#464872

Use Euclids division algorithm to find the HCF of:

(i) 135 and 225

(ii) 196 and 38220

(iii) 867 and 225

Solution

(i) 135 **and** 225

Step 1: First find which integer is larger.

225 > 135

Step 2: Then apply the Euclid's division algorithm to 225 and 135 to obtain

225 = 135 × 1 + 90

Repeat the above step until you will get remainder as zero.

Step 3: Now consider the divisor 135 and the remainder 90, and apply the division lemma to get

135 = 90 × 1 + 45

90 = 2 × 45 + 0

Since the remainder is zero, we cannot proceed further.

Step 4: Hence, the divisor at the last process is 45

So, the H.C.F. of 135 and 225 is 45.

(ii) 196 and 38220

Step 1: First find which integer is larger.

38220 > 196

Step 2: Then apply the Euclid's division algorithm to 38220 and 196 to obtain

38220 = 196 × 195 + 0

Since the remainder is zero, we cannot proceed further.

Step 3: Hence, the divisor at the last process is 196.

So, the H.C.F. of 196 and 38220 is 196.

(iii) 867 and 225

Step 1: First find which integer is larger.

867 > 255

Step 2: Then apply the Euclid's division algorithm to 867 and 255 to obtain

867 = 255 × 3 + 102

Repeat the above step until you will get remainder as zero.

Step 3: Now consider the divisor 225 and the remainder 102, and apply the division lemma to get

255 = 102 × 2 + 51

102 = 51 × 2 = 0

Since the remainder is zero, we cannot proceed further.

Step 4: Hence the divisor at the last process is 51.

So, the H.C.F. of 867 and 255 is 51.

#464877

Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

Solution

Using Euclid division algorithm, we know that $a = bq + r$, $0 \leq r \leq b$ ----(1)

Let a be any odd positive integer and $b = 6$.

Substitute b = 6 in equation (1)

$a = 6q + r$ where $0 \leq r \leq 6$ $r = 0, 1, 2, 3, 4, 5$

If $r = 0$, $a = 6q$, $6q$ is divisible by 6

If $r = 1$, $a = 6q + 1$, $6q + 1$ is not divisible by 2.

If $r = 2$, $a = 6q + 2$, $6q + 2$ is divisible by 2

If $r = 3$, $a = 6q + 3$, $6q + 3$ is not divisible by 2.

If $r = 4$, $a = 6q + 4$, $6q + 4$ is divisible by 2

If $r = 5$, $a = 6q + 5$, $6q + 5$ is not divisible by 2.

So, the numbers $6q, 6q + 2, 6q + 4$ are divisible by 2 and even numbers.

The remaining numbers $6q + 1, 6q + 3$ and $6q + 5$ are odd.

#464926

An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Solution

HCF $(616, 32)$ is the maximum number of columns in which they can march.

Step 1: First find which integer is larger.

616 > 32

Step 2: Then apply the Euclid's division algorithm to 616 and 32 to obtain

616 = 32 × 19 + 8

Repeat the above step until you will get remainder as zero.

Step 3: Now consider the divisor 32 and the remainder 8, and apply the division lemma to get

32 = 8 × 4 + 0

Since the remainder is zero, we cannot proceed further.

Step 4: Hence the divisor at the last process is 8

So, the H.C.F. of 616 and 32 is 8.

Therefore, 8 is the maximum number of columns in which they can march.

#464927

Use Euclids division lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m.

Solution

Using Euclid division algorithm, we know that $a = bq + r, \ 0 \le r \le b$ ....(1)

Let a be any positive integer, and $b = 3$.

Substitute $b = 3$ in equation (1)

$a = 3q + r$ where $0 \le r \le 6$ $r = 0, 1, 2$

If $r = 0, a = 3q$

On squaring we get, $a^2 = 3(3q^2)$, where $m = 3q^2$--- (2)

If $r = 1, a = 3q + 1$

On squaring we get,

$a^2 = 3(3q^2 + 2q) + 1$, where $m = 3q^2 + 2q$ ---(3)

If $r = 2, a = 3q + 2$

On squaring we get,

$a^2 = 3(3q^2 + 4q + 1) + 1$, where $m = 3q^2 + 4q + 1$ ---- (4)

From equation $2, 3$ and 4,

The square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m.

#464928

Use Euclids division lemma to show that the cube of any positive integer is of the form $9m, 9m + 1$ or $9m + 8$.

Solution

Using Euclid division algorithm, we know that $a = bq + r, \ 0 \le r \le b - - - - (1)$

Let a be any positive integer, and $b = 3$.

