

10

STRAIGHT LINES

We have read about lines, angles and rectilinear figures in geometry. Recall that a line is the join of two points in a plane continuing endlessly in both directions. We have also seen that graphs of linear equations, which came out to be straight lines

Interestingly, the reverse problem of the above is finding the equations of straight lines, under different conditions, in a plane. The analytical geometry, more commonly called coordinate geometry, comes to our help in this regard. In this lesson. We shall find equations of a straight line in different forms and try to solve problems based on those.



OBJECTIVES

After studying this lesson, you will be able to :

- derive equations of a line parallel to either of the coordinate axes;
- derive equations in different forms (slope-intercept, point-slope, two point, intercept, parametric and perpendicular) of a line;
- find the equation of a line in the above forms under given conditions;
- state that the general equation of first degree represents a line;
- express the general equation of a line into
(i) slope-intercept form (ii) intercept form and (iii) perpendicular form;
- derive the formula for the angle between two lines with given slopes;
- find the angle between two lines with given slopes;
- derive the conditions for parallelism and perpendicularity of two lines;
- determine whether two given lines are parallel or perpendicular;
- derive an expression for finding the distance of a given point from a given line;
- calculate the distance of a given point from a given line;

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- derive the equation of a line passing through a given point and parallel/perpendicular to a given line;
- write the equation of a line passing through a given point and:
 - (i) parallel or perpendicular to a given line (ii) with given x -intercept or y -intercept (iii) passing through the point of intersection of two lines; and
- prove various geometrical results using coordinate geometry.

EXPECTED BACKGROUND KNOWLEDGE

- Congruence and similarity of triangles

10.1 STRAIGHT LINE PARALLEL TO AN AXIS

If you stand in a room with your arms stretched, we can have a line drawn on the floor parallel to one side. Another line perpendicular to this line can be drawn intersecting the first line between your legs.

In this situation the part of the line in front of you and going behind you is the y -axis and the one being parallel to your arms is the x -axis.

The direction part of the y -axis in front of you is positive and behind you is negative.

The direction of the part x -axis to your right is positive and to that to your left is negative.

Now, let the side facing you be at b units away from you, then the equation of this edge will be $y = b$ (parallel to x -axis)

where b is equal in absolute value to the distance from the x -axis to the opposite side.

If $b > 0$, then the line lies in front of you, i.e., above the x -axis.

If $b < 0$, then the line lies behind you, i.e., below the x -axis.

If $b = 0$, then the line passes through you and is the x -axis itself.

Again, let the side of the right of you is at c units apart from you, then the equation of this line will be $x = c$ (parallel to y -axis)

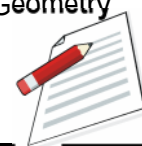
where c is equal in absolute value, to the distance from the y -axis on your right.

If $c > 0$, then the line lies on the right of you, i.e., to the right of y -axis.

If $c < 0$, then the line lies on the left of you, i.e., to the left of y -axis

If $c = 0$, then the line passes through you and is the y -axis.

Example 10.1 Find the equation of the lines passing through $(2, 3)$ and is
(i) parallel to x -axis. (ii) parallel to y -axis.



Solution :

- (i) The equation of any line parallel to x -axis is $y = b$
Since it passes through $(2, 3)$, hence $b = 3$
 $\therefore$ The required equation of the line is $y = 3$
- (ii) The equation of any line parallel to y -axis is $x = c$
Since it passes through $(2, 3)$, hence $c = 2$
 $\therefore$ The required equation of the line is $x = 2$.

Example 10.2 Find the equation of the line passing through $(-2, -3)$ and

- (i) parallel to x -axis (ii) parallel to y -axis

Solution :

- (i) The equation of any line parallel to x -axis is $y = b$
Since it passes through $(-2, -3)$, hence $-3 = b$
 $\therefore$ The required equation of the line is $y = -3$
- (ii) The equation of any line parallel to y -axis is $x = c$
Since it passes through $(-2, -3)$, hence $-2 = c$
 $\therefore$ The required equation of the line is $x = -2$



CHECK YOUR PROGRESS 10.1

- If we fold and press the paper then what will the crease look like
- Find the slope of a line which makes an angle of
 - 45° with the positive direction of x -axis.
 - 45° with the positive direction of y -axis.
 - 45° with the negative direction of x -axis.
- Find the slope of a line joining the points $(2, -3)$ and $(3, 4)$.
- Determine x so that the slope of the line through the points $(2, 5)$ and $(7, x)$ is 3.
- Find the equation of the line passing through $(-3, -4)$ and
 - parallel to x -axis.
 - parallel to y -axis.
- Find the equation of a line passing through $(5, -3)$ and perpendicular to x -axis.
- Find the equation of the line passing through $(-3, -7)$ and perpendicular to y -axis.

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10.2 DERIVATION OF THE EQUATION OF STRAIGHT LINE IN VARIOUS STANDARD FORMS

So far we have studied about the inclination, slope of a line and the lines parallel to the axes. Now the question is, can we find a relationship between x and y , where (x, y) is any arbitrary point on the line?

The relationship between x and y which is satisfied by the co-ordinates of arbitrary point on the line is called the equation of a straight line. The equation of the line can be found in various forms under the given conditions, such as

- When we are given the slope of the line and its intercept on y -axis.
- When we are given the slope of the line and it passes through a given point.
- When the line passes through two given points.
- When we are given the intercepts on the axes by the line.
- When we are given the length of perpendicular from origin on the line and the angle which the perpendicular makes with the positive direction of x -axis.
- When the line passes through a given point making an angle α with the positive direction of x -axis. (Parametric form).

We will discuss all the above cases one by one and try to find the equation of line in its standard forms.

(A) SLOPE-INTERCEPT FORM

Let AB be a straight line making an angle θ with x -axis and cutting off an intercept $OD = c$ from OY .

As the line makes intercept $OD = c$ on y -axis, it is called y -intercept.

Let AB intersect OX' at T .

Take any point $P(x, y)$ on AB . Draw $PM \perp OX$.

The $OM = x$, $MP = y$.

Draw $DN \perp MP$.

From the right-angled triangle DNP , we have

$$\begin{aligned}\tan \theta &= \frac{NP}{DN} = \frac{MP - MN}{OM} \\ &= \frac{y - OD}{OM} \\ &= \frac{y - c}{x}\end{aligned}$$

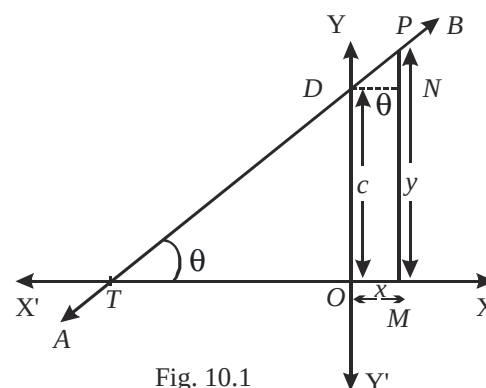


Fig. 10.1

$$\therefore y = x \tan \theta + c$$

$$\tan \theta = m \text{ (slope)}$$

$$\therefore y = mx + c$$

Since, this equation is true for every point on AB , and clearly for no other point in the plane, hence it represents the equation of the line AB .

Note : (1) When $c = 0$ and $m \neq 0 \Rightarrow$ the line passes through the origin and its equation is $y = mx$

(2) When $c = 0$ and $m = 0 \Rightarrow$ the line coincides with x -axis and its equation is of the form $y = 0$

(3) When $c \neq 0$ and $m = 0 \Rightarrow$ the line is parallel to x -axis and its equation is of the form $y = c$

Example 10.3 Find the equation of a line with slope 4 and y -intercept 0.

Solution : Putting $m = 4$ and $c = 0$ in the slope intercept form of the equation, we get $y = 4x$

This is the desired equation of the line.

Example 10.4 Determine the slope and the y -intercept of the line whose equation is $8x + 3y = 5$.

Solution : The given equation of the line is $8x + 3y = 5$

or,
$$y = -\frac{8}{3}x + \frac{5}{3}$$

Comparing this equation with the equation $y = mx + c$ (Slope intercept form) we get

$$m = -\frac{8}{3} \text{ and } c = \frac{5}{3}$$

Therefore, slope of the line is $-\frac{8}{3}$ and its

y -intercept is $\frac{5}{3}$.

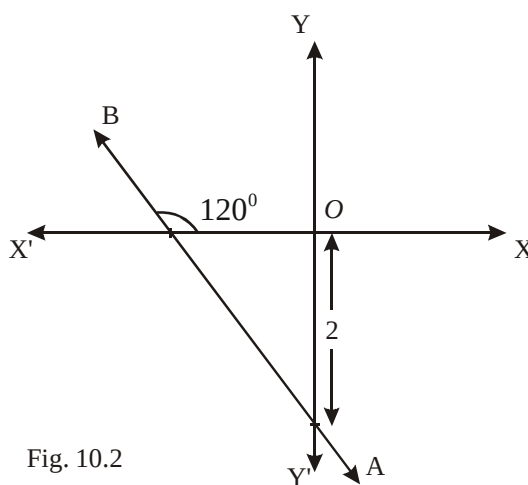


Fig. 10.2

Example 10.5 Find the equation of the line cutting off an intercept of length 2 from the negative direction of the axis of y and making an angle of 120° with the positive direction x -axis

Solution : From the slope intercept form of the line

$$\therefore y = x \tan 120^\circ + (-2)$$



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$$= -\sqrt{3}x - 2$$

$$\text{or, } y + \sqrt{3}x + 2 = 0$$

Here $m = \tan 120^\circ$, and $c = -2$, because the intercept is cut on the negative side of y -axis.

(b) POINT-SLOPE FORM

Here we will find the equation of a line passing through a given point $A(x_1, y_1)$ and having the slope m .

