#417855

Topic: Cartesian Product

If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$ find the values of x and y

Solution

It is given that $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$

Since the ordered pairs are equal the corresponding element will also be equal

Therefore $\frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$

$\Rightarrow y=1$

$\Rightarrow \frac{x}{3}+1=\frac{5}{3}$

$\Rightarrow x=2$

$\therefore x=2$ and $y=1$

#417856

Topic: Cartesian Product

If the set A has 3 elements and the set $B=\{3,4,5\}$ then find the number of elements in $(A \times B)$?

Solution

It is given that set A has 3 elements and the elements of set B are 3, 4 and 5

$\Rightarrow$ Number of elements in set $B=3$

Number of elements in $(A \times B)$

$=$ (Number of elements in A) $\times$ (Number of elements in B)

$=3 \times 3=9$

Thus the number of elements in $(A \times B)$ is 9

#417864

Topic: Cartesian Product

If $G=\{7,8\}$ and $H=\{5,4,2\}$, find $G \times H$ and $H \times G$.

Solution

$G=\{7,8\}$ and $H=\{5,4,2\}$

We know that the Cartesian product of $P \times Q$ of two non-empty sets P and Q is defined as

$P \times Q=\{(p, q): p \in P, q \in Q\}$

$\therefore G \times H=\{(7,5),(7,4),(7,2),(8,5),(8,4),(8,2)\}$

and $H \times G=\{(5,7),(5,8),(4,7),(4,8),(2,7),(2,8)\}$

#417877

Topic: Cartesian Product

State whether each of the following statements are true or false. If the statement is false rewrite the given statement correctly

(i) If $P=\{m, n\}$ and $Q=\{n, m\}$ then $P \times Q=\{(m, n),(n, m)\}$

(ii) If A and B are non-empty sets then $A \times B$ is a non-empty set of ordered pairs (x, y) such that $x \in A$ and $y \in B$

(iii) If $A=\{1,2\}, B=\{3,4\}$ then $A \times(B \cap \phi)=\phi$

Solution

(i) Given $P = \{m, n\}$ and $Q = \{n, m\}$ then

$P \times Q = \{(m, n), (m, m), (n, n), (n, m)\}$

So, given value of $P \times Q$ is incorrect.

Hence, the given statement (i) is false.

(ii) If A and B are non-empty sets, then $A \times B$ is a non-empty set of ordered pairs (x, y) such that $x \in A$ and $y \in B$

Hence, the given statement (ii) is true.

(iii) $A = \{1, 2\}$ and $B = \{3, 4\}$

$A \times (B \cap \phi) = A \times \phi = \phi$

So, (iii) is true.

#417884

Topic: Cartesian Product

If $= A\{-1, 1\}$ then find $A \times A \times A$.

Solution

It is known that for any non-empty set A, $A \times A \times A$ is defined as

$A \times A \times A = \{(a, b, c) : a, b, c \in A\}$

It is given that $A = \{-1, 1\}$

$A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1),$

$(1, 1, -1), (1, 1, 1)\}$

#417885

Topic: Cartesian Product

If $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$ Find A and B.

Solution

It is given that $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$

We know that the Cartesian product of two non-empty sets P and Q is defined as

$P \times Q = \{(p, q) : p \in P, q \in Q\}$

$\therefore$ A is the set of all first elements of $A \times B$ and B is the set of all second elements of $A \times B$

Thus $A = \{a, b\}$ and $B = \{x, y\}$

#417886

Topic: Cartesian Product

Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$ Verify that

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) $A \times C$ is a subset of $B \times D$

Solution

(i) To verify : $A \times (B \cap C) = (A \times B) \cap (A \times C)$

We have $B \cap C = \{1, 2, 3, 4\} \cap \{5, 6\} = \phi$

$\therefore$ L.H.S $= A \times (B \cap C) = A \times \phi = \phi$

$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\}$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$\therefore R.H.S. = (A \times B) \cap (A \times C) = \phi$

$\therefore L.H.S = R.H.S$

Hence $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) To verify: $A \times C$ is a subset of $B \times D$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$B \times D = \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7),$
$(3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$

We can observe that all the elements of set $A \times C$ are the elements of set $B \times D$

Therefore $A \times C$ is a subset of $B \times D$

#417887
Topic: Cartesian Product

Let $A = \{1, 2\}$ and $B = \{3, 4\}$. Write $A \times B$ and find how many subsets will $A \times B$ have? List them.

