#418041

Topic: Sets

Which of the following are sets ? Justify your answer.

(i) The collection of all the months of a year beginning with the letter J.

(ii) The collection of ten most talented writers of India.

(iii) A team of eleven best-cricket batsmen of the world.

(iv) The collection of all boys in your class.

(v) The collection of all natural numbers less than 100.

(vi) A collection of novels written by the writer Munshi Prem Chand.

(vii) The collection of all even integers.

(viii) The collection of questions in this Chapter.

(ix) A collection of most dangerous animals of the world

Solution

(i) The collection of all months of a year beginning with the letter J is a well-defined collection of objects because one can definitely identify a month that belongs to this collection.

Hence, this collection is a set.

(ii) The collection of ten most talented writers of India is not a well-defined collection because the criteria for determining a writer's talent may vary from person to person.

Hence, this collection is not a set.

(iii) A team of eleven best cricket batsmen of the world is not a well-defined collection because the criteria for determining a batsman's talent may vary from person to person.

Hence, this collection is not a set.

(iv) The collection of all boys in your class is a well-defined collection because, you can definitely identify a boy who belongs to this collection.

Hence, this collection is a set.

(v) The collection of all natural numbers less than 100 is a well-defined collection because one can definitely identify a number that belongs to this collection.

Hence, this collection is a set.

(vi) A collection of novels written by the writer Munshi Prem Chand is a well-defined collection because one can definitely identify a book that belongs to this collection.

Hence, this collection is a set.

(vii) The collection of all even integers is a well-defined collection because one can definitely identify an even integer that belongs to this collection.

Hence, this collection is a set.

(viii) The collection of questions in this chapter is a well-defined collection because one can definitely identify a question that belongs to this chapter.

Hence, this collection is a set.

(ix) The collection of most dangerous animals of the world is not a well-defined collection because that criteria for determining the dangerousness of an animal can vary from animal to animal.

Hence, this collection is not a set.

#418044

Topic: Sets

Let $A = \{1, 2, 3, 4, 5, 6\}$. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces:

(i) 5. . .A (ii) 8 . . . A (iii) 0. . .A (iv) 4. . . A (v) 2. . .A (vi) 10. . .A

Solution

If the element belongs to A we write $\in$,else $\notin$

(i) $5 \in A$, as 5 is the 5th element in set A.

(ii) $8 \notin A$, as 8 is not present in set A.

(iii) $0 \notin A$, as 0 is not present in set A.

(iv) $4 \in A$, as 4 is the 4th element in set A.

(v) $2 \in A$, as 2 is the 2nd element in set A.

(vi) $10 \notin A$ as 10 is not present in set A.

#418045

Topic: Sets

Write the following sets in roster form:

(i) $A = \{x : x$ is an integer and $-3 < x < 7\}$

(ii) $B = \{x : x$ is a natural number less than $6\}$

(iii) $C = \{x : x$ is a two-digit natural number such that the sum of its digits is $8\}$

(iv) $D = \{x : x$ is a prime number which is divisor of $60\}$

(v) $E =$ The set of all letters in the word TRIGONOMETRY

(vi) $F =$ The set of all letters in the word BETTER

Solution

(i) $A = \{x : x \text{ is an integer and } -3 < x < 7\}$

The elements of this set are $-2,\ -1, 0, 1, 2, 3, 4, 5,$ and 6 only.

Therefore, the given set can be written in roster form as

$A = \{-2,\ -1, 0, 1, 2, 3, 4, 5, 6\}$

(ii) $B = \{x : x \text{ is a natural number less than } 6\}$

The natural numbers less than 6 are $1, 2, 3, 4, 5$

So, the elements of this set are $1, 2, 3, 4,$ and 5 only.

Therefore, the given set can be written in roster from as

$B = \{1, 2, 3, 4, 5\}$

(iii) $C = \{x : x \text{ is a two-digit natural number such that the sum of its digits is } 8\}$

The elements of this set are $17, 26, 35, 44, 53, 62, 71$ and 80 only.

Therefore, this set can be written in roster form as

$C = \{17, 26, 35, 44, 53, 62, 71, 80\}$

(iv) $D = \{x : x \text{ is a prime number which is a divisor of } 60\}$

2 | 60

2 | 30

3 | 15

5 | 5

| 1

$\therefore 60 = 2 \times 2 \times 3 \times 5$

$\therefore$ The elements of this set are $2, 3,$ and 5 only.

Therefore, this set can be written in roster form as $D = \{2, 3, 5\}$.

(v) $E =$ The set of all letters in the word TRIGONOMETRY

There are 12 letters in the word TRIGONOMETRY, out of which the letters, T, R, and O are repeated. And we write the repeated letters once only.

Therefore, this set can be written in roster form as

$E = \{T, R, I, G, O, N, M, E, Y\}$

(vi) $F =$ The set of all letters in the word BETTER

There are 6 letters in the word BETTER, out of which letters E and T are repeated.

Therefore, this set can be written in roster form as

$F = \{B, E, T, R\}$

#418135
Topic: Sets

Write the following sets in the set-builder form

(i) $\{3, 6, 9, 12\}$

(ii) $\{2, 4, 8, 16, 32\}$

(iii) $\{5, 25, 125, 625\}$

(iv) $\{2, 4, 6, \ldots\}$

(v) $\{1, 4, 9, \ldots, 100\}$

Solution

(i) $\{3, 6, 9, 12\}$

$= \{x: x = 3n, n \epsilon N$ and $1 \leq n \leq 4\}$

Alternatively, $\{x: x$ is a multiple of 3 and $x \leq 12\}$

(ii) $\{2, 4, 8, 16, 32\}$

It can be seen that $2 = 2^1, 4 = 2^2, 8 = 2^3, 16 = 2^4$, and $32 = 2^5$.

$\therefore \{2, 4, 8, 16, 32\} = \{x: x = 2^n, n \epsilon N$ and $1 \leq n \leq 5\}$

(iii) $\{5, 25, 125, 625\}$

It can be seen that $5 = 5^1, 25 = 5^2, 125 = 5^3$, and $625 = 5^4$.

$\therefore \{5, 25, 125, 625\} = \{x: x = 5^n, n \epsilon N$ and $1 \leq n \leq 4\}$

(iv) $\{2, 4, 6, \ldots.\}$

It is a set of all even natural numbers.

$\therefore \{2, 4, 6, \ldots.\} = \{x: x$ is an even natural number$\}$

(v) $\{1, 4, 9, \ldots.. 100\}$

$\therefore \{1, 4, 9 \ldots 100\} = \{x: x = n^2, n \epsilon N$ and $1 \leq n \leq 10\}$

Alternatively, $\{x: x$ is a square of natural number and $x \leq 100\}$

#418173

Topic: Sets

List all the elements of the following sets:

(i) $A = \{x: x$ is an odd natural number$\}$

(ii) $B = \left\{x: x \text{ is an integer, } \frac{-1}{2} < x < \frac{9}{2}\right\}$

(iii) $C = \left\{x: x \text{ is an integer, } x^2 \leq 4\right\}$

(iv) $D = \{x: x$ is a letter in the word LOYAL$\}$

(v) $E = \{x: x$ is a month of a year not having 31 days$\}$

(vi) $F = \{x: x$ is a consonant in the English alphabet which precedes k$\}$

Solution

(i) $A = \{x : x \text{ is an odd natural number}\} = \{1, 3, 5, 7, 9.....\}$

(ii) $B = \left\{x : x \text{ is an integer}, -\frac{1}{2} < x < \frac{9}{2}\right\}$