Let $P(x, y)$ be any point other than A on given the line. Slope ($\tan \theta$) of the line joining $A(x_1, y_1)$ and $P(x, y)$ is given by

$$m = \tan \theta = \frac{y - y_1}{x - x_1}$$

The slope of the line AP is given to be m .

$$\therefore m = \frac{y - y_1}{x - x_1}$$

$\therefore$ The equation of the required line is

$$y - y_1 = m(x - x_1)$$

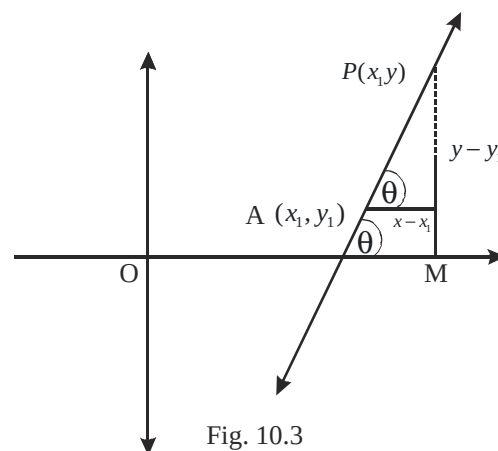


Fig. 10.3

Note : Since, the slope m is undefined for lines parallel to y -axis, the point-slope form of the equation will not give the equation of a line though $A(x_1, y_1)$ parallel to y -axis. However, this presents no difficulty, since for any such line the abscissa of any point on the line is x_1 . Therefore, the equation of such a line is $x = x_1$.

Example 10.6 Determine the equation of the line passing through the point $(2, -1)$ and having

$$\text{slope } \frac{2}{3}.$$

Solution : Putting $x_1 = 2, y_1 = -1$ and $m = \frac{2}{3}$ in the equation of the point-slope form of the line we get

$$y - (-1) = \frac{2}{3}(x - 2)$$

$$\Rightarrow y + 1 = \frac{2}{3}(x - 2)$$

$$\Rightarrow y = \frac{2}{3}x - \frac{7}{3}$$

which is the required equation of the line.

(c) TWO POINT FORM

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two given distinct points.

Slope of the line passing through these points is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad (x_2 \neq x_1)$$

From the equation of line in point slope form, we get

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

which is the required equation of the line in two-point form.

Example 10.7 Find the equation of the line passing through $(3, -7)$ and $(-2, -5)$.

Solution : The equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \quad \dots (i)$$

Since $x_1 = 3, y_1 = -7$ and $x_2 = -2$, and $y_2 = -5$, equation (i) becomes,

$$y + 7 = \frac{-5 + 7}{-2 - 3}(x - 3)$$

$$\text{or, } y + 7 = \frac{2}{-5}(x - 3)$$

$$\text{or, } 2x + 5y + 29 = 0$$

(d) INTERCEPT FORM

We want to find the equation of a line which cuts off given intercepts on both the co-ordinate axes.

Let PQ be a line meeting x -axis in A and y -axis in B . Let $OA = a$, $OB = b$.

Then the co-ordinates of A and B are $(a, 0)$ and $(0, b)$ respectively.

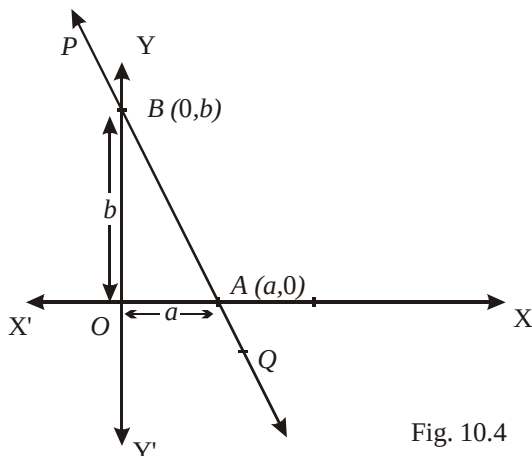
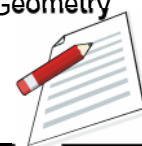


Fig. 10.4



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The equation of the line joining A and B is

$$y - 0 = \frac{b - 0}{0 - a} (x - a)$$

or, $y = -\frac{b}{a} (x - a)$

or, $\frac{y}{b} = -\frac{x}{a} + 1$

or, $\frac{x}{a} + \frac{y}{b} = 1$

This is the required equation of the line having intercepts a and b on the axes.

Example 10.8 Find the equation of a line which cuts off intercepts 5 and -3 on x and y axes respectively.

Solution : The intercepts are 5 and -3 on x and y axes respectively. i.e., $a = 5$, $b = -3$

The required equation of the line is

$$\frac{x}{5} + \frac{y}{-3} = 1$$

$$3x - 5y - 15 = 0$$

Example 10.9 Find the equation of a line which passes through the point $(3, 4)$ and makes intercepts on the axes equal in magnitude but opposite in sign.

Solution : Let the x -intercept and y -intercept be a and $-a$ respectively

$\therefore$ The equation of the line is

$$\frac{x}{a} + \frac{y}{-a} = 1$$

$$x - y = a \quad \dots (i)$$

Since (i) passes through $(3, 4)$

$$\therefore 3 - 4 = a \text{ or}$$

or, $a = -1$

Thus, the required equation of the line is

$$x - y = -1$$

$$\text{or } x - y + 1 = 0$$



Example 10.10 Determine the equation of the line through the point $(-1, 1)$ and parallel to x -axis.

Solution : Since the line is parallel to x -axis its slope is zero. Therefore from the point slope form of the equation, we get

$$y - 1 = 0 [x - (-1)]$$

$$y - 1 = 0$$

which is the required equation of the given line

Example 10.11 Find the intercepts made by the line

$$3x - 2y + 12 = 0 \text{ on the coordinate axes}$$

Solution : Equation of the given line is

$$3x - 2y = -12.$$

Dividing by -12 , we get

$$\frac{x}{-4} + \frac{y}{6} = 1$$

Comparing it with the standard equation of the line in intercept form, we find $a = -4$ and $b = 6$. Hence the intercepts on the x -axis and y -axis respectively are -4 and 6 .

Example 10.12 The segment of a line, intercepted between the coordinate axes is bisected at the point (x_1, y_1) . Find the equation of the line

Solution : Let $P(x_1, y_1)$ be the middle point or the segment CD of the line AB intercepted between the axes. Draw $PM \perp OX$

$$\therefore OM = x_1 \text{ and } MP = y_1$$

$$\therefore OC = 2x_1 \text{ and } OD = 2y_1$$

Now, from the intercept form of the line

$$\frac{x}{2x_1} + \frac{y}{2y_1} = 1$$

$$\text{or, } \frac{x}{x_1} + \frac{y}{y_1} = 2$$

which is the required equation of the line.

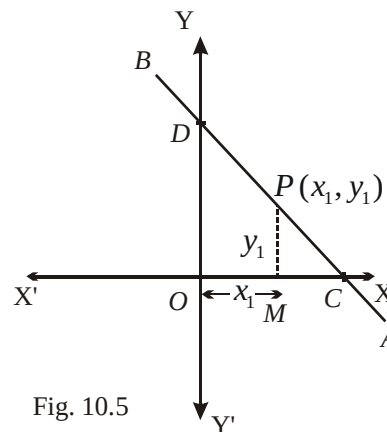


Fig. 10.5

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(e) PERPENDICULAR FORM (NORMAL FORM)

We now derive the equation of a line when p be the length of perpendicular from the origin on the line and α , the angle which this perpendicular makes with the positive direction of x -axis is given.

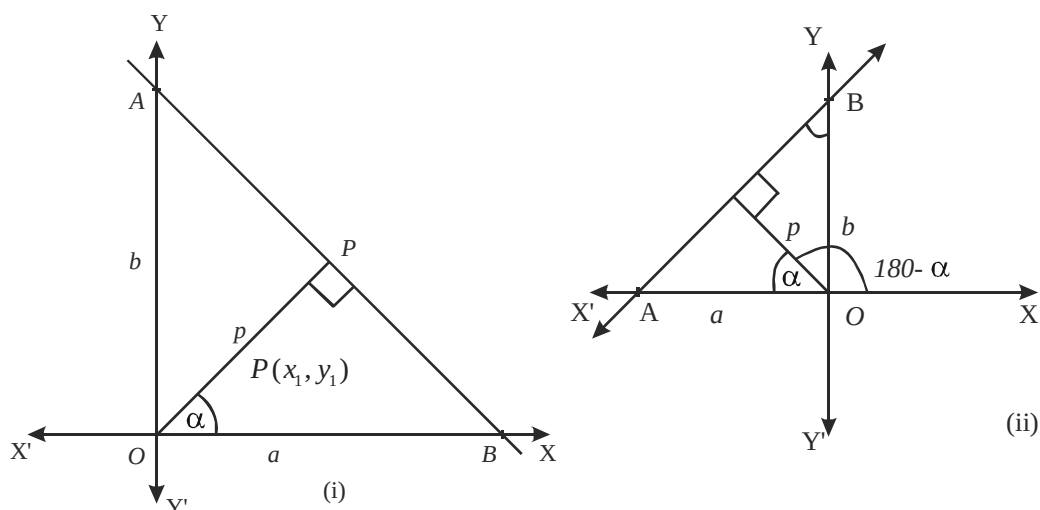


Fig. 10.6

- (i) Let AB be the given line cutting off intercepts a and b on x -axis and y -axis respectively. Let OP be perpendicular from origin O on AB and $\angle POB = \alpha$ (See Fig. 10.6 (i))

$$\therefore \frac{p}{a} = \cos \alpha \Rightarrow a = p \sec \alpha$$

$$\text{and, } \frac{p}{b} = \sin \alpha \Rightarrow b = p \operatorname{cosec} \alpha$$

$\therefore$ The equation of line AB is

$$\frac{x}{p \sec \alpha} + \frac{y}{p \operatorname{cosec} \alpha} = 1$$

$$\text{or, } x \cos \alpha + y \operatorname{cosec} \alpha = p$$

$$(ii) \quad \frac{p}{a} = \cos (180^\circ - \alpha) = -\cos \alpha \quad [\text{From Fig. 10.6 (ii)}]$$

$$\Rightarrow a = -p \sec \alpha$$

$$\text{similary, } b = p \operatorname{cosec} \alpha$$

$$\therefore \text{ The equation of the line } AB \text{ is } \frac{x}{-a} + \frac{y}{b} = 1$$

$$x \cos \alpha + y \sin \alpha = p$$



Note : 1. p is the length of perpendicular from the origin on the line and is always taken to be positive.

