 40 minutes. How long would it take the two of them to mow the field together?

9. One pump can fill a pool in 11 hours and a second pump can empty the same pool in 4 hours. While the pool is full both pumps are accidentally both turned on at the same time. How long will does it take to empty the pool?

10. How much pure acid should we add to a 32% acid solution to get 10 liters of a 60% acid solution.

11. We have 80 liters of a 2% saline solution. How much of a 10% saline solution should we add to this to increase the salinity to 4%?

12. We have 10 gallons of a 26% alcohol solution and we need 15 gallons of an 18% alcohol solution. What % alcohol solution should we add to the 26% solution to get the solution we want?

13. There is a field whose width is 6 meters less than its length. If both the length and width are doubled the perimeter will be 120 meters. What are the dimensions of the field?

14. A triangular piece of glass has been cut for a stained glass window. Two of the sides are the same length and the third side is 1 inch shorter than ½ the length of the other two sides. If the perimeter is 23 inches what are the lengths of the sides?

Section 2-4 : Equations With More Than One Variable

1. Solve $A = 3p(4-2r)$ for *p*.

2. Solve $A = 3p(4-2r)$ for *r*.

3. Solve $T = \frac{c}{3}\left(6p + \frac{3q}{c}\right) - 7p$ for *p*.

4. Solve $T = \frac{c}{3}\left(6p + \frac{3q}{c}\right) - 7p$ for *c*.

5. Solve $\frac{1}{n} = \frac{2}{m} - \frac{3}{q}$ for *n*.

6. Solve $\frac{1}{n} = \frac{2}{m} - \frac{3}{q}$ for *q*.

7. Solve $3A + 6C = 4A(B - 7C)$ for *C*.

8. Solve $3A + 6C = 4A(B - 7C)$ for *A*.

9. Solve $y = \frac{4-9x}{3}$ for *x*.

10. Solve $y = \frac{12}{1-x}$ for *x*.

11. Solve $y = \frac{7}{10x+9}$ for *x*.

12. Solve $y = \frac{8-5x}{9-7x}$ for *x*.

13. Solve $y = \frac{2+11x}{1+4x}$ for *x*.

14. Solve $y = \frac{9+2x}{4-x}$ for *x*.

Section 2-5 : Quadratic Equations - Part I

For problems 1 – 15 solve the quadratic equation by factoring.

1. $z^2 - 11z + 24 = 0$

2. $w^2 + 13w + 12 = 0$

3. $x^2 + 32 = 12x$

4. $y^2 = 6y + 27$

5. $u^2 - 4u - 20 = 3u + 24$

6. $z^2 - 36 = 0$

7. $144x^2 - 25 = 0$

8. $7x^2 + 19x = 6$

9. $4y^2 + 15y + 6 = 4y$

10. $6z^2 - 11z + 15 = 12z - 5$

11. $20v^2 + 3v = 5v^2 + 5v + 1$

12. $x^2 - 4x + 16 = 4x$

13. $9y^2 + 17y + 20 = 4 - 7y$

14. $7u^2 + 9u = 0$

15. $14x = 3x^2$

For problems 16 – 18 use factoring to solve the equation.

16. $3v^3 - 19v^2 - 14v = 0$

17. $y^6 + y^5 = 20y^4$

18. $z^4 + 2z^3 + z^2 = 0$

For problems 19 – 22 use factoring to solve the equation.

19. $1+\dfrac{2}{x-2}=\dfrac{12-x}{x^2+x-6}$

20. $\dfrac{t+1}{t+2}=\dfrac{4(t-5)}{t^2+2t}+\dfrac{4}{t}$

21. $\dfrac{w^2-1}{w+6}=\dfrac{5-5w}{w+6}-w$

22. $\dfrac{y-2}{y-9}+\dfrac{y^2-19y+34}{y^2-10y+9}=\dfrac{y-3}{y-1}$

For problems 23 – 31 use the Square Root Property to solve the equation.

23. $v^2-144=0$

24. $81x^2-25=0$

25. $4t^2+1=0$

26. $7y^2-3=0$

27. $14+2x^2=0$

28. $(3t-8)^2-16=0$

29. $(u+11)^2+6=0$

30. $4(2x-1)^2-36=0$

31. $(4-z)^2-121=0$

Section 2-6 : Quadratic Equations - Part II

For problems 1 – 6 complete the square.