Since, $-\frac{1}{2} = -0.5$ and $\frac{9}{2} = 4.5$

$\therefore B = \{0, 1, 2, 3, 4\}$

(iii) $C = \left\{x : x \text{ is an integer}; x^2 \leq 4\right\}$

It can be seen that

$(-1)^2 = 1 \leq 4$

$; (-2)^2 = 4 \leq 4$

$; (-3)^2 = 9 > 4$

$0^2 = 0 \leq 4$

$1^2 = 1 \leq 4$

$2^2 = 4 \leq 4$

$3^2 = 9 > 4$

$\therefore C = \{-2, -1, 0, 1, 2\}$

(iv) $D = \{x : x \text{ is a letter in the word LOYAL}\} = \{L, O, Y, A\}$

(v) $E = \{x : x$ is a month of a year not having 31 days$\}$

= {February, April, June, September, November}

(vi) $F = \{x : x$ is a consonant in the English alphabet which precedes k$\} = \{b, c, d, f, g, h, j\}$

#418174

Topic: Sets

Match each of the set on the left in the roster form with the same set on the right described in set-builder form:

(i) $\{1, 2, 3, 6\}$	(a) $\{x : x$ is a prime number and a divisor of 6$\}$
(ii) $\{2, 3\}$	(b) $\{x : x$ is an odd natural number less than 10$\}$
(iii) $\{M, A, T, H, E, I, C, S\}$	(c) $\{x : x$ is natural number and divisor of 6$\}$
(iv) $\{1, 3, 5, 7, 9\}$	(d) $\{x : x$ is a letter of the word MATHEMATICS$\}$

Solution

(i) All the elements of this set are natural number as well as the divisors of 6.

Therefore, (i) matches with (c).

(ii) It can be seen that 2 and 3 are prime numbers. They are also the divisors of 6.

Therefore, (ii) matches with (a).

(iii) All the elements of this set are letters of the word MATHEMATICS. Therefore, (iii) matches with (d).

(iv) All the elements of this set are odd natural numbers less than 10. Therefore, (iv) matches with (b).

#418274

Topic: Sets

Which of the following are examples of the null set?

(i) Set of odd natural numbers divisible by 2

(ii) Set of even prime numbers

(iii) $\{x : x$ is a natural numbers, $x < 5$ and $x > 7\}$

(iv) $\{y : y$ is a point common to any two parallel lines$\}$

Solution

(i) A set of odd natural numbers divisible by 2 is a null set because no odd number is divisible by 2.

(ii) $A = \{2\}$.

A set of even prime numbers is not a null set because 2 is an even prime number.

(iii) $\{x: x$ is a natural number, $x < 5$ and $x > 7\}$ is a null set because a number cannot be simultaneously less than 5 and greater than 7.

(iv) $\{y: y$ is a point common to any two parallel lines$\}$ is a null set because parallel lines do not intersect. Hence, they have no common point.

#418275

Topic: Sets

Which of the following sets are finite or infinite

(i) The set of months of a year

(ii) $\{1, 2, 3, \ldots\}$

(iii) $\{1, 2, 3, \ldots 99, 100\}$

(iv) The set of positive integers greater than 100

(v) The set of prime numbers less than 99

Solution

(i) The set of months of a year is a finite set because it has 12 elements.

(ii) $\{1, 2, 3 \ldots\}$ is an infinite set as it has infinite number of natural numbers.

(iii) $\{1, 2, 3, \ldots 99, 100\}$ is a finite set because the numbers from 1 to 100 are finite in number.

(iv) The set of positive integers greater than 100 is an infinite set because positive integers greater than 100 are infinite in number.

(v) The set of prime numbers less than 99 is a finite set because prime numbers less than 99 are finite in number.

#418277

Topic: Sets

State whether each of the following set is finite or infinite:

(i) The set of lines which are parallel to the x-axis

(ii) The set of letters in the English alphabet

(iii) The set of numbers which are multiple of 5

(iv) The set of animals living on the earth

(v) The set of circles passing through the origin (0,0)

Solution

(i) The set of lines which are parallel to the x-axis is an infinite set because lines parallel to the x-axis are infinite in number.

(ii) The set of letters in the English alphabet is a finite set because it has 26 elements.

(iii) The set of numbers which are multiple of 5 is an infinite set because multiples of 5 are infinite in number.

(iv) The set of animals living on the earth is a finite set because the number of animals living on the earth is finite (although it is quite a big number).

(v) The set of circles passing through the origin $(0, 0)$ is an infinite set because infinite number of circles can pass through the origin.

#418281

Topic: Types of Sets

In the following, state whether $A = B$ or not:

(i) $A = \{a, b, c, d\} B = \{d, c, b, a\}$

(ii) $A = \{4, 8, 12, 16\} B = \{8, 4, 16, 18\}$

(iii) $A = \{2, 4, 6, 8, 10\} B = \{x : x$ is a positive even integer and $x \leq 10\}$

(iv) $A = \{x : x$ is a multiple of $10\}$, $B = \{10, 15, 20, 25, 30, \ldots\}$

Solution

(i) $A = \{a, b, c, d\}$; $B = \{d, c, b, a\}$

The order in which the elements of a set are listed is not significant.

$\therefore A = B$

(ii) $A = \{4, 8, 12, 16\}$; $B = \{8, 4, 16, 18\}$

It can be seen that $12 \epsilon A$ but $12 \not\epsilon B$.

$\therefore A \neq B$

(iii) $A = \{2, 4, 6, 8, 10\}$

$B = \{x : x$ is a positive even integer and $x \leq 10\}$

$= \{2, 4, 6, 8, 10\}$

$\therefore A = B$

(iv) $A = \{x : x$ is a multiple of $10\}$

So, $A = \{10, 20, 30, 40....\}$

$B = \{10, 15, 20, 25, 30....\}$

It can be seen that $15 \epsilon B$ but $15 \not\epsilon A$.

$\therefore A \neq B$

#418282

Topic: Types of Sets

Are the following pair of sets equal ? Give reasons.

(i) $A = \{2, 3\}$, $B = \left\{x : x \text{ is solution of } x^2 + 5x + 6 = 0\right\}$

(ii) $A = \{x : x$ is a letter in the word FOLLOW$\}$

$B = \{y : y$ is a letter in the word WOLF$\}$

Solution

(i) $A = \{2, 3\}$

$B = \left\{x : x \text{ is a solution of } x^2 + 5x + 6 = 0\right\}$

$x^2 + 5x + 6 = 0$

$x(x + 3) + 2(x + 3) = 0$

$(x + 2)(x + 3) = 0$

$x = -2$ or $x = -3$

$\therefore B = \{-2, -3\}$

$\therefore A \neq B$

(ii) $A = \{x : x$ is a letter in the word $FOLLOW\} = \{F, O, L, W\}$

$B = \{y : y$ is a letter in the word WOLF$\} = \{W, O, L, F\}$

The order in which the elements of a set are listed is not significant.

$\therefore A = B$

#418373

Topic: Types of Sets

From the sets given below, select equal set:

$A = \{2, 4, 8, 12\}$,

$B = \{1, 2, 3, 4\}$,

$C = \{4, 8, 12, 14\}$,

$D = \{3, 1, 4, 2\}$

$E = \{-1, 1\}$,

$F = \{0, a\}$,

$G = \{-1, 1\}$,

$H = \{0, 1\}$

Solution

$A = \{2, 4, 8, 12\}$

$n(A) = 4$

$B = \{1, 2, 3, 4\}$

$n(B) = 4$

$C = \{4, 8, 12, 14\}$

$n(C) = 4$

$D = \{3, 1, 4, 2\}$

$n(D) = 4$

$E = \{-1, 1\}$

$n(E) = 2$

$F = \{0, a\}$

$n(F) = 2$

$G = \{1, -1\}$

$n(G) = 2$

$H = \{0, 1\}$

$n(H) = 2$

Number of elements in A,B,C,D are same i.e. all have 4 elements. So ,they are comparable.

