CALCULUS **DERIVATIVES AND LIMITS**

DERIVATIVE DEFINITION

$$\frac{d}{dx}\big(f(x)\big) = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

BASIC PROPERTIES

$$(cf(x))' = c(f'(x))$$
$$(f(x) \pm g(x))' = f'(x) \pm g'(x)$$
$$\frac{d}{dx}(c) = 0$$

MEAN VALUE THEOREM

If f is differentiable on the interval (a, b) and continuous at the end points there exists a c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

PRODUCT RULE

$$(f(x)g(x))' = f(x)'g(x) + f(x)g(x)'$$

QUOTIENT RULE

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

POWER RULE

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

CHAIN RULE

$$\frac{d}{dx}\Big(f\big(g(x)\big)\Big) = f'\big(g(x)\big)g'(x)$$

COMMON DERIVATIVES

COMMON DERIVATIVES
$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1 + x^2}$$

$$\frac{d}{dx}(a^x) = a^x \ln(a)$$

$$\frac{d}{dx}(a^{x}) = a^{x}$$

$$\frac{d}{dx}(e^{x}) = e^{x}$$

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, x > 0$$

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_a(x)) = \frac{1}{x \ln(a)}$$

LIMIT EVALUATION METHOD - FACTOR AND CANCEL

$$\lim_{x \to -3} \frac{x^2 - x - 12}{x^2 + 3x} = \lim_{x \to -3} \frac{(x+3)(x-4)}{x(x+3)} = \lim_{x \to -3} \frac{(x-4)}{x} = \frac{7}{3}$$

L'HOPITAL'S RULE

If
$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$$
 or $\frac{\pm \infty}{\pm \infty}$ then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

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CHAIN RULE AND OTHER EXAMPLES

$$\frac{d}{dx}([f(x)]^n) = n[f(x)]^{n-1}f'(x)$$

$$\frac{d}{dx}(e^{f(x)}) = f'(x)e^{f(x)}$$

$$\frac{d}{dx}(\ln[f(x)]) = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}(\sin[f(x)]) = f'(x)\cos[f(x)]$$

$$\frac{d}{dx}(\cos[f(x)]) = -f'(x)\sin[f(x)]$$

$$\frac{d}{dx}(\tan[f(x)]) = f'(x)\sec^2[f(x)]$$

$$\frac{d}{dx}(\sec[f(x)]) = f'(x)\sec[f(x)]\tan[f(x)]$$

$$\frac{d}{dx}(\tan^{-1}[f(x)]) = \frac{f'(x)}{1 + [f(x)]^2}$$

$$\frac{d}{dx}\left(f(x)^{g(x)}\right) = f(x)^{g(x)}\left(\frac{g(x)f'(x)}{f(x)} + \ln(f(x))g'(x)\right)$$

PROPERTIES OF LIMITS

These properties require that the limit of f(x) and g(x) exist

$$\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$$

$$\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$$

$$\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \lim_{x \to a} g(x)$$

$$\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \text{ if } \lim_{x \to a} g(x) \neq 0$$

$$\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$$

LIMIT EVALUATIONS AT +-∞

$$\lim_{x\to\infty} e^x = \infty$$
 and $\lim_{x\to-\infty} e^x = 0$

$$\lim_{x \to \infty} \ln(x) = \infty \text{ and } \lim_{x \to 0^+} \ln(x) = -\infty$$

If
$$r > 0$$
 then $\lim_{x \to \infty} \frac{c}{x^r} = 0$

If
$$r > 0$$
 & x^r is real for $x < 0$ then $\lim_{x \to -\infty} \frac{c}{x^r} = 0$

$$\lim_{x \to +\infty} x^r = \infty \text{ for even } r$$

$$\lim_{r \to \infty} x^r = \infty \& \lim_{r \to -\infty} x^r = -\infty \text{ for odd } r$$