Solution

$A = \{1, 2\}$ and $B = \{3, 4\}$

$\therefore A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$

$\Rightarrow n(A \times B) = 4$

We know that if $n(A \times B) = r$

Then, number of subsets of $A \times B$ is 2^r

Therefore the set $A \times B$ has $2^4 = 16$ subsets.

These are

$\{\phi, \{(1, 3)\}, \{(1, 4)\}, \{(2, 3)\}, \{(2, 4)\}, \{(1, 3), (1, 4)\}, \{(1, 3), (2, 3)\}, \{(1, 3), (2, 4)\},$

$\{(1, 4), (2, 3)\}, \{(1, 4), (2, 4)\}, \{(2, 3), (2, 4)\}, \{(1, 3), (1, 4), (2, 3)\}, \{(1, 3), (1, 4), (2, 4)\},$

$\{(1, 3), (2, 3), (2, 4)\}, \{(1, 4), (2, 3), (2, 4)\}, \{(1, 3), (1, 4), (2, 3), (2, 4)\}\}$

#417889
Topic: Cartesian Product

Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x, 1), (y, 2), (z, 1)$ are in $A \times B$ find A and B where x, y and z are distinct elements

Solution

It is given that $n(A) = 3$ and $n(B) = 2$ and $(x, 1), (y, 2), (z, 1)$ are in $A \times B$

We know that $A =$ Set of first elements of the ordered pair elements of $A \times B$

$B =$ Set of second elements of ordered pair elements of $A \times B$

$\therefore x, y$ and z are the elements of A and 1 and 2 are the elements of B

Since $n(A) = 3$ and $n(B) = 2$ it is clear that $A = \{x, y, z\}$ and $B = \{1, 2\}$

#417891
Topic: Cartesian Product

The cartesian product $A \times A$ has 9 elements among which are found $(-1, 0)$ and $(0, 1)$. Find the set A and the remaining elements of $A \times A$

Solution

We know that if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = n(A) \times n(B) = pq$

$\therefore n(A \times A) = n(A) \times n(A)$

It is given that $n(A \times A) = 9$

$\therefore n(A) \times n(A) = 9$

$\Rightarrow n(A) = 3$

The ordered pairs $(-1, 0)$ and $(0, 1)$ are two of the nine elements of $A \times A$

Now, $A \times A = \{(a, a) : a \in A\}$

Therefore -1, 0 and 1 are elements of A

Since $n(A) = 3$, so set $A = \{-1, 0, 1\}$

The remaining elements of set $A \times A$ are $(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0)$ and $(1, 1)$

#417893

Topic: Relations

Let $A = \{1, 2, 3, ..., 14\}$. Define a relation R from A to A by $R = \{(x, y) : 3x - y = 0$ where $x, y \in A\}$. Write down its domain, co-domain and range.

Solution

The relation R from A to A is given as

$R = \{(x, y) : 3x - y = 0; x, y \in A\}$

i.e., $R = \{(x, y) : 3x = y; x, y \in A\}$

$\therefore R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation

$\therefore$ Domain of $R = \{1, 2, 3, 4\}$

The whole set A is the co-domain of the relation R

$\therefore$ Codomain of $R = A = \{1, 2, 3, ..., 14\}$

The range of R is the set of all second elements of the ordered pairs in the relation.

$\therefore$ Range of $R = \{3, 6, 9, 12\}$

#417896

Topic: Relations

Define a relation R on the set N of natural numbers by $R = \{(x, y) : y = x + 5, x$ is a natural number less than 4; $x, y \in N\}$. Depict this relationship using roster form. Write down the domain and the range.