Substitute $b = 3$ in equation (1)

$a = 3q + r$ where $0 \le r \le 3, \ r = 0, 1, 2$

If $r = 0, a = 3q$

Cube the value, we get

$a^3 = 27q^3$

$a^3 = 9(3q^3)$, where m $= 3q^3 - - - (2)$

If $r = 1, a = 3q + 1$

Cube the value, we get

$a^3 = (3q + 1)^3$

$a^3 = (27q^3 + 27q^2 + 9q + 1)$

$a^3 = 9(3q^3 + 3q^2 + 1) + 1$, where m $= 3q^3 + 3q^2 + q - - - (3)$

If $r = 2, a = 3q + 2$

Cube the value, we get

$a^3 = (3q + 2)^3$

$a^3 = (27q^3 + 53q^2 + 36q + 8)$

$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$, where m $= 3q^3 + 6q^2 + 4q - - - - (4)$

From equation $2, 3$ and 4,

The cube of any positive integer is of the form $9m, 9m + 1$ or $9m + 8$.

#464929

Express each number as a product of its prime factors:

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

Solution

(I) 140

140 = 2 × 70

   = 2 × 2 × 35

   = 2 × 2 × 5 × 7

   = 2 × 2 × 5 × 7 × 1

(ii) 156

156 = 2 × 78

   = 2 × 2 × 39

   = 2 × 2 × 3 × 13

   = 2 × 2 × 3 × 13 × 1

(III) 3825

3825 = 3 × 1275

   = 3 × 3 × 425

   = 3 × 3 × 5 × 85

   = 3 × 3 × 5 × 5 × 17

   = 3 × 3 × 5 × 5 × 17 × 1

(iv) 5005

5005 = 5 × 1001

   = 5 × 7 × 143

   = 5 × 7 × 11 × 13

   = 5 × 7 × 11 × 13 × 1

(v) 7429

7429 = 17 × 437

   = 17 × 19 × 23

   = 17 × 19 × 23 × 1

#464936

Find the LCM and HCF of the following pairs of integers and verify that

LCM × HCF = product of the two numbers.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Solution

(i) 26 and 91

Factor of 26 = 2 × 13 × 1

Factor of 91 = 7 × 13 × 1

HCF of 26 and 91 = 13 × 1 = 13

LCM of 26 and 91 = 2 × 7 × 13 = 182

LCM × HCF = 182 × 13 = 2366

26 × 91 = 2366

So, LCM.HCF = product of the two numbers = 26 × 91.

Hence proved.

(ii) 510 × 92

Factor of 510 = 2 × 3 × 5 × 17 × 1

Factor of 92 = 2 × 2 × 23

HCF of 510 and 92 = 2 × 2 = 2

LCM of 510 and 92 = 2 × 2 × 3 × 5 × 17 × 23 = 23, 460

LCM × HCF = 23, 460 × 2 = 46, 920

510 × 92 = 46, 920

So, LCM.HCF = product of the two numbers = 510 × 92.

Hence proved.

(iii) 336 and 54

Factor of 336 = 2 × 2 × 2 × 2 × 7 × 3 × 1

Factor of 54 = 2 × 3 × 3 × 3

HCF of 336 and 54 = 2 × 3 = 6

LCM of 336 and 54 = 2 × 3 × 2 × 2 × 2 × 7 × 3 × 3 = 3, 024

LCM × HCF = 3, 024 × 6 = 18, 144

366 × 54 = 18, 144

So, LCM.HCF = product of the two numbers = 336 × 54.

Hence proved.

#464937

Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

Solution

Using prime factorisation method:

(I) 12, 15 and 21

Factor of $12 = 2 \times 2 \times 3$

Factor of $15 = 3 \times 5$

Factor of $21 = 3 \times 7$

HCF $(12, 15, 21) = 3$

LCM $(12, 15, 21) = 2 \times 2 \times 3 \times 5 \times 7 = 420$

(II) 17, 23 and 29

Factor of $17 = 1 \times 17$

Factor of $23 = 1 \times 23$

Factor of $29 = 1 \times 29$

HCF $(17, 23, 29) = 1$

LCM $(17, 23, 29) = 1 \times 17 \times 23 \times 29 = 11,339$

(III) 8, 9 and 25

Factor of $8 = 2 \times 2 \times 2 \times 1$

Factor of $9 = 3 \times 3 \times 1$

Factor of $25 = 5 \times 5 \times 1$

HCF $(8, 9, 25) = 1$

LCM $(8, 9, 25) = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1,800$

#464941

Given that HCF $(306$ and $657) = 9$, find LCM of $(306$ and $657)$

Solution

We know that,

LCM x HCF = Product of the two numbers

$$LCM = \frac{\text{Product of two numbers}}{HCF}$$

Product of the two numbers $= 306 \times 657 = 201042$

$$LCM = \frac{201042}{9}$$

LCM $= 22,338$

#464944

Check whether 6^n can end with the digit 0 for any natural number n.