Now, we see B and D has same elements.

So, B and D are equal sets.

Similarly, E,F,G, H are comparable as they all have same number of elements i.e. 2.

Clearly , E and G has same elements.

So, E and G are equal sets.

#418400

Topic: Subsets and Supersets

Make correct statements by filling in the symbols $\subset$ or $\not\subset$ in blank spaces:

(i) $\{2, 3, 4\}. \ldots \ldots \{1, 2, 3, 4, 5\}$

(ii) $\{a, b, c\}. \ldots \{b, c, d\}$

(iii) $\{x: x$ is a student of Class XI of your school$\}. \ldots \{x: x$ student of your school$\}$

(iv) $\{x: x$ is a circle in the plane$\} \ldots \{x: x$ is a circle in the same plane with radius 1 unit$\}$

(v) $\{x: x$ is a triangle in a plane$\} \ldots \{x: x$ is a rectangle in the plane$\}$

(vi) $\{x: x$ is an equilateral triangle in a plane$\} \ldots \{x: x$ is a triangle in the same plane$\}$

(vii) $\{x: x$ is an even natural number$\} \ldots \{x: x$ is an integer$\}$

Solution

(i) $\{2, 3, 4\} \subset \{1, 2, 3, 4, 5\}$

(ii) $\{a, b, c\} \not\subset \{b, c, d\}$

(iii) $\{x: x$ is a student of class XI of your school$\} \subset \{x: x$ is student of your school$\}$

(iv) $\{x: x$ is a circle in the plane$\} \not\subset \{x: x$ is a circle in the same plane with radius 1 unit$\}$

(v) $\{x: x$ is a triangle in a plane$\} \not\subset \{x: x$ is a rectangle in the plane$\}$

(vi) $\{x: x$ is an equilateral triangle in a plane$\} \subset \{x: x$ is a triangle in the same plane$\}$

(vii) $\{x: x$ is an even natural number$\} \subset \{x: x$ is an integer$\}$

#418406

Topic: Subsets and Supersets

Examine whether the following statements are true or false:

(i) $\{ a, b \} \not\subset \{ b, c, a \}$

(ii) $\{ a, e \} \subset \{ x: x$ is a vowel in the English alphabet$\}$

(iii) $\{ 1, 2, 3 \} \subset \{ 1, 3, 5 \}$

(iv) $\{ a \} \subset \{ a, b, c \}$

(v) $\{ a \} \epsilon \{ a, b, c \}$

(vi) $\{ x: x$ is an even natural number less than $6\} \subset \{ x$: x is a natural number which divides $36 \}$

Solution

(i) False. Each element of $\{a, b\}$ is also an element of $\{b, c, a\}$.

(ii) True. $\{a, e\}$ are two vowels of the English alphabet.

(iii) False. $2 \epsilon \{1, 2, 3\}$

However, $2 \notin \{1, 3, 5\}$

(iv) True. Each element of $\{a\}$ is also an element of $\{a, b, c\}$.

(v) False. The element of $\{a, b, c\}$ are a,b,c. Therefore, $a \subset a, b, c$

(vi) True. $\{x: x$is an even natural number less than $6\} = \{2, 4\}$

$\{x: x$is a natural number which divides $36\} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

Here, $\{2, 4\} \subset \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

#418414

Topic: Subsets and Supersets

Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following statements are incorrect and why?

(i) $\{3, 4\} \subset A$

(ii) $\{3, 4\} \epsilon A$

(iii) $\{\{3, 4\}\} \subset A$

(iv) $1 \epsilon A$

(v) $1 \subset A$

(vi) $\{1, 2, 5\} \subset A$

(vii) $\{1, 2, 5\} \epsilon A$

(viii) $\{1, 2, 3\} \subset A$

(ix) $\phi \epsilon A$

(x) $\phi \subset A$ (xi) $\{\phi\} \subset A$

Solution

Given $A = \{1, 2, \{3, 4\}, 5\}$

(i) The statement $\{3, 4\} \subset A$ is incorrect because $3 \in \{3, 4\}$; however, $3 \notin A$.

(ii) The statement $\{3, 4\} \in A$ is correct because $\{3, 4\}$ is an element of A.

(iii) The statement $\{\{3, 4\}\} \subset A$ is correct because $\{3, 4\} \in \{\{3, 4\}\}$ and $\{3, 4\} \in A$.

(iv) The statement $1 \in A$ is correct because 1 is an element of A.

(v) The statement $1 \subset A$ is incorrect because an element of a set can never be a subset of itself.

(vi) The statement $\{1, 2, 5\} \subset$ A is correct because each element of $\{1, 2, 5\}$ is also an element of A.

(vii) The statement $\{1, 2, 5\} \in A$ is incorrect because $\{1, 2, 5\}$ is not an element of A.

(viii) The statement $\{1, 2, 3\} \subset A$ is incorrect because $3 \in \{1, 2, 3\}$; however, $3 \notin A$.

(ix) The statement $\phi \in A$ is incorrect because ϕ is not an element of A.

(x) The statement $\phi \subset A$ is correct because ϕ is a subset of every set.

(xi) The statement $\{\phi\} \subset A$ is incorrect because $\phi \epsilon \{\phi\}$; however, $\phi \notin A$.

#418419

Topic: Subsets and Supersets

Write down all the subsets of the following sets

(i) $\{a\}$

(ii) $\{a, b\}$

(iii) $\{1, 2, 3\}$

(iv) ϕ

Solution

We know that number of subsets of any set is 2^n where n is the number of elements in that set.

Every non-empty set has two subsets ϕ and the set itself.

(i) The subsets of $\{a\}$ are ϕ and $\{a\}$.

(ii) The subsets of $\{a, b\}$ are $\phi, \{a\}, \{b\}, and \{a, b\}$.

(iii) The subsets of $\{1, 2, 3\}$ are $\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, and \{1, 2, 3\}$

(iv) The only subset of ϕ is ϕ.

#418426
Topic: Sets

Write the following as intervals :

(i) $\{x: x \epsilon R, 4 < x \leq 6\}$

(ii) $\{x: x \epsilon R, -12 < x < -10\}$

(iii) $\{x: x \epsilon R, 0 \leq x < 7\}$

(iv) $\{x: x \epsilon R, 3 \leq x \leq 4\}$

Solution

(i) $\{x: x\epsilon R, -4 < x \leq 6\} = (-4, 6]$

(ii) $\{x: x\epsilon R, -12 < x < -10\} = (-12, -10)$

(iii) $\{x: x\epsilon R, 0 \leq x < 7\} = [0, 7)$

(iv) $\{x: x\epsilon R, 3 \leq x \leq 4\} = [3, 4]$

#418432
Topic: Sets

Write the following intervals in set-builder form:

(i) (3, 0)

(ii) [6, 12]

(iii) (6, 12]

(iv) [23, 5)

Solution

(i) $(-3, 0) = \{x: x\epsilon R, -3 < x < 0\}$

(ii) $[6, 12] = \{x: x\epsilon R, 6 \leq x \leq 12\}$

(iii) $(6, 12] = \{x: x\epsilon R, 6 < x \leq 12\}$

(iv) $[-23, 5) = \{x: x\epsilon R, -23 \leq x < 5\}$

#418438
Topic: Subsets and Supersets

What universal set(s) would you propose for each of the following:

(i) The set of right triangles. (ii) The set of isosceles triangles

Solution

(i) For the set of right triangles, the universal set can be the set of triangles or the set of polygons.

(ii) For the set of isosceles triangles, the universal set can be the set of triangles or the set of polygons or the set of two-dimensional figures.