2. α is the angle between positive direction of x -axis and the line perpendicular from the origin to the given line.

Example 10.13 Determine the equation of the line with $\alpha = 135^\circ$ and perpendicular distance $p = \sqrt{2}$ from the origin.

Solution : From the standard equation of the line in normal form have

$$x \cos 135^\circ + y \sin 135^\circ = \sqrt{2}$$

$$\text{or, } -\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \sqrt{2}$$

$$\text{or, } -x + y - 2 = 0$$

$$\text{or, } x - y + 2 = 0$$

which is the required equation of the straight line.

Example 10.14 Find the equation of the line whose perpendicular distance from the origin is 6 units and the perpendicular from the origin to line makes an angle of 30° with the positive direction of x -axis.

Solution : Here $\alpha = 30^\circ$, $p = 6$

$\therefore$ The equation of the line is

$$x \cos 30^\circ + y \sin 30^\circ = 6$$

$$\text{or, } x \left(\frac{\sqrt{3}}{2} \right) + y \left(\frac{1}{2} \right) = 6$$

$$\text{or, } \sqrt{3}x + y = 12$$

(f) PARAMETRIC FORM

We now want to find the equation of a line through a given point $Q(x_1, y_1)$ which makes an angle α with the positive direction of x -axis in the form

$$\frac{x - x_1}{\cos \alpha} = \frac{y - y_1}{\sin \alpha} = r$$

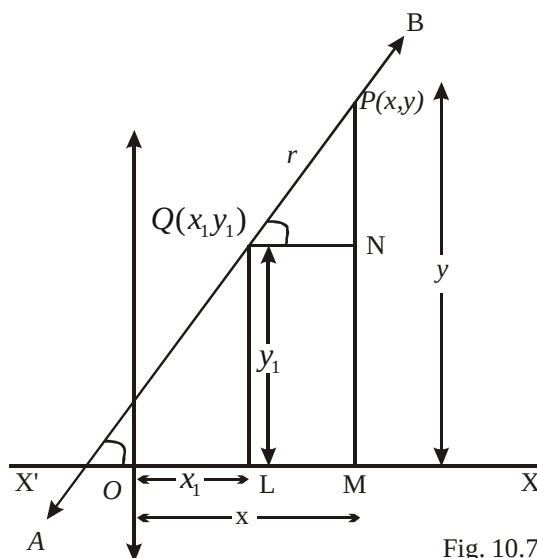


Fig. 10.7

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where r is the distance of any point $P(x, y)$ on the line from $Q(x_1, y_1)$.

Let AB be a line passing through a given point $Q(x_1, y_1)$ making an angle α with the positive direction of x -axis.

Let $P(x, y)$ be any point on the line such that $QP = r$ (say)

Draw QL, PM perpendiculars to x -axis and draw $QN \perp PM$.

From right angled ΔPNQ

$$\cos \alpha = \frac{QN}{QP}, \sin \alpha = \frac{PN}{QP}$$

But $QN = LM = OM - OL = x - x_1$

and $PN = PM - MN = PM - QL = y - y_1$

$$\therefore \cos \alpha = \frac{x - x_1}{r}, \sin \alpha = \frac{y - y_1}{r}$$

or, $\frac{x - x_1}{\cos \alpha} = \frac{y - y_1}{\sin \alpha} = r$, which is the required equation of the line

Note : 1. The co-ordinates of any point on this line can be written as $(x_1 + r \cos \alpha, y_1 + r \sin \alpha)$. Clearly coordinates of the point depend on the value of r . This variable r is called parameter.

2. The equation $x = x_1 + r \cos \alpha, y = y_1 + r \sin \alpha$ are called the parametric equations of the line.

3. The value of ' r ' is positive for all points lying on one side of the given point and negative for all points lying on the other side of the given point.

Example 10.15 Find the equation of a line passing through $A(-1, -2)$ and making an angle of 30° with the positive direction of x -axis, in the parametric form. Also find the coordinates of a point P on it at a distance of 2 units from the point A .

Solution : Here $x_1 = -1, y_1 = -2$ and $\alpha = 30^\circ$

$\therefore$ The equation of the line is

$$\frac{x + 1}{\cos 30^\circ} = \frac{y + 2}{\sin 30^\circ}$$

$$\text{or, } \frac{x + 1}{\frac{\sqrt{3}}{2}} = \frac{y + 2}{\frac{1}{2}}$$

Since $AP = 2$, $r = 2$

$$\therefore \frac{\frac{x+1}{\sqrt{3}}}{2} = \frac{\frac{y+2}{1}}{2} = 2$$

$$\Rightarrow x = \sqrt{3} - 1, y = -1$$

Thus the coordinates of the point P are $(\sqrt{3} - 1, -1)$

Example 10.16 Find the distance of the point $(1, 2)$ from the line $2x - 3y + 9 = 0$ measured along a line making an angle of 45° with x -axis.

Solution : The equation of any line through $A(1, 2)$ making an angle of 45° with x -axis is

$$\frac{x-1}{\cos 45^\circ} = \frac{y-2}{\sin 45^\circ} = r$$

$$\frac{x-1}{\frac{1}{\sqrt{2}}} = \frac{y-2}{\frac{1}{\sqrt{2}}} = r$$

Any point on it is $\left(1 + \frac{r}{\sqrt{2}}, 2 + \frac{r}{\sqrt{2}}\right)$

It lies on the given line $2x - 3y + 9 = 0$

$$\text{if } 2\left(1 + \frac{r}{\sqrt{2}}\right) - 3\left(2 + \frac{r}{\sqrt{2}}\right) + 9 = 0$$

$$\text{or, } 2 - 6 + 9 - \frac{r}{\sqrt{2}} = 0$$

$$\text{or, } r = 5\sqrt{2}$$

Hence the required distance is $5\sqrt{2}$.



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CHECK YOUR PROGRESS 10.2

- (a) Find the equation of a line with slope 2 and y -intercept is equal to -2 .

(b) Determine the slope and the intercepts made by the line on the axes whose equation is $4x + 3y = 6$.
- Find the equation of the line cutting off an intercept $\frac{1}{\sqrt{3}}$ on negative direction of axis of y and inclined at 120° to the positive direction of x -axis.
- Find the slope and y -intercept of the line whose equation is $3x - 6y = 12$.
- Determine the equation of the line passing through the point $(-7, 4)$ and having the slope $-\frac{3}{7}$.
- Determine the equation of the line passing through the point $(1, 2)$ which makes equal angles with the two axes.
- Find the equation of the line passing through $(2, 3)$ and parallel to the line joining the points $(2, -2)$ and $(6, 4)$.
- (a) Determine the equation of the line through $(3, -4)$ and $(-4, 3)$.

(b) Find the equation of the diagonals of the rectangle ABCD whose vertices are A $(3, 2)$, B $(11, 8)$, C $(8, 12)$ and D $(0, 6)$.
- Find the equation of the medians of a triangle whose vertices are $(2, 0)$, $(0, 2)$ and $(4, 6)$.
- Find the equation of the line which cuts off intercepts of length 3 units and 2 units on x -axis and y -axis respectively.
- Find the equation of a line such that the segment between the coordinate axes has its mid point at the point $(1, 3)$
- Find the equation of a line which passes through the point $(3, -2)$ and cuts off positive intercepts on x and y axes in the ratio of $4 : 3$.
- Determine the equation of the line whose perpendicular from the origin is of length 2 units and makes an angle of 45° with the positive direction of x -axis.
- If p is the length of the perpendicular segment from the origin, on the line whose intercept on the axes are a and b , then show that

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}.$$
- Find the equation of a line passing through A $(2, 1)$ and making an angle 45° with the positive direction of x -axis in parametric form. Also find the coordinates of a point P on it at a distance of 1 unit from the point A.

10.3 GENERAL EQUATION OF FIRST DEGREE

You know that a linear equation in two variables x and y is given by $Ax + By + C = 0 \dots (1)$

In order to understand its graphical representation, we need to take the following three cases.

Case-1: (When both A and B are equal to zero)

In this case C is automatically zero and the equation does not exist.

Case-2: (When $A = 0$ and $B \neq 0$)

In this case the equation (1) becomes $By + C = 0$.

or $y = -\frac{C}{B}$ and is satisfied by all points lying on a line which is parallel to x -axis and the

y -coordinate of every point on the line is $-\frac{C}{B}$. Hence this is the equation of a straight line. The case where $B = 0$ and $A \neq 0$ can be treated similarly.

Case-3: (When $A \neq 0$ and $B \neq 0$)

We can solve the equation (1) for y and obtain.

$$y = -\frac{A}{B}x - \frac{C}{B}$$

Clearly, this represents a straight line with slope $-\frac{A}{B}$ and y -intercept equal to $-\frac{C}{B}$.

10.3.1 CONVERSION OF GENERAL EQUATION OF A LINE INTO VARIOUS FORMS

If we are given the general equation of a line, in the form $Ax + By + C = 0$, we will see how this can be converted into various forms studied before.

10.3.2 CONVERSION INTO SLOPE-INTERCEPT FORM

We are given a first degree equation in x and y as $Ax + By + C = 0$

Are you able to find slope and y -intercept?

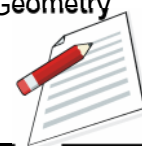
Yes, indeed, if we are able to put the general equation in slope-intercept form. For this purpose, let us re-arrange the given equation as.

$$Ax + By + C = 0 \text{ as}$$

$$By = -Ax - C$$

$$\text{or } y = -\frac{A}{B}x - \frac{C}{B} \text{ (Provided } B \neq 0)$$

which is the required form. Hence, the slope $= -\frac{A}{B}$, y -intercept $= -\frac{C}{B}$.