Solution

Given definition of R is

$R = \{(x, y) : y = x + 5, x$ is a natural number less than 4, $x, y \in N\}$

The natural numbers less than 4 are 1, 2 and 3

$\therefore R = \{(1, 6), (2, 7), (3, 8)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation

$\therefore$ Domain of $R = \{1, 2, 3\}$

The range of R is the set of all second elements of the ordered pairs in the relation

$\therefore$ Range of $R = \{6, 7, 8\}$

#417898

Topic: Relations

$A = \{1, 2, 3, 5\}$ and $B = \{4, 6, 9\}$. Define a relation R from A to B by $R = \{(x, y) :$ the difference between x and y is odd $x \in A, y \in B\}$. Write R in roster form

Solution

$A = \{1, 2, 3, 5\}$ and $B = \{4, 6, 9\}$

$R = \{(x, y) :$ the difference between x and y is odd $x \in A, y \in B\}$

$\therefore R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}$

#417901

Topic: Relations

Let $A = \{1, 2, 3, 4, 6\}$ and R be the relation on A defined by $\{(a, b): a, b \in A$, b is exactly divisible by $a\}$

(i) Write R in roster form

(ii) Find the domain of R

(iii) Find the range of R

Solution

$A = \{1, 2, 3, 4, 6\}$

$R = \{(a, b): a, b \in A, b$ is exactly divisible by $a\}$

(i) $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)\}$

(ii) Domain of $R = \{1, 2, 3, 4, 6\}$

(iii) Range of $R = \{1, 2, 3, 4, 6\}$

#417902
Topic: Relations

Determine the domain and range of the relation R defined by $R = \{(x, x + 5): x \in \{0, 1, 2, 3, 4, 5\}\}$

Solution

$R = \{(x, x + 5): x \in \{0, 1, 2, 3, 4, 5\}\}$

$\Rightarrow R = \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$

$\therefore$ Domain of $R = \{0, 1, 2, 3, 4, 5\}$

Range of $R = \{5, 6, 7, 8, 9, 10\}$

#417903
Topic: Relations

Write the relation $R = \{(x, x^3): x$ is a prime number less than $10\}$ in roster form

Solution

$R = \{(x, x^3): x$ is a prime number less than $10\}$

The prime numbers less than 10 are $2, 3, 5$ and 7

$\therefore R = \{(2, 8), (3, 27), (5, 125), (7, 343)\}$

#417904
Topic: Relations

Let $A = \{x, y, z\}$ and $B = \{1, 2\}$. Find the number of relations form A to B.

Solution

It is given that $A = \{x, y, z\}$ and $B = \{1, 2\}$

$\therefore A \times B = \{(x, 1), (x, 2), (y, 1), (y, 2), (z, 1), (z, 2)\}$

Since $n(A \times B) = 6$

The number of subsets of $A \times B$ is 2^6.

#417905
Topic: Functions

Which of the following relations are functions? Give reasons.

If it is a function determine its domain and range

(i) $\{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)\}$

(ii) $\{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}$

(iii) $\{(1, 3), (1, 5), (2, 5)\}$

Solution

(i) $\{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)\}$

It is a function as every input has a single output.

So, $2, 5, 8, 11, 14$ and 17 are the elements of the domain of the given relation.

Here domain $= \{2, 5, 8, 11, 14, 17\}$ and range $= \{1\}$

(ii) $\{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5)(12, 6), (14, 7)\}$

It is a function as every input has a single output.

So, $2, 4, 6, 8, 10, 12$ and 14 are the elements of the domain of the given relation.

Here domain $= \{2, 4, 6, 8, 10, 12, 14\}$ and range $= \{1, 2, 3, 4, 5, 6, 7\}$

(iii) $\{(1, 3), (1, 5), (2, 5)\}$

Since the element 1 corresponds to two different images i.e., 3 and 5 . So, this relation is not a function.

#417906

Topic: Functions

Find the domain and range of the following real function:

(i) $f(x) = -|x|$

(ii) $f(x) = \sqrt{9 - x^2}$

Solution

(i) $f(x) = -|x|,\ x \in R$

We know that $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$\therefore\ f(x) = -|x| = \begin{cases} -x, & x \geq 0 \\ x, & x < 0 \end{cases}$

Since, $f(x)$ is defined for $x \in R$

Domain of f is R

It can be observed that the range of $f(x) = -|x|$ is all real numbers except positive real numbers

$\therefore$ The range of f is $(-\infty, 0]$

(ii) $f(x) = \sqrt{9 - x^2}$

For this function to be defined,

$9 - x^2 \geq 0$

$\Rightarrow\ -3 \leq x \leq 3$

For any value of x such that $-3 \leq x \leq 3$ the value of $f(x)$ will lie between 0 and 3