#418445
Topic: Subsets and Supersets

Given the sets A = $\{1, 3, 5\}$, B = $\{2, 4, 6\}$ and C = $\{0, 2, 4, 6, 8\}$, which of the following may be considered as universal set (s) for all the three sets A, B and C

(i) $\{0, 1, 2, 3, 4, 5, 6\}$

(ii) ϕ

(iii) $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(iv) $\{1, 2, 3, 4, 5, 6, 7, 8\}$

Solution

(i) It can be seen that $A \subset \{0, 1, 2, 3, 4, 5, 6\}$

$B \subset \{0, 1, 2, 3, 4, 5, 6\}$

However, $C \not\subset \{0, 1, 2, 3, 4, 5, 6\}$

Therefore, the set $\{0, 1, 2, 3, 4, 5, 6\}$ cannot be the universal set for the sets A, B and C.

(ii) $A \not\subset \phi, B \not\subset \phi, C \not\subset \phi$

Therefore, ϕ cannot be the universal set for the sets A, B, and C.

(iii) $A \subset \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$B \subset \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$C \subset \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Therefore, the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is the universal set for the sets A, B and C.

(iv) $A \subset \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B \subset \{1, 2, 3, 4, 5, 6, 7, 8\}$

However, $C \not\subset \{1, 2, 3, 4, 5, 6, 7, 8\}$

Therefore, the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ cannot be the universal set for the sets A, B, and C.

#418970

Topic: Operations on Sets

Find the union of each of the following pairs of sets:

(i) $X = \{1, 3, 5\} Y = \{1, 2, 3\}$

(ii) $A = \{a, e, i, o, u\}, B = \{a, b, c\}$

(iii) $A = \{x: x$ is a natural number and multiple of $3\}$.

$B = \{x: x$ is a natural number less than $6\}$.

(iv) $A = \{x: x$ is a natural number and $1 < x \leq 6\}$

$B = \{x: x$ is a natural number and $6 < x < 10\}$.

(v) $A = \{1, 2, 3\}, B = \phi$

Solution

(i) $X = \{1, 3, 5\} Y = \{1, 2, 3\}$

$X \cup Y = \{1, 2, 3, 5\}$

(ii) $A = \{a, e, i, o, u\} B = \{a, b, c\}$

$A \cup B = \{a, b, c, e, i, o, u\}$

(iii) $A = \{x : x$ is a natural number and multiple of 3$\}$

$\Rightarrow A = \{3, 6, 9, 12, 15. \dots . \}$

$B = \{x : x$is a natural number less than 6$\}$

$\Rightarrow B = \{1, 2, 3, 4, 5, 6\}$

$\therefore A \cup B = \{x : x = 1, 2, 4, 5$ or a multiple of 3$\}$

(iv) $A = \{x : x$ is a natural number and $1 < x \leq 6\}$

$A = \{2, 3, 4, 5, 6\}$

$B = \{x : x$ is a natural number and $6 < x < 10\}$

$B = \{7, 8, 9\}$

$A \cup B = \{x : x \epsilon N$ and $1 < x < 10\}$

(v) $A = \{1, 2, 3\}, B = \phi$

$A \cup B = \{1, 2, 3\}$

#418975
Topic: Operations on Sets

Let $A = \{a, b\}, B = \{a, b, c\}$. Is $A \subset B$? What is $A \cup B$?

Solution

Here, $A = \{a, b\}$ and $B = \{a, b, c\}$

Since, every element of set A is in set B. So, $A \subset B$

$A \cup B = \{a, b, c\} = B$

#418977
Topic: Operations on Sets

If A and B are two sets such that $A \subset B$, then what is $A \cup B$?

Solution

Let $x \in A \cup B$

$\Rightarrow x \in A or x \in B$

Since, $A \subset B$

$\Rightarrow x \in B$

But x is an arbitrary element of set $A \cup B$.

$A \cup B \subset B$ ....(1)

Let $y \in B$

$\Rightarrow y \in A \cup B$

But y is an arbitrary element of set B

$B \subset A \cup B$ ....(2)

From (1) and (2), we get

$A \cup B = B$

#418983

Topic: Operations on Sets

If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$; find

(i) $A \cup B$

(ii) $A \cup C$

(iii) $B \cup C$

(iv) $B \cup D$

(v) $A \cup B \cup C$

(vi) $A \cup B \cup D$

(vii) $B \cup C \cup D$

Solution

$A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$

(i) $A \cup B = \{1, 2, 3, 4, 5, 6\}$

(ii) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(iii) $B \cup C = \{3, 4, 5, 6, 7, 8\}$

(iv) $B \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

(v) $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

(vi) $A \cup B \cup D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(vii) $B \cup C \cup D = \{3, 4, 5, 6, 7, 8, 9, 10\}$

#419036

Topic: Operations on Sets

Find the intersection of each pair of sets:

(i) X={1, 3, 5} Y={1, 2, 3}

(ii) A={a, e, i, o, u} B={a, b, c}

(iii) A={x: x is a natural number and multiple of 3}

B={x: x is a natural number less than 6}

(iv) A={x:x is a natural number and $1 < x \leq 6$}

B={x:x is a natural number and 6 < x < 10}

(v) A={1,2,3}, $B = \phi$

Solution

(i) $X = \{1, 3, 4\}$, $Y = \{1, 2, 3\}$

$X \cap Y = \{1, 3\}$

(ii) $A = \{a, e, i, o, u\}$, $B = \{a, b, c\}$

$A \cap B = \{a\}$

(iii) $A = \{x: x$ is a natural number and multiple of $3\} = \{3, 6, 9. \dots\}$

$B = \{x: x$ is a natural number less than $6\} = \{1, 2, 3, 4, 5\}$

$\therefore A \cap B = \{3\}$

(iv) $A = \{x: x$ is a natural number and $1 < x \leq 6\} = \{2, 3, 4, 5, 6\}$

$B = \{x: x$ is a natural number and $6 < x < 10\} = \{7, 8, 9\}$

$A \cap B = \phi$

(v) $A = \{1, 2, 3\}$, $B = \phi$

$A \cap B = \phi$

#419043

Topic: Operations on Sets

If $A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ and $D = \{15, 17\}$; find

(i) $A \cap B$

(ii) $B \cap C$

(iii) $A \cap C \cap D$

(iv) $A \cap C$

(v) $B \cap D$

(vi) $A \cap (B \cup C)$

(vii) $A \cap D$

(viii) $A \cap (B \cup D)$

(ix) $(A \cap B) \cap (B \cap C)$

(x) $(A \cup D) \cap (B \cup C)$

Solution

$A = \{3, 5, 7, 9, 11\}$, $B = \{7, 9, 11, 13\}$, $C = \{11, 13, 15\}$ and $D = \{15, 17\}$

(i) $A \cap B = \{7, 9, 11\}$

(ii) $B \cap C = \{11, 13\}$

(iii) $A \cap C \cap D = (A \cap C) \cap D = \{11\} \cap \{15, 17\} = \phi$

(iv) $A \cap C = \{11\}$

(v) $B \cap D = \phi$

(vi) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$= \{7, 9, 11\} \cup \{11\} = \{7, 9, 11\}$

(vii) $A \cap D = \phi$

(viii) $A \cap (B \cup D) = (A \cap B) \cup (A \cap D)$

$= \{7, 9, 11\} \cup \phi = \{7, 9, 11\}$

(ix) $(A \cap B) \cap (B \cup C) = \{7, 9, 11\} \cap \{7, 9, 11, 13, 15\} = \{7, 9, 11\}$

(x) $(A \cup D) \cap (B \cup C) = \{3, 5, 7, 9, 11, 15, 17\} \cap \{7, 9, 11, 13, 15\}$