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Example 10.17 Reduce the equation $x + 7y - 4 = 0$ to the slope – intercept form.**Solution :** The given equation is

$$x + 7y - 4 = 0$$

$$\text{or } 7y = -x + 4$$

$$\text{or } y = -\frac{1}{7}x + \frac{4}{7}$$

which is the converted form of the given equation in slope-intercept form

Example 10.18 Find the slope and y intercept of the line $x + 4y - 3 = 0$.**Solution :** The given equation is

$$x + 4y - 3 = 0$$

$$\text{or } 4y = -x + 3$$

$$\text{or } y = -\frac{1}{4}x + \frac{3}{4}$$

Comparing it with slope-intercept form, we have

$$\text{slope} = -\frac{1}{4}, y\text{-intercept} = \frac{3}{4}.$$

10.3.3 CONVERSION INTO INTERCEPT FORMSuppose the given first degree equation in x and y is

$$Ax + By + C = 0. \quad \dots(i)$$

In order to convert (i) in intercept form, we re arrange it as

$$Ax + By = -C \text{ or } \frac{Ax}{-C} + \frac{By}{-C} = 1$$

$$\text{or } \frac{\frac{x}{(-\frac{C}{A})}}{(-\frac{C}{A})} + \frac{\frac{y}{(-\frac{C}{B})}}{(-\frac{C}{B})} = 1 \quad (\text{Provided } A \neq 0 \text{ and } B \neq 0)$$

which is the requied converted form. It may be noted that intercept on x – axis = $\frac{-C}{A}$ and

$$\text{intercept on } y\text{ – axis} = \frac{-C}{B}$$



Example 10.19 Reduce $3x + 5y = 7$ into the intercept form and find its intercepts on the axes.

Solution : The given equation is

$$3x + 5y = 7$$

$$\text{or, } \frac{3}{7}x + \frac{5}{7}y = 1$$

$$\text{or, } \frac{x}{\frac{7}{3}} + \frac{y}{\frac{7}{5}} = 1$$

$$\therefore \text{ The } x\text{-intercept} = \frac{7}{3} \quad \text{and, } y\text{-intercept} = \frac{7}{5}$$

Example 10.20 Find x and y -intercepts for the line $3x - 2y = 5$.

Solution : The given equation is

$$3x - 2y = 5$$

$$\text{or, } \frac{3}{5}x - \frac{2}{5}y = 1$$

$$\text{or, } \frac{x}{\frac{5}{3}} + \frac{y}{\frac{-5}{2}} = 1$$

$$\text{Thus, the required } x\text{-intercept} = \frac{5}{3}$$

$$\text{and } y\text{-intercept} = -\frac{5}{2}$$

10.3.4 CONVERSION INTO PERPENDICULAR FORM

Let the general first degree equation in x and y be

$$Ax + By + C = 0 \quad \dots (i)$$

We will convert this general equation in perpendicular form. For this purpose let us re-write the given equation (i) as $Ax + By = -C$

Multiplying both sides of the above equation by λ , we have

$$\lambda Ax + \lambda By = -\lambda C \quad \dots (ii)$$

Let us choose λ such that $(\lambda A)^2 + (\lambda B)^2 = 1$

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or $\lambda = \frac{1}{\sqrt{(A^2 + B^2)}} \quad (\text{Taking positive sign})$

Substituting this value of λ in (ii), we have

$$\frac{Ax}{\sqrt{(A^2 + B^2)}} + \frac{By}{\sqrt{(A^2 + B^2)}} = -\frac{C}{\sqrt{(A^2 + B^2)}} \quad \dots (iii)$$

This is required conversion of (i) in perpendicular form. Two cases arise according as C is negative or positive.

(i) If $C < 0$, the equation (ii) is the required form.

(ii) If $C > 0$, the R. H. S. of the equation of (iii) is negative.

$\therefore$ We shall multiply both sides of the equation of (iii) by -1 .

$\therefore$ The required form will be

$$-\frac{Ax}{\sqrt{(A^2 + B^2)}} - \frac{By}{\sqrt{(A^2 + B^2)}} = \frac{C}{\sqrt{(A^2 + B^2)}}$$

Thus, length of perpendicular from the origin = $\frac{|C|}{\sqrt{(A^2 + B^2)}}$

Inclination of the perpendicular with the positive direction of x-axis is

is given by $\cos \theta = \mp \frac{A}{\sqrt{A^2 + B^2}}$

or $\sin \theta = \left(\mp \frac{B}{\sqrt{(A^2 + B^2)}} \right)$

where the upper sign is taken for $C > 0$ and the lower sign for $C < 0$. If $C = 0$, the line passes through the origin and there is no perpendicular from the origin on the line.

With the help of the above three cases, we are able to say that

"The general equation of first degree in x and y always represents a straight line provided A and B are not both zero simultaneously."

Is the converse of the above statement true? **The converse of the above statement is that every straight line can be expressed as a general equation of first degree in x and y .**



In this lesson we have studied about the various forms of equation of straight line. For example, let us take some of them as $y = mx + c$, $\frac{x}{a} + \frac{y}{b} = 1$ and $x \cos \alpha + y \sin \alpha = p$. Obviously, all are linear equations in x and y . We can re-arrange them as $y - mx - c = 0$, $bx + ay - ab = 0$ and $x \cos \alpha + y \sin \alpha - p = 0$ respectively. Clearly, these equations are nothing but a different arrangement of general equation of first degree in x and y . Thus, we have established that

"Every straight line can be expressed as a general equation of first degree in x and y ".

Example 10.21 Reduce the equation $x + \sqrt{3}y + 7 = 0$ into perpendicular form.

Solution : The equation of given line is $x + \sqrt{3}y + 7 = 0$... (i)

Comparing (i) with general equation of straight line, we have

$$A = 1 \text{ and } B = \sqrt{3}$$

$$\therefore \sqrt{A^2 + B^2} = 2$$

Dividing equation (i) by 2, we have

$$\frac{x}{2} + \frac{\sqrt{3}}{2}y + \frac{7}{2} = 0$$

$$\text{or } \left(-\frac{1}{2}\right)x + \left(-\frac{\sqrt{3}}{2}\right)y - \frac{7}{2} = 0$$

$$\text{or } x \cos \frac{4\pi}{3} + y \sin \frac{4\pi}{3} = \frac{7}{2}$$

($\cos \theta$ and $\sin \theta$ being both negative in the third quadrant, value of θ will lie in the third quadrant).

This is the representation of the given line in perpendicular form.

Example 10.22 Find the perpendicular distance from the origin on the line $\sqrt{3}x - y + 2 = 0$. Also, find the inclination of the perpendicular from the origin.

Solution : The given equation is $\sqrt{3}x - y + 2 = 0$

Dividing both sides by $\sqrt{(\sqrt{3})^2 + (-1)^2}$ or 2, we have

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$$\frac{\sqrt{3}}{2}x - \frac{1}{2}y + 1 = 0$$

$$\text{or, } \frac{\sqrt{3}}{2}x - \frac{1}{2}y = -1$$

 Multiplying both sides by -1 , we have

$$-\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 1$$

or, $x \cos \frac{5\pi}{6} + y \sin \frac{5\pi}{6} = 1$ ($\cos \theta$ is $-ve$ in second quadrant and $\sin \theta$ is $+ve$ in second quadrant, so value of θ lies in the second quadrant).

Thus, inclination of the perpendicular from the origin is 150° and its length is equal to 1.

Example 10.23 Find the equation of a line which passes through the point $(3, 1)$ and bisects the portion of the line $3x + 4y = 12$ intercepted between coordinate axes.

Solution : First we find the intercepts on coordinate axes cut off by the line whose equation is from. $3x + 4y = 12$

$$\text{or } \frac{3x}{12} + \frac{4y}{12} = 1$$

$$\text{or } \frac{x}{4} + \frac{y}{3} = 1$$

Hence, intercepts on x -axis and y -axis are 4 and 3 respectively.

Thus, the coordinates of the points where the line meets the coordinate axes are $A(4, 0)$ and $B(0, 3)$.

$$\therefore \text{Mid-point of } AB \text{ is } \left(2, \frac{3}{2}\right).$$

Hence the equation of the line through $(3, 1)$

and $\left(2, \frac{3}{2}\right)$ is

$$y - 1 = \frac{\frac{3}{2} - 1}{2 - 3}(x - 3)$$

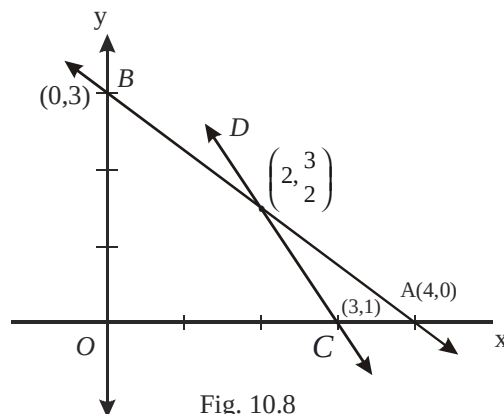
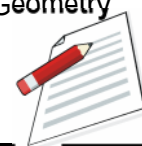


Fig. 10.8



$$\text{or } y - 1 = -\frac{1}{2}(x - 3)$$

$$\text{or } 2(y - 1) + (x - 3) = 0$$

$$\text{or } 2y - 2 + x - 3 = 0$$

$$\text{or } x + 2y - 5 = 0$$

Example 10.24 Prove that the line through (8, 7) and (6, 9) cuts off equal intercepts on coordinate axes.

Solution : The equation of the line passing through (8, 7) and (6, 9) is

$$y - 7 = \frac{9 - 7}{6 - 8}(x - 8)$$

$$\text{or } y - 7 = -(x - 8)$$

$$\text{or } x + y = 15$$

$$\text{or } \frac{x}{15} + \frac{y}{15} = 1$$

Hence, intercepts on both axes are 15 each.

Example 10.25 Find the ratio in which the line joining (−5, 1) and (1, −3) divides the join of (3, 4) and (7, 8).

Solution : The equation of the line joining C (−5, 1) and D (1, −3) is

$$y - 1 = \frac{-3 - 1}{1 + 5}(x + 5)$$

$$\text{or } y - 1 = -\frac{4}{6}(x + 5)$$

$$\text{or } 3y - 3 = -2x - 10$$

$$\text{or } 2x + 3y + 7 = 0 \quad \dots (i)$$

Let line (i) divide the join of A (3, 4) and B (7, 8) at the point P.

