#418557

Topic: Equations of Ellipse

Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1.$

Solution

Given equation of ellipse is

$\frac{x^2}{16}+\frac{y^2}{9}=1$

On comparing the given equation with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, we get

$a=4, b=3$

Since, $a>b$,therefore the major axis is along the x-axis while the minor axis is along the y-axis.

Length of major axis $=2a=8$

Length of minor axis $=2b=6$

Eccentricity $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{\frac{16-9}{16}}==\frac{\sqrt{7}}{4}$

Coordinates of the foci are $(\sqrt{7},0)$ and $(-\sqrt{7},0)$

The coordinates of the vertices are $(4,0)$ and $(-4,0)$

Length of latus rectum = $\frac{2b^2}{a}=\frac{2\times 9}{4}=\frac{9}{2}$

#418867

Topic: Equations of Ellipse

Find the equation for the ellipse that satisfies the given condition:

Vertices $(\pm 5,0)$, foci $(\pm 4,0)$.

Solution

It is given that vertices $(\pm 5,0)$ foci $(\pm 4,0)$

Clearly, the vertices are on the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

where a is the semi-major axis and $a=5, ae=4$

$\Rightarrow e=\frac{4}{5}$, where e is eccentricity of the ellipse.

Also, we know that $b^2=a^2(1-e^2)=a^2-a^2e^2$

$\therefore b^2=5^2-4^2=9=3^2$

$\Rightarrow b=3$

Thus, the equation of the ellipse is $\frac{x^2}{5^2}+\frac{y^2}{3^2}=1$ or $\frac{x^2}{25}+\frac{y^2}{9}=1$

#418878

Topic: Equations of Ellipse

Find the equation for the ellipse that satisfies the given conditions: Vertices $(0,\pm 13)$, foci $(0,\pm 5)$

Solution

We have vertices $(0,\pm 13)$, foci $(0,\pm 5)$

Clearly here the vertices are on the y-axis

Therefore the equation of the ellipse will be of the form $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

where a is the semi-major axis and $\Rightarrow a=13, ae=5\Rightarrow e=\frac{5}{13}$

Now, we know that $b^2=a^2(1-e^2)=a^2-a^2e^2$

$\Rightarrow b^2=13^2-5^2=144\Rightarrow b=12$

Thus the equation of the ellipse is $\frac{x^2}{12^2}+\frac{y^2}{13^2}=1$ or $\frac{x^2}{144}+\frac{y^2}{169}=1$

#418915

Topic: Equations of Ellipse

Find the equation for the ellipse that satisfies the given conditions:

Vertices $(\pm 6, 0)$, foci $(\pm 4, 0)$.

Solution

It is given that, vertices $(\pm 6, 0)$, foci $(\pm 4, 0)$

Clearly, the vertices are on the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a is the semi-major axis.

Also, $a = 6, ae = 4 \Rightarrow e = \frac{2}{3}$

We know that $b^2 = a^2(1 - e^2) = a^2 - a^2e^2 = 36 - 16 = 20$

Thus, the equation of the ellipse is $\frac{x^2}{36} + \frac{y^2}{20} = 1$

#418925

Topic: Equations of Ellipse

Find the equation for the ellipse that satisfies the given conditions: Ends of major axis $(\pm 3, 0)$ ends of minor axis $(0, \pm 2)$.

Solution

It is given that, ends of major axis $(\pm 3, 0)$ and ends of minor axis $(0, \pm 2)$

Clearly, here the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a is the semi-major axis.

Accordingly $a = 3$ and $b = 2$

Thus, the equation of the ellipse is $\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1\, i.e.,\ \frac{x^2}{9} + \frac{y^2}{4} = 1$

#418939

Topic: Equations of Ellipse

Find the equation for the ellipse that satisfies the given conditions: Ends of major axis $(0, \pm\sqrt{5})$ ends of minor axis $(\pm 1, 0)$.

Solution

It is given that, ends if major axis $(0, \pm\sqrt{5})$ and ends of minor axis $(\pm 1, 0)$

Clearly, here the major axis is along the y-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ where a is the semi-major axis

Accordingly $a = \sqrt{5}$ and $b = 1$

Thus, the equation of the ellipse is $\frac{x^2}{1^2} + \frac{y^2}{(\sqrt{5})^2} = 1$

$\Rightarrow \frac{x^2}{1} + \frac{y^2}{5} = 1$

#418957

Topic: Equations of Ellipse

Find the equation for the ellipse that satisfies the given conditions: Length of major axis $= 26$, foci $(\pm 5, 0)$.

Solution

It is given that, length of major axis 26, foci $(\pm 5, 0)$.

Since, the foci are on the x-axis the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where a is the semi-major axis.

$\Rightarrow 2a = 26 \Rightarrow a = 13$ and $ae = 5$

We know that, $b^2 = a^2 - a^2e^2 = 169 - 25 = 144$

Thus, the equation of the ellipse is $\frac{x^2}{169} + \frac{y^2}{144} = 1$

#418964

Topic: Equations of Ellipse

Find the equation for the ellipse that given that satisfies the given conditions:

Length of minor axis $= 16$, foci $(0, \pm 6)$.