$= \{7, 9, 11, 15\}$

#419052

Topic: Operations on Sets

If $A = \{x : x$ is a natural number $\}$, $B = \{x : x$ is an even natural number$\}$ $C = \{x : x$ is an odd natural number$\}$ and $D = \{x : x$ is a prime number $\}$, find

(i) $A \cap B$

(ii) $A \cap C$

(iii) $A \cap D$

(iv) $B \cap C$

(v) $B \cap D$

(vi) $C \cap D$

Solution

A = {x: $x$is a natural number} = {1, 2, 3, 4, 5.....}

B = {x: x is an even natural number} = {2, 4, 6, 8. . . . . }

C = {x: x is an odd natural number} = {1, 3, 5, 7, 9. . . . . . . }

D = {x: $x$is a prime number} = {2, 3, 5, 7. . . . }

(i) $A \cap B$ = {x: x is a even natural number} = B

(ii) $A \cap C$ = {x: x is an odd natural number} = C

(iii) $A \cap D$ = {x: x is a prime number} = D

(iv) $B \cap C = \phi$

(v) $B \cap D$ = {2}

(vi) $C \cap D$ = {x: $x$is odd prime number}

#419057

Topic: Types of Sets

Which of the following pairs of sets are disjoint

(i) {1, 2, 3, 4} and {x: x is a natural number and $4 \leq x \leq 6$}

(ii) { a, e, i, o, u } and { c, d, e, f }

(iii) {x: x is an even integer } and {x: x is an odd integer}

Solution

(i) A = {1, 2, 3, 4}

B = {x: x is a natural number and $4 \leq x \leq 6$} = {4, 5, 6}

Now, $A \cap B$ = {1, 2, 3, 4} $\cap$ {4, 5, 6} = {4}

Therefore, this pair of sets are not disjoint.

(ii) A = {a, e, i, o, u}

B = {c, d, e, f}

$A \cap B$ = {e}

Therefore, this pair of sets are not disjoint.

(iii) A = {x: x is an even integer}

B = {x: $x$is an odd integer}

$A \cap B = \phi$

Therefore, this pair of sets is disjoint.

#419062

Topic: Operations on Sets

If A = {3, 6, 9, 12, 15, 18, 21}, B = {4, 8, 12, 16, 20},

C = {2, 4, 6, 8, 10, 12, 14, 16}, D = {5, 10, 15, 20}; find

(i) $A - B$

(ii) $A - C$

(iii) $A - D$

(iv) $B - A$

(v) $C - A$

(vi) $D - A$

(vii) $B - C$

(viii) $B - D$

(ix) $C - B$

(x) $D - B$

(xi) $C - D$

(xii) $D - C$

Solution

A = {3, 6, 9, 12, 15, 18, 21}

B = {4, 8, 12, 16, 20}

C = {2, 4, 6, 8, 10, 12, 14, 16}

D = {5, 10, 15, 20}

(i) $A - B$ = {3, 6, 9, 15, 18, 21}

(ii) $A - C$ = {3, 9, 15, 18, 21}

(iii) $A - D$ = {3, 6, 9, 12, 18, 21}

(iv) $B - A$ = {4, 8, 16, 20}

(v) $C - A$ = {2, 4, 8, 10, 14, 16}

(vi) $D - A$ = {5, 10, 20}

(vii) $B - C$ = {20}

(viii) $B - D$ = {4, 8, 12, 16}

(ix) $C - B$ = {2, 6, 10, 14}

(x) $D - B$ = {5, 10, 15}

(xi) $C - D$ = {2, 4, 6, 8, 12, 14, 16}

(xii) $D - C$ = {5, 15, 20}

#419064

Topic: Operations on Sets

If X = {a, b, c, d} and Y = {f, b, d, g}, find

(i) $X - Y$ (ii) $Y - X$ (iii) $X \cap Y$

Solution

X = {a, b, c, d}

Y = {f, b, d, g}

(i) $X - Y$ = {a, c}

(ii) $Y - X$ = {f, g}

(iii) $X \cap Y$ = {b, d}

#419066

Topic: Operations on Sets

If R is the set of real numbers and Q is the set of rational numbers, then what is $R - Q$?

Solution

R: Set of real numbers i.e it has both rational and irrational numbers.

Q: Set of rational numbers

Therefore, $R - Q$ is a set of irrational numbers.

#419075
Topic: Types of Sets

State whether each of the following statement is true or false. Justify your answer.

(i) $\{2, 3, 4, 5\}$ and $\{3, 6\}$ are disjoint sets.

(ii) $\{a, e, i, o, u\}$ and $\{a, b, c, d\}$ are disjoint sets

(iii) $\{2, 6, 10, 14\}$ and $\{3, 7, 11, 15\}$ are disjoint sets

(iv) $\{2, 6, 10\}$ and $\{3, 7, 11\}$ are disjoint sets

Solution

(i) False

$A = \{2, 3, 4, 5\}$ $B = \{3, 6\}$

$\Rightarrow A \cap B = \{3\}$

So, A and B are not disjoint sets.

(ii) False

$A = \{a, e, i, o, u\}$, $B = \{a, b, c, d\}$

$A \cap B = \{a\}$

Hence, A and B are not disjoint sets.

(iii) True

$A = \{2, 6, 10, 14\}$,$B = \{3, 7, 11, 15\}$

$A \cap B = \phi$

Hence, A and B are disjoint sets.

(iv) True

$A = \{2, 6, 10\}$,$B = \{3, 7, 11\}$

$A \cap B = \phi$

Hence, A and B are disjoint sets.

#419629
Topic: Operations on Sets

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$. Find (i) A' (ii) B' (iii) $(A \cup C)'$ (iv) $(A \cup B)'$ (v) $(A')'$ (vi) $(B - C)'$

Solution

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 2, 3, 4\}$

$B = \{2, 4, 6, 8\}$

$C = \{3, 4, 5, 6\}$

(i) $A' = U - A = \{5, 6, 7, 8, 9\}$

(ii) $B' = \{1, 3, 5, 7, 9\}$

(iii) $A \cup C = \{1, 2, 3, 4, 5, 6\}$

$\therefore (A \cup C)' = \{7, 8, 9\}$

(iv) $A \cup B = \{1, 2, 3, 4, 6, 8\}$

$(A \cup B)' = \{5, 7, 9\}$

(v) We know $(A')' = A$

$\Rightarrow (A')' = \{1, 2, 3, 4\}$

(vi) $B - C = \{2, 8\}$

$\therefore (B - C)' = \{1, 3, 4, 5, 6, 7, 9\}$

#419633

Topic: Operations on Sets

If $U = \{a, b, c, d, e, f, g, h\}$, find the complements of the following sets :

(i) $A = \{a, b, c\}$

(ii) $B = \{d, e, f, g\}$

(iii) $C = \{a, c, e, g\}$

(iv) $D = \{f, g, h, a\}$

Solution

We know that complement of set A is

$A' = U - A$

Given $U = \{a, b, c, d, e, f, g, h\}$

(i) $A = \{a, b, c\}$

$A' = \{d, e, f, g, h\}$

(ii) $B = \{d, e, f, g\}$

$\therefore B' = \{a, b, c, h\}$

(iii) $C = \{a, c, e, g\}$

$\therefore C' = \{b, d, f, h\}$

(iv) $D = \{f, g, h, a\}$

$\therefore D' = \{b, c, d, e\}$

#419644

Topic: Operations on Sets

Taking the set of natural numbers as the universal set, write down the complements of the following sets:

(i) $\{x: x$ is an even natural number$\}$

(ii) $\{x: x$ is an odd natural number$\}$

(iii) $\{x: x$ is a positive multiple of $3\}$

(iv) $\{\ x: x$ is a prime number $\}$

(v) $\{x: x$ is a natural number divisible by 3 and $5\}$

(vi) $\{x: x$ is a perfect square $\}$

(vii) $\{\ x: x$ is a perfect cube$\}$

(viii) $\{x: x+5=8\}$

(ix) $\{x: 2x+5=9\}$

(x) $\{x: x \geq 7\}$

(xi) $\{x: x \epsilon N \ and \ 2x+1>10\}$

Solution

U= N: Set of natural numbers

(i) $\{x: x$is an even natural number$\}'$ = $\{x: x$ is an odd natural number$\}$

(ii) $\{x: x$is an odd natural number$\}'$ = $\{x: x$is an even natural number$\}$

(iii) $\{x: x$ is a positive multiple of $3\}'$ = $\{x: x \in N$ and x is not a multiple of 3$\}$

(iv) $\{x: x$is a prime number$\}'$ = $\{x: x$is a positive composite number and x=1$\}$

(v) $\{x: x$is a natural number divisible by 3 and $5\}'$

= $\{x: x$is a natural number that is not divisible by 3 or 5$\}$

(vi) $\{x: x$is a perfect square$\}'$ = $\{x: x \in N$and x is not a perfect square$\}$

(vii) $\{x: x$is a perfect cube$\}'$ = $\{x: x \in N$and x is not a perfect cube$\}$

(viii) $\{x: x+5=8\}' = \{x: x \in N \text{and } x \neq 3\}$

(ix) $\{x: 2x+5=9\}' = \{x: x \in N \text{and} x \neq 2\}$

(x) $\{x: x \geq 7\}' = \{x: x \in N \text{and} x < 7\}$

(xi) $\{x: x \epsilon N \text{and} 2x+1>10\}' = \left\{x: x \in \ N and x \leq \frac{9}{2}\right\}$

#419649
Topic: Operations on Sets

If $U=\{1,2,3,4,5,6,7,8,9\}$, $A=\{2,4,6,8\}$ and $B=\{2,3,5,7\}$. Verify that

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Solution

$U=\{1,2,3,4,5,6,7,8,9\}$

$A=\{2,4,6,8\}$

$B=\{2,3,5,7\}$

(i) $(A \cup B)' = \{2,3,4,5,6,7,8\}' = \{1,9\}$

$A' \cap B' = \{1,3,5,7,9\} \cap \{1,4,6,8,9\} = \{1,9\}$

$\therefore (A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = \{2\}' = \{1,3,4,5,6,7,8,9\}$

$A' \cup B' = \{1,3,5,7,9\} \cup \{1,4,6,8,9\} = \{1,3,4,5,6,7,8,9\}$

$\therefore (A \cap B)' = A' \cup B'$

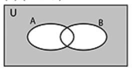
#419650
Topic: Venn Diagrams

Draw appropriate Venn diagram for each of the following:

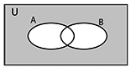
(i) $(A \cup B)'$, (ii) $A' \cap B'$, (iii) $(A \cap B)'$, (iv) $A' \cup B'$

Solution

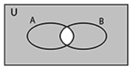
(i) (A ∪ B)'



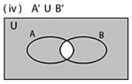
(ii) A' ∩ B'



(iii) (A ∩ B)'



(iv) A' ∪ B'



#419651

Topic: Sets

Let U be the set of all triangles in a plane. If A is the set of all triangles with at least one angle different from 60^o, what is A'?

Solution

U- Set of all triangles in a plane

A- Set of triangles with at least one angle different from 60^0 i.e. the set cannot have equilateral triangles

$A' = U - A$

So, A' is the set of all equilateral triangles.

#419652

Topic: Operations on Sets

Fill in the blanks to make each of the following a true statement:

(i) $A \cup A' = \ldots$ (ii) $\phi' \cap A = \ldots$ (iii) $A \cap A' = \ldots$ (iv) $U' \cap A = \ldots$

Solution

(i) It can be clearly seen from the Venn diagram that

$A \cup A' = U$

(ii) $\phi' \cap A = U \cap A = A$

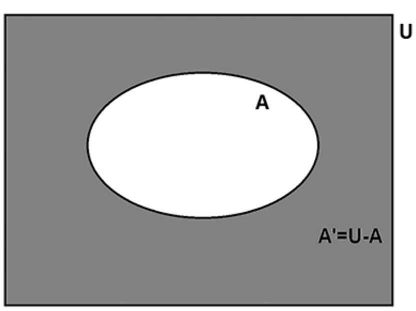
$\therefore \phi' \cap A = A$

(iii) It can be clearly seen from the Venn diagram , there is no common portion between A and A' .

Hence, $A \cap A' = \phi$

(iv) $U' \cap A = \phi \cap A = \phi$

$\therefore U' \cap A = \phi$



#419667

Topic: Sets

If X and Y are two sets such that $n(X) = 17$, $n(Y) = 23$ and $n(X \cup Y) = 38$, find $n(X \cap Y)$

Solution

It is given that:

$n(X) = 17, n(Y) = 23, n(X \cup Y) = 38$

$n(X \cap Y) = ?$

Using the formula

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$\therefore 38 = 17 + 23 - n(X \cap Y)$

$\Rightarrow n(X \cap Y) = 40 - 38 = 2$

$\therefore n(X \cap Y) = 2$

#419668

Topic: Sets

If X and Y are two sets such that $X \cup Y$ has 18 elements, X has 8 elements and Y has 15 elements; how many elements does $X \cap Y$ have?

Solution

It is given that:

$n(X \cup Y) = 18, n(X) = 8, n(Y) = 15$

$n(X \cap Y) = ?$

We know that:

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$\therefore 18 = 8 + 15 - n(X \cap Y)$

$\Rightarrow n(X \cap Y) = 23 - 18 = 5$

$\therefore n(X \cap Y) = 5$

#419671

Topic: Sets

In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?

Solution

Let H be the set of people who speak Hindi, and E be the set of people who speak English

$\therefore n(H \cup E) = 400, n(H) = 250, n(E) = 200$

$n(H \cap E) = ?$

We know that:

$n(H \cup E) = n(H) + n(E) - n(H \cap E)$

$\therefore 400 = 250 + 200 - n(H \cap E)$

$\Rightarrow 400 = 450 - n(H \cap E)$

$\Rightarrow n(H \cap E) = 450 - 400$

$\therefore n(H \cap E) = 50$

Thus, 50 people can speak both Hindi and English.

#419676

Topic: Sets

If S and T are two sets such that S has 21 elements, T has 32 elements, and $S \cap T$ has 11 elements, how many elements does $S \cup T$ have?

Solution

Given

$n(S) = 21, n(T) = 32, n(S \cap T) = 11$

We know that:

$n(S \cup T) = n(S) + n(T) - n(S \cap T)$

$\therefore n(S \cup T) = 21 + 32 - 11 = 42$

Thus, the set $(S \cup T)$ has 42 elements.

#419679

Topic: Sets

If X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, how many elements does Y have?

Solution

Given

$n(X) = 40, n(X \cup Y) = 60, n(X \cap Y) = 10$

We know that:

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$\therefore 60 = 40 + \; + n(Y) - 10$

$\therefore n(Y) = 60 - (40 - 10) = 30$

Thus, the set Y has 30 elements.

#419681

Topic: Sets

In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea?