1. $w^2 + 3w$

2. $x^2 - 10x$

3. $y^2 + 14y$

4. $3u^2 - 36u$

5. $2t^2 - 9t$

6. $18x - x^2$

For problems 7 – 16 solve the quadratic equation by completing the square.

7. $x^2 + 3x - 10 = 0$

8. $z^2 - 12z + 40 = 0$

9. $t^2 - 7t + 2 = 0$

10. $u^2 + 5u + 9 = 0$

11. $4x^2 - 4x + 5 = 0$

12. $16w^2 + 8w + 1 = 0$

13. $4y^2 - 24y + 29 = 0$

14. $81z^2 + 54z + 10 = 0$

15. $9t^2 - 12t - 14 = 0$

16. $5v^2 - 14v + 11 = 0$

For problems 17 – 26 use the quadratic formula to solve the quadratic equation.

17. $w^2 - 14w + 245 = 0$

18. $3t^2 + 20t + 31 = 0$

19. $6x + 61 + 18x^2 = 0$

20. $x^2 = 4x - 23$

21. $y^2 + 20y = 4y - 64$

22. $33 = 8z + z^2$

23. $2t^2 + 49 = 32t - 2t^2$

24. $40u + 25u^2 = 10u - 11$

25. $10x^2 - 10x = 4x^2 - 3x + 10$

26. $16z^2 + 4z - 40 = 140z + 19$

Section 2-7 : Quadratic Equations : A Summary

For problems 1 – 7 use the discriminant to determine the type of roots for the equation. Do not find any roots.

1. $25x^2 - 120x + 619 = 0$

2. $104x^2 - 75x - 14 = 0$

3. $2x^2 + 60x + 450 = 0$

4. $\frac{1}{6}x^2 - 43 = 0$

5. $97 + 136x + 289x^2 = 0$

6. $10x^2 - 7x = 0$

7. $\frac{49}{9}x^2 + \frac{14}{15}x + \frac{1}{25} = 0$

Section 2-8 : Applications of Quadratic Equations

1. The length of a rectangle is 4 feet more than the width. If the area of the rectangle is 136 ft^2 what are the dimensions of the rectangle?

2. The area of some rectangle is 35 in^2. Four times the width of this rectangle is the same as 3 inches more than twice the length. What are the dimensions of the rectangle?

3. The area of a triangle is 28 m^2 and the height of the triangle is 2 meters less than 5 times the base. What are the height and base of this triangle?

4. Two cars start out at the same spot. One car starts to drive north at 18 mph 5 hours before the second car starts driving to the east at 32 mph. How long after the first car starts driving does it take for the two cars to be 350 miles apart?

5. Two cars start out at the same point and at the same time one starts driving north while the other starts driving east at a speed that is 4 mph faster than the car driving north. Twelve hours after the cars start driving they are 600 miles apart. What was the speed of each car?

6. Two people can paint a house in 21 hours. Working individually one of the people can paint the house in 6 hours more than it takes the other person to paint the house. How long would it take each person working individually to paint the house?

Section 2-9 : Equations Reducible to Quadratic in Form

Solve each of the following equations.

1. $8x^6 + 215x^3 - 27 = 0$

2. $x^{\frac{4}{3}} - 13x^{\frac{2}{3}} + 36 = 0$

3. $32x^{-10} - 31x^{-5} - 1 = 0$

4. $x - 8\sqrt{x} + 15 = 0$

5. $x^{-\frac{1}{2}} - 13x^{-\frac{1}{4}} + 30 = 0$

6. $x^{-6} - 3x^{-3} - 28 = 0$

7. $x^{10} - 1024 = 0$

8. $x^4 - 8x^2 + 5 = 0$

9. $\frac{1}{x^4} + \frac{10}{x^2} + 22 = 0$

Section 2-10 : Equations with Radicals

Solve each of the following equations.