$\therefore$ The range of f is $[0, 3]$

#417907

Topic: Functions

A function f is defined by $f(x) = 2x - 5$. Write down the values of

(i) $f(0)$

(ii) $f(7)$

(iii) $f(-3)$

Solution

f is given by $f(x) = 2x - 5$

Then, we have

(i) $f(0) = 2(0) - 5 = -5$

(ii) $f(7) = 2(7) - 5 = 9$

(iii) $f(-3) = 2(-3) - 5 = -11$

#417908

Topic: Functions

The function t which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32^o$

Find :

(i) $t(0^o)$

(ii) $t(28^o)$

(iii) $t(-10^o)$

(iv) The value of C when $t(C) = 212^oF$

Solution

The given function is

$t(C) = \frac{9C}{5} + 32^o$

(i) $t(0^o) = \frac{9 \times 0}{5} + 32^o = 0 + 32^o = 32^oF$

(ii) $t(28^o) = \frac{9 \times 28^o}{5} + 32^o = \frac{252^o + 160^o}{5} = \frac{412^o}{5} = 82.4^oF$

(iii) $t(-10^o) = \frac{9 \times \left(-10^o\right)}{5} + 32^o = 9 \times \left(-2^o\right) + 32^o = -18^o + 32^o = 14^oF$

(iv) It is given that $t(C) = 212^oF$

$\therefore 212^o = \frac{9C}{5} + 32^o$

$\Rightarrow \frac{9C}{5} = 212^o - 32^o$

$\Rightarrow \frac{9C}{5} = 180^o$

$\Rightarrow 9C = 180^o \times 5$

$\Rightarrow C = \frac{180^o \times 5}{9} = 100^o$

Thus the value of Celsius temperature is 100^o when Fahrenheit temperature is 212^o.

#417910

Topic: Functions

Find the range of each of the following functions

(i) $f(x) = 2 - 3x,\ x \in R, x > 0$

(ii) $f(x) = x^2 + 2$, x is a real number

(iii) $f(x) = x$, x is a real number

Solution

(i) Given $x > 0$

$\Rightarrow 3x > 0$

$\Rightarrow -3x < 0$

$\Rightarrow 2 - 3x < 2$

$\Rightarrow f(x) < 2$

$\therefore$ Range of $f = (-\infty, 2)$

(ii) Since, for any real number x, $x^2 \geq 0$

$\Rightarrow x^2 + 2 \geq 0 + 2$

$\Rightarrow x^2 + 2 \geq 2$

$\Rightarrow f(x) \geq 2$

$\therefore$ Range of $f = [2, \infty)$

(iii) $f(x) = x$, x is a real number

It is clear that the range of f is the set of all real numbers

$\therefore$ Range of $f = R$

#417913

Topic: Functions

The relation f is defined by $f(x) = \begin{cases} x^2, 0 \leq x \leq 3 \\ 3x, 3 \leq x \leq 10 \end{cases}$

The relation g is defined by $g(x) = \begin{cases} x^2, 0 \leq x \leq 2 \\ 3x, 2 \leq x \leq 10 \end{cases}$

Show that f is a function and g is not a function

Solution

The relation f is defined as $f(x) = \begin{cases} x^2, 0 \leq x \leq 3 \\ 3x, 3 \leq x \leq 10 \end{cases}$

It is observed that for

$0 \leq x \leq 3$, $f(x) = x^2$

$3 \leq x \leq 10$, $f(x) = 3x$

Also at $x = 3$, $f(x) = 3^2 = 9$ or $f(x) = 3 \times 3 = 9$

i.e., at $x = 3$, $f(x) = 9$

Therefore for $0 \leq x \leq 10$, the images of $f(x)$ are unique

Thus the given relation is a function

The relation g is defined as $g(x) = \begin{cases} x^2, 0 \leq x \leq 2 \\ 3x, 2 \leq x \leq 10 \end{cases}$

It can be observed that for $x = 2$, $g(x) = 2^2 = 4$ and $g(x) = 3 \times 2 = 6$

Hence element 2 of the domain of the relation g corresponds to two different images i.e., 4 and 6.

Hence, this relation is not a function.