Solution

Let C denote the set of people who like coffee, and T denote the set of people who like tea

$n(C \cup T) = 70, n(C) = 37, n(T) = 52$

We know that:

$n(C \cup T) = n(C) + n(T) - n(C \cap T)$

$\therefore 70 = 37 + 52 - n(C \cap T)$

$\Rightarrow 70 = 89 - n(C \cap T)$

$\Rightarrow n(C \cap T) = 89 - 70 = 19$

Thus, 19 people like both coffee and tea

#419683

Topic: Sets

In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

Solution

Let C denote the set the people like cricket, and T denote the set of people who like tennis

$\therefore n(C \cup T) = 65, n(C) = 40, n(C \cap T) = 10$

We know that

$n(C \cup T) = n(C) + n(T) - n(C \cap T)$

$\therefore 65 = 40 + n(T) - 10$

$\Rightarrow 65 = 30 + n(T)$

$\Rightarrow n(T) = 65 - 30 = 35$

Therefore, 35 people like tennis.

Now,

$n(T - C) = n(T) - n(T \cap C)$

$\Rightarrow n(T - C) = 35 - 10 = 25$

Thus, 25 people like only tennis.

#419684

Topic: Sets

In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?

Solution

Let F be the set of people in the committee who speak French, and S be the set of people in the committee who speak Spanish.

$\therefore n(F) = 50, n(S) = 20, n(S \cap F) = 10$

We know that:

$n(S \cup F) = n(S) + n(F) - n(S \cap F)$

$= 20 + 50 - 10$

$= 70 - 10 = 60$

Thus, 60 people in the committee speak at least one of the two languages.

#419686

Topic: Subsets and Supersets

Decide, among the following sets, which sets are subsets of one and another:

$A = \{x: x \in$ R and x satisfy $x^2 - 8x + 12 = 0\}$,

$B = \{2, 4, 6\}$,

$C = \{2, 4, 6, 8, \ldots\}$, $D = \{6\}$

Solution

$A = \left\{x: x \in R \text{and x satisfies} x^2 - 8x + 12 = 0\right\}$

2 and 6 are the only solutions of $x^2 - 8x + 12 = 0$.

$\therefore A = \{2, 6\}$

$B = \{2, 4, 6\}$

$C = \{2, 4, 6, 8, \ldots.\}$

$D = \{6\}$

Clearly, $D \subset A \subset B \subset C$

Hence, $A \subset B, A \subset C, B \subset C, D \subset A, D \subset B, D \subset C$

#419693

Topic: Subsets and Supersets

In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example.

(i) If $x \in A$ and $A \in B$, then $x \in B$

(ii) If $A \subset B$ and $B \in C$, then $A \in C$

(iii) If $A \subset B$ and $B \subset C$, then AC

(iv) If $A \not\subset B$ and $B \not\subset C$, then $A \not\subset C$

(v) If $x \in A$ and $A \not\subset B$, then $x \in B$

(vi) If $A \subset B$ and $x \notin B$, then $x \notin A$

Solution

(i) False

Let $A = \{1, 2\}$ and $B = \{1, \{1, 2\}, \{3\}\}$

$2 \in \{1, 2\}$ and $\{1, 2\} \in \{\{3\}, 1, \{1, 2\}\}$

Now,

$\therefore A \in B$

However, $2 \notin \{\{3\}, 1, \{1, 2\}\}$

(ii) False.

As $A \subset B, B \in C$

Let $A = \{2\}, B = \{0, 2\}$, and $C = \{1, \{0, 2\}, 3\}$

However, $A \notin C$

(iii) True

Let $A \subset B$ and $B \subset C$.

Let $x \in A$

$\Rightarrow x \in B \quad [\because A \subset B]$

$\Rightarrow x \in C \quad [\because B \subset C]$

$\therefore A \subset C$

(iv) False

As, $A \notin B$ and $B \notin C$

Let $A = \{1, 2\}, B = \{0, 6, 8\}$, and $C = \{0, 1, 2, 6, 9\}$

However, $A \subset C$

(v) False

Let $A = \{3, 5, 7\}$ and $B = \{3, 4, 6\}$

Now, $5 \in A$ and $A \notin B$

However, $5 \notin B$

(vi) True

Let $A \subset B$ and $x \notin B$.

To show: $x \notin A$

If possible, suppose $x \in A$.

Then, $x \in B$, which is a contradiction as $x \notin B$

$\therefore x \notin A$

#419695

Topic: Operations on Sets

Let $A, B,$ and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that $B = C$

Solution

Let, A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$.

To show: $B = C$

Let $x \in B$

$\Rightarrow x \in A \cup B$ (By def of union of sets)

$\Rightarrow x \in A \cup C$ $(A \cup B = A \cup C)$

$\Rightarrow x \in A$ or $x \in C$

Case I

$x \in A$

Also, $x \in B$

$\therefore x \in A \cap B$

$\Rightarrow x \in A \cap C$ $(\because A \cap B = A \cap C)$

$\therefore x \in A$ and $x \in C$

$\therefore x \in C$

But x is an arbitrary element in B.

$\therefore B \subset C$ .....(1)

Now, we will show that $C \subset B$.

Let $y \in C$

$\Rightarrow y \in A \cup C$ (by def of union of sets)

$\Rightarrow y \in A \cup B$ $(A \cup B = A \cup C)$

$\Rightarrow y \in A$ or $y \in B$

Case I : When $y \in A$

Also, $y \in C$

$\Rightarrow y \in A \cap C$

$\Rightarrow y \in A \cap B$

$\Rightarrow y \in A$ and $y \in B$

$\Rightarrow y \in B$

But y is an arbitrary element of C.

Hence, $C \subset B$ .....(2)

From (1) and (2), w eget

$\therefore B = C$.

#419699

Topic: Types of Sets

Show that the following four conditions are equivalent:

(i) $A \subset B$ (ii) $A - B = \phi$ (iii) $A \cup B = B$ (iv) $A \cap B = A$

Solution

First, we have to show that (i) $\Leftrightarrow$ (ii).

$(i) \Rightarrow$ (ii).

Let $A \subset B$

To show: $A - B = \phi$

If possible, suppose $A - B \neq \phi$

This means that there exists $x \in A, x \notin B$, which is not possible as $A \subset B$.

$\therefore A - B = \phi$

$\therefore A \subset B \Rightarrow A - B = \phi$

$(ii) \Rightarrow$ (i).

Let $A - B = \phi$

To show: $A \subset B$

Let $x \epsilon A$

Clearly, $x \epsilon B$ because if $x \notin B$, then $A - B \neq \phi$

$\therefore A - B = \phi \Rightarrow A \subset B$

Hence, $(ii) \Leftrightarrow (i)$

$(i) \Rightarrow$ (iii).

Let $A \subset B$

To show: $A \cup B = B$

Clearly, $B \subset A \cup B$

Let $x \epsilon A \cup B$

$\Rightarrow x \epsilon A$ or $x \epsilon B$

Case I: $x \epsilon A$

$\Rightarrow x \epsilon B \quad [\because A \subset B]$

$\therefore A \cup B \subset B$

Case II: $x \epsilon B$

Then $A \cup B \subset B$

So, $A \cup B = B$

$(iii) \Rightarrow$ (i).

Conversely, let $A \cup B = B$

To show : $A \subset B$

Let $x \epsilon A$

$\Rightarrow x \epsilon A \cup B \quad [\because A \subset A \cup B]$

$\Rightarrow x \epsilon B \quad [\because A \cup B = B]$

$\therefore A \subset B$

Hence, $(iii) \Leftrightarrow (i)$

Now, we have to show that $(i) \Leftrightarrow (iv)$.