Solution

It is given that, length of minor axis $= 16$; foci = $(0, \pm 6)$

Since the foci are on the y-axis the major axis is along the y-axis

Therefore, the equation of the ellipse will be of the form $\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1$ where a is the semi-major axis

Accordingly $2b = 16, \Rightarrow b = 8$ and $ae = 6$

We know that, $a^2 = b^2 + a^2e^2$

$\therefore a^2 = 8^2 + 6^2 = 64 + 36 = 100$

$\Rightarrow a = \sqrt{100} = 10$

Thus, the equation of the ellipse is $\dfrac{x^2}{8^2} + \dfrac{y^2}{10^2} = 1$ or $\dfrac{x^2}{64} + \dfrac{y^2}{100} = 1$

#418972

Topic: Equations of Ellipse

Find the equation for the ellipse that satisfies the given conditions: Foci $(\pm 3, 0)$, $a = 4$.

Solution

It is given that, foci $(\pm 3, 0)$, $a = 4$

Since the foci are on the x-axis the major axis is along the$x$-axis.

Therefore, the equation of the ellipse will be of the form $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ where a is the semi-major axis

Accordingly $ae = 3$ and $a = 4$

We know that $a^2 = b^2 + a^2e^2$

$\therefore 4^2 = b^2 + 3^2$

$\Rightarrow 16 = b^2 + 9$

$\Rightarrow b^2 = 16 - 9 = 7$

Thus, the equation of the ellipse is $\dfrac{x^2}{16} + \dfrac{y^2}{7} = 1$

#418981

Topic: Equations of Ellipse

Find the equation for the ellipse that satisfies the given conditions: $b = 3, ae = 4$centre at the origin; foci on the x- axis.

Solution

It is given that $b = 3, ae = 4$ centre at the origin; foci on the x- axis.

Since the foci are on the x-axis $\Rightarrow$ the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ where a is the semi-major axis.

Accordingly $b = 3, ae = 4.$

It is known that $a^2 = b^2 + ae^2$

$\therefore a^2 = 3^2 + 4^2 = 9 + 16 = 25$

$\Rightarrow a = 5$

Thus, the equation of the ellipse is $\dfrac{x^2}{5^2} + \dfrac{y^2}{3^2} = 1$

$\Rightarrow \dfrac{x^2}{25} + \dfrac{y^2}{9} = 1.$

#419007

Topic: Equations of Ellipse

Find the equation for the ellipse that satisfies the given conditions: Centre at $(0, 0)$, major axis on the y-axis and passes through the points $(3, 2)$ and $(1, 6)$.

Solution

Since the centre is at $(0, 0)$ and the major axis is on the y-axi,s the equation of the ellipse will be of the form,

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \ldots (1)$$

where a is the semi-major axis.

The ellipse passes through points $(3, 2)$ and $(1, 6)$.

Hence, $\frac{9}{b^2} + \frac{4}{a^2} = 1 \ldots (2)$

$$\frac{1}{b^2} + \frac{36}{a^2} = 1 \ldots (3)$$

On solving equations (2) and (3), we obtain $b^2 = 10$ and $a^2 = 40$

Thus the equation of the ellipse is $\frac{x^2}{10} + \frac{y^2}{40} = 1$ or $4x^2 + y^2 = 40$

#419023

Topic: Equations of Ellipse

Find the equation for the ellipse that satisfies the given conditions: Major axis on the x-axis, centre is origin and passes through the points $(4, 3)$ and $(6, 2)$.

Solution

Since the major axis is on the x-axis, the equation of the ellipse will be of the form,

$$\frac{x^2}{a} + \frac{y^2}{b^2} = 1 \ldots (1)$$

Where a is the semi-major axis.

The ellipse passes through points $(4, 3)$ and $(6, 2)$. Hence,

$$\frac{16}{a^2} + \frac{9}{b^2} = 1 \ldots (2)$$

$$\frac{36}{a^2} + \frac{4}{b^2} = 1 \ldots (3)$$

On solving equations (2) and (3) we obtain $a^2 = 52 \text{ and } b^2 = 13$

Thus the equation of the ellipse is $\frac{x^2}{52} + \frac{y^2}{13} = 1$ or $x^2 + 4y^2 = 52$

#419555

Topic: Equations of Ellipse

An arch is in the form of a semi-ellipse. It is 8 m wide and 2 m high at the centre. Find the height of the arch at a point 1.5 m from one end.

Solution

Since the height and width of the arc from the centre is 2 m and 8 m respectively. It is clear that the length of the major axis is 8 m, while the length of the semi-minor axis is 2 r

The origin of the coordinate plane is taken as the centre of the ellipse while the major axis is taken along the x-axis. Hence the semi-ellipse can be diagrammatically represented as,

The equation of the semi-ellipse will be of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, y\geq 0$where a is the semi-major axis.

Accordingly, $2a=8$

$a=4, b=2$

Therefore, the equation of the semi-ellipse is $\frac{x^2}{16}+\frac{y^2}{4}=1, y\geq 0$ ...(1)

Let B be a point on the major axis such that $AB=1.5$ m

Draw $BC\perp OA$

$OB=(4-1.5)m=2.5\ m$

The x-coordinate of point C is $-2.5\ m$.