If the required ratio is $\lambda : 1$ in which line (i) divides the join of A (3, 4) and B (7, 8), then the coordinates of P are

$$\left(\frac{7\lambda + 3}{\lambda + 1}, \frac{8\lambda + 4}{\lambda + 1} \right)$$

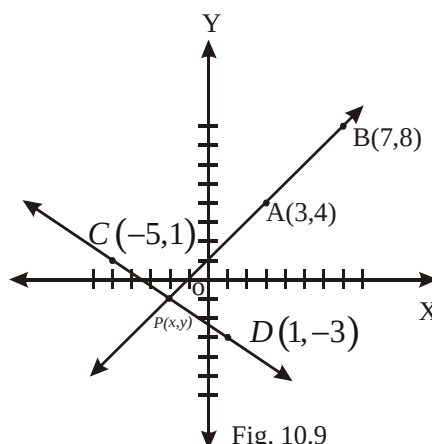


Fig. 10.9

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Since P lies on the line (i), we have

$$2\left(\frac{7\lambda + 3}{\lambda + 1}\right) + 3\left(\frac{8\lambda + 4}{\lambda + 1}\right) + 7 = 0$$

$$\Rightarrow 14\lambda + 6 + 24\lambda + 12 + 7\lambda + 7 = 0$$

$$\Rightarrow 45\lambda + 25 = 0 \Rightarrow \lambda = -\frac{5}{9}$$

Hence, the line joining $(-5, 1)$ and $(1, -3)$ divides the join of $(3, 4)$ and $(7, 8)$ externally in the ratio $5 : 9$.

**CHECK YOUR PROGRESS 10.3**

- Under what condition, the general equation $Ax + By + C = 0$ of first degree in x and y represents a line?
- Reduce the equation $2x + 5y + 3 = 0$ to the slope intercept form.
- Find the x and y intercepts for the following lines :
(a) $y = mx + c$ (b) $3y = 3x + 8$ (c) $3x - 2y + 12 = 0$
- Find the length of the line segment AB intercepted by the straight line $3x - 2y + 12 = 0$ between the two axes.
- Reduce the equation $x \cos \alpha + y \sin \alpha = p$ to the intercept form of the equation and also find the intercepts on the axes.
- Reduce the following equations into normal form.
(a) $3x - 4y + 10 = 0$ (b) $3x - 4y = 0$
- Which of the lines $2x - y + 3 = 0$ and $x - 4y - 7 = 0$ is nearer from the origin?

We will here develop a formula for finding angle between two given lines

10.4 ANGLE BETWEEN TWO LINES

We will now try to find out the angle between the arms of the divider or clock when equation of the arms are known. Two methods are discussed below.

10.4.1 When two lines with slopes m_1 and m_2 are given

Let AB and CD be two straight lines whose equations are

$$y = m_1 x + c_1 \text{ and } y = m_2 x + c_2 \text{ respectively.}$$

Let P be the point of intersection of AB and CD .

... $\angle XEB = \theta_1$ and $\angle XFD = \theta_2$. Then

$$\tan \theta_1 = m_1 \text{ and } \tan \theta_2 = m_2$$

Let θ be the angle $\angle EPF$ between AB and CD .

$$\text{Now } \theta_1 = \theta_2 + \theta$$

$$\text{or } \theta = \theta_1 - \theta_2$$

$$\therefore \tan \theta = \tan (\theta_1 - \theta_2)$$

$$\text{or } \tan \theta = \frac{\tan \theta_1 - \tan \theta_2}{1 + \tan \theta_1 \tan \theta_2}$$

$$\text{Hence, } \tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

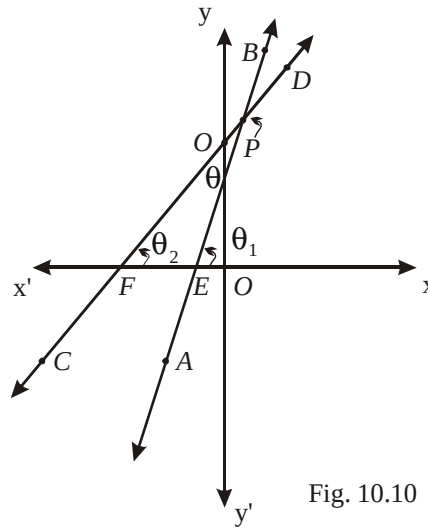


Fig. 10.10

which gives tangent of the angle θ between two given lines in terms of their slopes.

From this result we can find the angle between any two given lines when their slopes are given.

Note : (i) If $\frac{m_1 - m_2}{1 + m_1 m_2} > 0$, then angle between lines is **acute**.

(ii) If $\frac{m_1 - m_2}{1 + m_1 m_2} < 0$, then angle between lines is **obtuse**.

(iii) If $\frac{m_1 - m_2}{1 + m_1 m_2} = 0$,

i.e., $m_1 = m_2$, then lines are either **coincident or parallel**.

i.e., Two lines are parallel if they differ only by a constant term.

(iv) If the denominator of $\frac{m_1 - m_2}{1 + m_1 m_2}$ is zero.

i.e., $m_1 m_2 = -1$, then lines are **perpendicular**.

Example 10.26 Find the angle between the lines $7x - y = 1$ and $6x - y = 11$.

Solution : Let θ be the angle between the lines $7x - y = 1$ and $6x - y = 11$. The equations of these lines can be written as $y = 7x - 1$ and $y = 6x - 11$.

Here, $m_1 = 7$, $m_2 = 6$. where m_1 and m_2 are respective slopes of the given lines.

$$\therefore \tan \theta = \frac{7 - 6}{1 + 42} = \frac{1}{43}$$



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The angle θ between the given lines is given by $\tan \theta = \frac{1}{43}$

Example 10.27 Find the angle between the lines $x + y + 1 = 0$ and $2x - y - 1 = 0$.

Solution : Putting the equations of the given lines in slope intercept form, we get $y = -x - 1$ and $y = 2x - 1$

$\therefore m_1 = -1$ and $m_2 = 2$ where m_1 , and m_2 are respectively slopes of the given lines.

$$\therefore \tan \theta = \frac{-1-2}{1+(-1)(2)} = 3$$

The angle θ between the given lines is given by $\tan \theta = 3$

Example 10.28 Are the straight lines $y = 3x - 5$ and $3x + 4 = y$, parallel or perpendicular?

Solution : For the first line $y = 3x - 5$, the slope $m_1 = 3$ and for the second line $3x + 4 = y$, the slope $m_2 = 3$. Since the slopes of both the lines are same for both the lines, hence the lines are parallel.

Example 10.29 Are the straight lines $y = 3x$ and $y = -\frac{1}{3}x$, parallel or perpendicular?

Solution : The slope m_1 of the line $y = 3x$ is 3 and slope m_2 of the second line $y = -\frac{x}{3}$ is $-\frac{1}{3}$.

Since, $m_1 m_2 = -1$, the lines are perpendicular.

10.4.2 When the Equation of two lines are in general form

Let the general equations of two given straight lines be given by

$$A_1 x + B_1 y + C_1 = 0 \quad \dots (i)$$

$$\text{and } A_2 x + B_2 y + C_2 = 0 \quad \dots (ii)$$

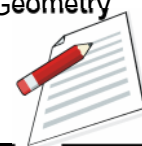
Equations (i) and (ii) can be written into their slope intercept form as

$$y = -\frac{A_1}{B_1}x - \frac{C_1}{B_1} \quad \text{and} \quad y = -\frac{A_2}{B_2}x - \frac{C_2}{B_2}$$

Let m_1 and m_2 be their respective slopes and using the formula, we get.

$$\text{i.e., } \tan \theta = \frac{A_2 B_1 - A_1 B_2}{A_1 A_2 + B_1 B_2}$$

Thus, by using this formula we can calculate the angle between two lines when they are in general form.



Note : (i) If $A_2 B_1 - A_1 B_2 = 0$

or $A_1 B_2 - A_2 B_1 = 0$

i.e. $\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix} = 0$, then the given lines are parallel.

(ii) If $A_1 A_2 + B_1 B_2 = 0$, then the given lines are perpendicular.

Example 10.30 Are the straight lines $x - 7y + 12 = 0$ and $x - 7y + 6 = 0$, parallel?

Solution : Here $A = 1$, $B_1 = -7$; $A_2 = 1$ and $B_2 = -7$

$$\text{Since } \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix} = \begin{vmatrix} 1 & -7 \\ 1 & -7 \end{vmatrix} = 0$$

Example 10.31 Determine whether the straight lines $2x + 5y - 8 = 0$ and $5x - 2y - 3 = 0$ are parallel or perpendicular?

Here $A_1 = 2$, $B_1 = 5$

$A_2 = 5$ and $B_2 = -2$

Solution : Here $A_1 A_2 + B_1 B_2 = (2) \times (5) + (5) \times (-2) = 0$.

Hence the given lines are perpendicular.



CHECK YOUR PROGRESS 10.4

- What is the condition for given lines $y = m_1 x + c_1$ and $Ax + By + C = 0$
 - to be parallel?
 - to be perpendicular?
- Find the angle between the lines $4x + y = 3$ and $\frac{x}{2} + y = \frac{4}{7}$.
- Write the condition that the two lines $A_1 x + B_1 y + C_1 = 0$ and $A_2 x + B_2 y + C_2 = 0$ are
 - parallel.
 - perpendicular.
 - Are the straight lines $x - 3y = 7$ and $2x - 6y - 16 = 0$, parallel?
 - Are the straight lines $x = y + 1$ and $x = -y + 1$ perpendicular?

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10.5 DISTANCE OF A GIVEN POINT FROM A GIVEN LINE

In this section, we shall discuss the concept of finding the distance of a given point from a given line or lines.

Let $P(x_1, y_1)$ be the given point and l be the line $Ax + By + C = 0$.

Let the line l intersect x axis and y axis R and Q respectively.

Draw $PM \perp l$ and let $PM = d$.