Let $A \subset B$

Clearly $A \cap B \subset A$

Let $x \epsilon A$

We have to show that $x \epsilon A \cap B$

As $A \subset B, x \epsilon B$

$\therefore x \epsilon A \cap B$

$\therefore A \subset A \cap B$

Hence, $A = A \cap B$

Conversely, suppose $A \cap B = A$

Let $x \epsilon A$

$\Rightarrow x \epsilon A \cap B$

$\Rightarrow x \epsilon A$ and $x \epsilon B$

$\Rightarrow x \epsilon B$

$\therefore A \subset B$

Hence, $(i) \Leftrightarrow (iv)$.

#419701

Topic: Operations on Sets

Show that if $A \subset B$, then $C - B \subset C - A$

Solution

Let $A \subset B$

To show: $C - B \subset C - A$

Let $x \in C - B$

$\Rightarrow x \in C$ and $x \notin B$

$\Rightarrow x \in C$ and $x \notin A$ $(A \subset B)$

$\Rightarrow x \in C - A$

$\therefore C - B \subset C - A$

#420084

Topic: Operations on Sets

Show that for any sets A and B,

$A = (A \cap B) \cup (A - B)$ and $A \cup (B - A) = (A \cup B)$

Solution

(i) $A = (A \cap B) \cup (A - B)$

Consider $RHS = (A \cap B) \cup (A - B)$

$= (A \cap B) \cup (A \cap B')$ (by def of difference of sets, $A - B = A \cap B'$)

$= A \cap (B \cup B')$ (by distributive)

$= A \cap U$ $(\because A \cup A' = U)$

$= A$

$= LHS$

Hence, $A = (A \cap B) \cup (A - B)$

(ii) $A \cup (B - A) = A \cup B$

Consider, $A \cup (B - A)$

$= A \cup (B \cap A')$ (by def of difference of sets, $A - B = A \cap B'$)

$= (A \cup B) \cap (A \cup A')$ (by distributive property)

$= (A \cup B) \cap U$ $(\because A \cup A' = U)$

$= A \cup B$

#420090

Topic: Operations on Sets

Using properties of sets, show that

(i) $A \cup (A \cap B) = A$ (ii) $A \cap (A \cup B) = A$

Solution

(i) To show: $A \cup (A \cap B) = A$

We know that

$A \subset A$

$\Rightarrow A \cap B \subset A$

$\therefore A \cup (A \cap B) \subset A$ ......(1)

Also, $A \subset A \cup (A \cap B)$ ....(2)

$\therefore$ From (1) and (2),

$A \cup (A \cap B) = A$

(ii) To show: $A \cap (A \cup B) = A$

$A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$

$= A \cup (A \cap B)$

$= A$ (by (i))

$\Rightarrow A \cap (A \cup B) = A$

#420091

Topic: Operations on Sets

Show that $A \cap B = A \cap C$ need not imply $B = C$

Solution

Let $A = \{0, 1\}$

$B = \{0, 2, 3\}$

$C = \{0, 4, 5\}$

So, $A \cap B = \{0\}$

and $A \cap C = \{0\}$

Here, $A \cap B = A \cap C = \{0\}$

However, $B \neq C$ as $2 \in B$ and $2 \notin C$

#420095

Topic: Operations on Sets

Let A and B be sets. If $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X, show that $A = B$

Solution

Let A and B be two sets such that $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X.

To show: $A = B$

It can be seen that

$A = A \cap (A \cup X)$

$= A \cap (B \cup X)$ $(A \cup X = B \cup X)$

$= (A \cap B) \cup (A \cap X)$ (Distributive law)

$= (A \cap B) \cup \phi$ $(\because A \cap X = \phi)$

$= A \cap B$ ......(1)

Now, $B = B \cap (B \cup X)$

$= B \cap (A \cup X)$ $(\because A \cup X = B \cup X)$

$= (B \cap A) \cup (B \cap X)$ (Distributive law)

$= (B \cap A) \cup \phi$ $(\because B \cap X = \phi)$

$= B \cap A$

$= A \cap B$ ......(2)

Hence, from (1) and (2), we get

$A = B$.

#420097

Topic: Operations on Sets

Find sets A, B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \phi$

Solution

Let $A = \{0, 1\}$, $B = \{1, 2\}$, and $C = \{2, 0\}$

Accordingly,

$A \cap B = \{1\}$

$B \cap C = \{2\}$

$A \cap C = \{0\}$.

$\therefore$ $A \cap B$, $B \cap C$, and $A \cap C$ are non-empty.

However, $A \cap B \cap C = \phi$

#420104

Topic: Sets

In a survey of 600 students in a school, 150 students were found to be taking tea and 225 taking coffee, 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee?

Solution

Let U be the set of all students who took part in the survey.

Let T be the set of students taking tea.

Let C be the set of students taking coffee.

$n(U) = 600$, $n(T) = 150$, $n(C) = 225$, $n(T \cap C) = 100$

To find: Number of student taking neither tea nor coffee i.e., we have to find $n(T' \cap C')$.

$n(T' \cap C') = n(T \cup C)'$

$= n(U) - n(T \cup C)$

$= n(U) - [n(T) + n(C) - n(T \cap C)]$

$= 600 - [150 + 225 - 100]$

$= 600 - 275$

$= 325$

Hence, 325 students were taking neither tea nor coffee.

#420109

Topic: Sets

In a group of students, 100 students know Hindi,50 know English and 25 know both. Each of the students knows either Hindi or English. How many students are there in the group?

Solution

Let U be the set of all students in the group.

Let E be the set of all students who know English.

Let H be the set of all students who know Hindi.

$\therefore H \cup E = U$

Given $n(H) = 100$ and $n(E) = 50$

$n(H \cap E) = 25$

$n(U) = n(H) + n(E) - n(H \cap E)$

$= 100 + 50 - 25$

Hence, there are 125 students in the group.

#420128

Topic: Sets

In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T,8 read both T and I, 3 read all three newspapers. Find:

(i) the number of people who read at least one of the newspapers.

(ii) the number of people who read exactly one newspaper.

Solution

Let A be the set of people who read newspaper H.

Let B be the set of people who read newspaper T.

Let C be the set of people who read newspaper I.

Given $n(A) = 25$, $n(B) = 26$, and $n(C) = 26$

$n(A \cap C) = 9$, $n(A \cap B) = 11$, and $(B \cap C) = 8$

$n(A \cap B \cap C) = 3$

Let U be the set of people who took part in the survey.

(i) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$= 25 + 26 + 26 - 11 - 8 - 9 + 3$

$= 52$

Hence, 52 people read at least one of the newspaper.

(ii) Let a be the number of people who read newspapers H and T only.

Let b denote the number of people who read newspapers I and H only.

Let c denote the number of people who read newspaper T and I only.

Let d denote the number of people who read all three newspaper.

Accordingly, $d = n(A \cap B \cap C) = 3$

Now, $n(A \cap B) = a + d$

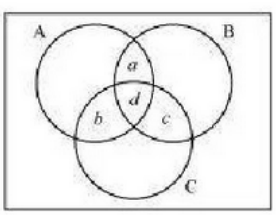
$n(B \cap C) = c + d$

$n(C \cap A) = b + d$

$\therefore a + d + c + d + b + d = 11 + 8 + 9 = 28$

$\Rightarrow a + b + c + d = 28 - 2d = 28 - 6 = 22$

Hence, $(52 - 22) = 30$ people read exactly one newspaper.



#420132

Topic: Sets

In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products.

Find how many liked product C only

Solution

Let A, B, and C be the set of people who like product A, product B, and product C respectively.

Accordingly, $n(A) = 21$, $n(B) = 26$, $n(C) = 29$, $n(A \cap B) = 14$, $n(C \cap A) = 12$, $n(B \cap C) = 14$, $n(A \cap B \cap C) = 8$

The Venn diagram for the given problem can be drawn as

It can be seen that number of people who like product C only is

$\{29 - (4 + 8 + 6)\} = 11$