#417914

Topic: Functions

If $f(x) = x^2$ find $\frac{f(1.1) - f(1)}{(1.1 - 1)}$

Solution

Given, $f(x) = x^2$

$$\therefore \frac{f(1.1) - f(1)}{(1.1 - 1)} = \frac{(1.1)^2 - (1)^2}{(1.1 - 1)} = \frac{1.21 - 1}{0.1} = \frac{0.21}{0.1} = 2.1$$

#417915

Topic: Functions

Find the domain of the function $f(x) = \dfrac{x^2+2x+1}{x^2-8x+12}$

Solution

The given function is, $f(x) = \dfrac{x^2+2x+1}{x^2-8x+12}$

$\Rightarrow f(x) = \dfrac{x^2+2x+1}{(x-6)(x-2)}$

It can be seen that function f is defined for all real numbers except at $x = 6$ and $x = 2$

Hence the domain of f is $R - \{2, 6\}$

#417917

Topic: Functions

Find the domain and the range of the real function f defined by $f(x) = \sqrt{(x-1)}$

Solution

The given real function is $f(x) = \sqrt{x-1}$

It can be seen that $\sqrt{x-1}$ is defined for $(x-1) \geq 0$

i.e., $f(x) = \sqrt{(x-1)}$ is defined for $x \geq 1$

Therefore the domain of f is the set of all real numbers greater than or equal to 1 i.e.,

the domain of f $= [1, \infty)$

As $x \geq 1$

$\Rightarrow (x-1) \geq 0$

$\Rightarrow \sqrt{x-1} \geq 0$

$f(x) \geq 0$

Therefore the range of f is the set of all real numbers greater than or equal to 0

i.e., the range of f $= [0, \infty)$

#417918

Topic: Functions

Find the domain and range of the real function f defined by $f(x) = |x-1|$

Solution

The given real function is $f(x) = |x-1|$

It is clear that $|x-1|$ is defined for all real numbers

$\therefore$ Domain of $f = R$

Thus for $x \in R, |x-1|$ assumes all non-negative real numbers.

Hence the range of f is the set of all non-negative real numbers=$(0, \infty)$

#417920

Topic: Functions

Let $f = \left\{ \left(x, \dfrac{x^2}{1+x^2} \right) : x \in R \right\}$ be a function from R into R. Determine the range of f.

Solution

Let $y = \frac{x^2}{1+x^2}$

$\Rightarrow y + x^2y = x^2$

$\Rightarrow y = x^2(1-y)$

$\Rightarrow x^2 = \frac{y}{1-y}$

$\Rightarrow x = \sqrt{\frac{y}{1-y}}$

Since, x is real

$\Rightarrow \frac{y}{1-y} \geq 0$

$\Rightarrow \frac{y(1-y)}{(1-y)^2} \geq 0$

$\Rightarrow y(1-y) \geq 0$ and $(1-y)^2 > 0$

$\Rightarrow 0 \leq y \leq 1$ and $-y > -1$

$\Rightarrow 0 \leq y \leq 1$ and $y < 1$

Hence, $0 \leq y < 1$

Range of f is $[0, 1)$

#417921

Topic: Algebra of Real Functions

Let $f, g : R \rightarrow R$ be defined respectively by $f(x) = x + 1$, $g(x) = 2x - 3$. Find $f + g$, $f - g$ and $\frac{f}{g}$

Solution

$f, g : R \rightarrow R$ is defined as

$f(x) = x + 1$

$g(x) = 2x - 3$

Now, $(f+g)(x) = f(x) + g(x) = (x+1) + (2x-3) = 3x - 2$

$\therefore (f+g)(x) = 3x - 2$

Now, $(f-g)(x) = f(x) - g(x) = (x+1) - (2x-3) = x + 1 - 2x + 3 = -x + 4$

$\therefore (f-g)(x) = -x + 4$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0, x \in R$

$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}, 2x - 3 \neq 0 \text{ or } 2x \neq 3$

$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}, x \neq \frac{3}{2}$

#417922

Topic: Functions

Let $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$ be a function from Z to Z defined by $f(x) = ax + b$ for some integers a, b. Determine a, b

Solution

$f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$

$f(x) = ax + b$

$(1, 1) \in f$

$\Rightarrow f(1) = 1$

$\Rightarrow a \times 1 + b = 1$

$\Rightarrow a + b = 1$ .....(1)

$(0, -1) \in f$

$\Rightarrow f(0) = -1$

$\Rightarrow a \times 0 + b = -1$

$\Rightarrow b = -1$

On substituting $b = -1$ in eqn (1), we get

$a + (-1) = 1$

$\Rightarrow a = 1 + 1 = 2$

Thus the respective values of a and b are 2 and -1

#417923

Topic: Relations

Let R be a relation from N to N defined by $R = \{(a, b): a, b \in N$ and $a = b^2\}$. Are the following true?