Let the coordinates of M be (x_2, y_2)

$$d = \sqrt{\{(x_1 - x_2)^2 + (y_1 - y_2)^2\}} \quad \dots(i)$$

$\therefore M$ lies on l

$$\therefore Ax_2 + By_2 + C = 0$$

$$\text{or } C = -(Ax_2 + By_2) \quad \dots(ii)$$

The coordinates of R and Q are $\left(-\frac{C}{A}, 0\right)$ and $\left(0, -\frac{C}{B}\right)$ respectively.

$$\text{The slope of } QR = \frac{0 + \frac{C}{B}}{-\frac{C}{A} - 0} = -\frac{A}{B} \text{ and,}$$

$$\text{the slope of } PM = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{As } PM \perp QR \Rightarrow \frac{y_2 - y_1}{x_2 - x_1} \times \left(-\frac{A}{B}\right) = -1.$$

$$\text{or } \frac{y_1 - y_2}{x_1 - x_2} = \frac{B}{A} \quad \dots(iii)$$

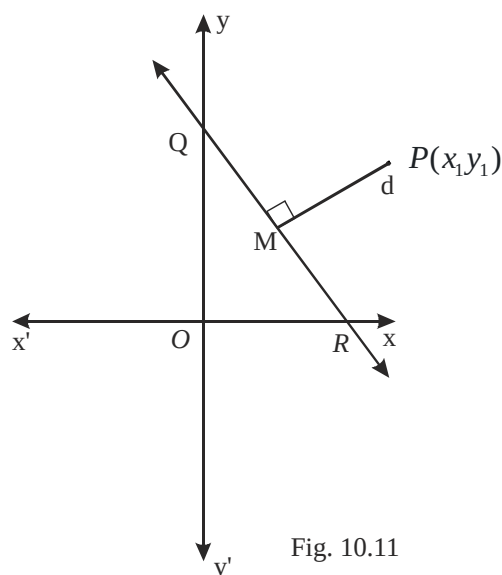


Fig. 10.11



$$\text{From (iii)} \quad \frac{x_1 - x_2}{A} = \frac{y_1 - y_2}{B} = \frac{\sqrt{\{(x_1 - x_2)^2 + (y_1 - y_2)^2\}}}{\sqrt{(A^2 + B^2)}} \quad \dots(\text{iv})$$

(Using properties of Ratio and Proportion)

$$\text{Also} \quad \frac{x_1 - x_2}{A} = \frac{y_1 - y_2}{B} = \frac{A(x_1 - x_2) + B(y_1 - y_2)}{A^2 + B^2} \quad \dots(\text{v})$$

From (iv) and (v), we get

$$\frac{\sqrt{\{(x_1 - x_2)^2 + (y_1 - y_2)^2\}}}{\sqrt{(A^2 + B^2)}} = \frac{A(x_1 - x_2) + B(y_1 - y_2)}{A^2 + B^2}$$

$$\text{or} \quad \frac{d}{\sqrt{A^2 + B^2}} + \frac{Ax_1 + By_1 - (Ax_2 + By_2)}{A^2 + B^2} \quad [\text{Using (i)}]$$

$$\text{or} \quad \frac{Ax_1 + By_1 + C}{\sqrt{(A^2 + B^2)}} \quad [\text{Using (ii)}]$$

Since the distance is always positive, we can write

$$d = \left| \frac{Ax_1 + By_1 + C}{\sqrt{(A^2 + B^2)}} \right|$$

Note : The perpendicular distance of the origin (0, 0) from $Ax + By + C = 0$ is

$$\frac{A(0) + B(0) + C}{\sqrt{(A^2 + B^2)}} = \frac{C}{\sqrt{(A^2 + B^2)}}$$

Example 10.32 Find the points on the x-axis whose perpendicular distance from the straight

$$\text{line } \frac{x}{a} + \frac{y}{b} = 1 \text{ is } a.$$

Solution : Let $(x_1, 0)$ be any point on x-axis.

Equation of the given line is $bx + ay - ab = 0$. The perpendicular distance of the point $(x_1, 0)$ from the given line is

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$$a = \pm \frac{bx_1 + a \cdot 0 - ab}{\sqrt{(a^2 + b^2)}}$$

$$\therefore x_1 = \frac{a}{b} \left\{ b \pm \sqrt{(a^2 + b^2)} \right\}$$

Thus, the point on x-axis is $\left(\frac{a}{b} (b \pm \sqrt{a^2 + b^2}), 0 \right)$



CHECK YOUR PROGRESS 10.5

- Find the perpendicular distance of the point (2, 3) from $3x + 2y + 4 = 0$.
- Find the points on the axis of y whose perpendicular distance from the straight line $\frac{x}{a} + \frac{y}{b} = 1$ is b.
- Find the points on the axis of y whose perpendicular distance from the straight line $4x + 3y = 12$ is 4.
- Find the perpendicular distance of the origin from $3x + 7y + 14 = 0$
- What are the points on the axis of x whose perpendicular distance from the straight line $\frac{x}{4} + \frac{y}{4} = 1$ is 3?

10.6 EQUATION OF PARALLEL (OR PERPENDICULAR) LINES

Till now, we have developed methods to find out whether the given lines are parallel or perpendicular. In this section, we shall try to find, the equation of a line which is parallel or perpendicular to a given line.

10.6.1 EQUATION OF A STRAIGHT LINE PARALLEL TO THE GIVEN LINE

$$Ax + By + c = 0$$

Let $A_1x + B_1y + C_1 = 0$... (i)

be any line parallel to the given line

$$Ax + By + C = 0$$

The condition for parallelism of (i) and (ii) is ... (ii)

$$\frac{A_1}{A} = \frac{B_1}{B} = K_1 \quad (\text{say})$$

$$\Rightarrow A_1 = AK_1, B_1 = BK_1$$



with these values of A_1 and B_1 , (i) gives

$$AK_1x + BK_1y + C_1 = 0$$

or $Ax + By + \frac{C_1}{K_1} = 0$

or $Ax + By + K = 0$, where $K = \frac{C_1}{K_1}$... (iii)

This is a line parallel to the given line. From equations (ii) and (iii) we observe that

(i) coefficients of x and y are same

(ii) constants are different, and are to be evaluated from given conditions.

Example 10.33 Find equation of the straight line, which passes through the point $(1, 2)$ and which is parallel to the straight line $2x + 3y + 6 = 0$.

Solution : Equation of any straight line parallel to the given equation can be written if we put

(i) the coefficients of x and y as same as in the given equation.

(ii) constant to be different from the given equation, which is to be evaluated under given condition.

Thus, the required equation of the line will be

$$2x + 3y + K = 0 \text{ for some constant } K$$

Since it passes through the point $(1, 2)$ hence

$$2 \times 1 + 3 \times 2 + K = 0$$

or $K = -8$

$\therefore$ Required equation of the line is $2x + 3y = 8$.

10.7 STRAIGHT LINE PERPENDICULAR TO THE GIVEN LINE

$$Ax + By + C = 0$$

Let $A_1x + B_1y + C_1 = 0$

be any line perpendicular to the given line ... (i)

$$Ax + By + C = 0$$

Condition for perpendicularity of lines (i) and (ii) is ... (ii)

$$AA_1 + BB_1 = 0$$

$$\Rightarrow \frac{A_1}{B} = -\frac{B_1}{A} = K_1 \text{ (say)}$$

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$$\Rightarrow A_1 = BK_1 \text{ and } B_1 = -AK_1$$

With these values of A_1 and B_1 , (i) gives

$$Bx - Ay + \frac{C_1}{K_1} = 0 = 0$$

$$\text{or } Bx - Ay + K = 0 \text{ where } K = \frac{C_1}{K_1} \quad \dots \text{(iii)}$$

Hence, the line (iii) is perpendicular to the given line (ii)

We observe that in order to get a line perpendicular to the given line we have to follow the following procedure :

- (i) Interchange the coefficients of x and y
- (ii) Change the sign of one of them.
- (iii) Change the Constant term to a new constant K (say), and evaluate it from given condition.

Example 10.34 Find the equation of the line which passes through the point $(1, 2)$ and is perpendicular to the line $2x + 3y + 6 = 0$.

Solution : Following the procedure given above, we get the equation of line perpendicular to the given equation as $3x - 2y + K = 0$...(i)

(i) passes through the point $(1, 2)$, hence

$$3 \times 1 - 2 \times 2 + K = 0 \text{ or } K = 1$$

$\therefore$ Required equation of the straight line is $3x - 2y + 1 = 0$.

Example 10.35 Find the equation of the line which passes through the point (x_2, y_2) and is perpendicular to the straight line $yy_1 = 2a(x + x_1)$.

Solution : The given straight line is $yy_1 - 2ax - 2ax_1 = 0$...(i)

Any straight line perpendicular to (i) is $2ay + xy_1 + C = 0$

This passes through the point (x_2, y_2)

$$\therefore 2ay_2 + x_2 y_1 + C = 0$$

$$\Rightarrow C = -2ay_2 - x_2 y_1$$

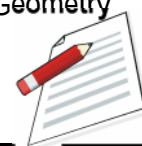
$\therefore$ Required equation of the straight line is

$$2a(y - y_2) + y_1(x - x_2) = 0$$



CHECK YOUR PROGRESS 10.6

- Find the equation of the straight line which passes through the point $(0, -2)$ and is parallel to the straight line $3x + y = 2$.
- Find the equation of the straight line which passes through the point $(-1, 0)$ and is parallel to the straight line $y = 2x + 3$.
- Find the equation of the straight line which passes through the point $(0, -3)$ and is perpendicular to the straight line $x + y + 1 = 0$.
- Find the equation of the line which passes through the point $(0, 0)$ and is perpendicular to the straight line $x + y = 3$.
- Find the equation of the straight line which passes through the point $(2, -3)$ and is perpendicular to the given straight line $2a(x + 2) + 3y = 0$.
- Find the equation of the line which has x -intercept -8 and is perpendicular to the line $3x + 4y - 17 = 0$.
- Find the equation of the line whose y -intercept is 2 and is parallel to the line $2x - 3y + 7 = 0$.
- Prove that the equation of a straight line passing through $(a \cos^3 \theta, a \sin^3 \theta)$ and perpendicular to the line $x \sec \theta + y \operatorname{cosec} \theta = a$ is $x \cos \theta - y \sin \theta = a \cos 2\theta$.



