#418993

Topic: Algebra of Derivative of Functions

Differentiate the function with respect to x

$\cos x^3 . \sin^2(x^5)$

Solution

Let $f(x) = \cos x^3 . \sin^2(x^5)$

$\therefore f'(x) = \frac{d}{dx}[\cos x^3 . \sin^2(x^5)] = \sin^2(x^5) \times \frac{d}{dx}(\cos x^3) + \cos x^3 \times \frac{d}{dx}[\sin^2(x^5)]$

$= \sin^2(x^5) \times (-\sin x^3) \times \frac{d}{dx}(x^3) + \cos x^3 \times 2\sin(x^5). \frac{d}{dx}[\sin x^5]$

$= -\sin x^3 \sin^2(x^5) \times 3x^2 + 2\sin x^5 \cos x^3 . \cos x^5 \times \frac{d}{dx}(x^5)$

$= -3x^2 \sin x^3 . \sin^2(x^5) + 2\sin x^5 \cos x^5 \cos x^3 \times 5x^4$

$= 10x^4 \sin x^5 \cos x^5 \cos x^3 - 3x^2 \sin x^3 \sin^2(x^5)$

#419072

Topic: Algebra of Derivative of Functions

Find $\frac{dy}{dx}$ of $2x + 3y = \sin x$

Solution

$2x + 3y = sinx$

Differentiating both sides w.r.t. x, we obtain

$\frac{d}{dx}(2x + 3y) = \frac{d}{dx}(\sin x)$

$\Rightarrow \frac{d}{dx}(2x) + \frac{d}{dx}(3y) = \cos x$

$\Rightarrow 2 + 3\frac{dy}{dx} = \cos x$

$\Rightarrow 3\frac{dy}{dx} = \cos x - 2$

$\therefore \frac{dy}{dx} = \frac{\cos x - 2}{3}$

#419896

Topic: Algebra of Derivative of Functions

Differentiate the given function w.r.t. x.

$\frac{e^x}{\sin x}$

Solution

Let $y = \frac{e^x}{\sin x}$

Thus by using the quotient rule, we obtain

$\frac{dy}{dx} = \frac{\sin x \frac{d}{dx}(e^x) - e^x \frac{d}{dx}(\sin x)}{\sin^2 x}$

$= \frac{\sin x . (e^x) - e^x . (\cos x)}{\sin^2 x}$

$= \frac{e^x(\sin x - \cos x)}{\sin^2 x}, x \neq n\pi, n \in Z$

#420863

Topic: Algebra of Derivative of Functions

Differentiate the given function w.r.t. x.

$y = e^x + e^{x^2} + . . . + e^{x^5}$

Solution

$\frac{d}{dx}(e^x + e^{x^2} + \ldots + e^{x^5})$

$= \frac{d}{dx}(e^x) + \frac{d}{dx}(e^{x^2}) + \frac{d}{dx}(e^{x^3}) + \frac{d}{dx}(e^{x^4}) + \frac{d}{dx}(e^{x^5})$

$= e^x + \left[e^{x^2} \cdot \frac{d}{dx}(x^2)\right] + \left[e^{x^3} \cdot \frac{d}{dx}(x^3)\right] + \left[e^{x^4} \cdot \frac{d}{dx}(x^4)\right] + \left[e^{x^5} \cdot \frac{d}{dx}(x^5)\right]$

$= e^x + (e^{x^2} \times 2x) + (e^{x^3} \times 3x^2) + (e^{x^4} \times 4x^3) + (e^{x^5} \times 5x^4)$

$= e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}$

#420955

Topic: Algebra of Derivative of Functions

Differentiate the given function w.r.t. x.

$\frac{\cos x}{\log x}, x > 0$

Solution

Let $y = \frac{\cos x}{\log x}, x > 0$

Thus using quotient rule,

$\frac{dy}{dx} = \frac{\frac{d}{dx}(\cos x) \times \log x - \cos x \times \frac{d}{dx}(\log x)}{(\log x)^2}$

$= \frac{-\sin x \log x - \cos x \times \frac{1}{x}}{(\log x)^2}$

$= \frac{-[x \log x . \sin x + \cos x]}{x(\log x)^2}, x > 0$

#421877

Topic: Algebra of Derivative of Functions

Differentiate the given function w.r.t. x:

$\sin^3 x + \cos^6 x$

Solution

Let $y = \sin^3 x + \cos^6 x$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sin^3 x) + \frac{d}{dx}(\cos^6 x)$

$= 3\sin^2 x . \frac{d}{dx}(\sin x) + 6\cos^5 x . \frac{d}{dx}(\cos x)$

$= 3\sin^2 x . \cos x + 6\cos^5 x . (-\sin x)$

$= 3\sin x \cos x(\sin x - 2\cos^4 x)$

#422262

Topic: Algebra of Derivative of Functions

Differentiate the given function w.r.t. x:

$\frac{\cos^{-1}\frac{x}{2}}{\sqrt{2x+7}}, -2 < x < 2$

Solution

Let $y = \frac{\cos^{-1}\frac{x}{2}}{\sqrt{2x+7}}$

Thus using quotient rule, we obtain

$$\frac{dy}{dx} = \frac{\sqrt{2x+7}\frac{d}{dx}\left(\cos^{-1}\frac{x}{2}\right) - \left(\cos^{-1}\frac{x}{2}\right)\frac{d}{dx}(\sqrt{2x+7})}{(\sqrt{2x+7})^2}$$

$$= \frac{\sqrt{2x+7}\left[\frac{-1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}.\frac{d}{dx}\left(\frac{x}{2}\right)\right] - \left(\cos^{-1}\frac{x}{2}\right).\frac{1}{2\sqrt{2x+7}}\frac{d}{dx}(2x+7)}{2x+7}$$

$$= \frac{\sqrt{2x+7}\frac{-1}{\sqrt{4-x^2}} - \left(\cos^{-1}\frac{x}{2}\right)\frac{1}{2\sqrt{2x+7}} \times 2}{2x+7}$$

$$= -\left[\frac{1}{\sqrt{4-x^2}\sqrt{2x+7}} + \frac{\cos^{-1}\frac{x}{2}}{(2x+7)^{\frac{3}{2}}}\right]$$

#422844

Topic: Algebra of Derivative of Functions

Find $\frac{dy}{dx}$, if $y = \sin^{-1}x + \sin^{-1}\sqrt{1-x^2}$, $-1 \le t \le 1$

Solution

$y = \sin^{-1}x + \sin^{-1}\sqrt{1-x^2}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}[\sin^{-1}x + \sin^{-1}\sqrt{1-x^2}]$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\sin^{-1}x) + \frac{d}{dx}(\sin^{-1}\sqrt{1-x^2})$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-(\sqrt{1-x^2})^2}}\cdot\frac{d}{dx}(\sqrt{1-x^2})$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \frac{1}{x}\cdot\frac{1}{2\sqrt{1-x^2}}\cdot\frac{d}{dx}(1-x^2)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \frac{1}{2x\sqrt{1-x^2}}(-2x)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = 0$