(i) $(a, a) \in R$ for all $a \in N$.

(ii) $(a, b) \in R$, implies $(b, a) \in R$.

(iii)$(a, b) \in R, (b, c) \in R$ implies $(a, c) \in R$.

Justify your answer in each case

Solution

$R = \{(a, b): a, b \in N; a = b^2\}$

(i) It can be seen that $2 \in N$; however $2 \neq 2^2 = 4$

Therefore the statement $(a, a) \in R$ for all $a \in N$ is not true

(ii) It can be seen that $(9, 3) \in N$ because $9, 3 \in N$ and $9 = 3^2$

Now $3 \neq 9^2 = 81$

$\therefore (3, 9) \notin N$

Therefore the statement $(a, b) \in R$ implies $(b, a) \in R$ is not true.

(iii) It can be seen that $(9, 3) \in R, (16, 4) \in R$ because $9, 3, 16, 4 \in N$ and $9 = 3^2$ and $16 = 4^2$

Now $9 \neq 4^2 = 16$

$\therefore (9, 4) \notin N$

Therefore the statement $(a, b) \in R, (b, c) \in R$ implies $(a, c) \in R$ is not true

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#418034

Topic: Functions

Let $A = \{1, 2, 3, 4\}, B = \{1, 5, 9, 11, 15, 16\}$ and $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$ Are the following true?

(i) f is a relation from A to B

(ii) f is a function from A to B

Justify your answer in each case

Solution

$A = \{1, 2, 3, 4\}$ and $B = \{1, 5, 9, 11, 15, 16\}$

$\therefore A \times B = \{(1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), (2, 11), (2, 15), (2, 16),$

$(3, 1), (3, 5), (3, 9), (3, 11), (3, 15), (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16)\}$

It is given that $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$

(i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product $A \times B$

It is observed that f is a subset of $A \times B$

Thus f is a relation from A to B.

(ii) Since the element 2 corresponds to two different images i.e., 9 and 11. So, relation f is not a function

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#418054

Topic: Functions

Let f be the subset of $Z \times Z$ defined by $f = \{(ab, a + b) : a, b \in Z\}$. Is f a function from Z to Z: justify your answer

Solution

The relation f is defined as

$f = \{(ab, a + b) : a, b \in Z\}$

We know that a relation f from set A to set B is said to be a function if every element of set A has unique images in set B.

Since $2, 6 \in Z$,

$\Rightarrow (2 \times 6, 2 + 6) \in f$

Again since, $-2, -6 \in Z$,

$\Rightarrow (-2 \times -6, -2 + (-6)) \in f$

i.e., $(12, 8), (12, -8) \in f$

It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and -8.

Thus relation f is not a function

#418064

Topic: Functions

Let $A = \{9, 10, 11, 12, 13\}$ and let $f: A \rightarrow N$ be defined by $f(n) =$ the highest prime factor of n. Find the range of f

Solution

$A = \{9, 10, 11, 12, 13\}$

$f: A \rightarrow N$ is defined as

$f(n) =$ The highest prime factor of n

Prime factor of $9 = 3$

Prime factors of $10 = 2, 5$

Prime factors of $11 = 11$

Prime factors of $12 = 2, 3$

Prime factors of $13 = 13$

$\therefore f(9) =$ The highest prime factor of $9 = 3$

$f(10) =$ The highest prime factor of $10 = 5$

$f(11) =$ The highest prime factor of $11 = 11$

$f(12) =$ The highest prime factor of $12 = 3$

$f(13) =$ The highest prime factor of $13 = 13$

Thus range of f is the set of all $f(n)$ where $n \in A$

is $= \{3, 5, 11, 13\}$

#418423

Topic: Functions

How many elements has $P(A)$, if $A = \phi$?