1. $x = \sqrt{4x-3}$

2. $2x = -\sqrt{3-x}$

3. $4 - \sqrt{x+6} = -x$

4. $x + 3 = \sqrt{11(x+3)}$

5. $x = 8 + \sqrt{22-3x}$

6. $2 - x = \sqrt{8-7x}$

7. $\sqrt{1+3x} = 4 + \sqrt{5-x}$

8. $\sqrt{x-3} + \sqrt{x+1} = 2$

Section 2-11 : Linear Inequalities

For problems 1 – 6 solve each of the following inequalities. Give the solution in both inequality and interval notations.

1. $7x+2(4-x)<12-3(5+6x)$

2. $10(3+w)\geq 9(2-4w)$

3. $2(4+5y)\leq 12y-6(1-3y)$

4. $2\left(\frac{1}{3}-\frac{1}{6}z\right)>\frac{1}{9}z+4\left(2-\frac{7}{18}z\right)$

5. $2\leq 2+4(3-x)\leq 6$

6. $-4<7x+8\leq 1$

7. $\frac{1}{2}<2\left(\frac{1}{4}+\frac{1}{8}t\right)<\frac{3}{4}$

8. $-12\leq 4-11m\leq 3$

9. $0\leq\frac{3}{7}-\frac{5}{14}x<\frac{1}{2}$

10. $-8<2(3+4x)-4(1+3x)\leq 3$

11. If $-7<x\leq 6$ determine *a* and *b* for the inequality : $a\leq 3x+8<b$

12. If $-3\leq x\leq -1$ determine *a* and *b* for the inequality : $a\leq 6-2x<b$

Section 2-12 : Polynomial Inequalities

Solve each of the following inequalities.

1. $z^2 - 11z + 24 < 0$

2. $2x^2 - 3 \geq 5x$

3. $t^2 > 30 - 7t$

4. $m^2 - 7m \leq 8$

5. $x^2 + 6x \geq -9$

6. $u^2 + u \leq 1$

7. $w^2 + 4w - 12 > 0$

8. $x^2 + 49 > 14x$

9. $t^2 \leq t$

10. $x^2 - 8x > -14$

11. $9u^2 - 6u + 1 < 0$

12. $z^6 + 8z^5 + 12z^4 \geq 0$

13. $2w^3 - 3w^2 > 14w$

Section 2-13 : Rational Inequalities

Solve each of the following inequalities.

1. $\dfrac{t+6}{t-1} < 0$

2. $\dfrac{4x+2}{3-x} \ge 0$

3. $\dfrac{2u+3}{u+6} > 0$

4. $\dfrac{3-z}{z+1} < -2$

5. $\dfrac{w+9}{w+2} \le 3$

6. $\dfrac{x^2+9x+14}{x-1} > 0$

7. $\dfrac{4z^2+3z-10}{z} \ge 0$

8. $\dfrac{t^2+10t+16}{t^2-4t+3} \le 0$

9. $\dfrac{z^2-6z+4}{z-5} < 4$

10. $w-5 \ge \dfrac{3-w^2}{w}$

11. $\dfrac{x^2+8x+16}{x} > 0$

12. $\dfrac{u-8}{3u^4-u^5} \le 0$

13. $\dfrac{2}{x^2-2x+1} \ge 0$

Section 2-14 : Absolute Value Equations

For problems 1 – 10 solve each of the equation.

1. $|2x+9|=7$

2. $|5w-2|=3$

3. $|6-7t|=10$

4. $2=\left|\frac{1}{4}m-\frac{1}{3}\right|$

5. $|8u+9|=9$

6. $|x+3|=4x+1$

7. $|2z-7|=3z-10$

8. $|3y+9|=10-y$

9. $|6w+12|=1+w$

10. $|8x+3|=0$

For problems 11 – 13 find all the real valued solutions to the equation.

11. $|x^2+1|=-4$

12. $|u^2-7u|=12$

13. $|z^2-6|=z$

Section 2-15 : Absolute Value Inequalities

Solve each of the following inequalities.

1. $|3x+1| \le 9$

2. $|10w-4| < 2$

3. $|8t-5| \le 0$

4. $|9-z| < 14$

5. $|2-7u| \le 20$

6. $|4x+2| < -1$

7. $|1-4z| > 1$

8. $|3w+15| \ge 4$

9. $|6t-10| > 12$

10. $|8-2x| \ge 5$

11. $|4u-1| > -1$