Solution

If any digit has the last digit 10 that means it divisible by 10.

The factor of $10 = 2 \times 5$,

So value of $6n$ should be divisible by 2 and 5.

Both $6n$ is divisible by 2 but not divisible by 5.

So, it can not end with 0.

#464948

Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Solution

$7 \times 11 \times 13 + 13$

Take 13 common, we get

$= 13(7 \times 11 + 1)$

$= 13(77 + 1)$

$= 13(78)$

It is product of two numbers and both numbers are more than 1, so it is a composite number.

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

Take 5 common,

$= 5(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 1)$

$= 5(1008 + 1)$

$= 5(1009)$

It is product of two numbers and both numbers are more than 1, so it is a composite number.

#464952

There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Solution

Sonia and Ravi drive one round of the field in same point, same time and same direction.

So, they will meet again after LCM of both values at the starting point.

To get the LCM we have factorize the number.

Factor of $18 = 2 \times 3 \times 3$

Factor of $12 = 2 \times 2 \times 3$

LCM $= 2 \times 2 \times 3 \times 3 = 36$

Hence, they will meet together at the starting point after 36 minutes.

#464961

Prove that $\sqrt{5}$ is irrational.

Solution

Let us assume, to the contrary, that $\sqrt{5}$ is rational.

So, we can find the integers r and s $\neq 0$ such that $\sqrt{5} = \dfrac{r}{s}$

Suppose r and s have a common factor other than 1. Then, we divide by the common factor to get $\sqrt{5} = \dfrac{a}{b}$, where a and b are co-prime.

So, $b\sqrt{5} = a$

Squaring on both the sides, we get

$5b^2 = a^2$ ---(1)

Therefore, 5 divides a^2.

Now, according to the theorem of rational number, for any prime number p which divides a^2 then it will divide a also.

That means 5 will divide a. So we can write

$a = 5c$

Subtitute the value of a in equation (1)

$5b^2 = (5c)^2$

$5b^2 = 25c^2$

Divide by 25 we get

$\dfrac{b^2}{5} = c^2$

Again using the same theorem we get that be will be divide by 5 and we have already get that a is divide by 5 but a and b are co-prime number.

This contradiction has arised because of our incorrect assumption that $\sqrt{5}$ is irrational.

So, we conclude that $\sqrt{5}$ is irrational.

#464968

Prove that $3 + 2\sqrt{5}$ is irrational.

Solution

Let us assume $3 + 2\sqrt{5} +$ is rational.

So we can write this number as

$3 + 2\sqrt{5} = \dfrac{a}{b}$ ---- (1)

Here a and b are two co-prime number and b is not equal to zero.

Simplify the equation (1) subtract 3 both sides, we get

$2\sqrt{5} = \dfrac{a}{b} - 3$

$2\sqrt{5} = \dfrac{a - 3b}{b}$

Now divide by 2 we get

$\sqrt{5} = \dfrac{a - 3b}{2b}$

Here a and b are integer so $\dfrac{a - 3b}{2b}$ is a rational number, so $\sqrt{5}$ should be a rational number.

But $\sqrt{5}$ is a irrational number, so it is contradict.

Therefore, $3 + 2\sqrt{5}$ is irrational number.

#464974

Prove that the following are irrational:

(i) $\dfrac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

Solution

(I) $\dfrac{1}{\sqrt{2}}$

Let us assume $\dfrac{1}{\sqrt{2}}$ is rational.

So we can write this number as

$\dfrac{1}{\sqrt{2}} = \dfrac{a}{b}$ ---- (1)

Here, a and b are two co-prime numbers and b is not equal to zero.

Simplify the equation (1) multiply by $\sqrt{2}$ both sides, we get

$1 = \dfrac{a\sqrt{2}}{b}$

Now, divide by b, we get

$b = a\sqrt{2}$ or $\dfrac{b}{a} = \sqrt{2}$

Here, a and b are integers so, $\dfrac{b}{a}$ is a rational number, so $\sqrt{2}$ should be a rational number.

But $\sqrt{2}$ is a irrational number, so it is contradictory.

Therefore, $\dfrac{1}{\sqrt{2}}$ is irrational number.

(II) $7\sqrt{5}$

Let us assume $7\sqrt{5}$ is rational.

So, we can write this number as

$7\sqrt{5} = \dfrac{a}{b}$ ---- (1)

Here, a and b are two co-prime numbers and b is not equal to zero.

Simplify the equation (1) divide by 7 both sides, we get

$\sqrt{5} = \dfrac{a}{7b}$

Here, a and b are integers, so $\dfrac{a}{7b}$ is a rational number, so $\sqrt{5}$ should be a rational number.