Notes

10.8 APPLICATION OF COORDINATE GEOMETRY

Let us take some examples to show how coordinate geometry can be gainfully used to prove some geometrical results.

Example 10.36 Prove that the diagonals of a rectangle are equal.

Solution : Let us take origin as one vertex of the rectangle without any loss of generality and take one side along x -axis of length a and another side of length b along y -axis, as shown in Fig. 10.12. Then the coordinates of the points A, B and C are $(a, 0)$, (a, b) and $(0, b)$ respectively

$$\text{Length of diagonal AC} = \sqrt{(a - 0)^2 + (0 - b)^2} = \sqrt{a^2 + b^2}$$

$$\text{Length of diagonal OB} = \sqrt{(a - 0)^2 + (0 - b)^2} = \sqrt{a^2 + b^2}$$

$$\therefore AC = OB$$

i.e., the diagonals of the rectangle OA BC are equal. Thus, in general, we can say that the length of the diagonals of a rectangle are equal.

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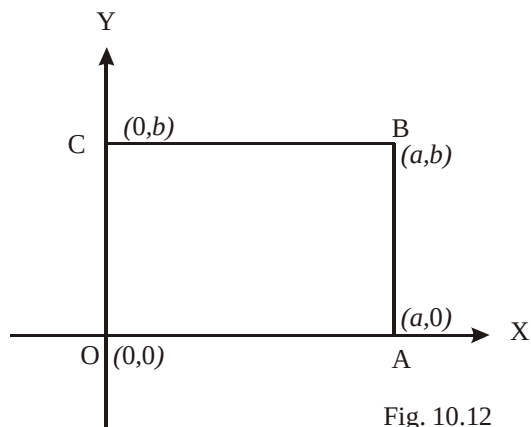


Fig. 10.12

Example 10.37 Prove that angle in a semi circle is a right angle.

Solution : Let O (0,0) be the centre of the circle and $(-a,0)$ and $(a,0)$ be the end- points of one of its diameter. Let the circle intersects the y-axis at $(0, a)$ [see Fig. 10.13]

$$\therefore \text{Slope of BQ} = \frac{a-0}{0-a} = -1 = m_1$$

$$\text{Slope of AQ} = \frac{0-a}{a-0} = -1 = m_2$$

As $m_1 m_2 = -1 \Rightarrow$ AQ is perpendicular to BQ

$\therefore$ BQA is a right angle.

Again, let $P(a \cos \theta, a \sin \theta)$ be a general point on the circle.

$$\therefore \text{Slope of PB} = \frac{a \sin \theta}{a \cos \theta + a} = \frac{\sin \theta}{1 + \cos \theta} = m_1 \text{ (say)}$$

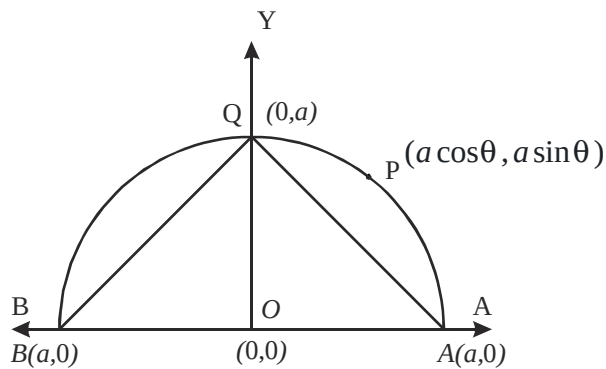


Fig. 10.13



$$\text{Slope of QA} = \frac{0 - a \sin \theta}{a - a \cos \theta} = \frac{\sin \theta}{1 - \cos \theta} = m_2 \text{ (say)}$$

$$\text{and } m_1 \cdot m_2 = \left(\frac{\sin \theta}{1 + \cos \theta} \right) \left(-\frac{\sin \theta}{1 - \cos \theta} \right) = -\frac{\sin^2 \theta}{1 - \cos^2 \theta} = -\frac{\sin^2 \theta}{1 - \cos^2 \theta} = -1$$

$\therefore$ PA is perpendicular to PB.

$\angle BPA$ is a right angle.

Example 10.38 Medians drawn from two vertices to equal sides of an isosceles triangle are equal.

Solution : Let OAB be an equilateral triangle with vertices $(0,0)$, $(a,0)$ and $\left(\frac{a}{2}, a\right)$

Let C and D be the mid-points of AB and OB respectively.

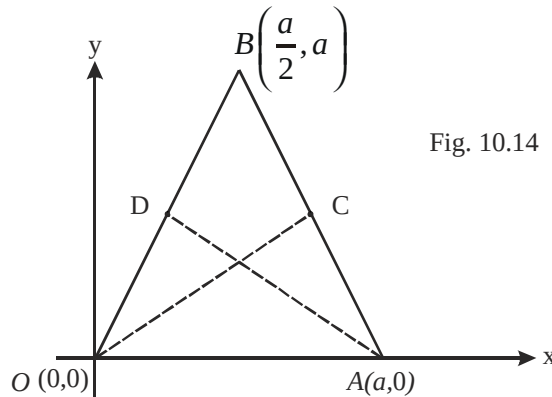


Fig. 10.14

$\therefore$ C has coordinates $\left(\frac{3a}{4}, \frac{a}{2}\right)$ and

D has coordinates $\left(\frac{a}{4}, \frac{a}{2}\right)$

$$\therefore \text{Length of OC} = \sqrt{\left(\frac{3a}{4}\right)^2 + \left(\frac{a}{2}\right)^2} = \sqrt{\frac{9a^2}{16} + \frac{a^2}{4}} = \frac{a}{4} \sqrt{13}$$

$$\text{and Length of AD} = \sqrt{\left(a - \frac{a}{4}\right)^2 + \left(0 - \frac{a}{2}\right)^2} = \frac{a}{4} \sqrt{13}$$

$\therefore$ Length of OC = length of AD

Hence the proof.

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Example 10.39 The line segment joining the mid-points of two sides of a triangle is parallel to the third side and is half of it.

Solution : Let the coordinates of the vertices be O (0, 0), A (a, 0) and B (b, c). Let C and D be the mid-points of OB and AB respectively

$\therefore$ The coordinates of C are $\left(\frac{b}{2}, \frac{c}{2}\right)$

The coordinates of D are $\left(\frac{a+b}{2}, \frac{c}{2}\right)$

$$\therefore \text{Slope of CD} = \frac{\frac{c}{2} - \frac{c}{2}}{\frac{a+b}{2} - \frac{b}{2}} = 0$$

Slope of OA, i.e, x-axis = 0

$\therefore$ CD is parallel to OA

$$\text{Length of CD} = \sqrt{\left(\frac{a+b}{2} - \frac{b}{2}\right)^2} + \sqrt{\left(\frac{c}{2} - \frac{c}{2}\right)^2} = \frac{a}{2}$$

Length of OA = a

$$\therefore \text{CD} = \frac{1}{2} \text{OA}$$

which proves the above result.

Example 10.40 If the diagonals of a quadrilateral are perpendicular and bisect each other, then the quadrilateral is a rhombus.

Solution : Let the diagonals be represented along OX and OY as shown in Fig. 10.16 and suppose that the lengths of their diagonals be 2a and 2b respectively.

$$\text{OA} = \text{OC} = a \text{ and } \text{OD} = \text{OB} = b$$

$$\text{AD} = \sqrt{\text{OA}^2 + \text{OD}^2} = \sqrt{a^2 + b^2}$$

$$\text{AB} = \sqrt{\text{OB}^2 + \text{OA}^2} = \sqrt{a^2 + b^2}$$

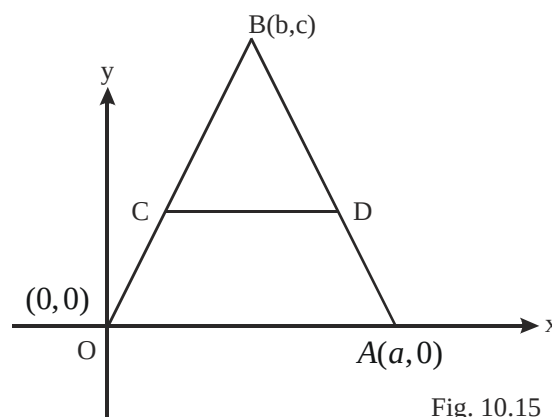


Fig. 10.15

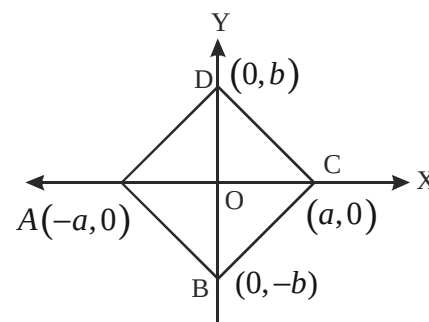


Fig. 10.16

Similarly $BC = \sqrt{a^2 + b^2} = CD$

As $AB = BC = CD = DA$

$\therefore$ A BCD is a rhombus.



CHECK YOUR PROGRESS 10.7

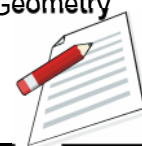
Using coordinate geometry, prove the following geometrical results:

1. If two medians of a triangle are equal, then the triangle is isosceles.
2. The diagonals of a square are equal and perpendicular to each other.
3. If the diagonals of a parallelogram are equal, then the parallelogram is a rectangle.
4. If D is the mid-point of base BC of ΔABC , then $AB^2 + AC^2 = 2(AD^2 + BD^2)$
5. In a triangle, if a line is drawn parallel to one side, it divides the other two sides proportionally.
6. If two sides of a triangle are unequal, the greater side has a greater angle opposite to it.
7. The sum of any two sides of a triangle is greater than the third side.
8. In a triangle, the medians pass through the same point.
9. The diagonal of a parallelogram divides it into two triangles of equal area.
10. In a parallelogram, the pairs of opposite sides are of equal length.