On substituting the value of x with -2.5 in equation (1), we obtain

$\frac{(-2.5)^2}{16}+\frac{y^2}{4}=1$

$\Rightarrow \frac{6.25}{16}+\frac{y^2}{4}=1$

$\Rightarrow y^2=4\left(1-\frac{6.25}{16}\right)$

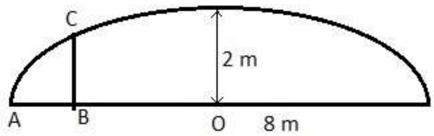
$\Rightarrow y^2=4\left(\frac{9.75}{16}\right)$

$\Rightarrow y^2=2.4375$

$\Rightarrow y=1.56$(approx) $(\because y\geq 0)$

$\therefore AC=1.56m$

Thus the height of the arch at a point 1.5 m from one end is approximately $1.56\ m$.



#419608

Topic: Equations of Ellipse

A man running a racecourse notes that the sum of the distance from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m. Find the equation of the posts traced by the man.

Solution

Let A and B be the positions of the two flag posts and $P(x, y)$ be the position of the man such that

$PA + PB = 10.$

We know that " If a point moves in a plane in such a way that the sum of its distance from two fixed points is constant then the path is an ellipse and this constant value is equa

the length of the major axis of the ellipse".

Therefore, the path traced by the man is an ellipse.

Length of the major axis $= 10m$

Points A and B are the foci.

Taking the origin of the coordinate plane as the centre of the ellipse while taking the major axis along the x-axis the ellipse.

The equation of then ellipse will be of the form $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ where a is the semi-major axis.

$2a = 10$

$\Rightarrow a = 5$

Distance between the foci $(2c) = 8$

$\Rightarrow c = ae = 4$

Since, $c = \sqrt{a^2 - b^2}$, we get

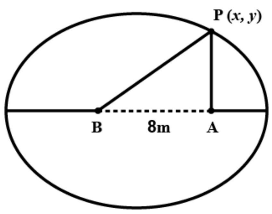
$4 = \sqrt{25 - b^2}$

$\Rightarrow 16 = 25 - b^2$

$\Rightarrow b^2 = 25 - 16 = 9$

$\Rightarrow b = 3$

So, the equation of the path traced by the man is $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$



#459609

Topic: Equation of Parabola

The point on the curve $x^2 = 2y$ which is nearest to the point $(0, 5)$ is

A $(2\sqrt{2}, 4)$

B $(2\sqrt{2}, 0)$

C $(0, 0)$

D $(2, 2)$

E $(2\sqrt{2}, 2)$

Solution

Point on a parabola can be taken to be $(2at, at^2)$.

Here $a = \dfrac{1}{2}$

So the point is $(t, \dfrac{t^2}{2})$

Distance of this point from $(0, 5)$ will be $\sqrt{t^2 + \left(\dfrac{t^2}{2} - 5\right)^2}$

Lets minnimise D^2 and we will get minimum distance automatically.

First derivative of D^2 is $t^3 - 8t$

Putting this to zero we get $t = 0$ and $t = \pm 2\sqrt{2}$

Now let's do double derivative, it comes out to be $3t^2 - 8$.

So, we can see that for $t = 0$, distance is going to be maximum.

So, distance is minimum from $(2\sqrt{2}, 4)$.

#459621

Topic: Equations of Ellipse

Find the maximum area of an isosceles triangle inscribed in the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ with its vertex at one end of the major axis.

Solution

Solution:

Consider the isosceles triangle ABC :

$A(a, 0), B(a\cos\theta, b\sin\theta)$ and $C(a\cos\theta, -b\sin\theta)$

Area of $\triangle ABC = \dfrac{1}{2} \times BC \times$ height of $\triangle ABC$

Height of $\triangle ABC = a(1 + \cos\theta)$

$BC = 2b\sin\theta$

or, $\triangle = \dfrac{1}{2} \times 2b\sin\theta \times a(1 + \cos\theta) = ab\sin\theta(1 + \cos\theta)$

For maximum area of the triangle,

$\dfrac{d\triangle}{d\theta} = ab\cos\theta(1 + \cos\theta) - absin^2\theta = 0$

or, $\cos\theta(1 + \cos\theta) - sin^2\theta = 0$

or, $\cos\theta(1 + \cos\theta) - (1 + \cos\theta)(1 - \cos\theta) = 0$

or, $(1 + \cos\theta)(\cos\theta - 1 + \cos\theta) = 0$

or, $(1 + \cos\theta)(2\cos\theta - 1) = 0$

or, $\cos\theta = -1$ or, $\cos\theta = \dfrac{1}{2}$

For maximum value, we take

$\cos\theta = \dfrac{1}{2}$ and $\sin\theta = \dfrac{\sqrt{3}}{2}$

or, Maximum area $= ab \times \dfrac{\sqrt{3}}{2}\left(1 + \dfrac{1}{2}\right) = \dfrac{3\sqrt{3}}{4}ab$