But $\sqrt{5}$ is a irrational number, so it is contradictory.

Therefore, $7\sqrt{5}$ is irrational number.

(III) $6 + \sqrt{2}$

Let us assume $6 + \sqrt{2}$ is rational.

So we can write this number as

$6 + \sqrt{2} = \dfrac{a}{b}$ ---- (1)

Here, a and b are two co-prime number and b is not equal to zero.

Simplify the equation (1) subtract 6 on both sides, we get

$\sqrt{2} = \dfrac{a}{b} - 6$

$\sqrt{2} = \dfrac{a - 6b}{b}$

Here, a and b are integers so, $\dfrac{a-6b}{b}$ is a rational number, so $\sqrt{2}$ should be a rational number.

But $\sqrt{2}$ is a irrational number, so it is contradictory.

Therefore, $6 + \sqrt{2}$ is irrational number.

#464981

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\dfrac{13}{3125}$ (ii) $\dfrac{17}{8}$ (iii) $\dfrac{64}{455}$ (iv) $\dfrac{15}{1600}$ (v) $\dfrac{29}{343}$

(vi) $\dfrac{23}{2^3 5^2}$ (vii) $\dfrac{129}{2^2 5^7 7^5}$ (viii) $\dfrac{6}{15}$ (ix) $\dfrac{35}{50}$ (x) $\dfrac{77}{210}$

Solution

Theorem: Let $x = \dfrac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then, x has a decimal expansion which terminates.

(I) $\dfrac{13}{3125}$

Factorise the denominator, we get

$3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5$

So, denominator is in form of 5^m so, $\dfrac{13}{3125}$ is terminating.

(II) $\dfrac{17}{8}$

Factorise the denominator, we get

$8 = 2 \times 2 \times 2 = 2^3$

So, denominator is in form of 2^n so, $\dfrac{17}{8}$ is terminating.

(III) $\dfrac{64}{455}$

Factorise the denominator, we get

$455 = 5 \times 7 \times 13$

So, denominator is not in form of $2^n 5^m$ so, $\dfrac{64}{455}$ is not terminating.

(iv) $\dfrac{15}{1600}$

Factorise the denominator, we get

$1600 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 2^6 5^2$

So, denominator is in form of $2^n 5^m$ so, $\dfrac{15}{1600}$ is terminating.

(v) $\dfrac{29}{343}$

Factorise the denominator, we get

$343 = 7 \times 7 \times 7 = 7^3$

So, denominator is not in form of $2^n 5^m$ so, $\dfrac{29}{343}$ is not terminating.

(vi) $\dfrac{23}{2^3 5^2}$

Here, the denominator is in form of $2^n 5^m$ so, $\dfrac{23}{2^3 5^2}$ is terminating.

(vii) $\dfrac{129}{2^2 5^7 7^5}$

Here, the denominator is not in form of $2^n 5^m$ so, $\dfrac{129}{2^2 5^7 7^5}$ is not terminating.

(viii) $\dfrac{6}{15}$

Divide nominator and denominator both by 3 we get $\dfrac{3}{15}$

So, denominator is in form of 5^m so, $\dfrac{6}{15}$ is terminating.

(lx) $\dfrac{35}{50}$

Divide nominator and denominator both by 5 we get $\dfrac{7}{10}$

Factorise the denominator, we get

$10 = 2 \times 5$

So, denominator is in form of $2^n 5^m$ so, $\dfrac{35}{50}$ is terminating.

(x) $\dfrac{77}{210}$

Divide nominator and denominator both by 7 we get $\dfrac{11}{30}$

Factorise the denominator, we get

$30 = 2 \times 3 \times 5$

So, denominator is not in the form of $2^n 5^m$ so $\dfrac{6}{15}$ is not terminating.

#464989

The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational and of the form p, you say about

the prime factors of q?

(i) 43.123456789

(ii) 0.120120012000120000...

(iii) $43.\overline{123456789}$

Solution

(I) 43.123456789

It has certain number of digits, so they can be represented in form of $\dfrac{p}{q}$.

Hence they are rational number.

As they have certain number of digits and the number which has certain number of digits is always terminating number and for terminating number denominator has prime fact

2 and 5 only.

(ii) 0.120120012000120000...

Here the prime factor of denominator Q will has a value which is not equal to 2 or 5.

So, it is an irrational number as it is non-terminating and non-repeating.

(iii) $43.\overline{123456789}$

Here the prime factor of denominator Q will has a value which is apart from 2 or 5, some other factor also.

So, it is an rational number, 0.123456789 repeating again and again. It is non- terminating.