LET US SUM UP

- The equation of a line parallel to y-axis is $x = a$ and parallel to x-axis is $y = b$.
- The equation of the line which cuts off intercept c on y-axis and having slope m is $y = mx + c$
- The equation of the line passing through $A(x_1, y_1)$ and having the slope m is $y - y_1 = m(x - x_1)$
- The equation of the line passing through two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$
- The equation of the line which cuts off intercepts a and b on x-axis and y-axis respectively is $\frac{x}{a} + \frac{y}{b} = 1$
- The equation of the line in normal or perpendicular form is $x \cos \alpha + y \sin \alpha = p$



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where p is the length of perpendicular from the origin to the line and α is the angle which this perpendicular makes with the positive direction of the x -axis.

- The equation of a line in the parametric form is $\frac{x - x_1}{\cos \alpha} = \frac{y - y_1}{\sin \alpha} = r$

where r is the distance of any point $P(x, y)$ on the line from a given point $A(x_1, y_1)$ on the given line and α is the angle which the line makes with the positive direction of the x -axis.

- The general equation of first degree in x and y always represents a straight line provided A and B are not both zero simultaneously.
- From general equation $Ax + By + C = 0$ we can evaluate the following :

$$(i) \text{ Slope of the line} = -\frac{A}{B} \quad (ii) \text{ x-intercept} = -\frac{C}{A} \quad (iii) \text{ y-intercept} = -\frac{C}{B}$$

$$(iv) \text{ Length of perpendicular from the origin to the line} = \frac{|C|}{\sqrt{(A^2 + B^2)}}$$

(v) Inclination of the perpendicular from the origin is given by

$$\cos \alpha = \frac{\mp A}{\sqrt{(A^2 + B^2)}} ; \sin \alpha = \frac{\mp B}{\sqrt{(A^2 + B^2)}}$$

where the upper sign is taken for $C > 0$ and the lower sign for $C < 0$; but if $C = 0$ then either only the upper sign or only the lower sign are taken.

- If θ be the angle between two lines with slopes m_1, m_2 then $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

(i) Lines are parallel if $m_1 = m_2$. (ii) Lines are perpendicular if $m_1 m_2 = -1$.

(B) Angle θ between two lines of general form is:

$$\tan \theta = \frac{A_2 B_1 - A_1 B_2}{A_1 A_2 + B_1 B_2}$$

(i) Lines are parallel if $A_1 B_2 - A_2 B_1 = 0$ or $\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix} = 0$

(ii) Lines are perpendicular if $A_1 A_2 + B_1 B_2 = 0$.

- Distance of a given point (x_1, y_1) from a given line $Ax + By + C = 0$ is $d = \left| \frac{Ax_1 + By_1 + C}{\sqrt{(A^2 + B^2)}} \right|$
- Equation of a line parallel to the line $Ax + By + C = 0$ is $Ax + By + k = 0$
- Equation of a line perpendicular to the line $Ax + By + C = 0$ is $Bx - Ay + k = 0$

**SUPPORTIVE WEB SITES**

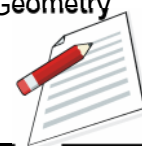
<http://www.wikipedia.org>

<http://mathworld.wolfram.com>

**TERMINAL EXERCISE**

- Find the equation of the straight line whose y-intercept is -3 and which is:
 - parallel to the line joining the points $(-2, 3)$ and $(4, -5)$.
 - perpendicular to the line joining the points $(0, -5)$ and $(-1, 3)$.
- Find the equation of the line passing through the point $(4, -5)$ and
 - parallel to the line joining the points $(3, 7)$ and $(-2, 4)$.
 - perpendicular to the line joining the points $(-1, 2)$ and $(4, 6)$.
- Show that the points $(a, 0)$, $(0, b)$ and $(3a, -2b)$ are collinear. Also find the equation of the line containing them.
- A $(1, 4)$, B $(2, -3)$ and C $(-1, -2)$ are the vertices of triangle ABC. Find
 - the equation of the median through A.
 - the equation of the altitude through A.
 - the right bisector of the side BC.
- A straight line is drawn through point A $(2, 1)$ making an angle of $\frac{\pi}{6}$ with the positive direction of x-axis. Find the equation of the line.
- A straight line passes through the point $(2, 3)$ and is parallel to the line $2x + 3y + 7 = 0$. Find its equation.
- Find the equation of the line having a and b as x-intercept and y-intercepts respectively.
- Find the angle between the lines $y = (2 - \sqrt{3})x + 5$ and $y = (2 + \sqrt{3})x - d$.
- Find the angle between the lines $2x + 3y = 4$ and $3x - 2y = 7$
- Find the length of the perpendicular drawn from the point $(3, 4)$ on the straight line $12(x + 6) = 5(y - 2)$.
- Find the length of the perpendicular from $(0, 1)$ on $3x + 4y + 5 = 0$.
- Find the distance between the lines

$$2x + 3y = 4 \text{ and } 4x + 6y = 20$$



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13. Find the length of the perpendicular drawn from the point $(-3, -4)$ on the line $4x - 3y = 7$.
14. Show that the product of the perpendiculars drawn from the points on the straight line $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$ is b^2 .
15. Prove that the equation of the straight line which passes through the point $(a \cos^3 \theta, b \sin^3 \theta)$ and is perpendicular to $x \sec \theta + y \operatorname{cosec} \theta = a$ is $x \cos \theta - y \operatorname{cosec} \theta = a \cos 2\theta$
16. Prove the following geometrical results :
 - (a) The altitudes of a triangle are concurrent.
 - (b) The medians of an equilateral triangle are equal.
 - (c) The area of an equilateral triangle of side a is $\frac{\sqrt{3} a^2}{4}$
 - (d) If G is the centroid of ΔABC , and O is any point, then $OA^2 + OB^2 + OC^2 = GA^2 + GB^2 + GC^2 + 3OG^2$.
 - (e) The perpendicular drawn from the centre of a circle to a chord bisects the chord.



ANSWERS



Notes

CHECK YOUR PROGRESS 10.1

1. Straight line 2. (a) 1 (b) -1 (c) 1
3. 7 4. 20 5. (a) $y = -4$ (b) $x = -3$
6. $x = 5$ 7. $y + 7 = 0$

CHECK YOUR PROGRESS 10.2

1. (a) $y = 2x - 2$ (b) Slope = $\frac{-4}{3}$, y -intercept = 2
2. $\sqrt{3}y = -3x - 1$ 3. Slope = $\frac{1}{2}$, y -intercept = -2
4. $3x + 7y = 7$ 5. $y = x + 1$; $x + y - 3 = 0$ 6. $3x - 2y = 0$
7. (a) $x + y = -1$ (b) Equation of the diagonal AC = $2x - y - 4 = 0$
Equation of the diagonal BD = $2x - 11y + 66 = 0$
8. $x - 2 = 0$, $x - 3y + 6 = 9$ and $5x - 3y - 2 = 0$
9. $2x + 3y = 6$ 10. $3x + y = 6$ 11. $3x + 4y = 1$ 12. $x + y = 2\sqrt{2}$
14. $\frac{x-2}{\frac{1}{\sqrt{2}}} = \frac{y-1}{\frac{1}{\sqrt{2}}}$ and the co-ordinates of the point are $\left(2 + \frac{1}{\sqrt{2}}, 1 + \frac{1}{\sqrt{2}}\right)$

CHECK YOUR PROGRESS 10.3

1. A and B are not both simultaneously zero 2. $y = \frac{-2}{5}x - \frac{3}{5}$
3. (a) x -intercept = $\frac{-c}{m}$; y -intercept = c (b) x -intercept = $\frac{-8}{3}$; y -intercept = $\frac{8}{3}$
(c) x -intercept = -4; y -intercept = 6
4. $2\sqrt{13}$ units 5. $\frac{x}{p \sec \alpha} + \frac{x}{p \operatorname{cosec} \alpha} = 1$

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6. (a) $\frac{-3}{5}x + \frac{4}{5}y - 2 = 0$ (b) $\frac{-3}{5}x + \frac{4}{5}y = 0$

7. The first line is nearer from the origin.

CHECK YOUR PROGRESS 10.4

1. (a) $m_1 = \frac{-A}{B}$ (b) $Am_1 = B$
2. $\theta = \tan^{-1}\left(\frac{7}{6}\right)$
3. (a) (i) $\frac{A_1}{B_1} = \frac{A_2}{B_2}$ (ii) $A_1 A_2 + B_1 B_2 = 0$
(b) Parallel (c) Perpendicular.

CHECK YOUR PROGRESS 10.5

1. $d = \frac{16}{\sqrt{13}}$ 2. $\left(0, \frac{b}{a}(a \pm \sqrt{a^2 + b^2})\right)$ 3. $\left(0, \frac{32}{3}\right)$
4. $\frac{14}{\sqrt{58}}$ 5. $\left(\frac{3}{4}(4 \pm 5), 0\right)$

CHECK YOUR PROGRESS 10.6

1. $3x + y + 2 = 0$ 2. $y = 2x + 2$ 3. $x - y = 3$ 4. $y = x$
5. $3x - 2ay = 6(a - 1)$ 6. $4x - 3y + 32 = 0$ 7. $2x - 3y + 6 = 0$

TERMINAL EXERCISE

1. (a) $4x + 3y + 9 = 0$ (b) $x - 8y - 24 = 0$
2. (a) $3x - 5y - 37 = 0$ (b) $5x - 8y - 60 = 0$
4. (a) $13x - y - 9 = 0$ (b) $3x - y + 1 = 0$
(c) $3x - y - 4 = 0$
5. $x - \sqrt{3}y = 2 - \sqrt{3}$ 6. $2x + 3y + 13 = 0$
7. $bx + ay = ab$ 8. $\frac{\pi}{2}$ 9. $\frac{\pi}{2}$ 10. $\frac{98}{13}$
11. $\frac{9}{5}$ 12. $\frac{6}{\sqrt{13}}$ 13. $\frac{7}{5}$