Solution

We know that if A is a set with m elements i.e., $n(A) = m$, then $n[P(A)] = 2^m$.

If $A = \phi$, then $n(A) = 0$

$\therefore n[P(A)] = 2^o = 1$

Hence, P(A) has one element.

#419705

Topic: Functions

Assume that $P(A) = P(B)$. Show that $A = B$

Solution

Let $P(A) = P(B)$

To show: $A = B$

Let $x \in A$

$A \in P(A) = P(B)$

$\therefore x \epsilon C$, for some $C \epsilon P(B)$

Now, $C \subset B$

$\therefore x \epsilon B$

But x is an arbitrary element in A

$\therefore A \subset B$ ....(1)

Now, let $y \in B$

$B \in P(B) = P(A)$

$\Rightarrow y \in D$ for some $D \in P(A)$

$D \subset A$

$\Rightarrow y \in A$

But y is an arbitrary element in B.

Hence, $B \subset A$ .....(2)

From (1) and (2), we get

$A = B$

#419708

Topic: Functions

Is it true that for any sets A and B, $P(A) \cup P(B) = P(A \cup B)$? Justify your answer

Solution

False

Let $A = \{0, 1\}$ and $B = \{1, 2\}$

$\therefore A \cup B = \{0, 1, 2\}$

$P(A) = \{\phi, \{0\}, \{1\}, \{0, 1\}\}$

$P(B) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

$P(A \cup B) = \{\phi, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{1, 2\}, \{0, 2\}, \{0, 1, 2\}\}$

$P(A) \cup P(B) = \{\phi, \{0\}, \{1\}, \{0, 1\}, \{2\}, \{1, 2\}\}$

$\therefore P(A) \cup P(B) \neq P(A \cup B)$

#446760

Topic: Relations

Let R be the relation on Z defined by $R = \{(a, b) : a, b \in Z, a - b$ is an integer$\}$. Find the domain and range of R.

Solution

Given $R = \{(a, b) : a, b \in Z, a - b$ is an integer$\}$

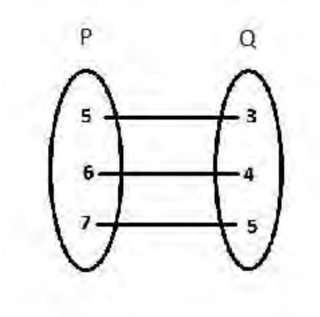
$a \in Z$ So, Domain of R is Z.

R is such that $a - b \in Z$ and we have $a \in Z$ so $b \in Z$.

So, Range of R is Z.

$\therefore$ Domain of $R = Z$ and Range of $R = Z$.

#458289
Topic: Relations



The figure shows a relationship between the sets P and Q. Write this relation in

(i) in set-builder form (ii) roster form

Solution

The relation mentioned in the figure shows, P a domain and Q as range.

Let the relation be R

In roster form $R = \{(5, 3), (6, 4), (7, 5)\}$

In set builder form $R = \{(x, y) : x \in P, y \in Q, y = x - 2\}$

#459565
Topic: Special Functions

Find the maximum and minimum values, if any of the following function given by:

$f(x) = |x + 2| - 1$

Solution

$f(x) = |x + 2| - 1$

Here, $|x + 2|$ is always greater than 0 (property of mod)

$|x + 2| \geq 0$

$\Rightarrow f(x) = |x + 2| - 1 \geq -1$

$\Rightarrow$ function has a minimum value of -1 and this happenes when $|x + 2| = 0$

i.e. at $x = -2$

#459566
Topic: Special Functions

Find the maximum and minimum values, if any of the following function given by:

$g(x) = -|x + 1| + 3$

Solution

$g(x) = -|x + 1| + 3$

As we know from property of mod that $|x + 1| \geq 0$

$\Rightarrow -|x + 1| \leq 0$(When an inequality is multiplied from -1 then it get inverted)

$\Rightarrow -|x + 1| + 3 \leq 3$

$\Rightarrow g(x) \leq 3$

This shows that $g(x)$ will attain maximum value of 3 and that to when $-|x + 1| = 0$

i.e. at $x = -1$