**UNIT-4**

# QUADRATIC EQUATIONS

**For the things of this world cannot be made known without a knowledge of mathematics.**

1. Solve by factorization

**a. $4x^2 - 4a^2x + (a^4 - b^4) = 0$**

**Ans:** $4x^2 - 4a^2x + (a^4 - b^4) = 0.$

$4x^2 - [2(a^2 + b^2) + 2(a^2 - b^2)]x + (a^2 - b^2)(a^2 + b^2) = 0.$

$\Rightarrow 2x[2x - (a^2 + b^2)] - (a^2 - b^2)[2x - (a^2 + b^2)] = 0.$

$$\Rightarrow x = \frac{a^2 + b^2}{2} \quad x = \frac{a^2 - b^2}{2}$$

**b. $x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$**

**Ans:** $x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1$

$$\Rightarrow x^2 + \left(\frac{a}{a+b}x + \frac{a+b}{a}x + \frac{a}{a+b}\cdot\frac{a+b}{a}\right)$$

$$\Rightarrow x\left[x + \frac{a}{a+b}\right] + \frac{a+b}{a}\left[x + \frac{a}{a+b}\right] = 0$$

$$\Rightarrow x = \frac{-a}{a+b} \quad x = \frac{(-a+b)}{a} \qquad a+b \neq 0.$$

**c. $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x} \qquad a + b \neq 0$**

**Ans:** $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

$$\Rightarrow \frac{1}{a+b+x} - \frac{1}{x} = +\frac{1}{a} + \frac{1}{b}$$

$$\Rightarrow \frac{x - (a+b+x)}{x(a+b+x)} = +\frac{a+b}{ab}$$

$\Rightarrow (a+b)\{x(a+b+x)+ab\}=0$

$\Rightarrow x(a+b+x)+ab=0$

$\Rightarrow x^2 + ax + bx + ab = 0$

$\Rightarrow (x+a)(x+b)=0$

$\Rightarrow x=-a \quad x=-b$

d. $(x-3)(x-4) = \frac{34}{33^2}$

**Ans :** $(x-3)(x-4) = \frac{34}{33^2}$

$$\Rightarrow x^2 - 7x + 12 = \frac{34}{33^2}$$

$$x^2 - 7x + \frac{13034}{33^2} = 0$$

$$x^2 - 7x + \frac{98}{33} x \frac{133}{33} = 0$$

$$x^2 - \frac{231}{33} x + \frac{98}{33} x \frac{133}{33} = 0$$

$$x^2 - \left(\frac{98}{33} + \frac{133}{33}\right) x + \frac{98}{33} x \frac{133}{33} = 0$$

$$\Rightarrow \left(x - \frac{98}{33}\right)\left(x - \frac{133}{33}\right) = 0$$

$$\Rightarrow x = \frac{98}{33} \text{ or } x = \frac{133}{33}$$

**e.** $\mathbf{x} = \cfrac{1}{2 - \cfrac{1}{2 - \cfrac{1}{2-x}}}$ $\quad \mathbf{x \neq 2}$

**Ans:** $x = \cfrac{2}{2 - \cfrac{1}{2 - \cfrac{1}{2-x}}}$ $\quad x \neq 2$

$$x = \cfrac{1}{2 - \cfrac{1}{2 - \cfrac{1}{2-x}}}$$

$$x = \cfrac{1}{2 - \cfrac{1}{2 - \cfrac{(2-x)}{4-2x-1}}}$$

$$x = \frac{1}{2 - \frac{2-x}{3-2x}}$$

$$\Rightarrow x = \frac{3-2x}{2(3-2x)-(2-x)}$$

$$\Rightarrow x = \frac{3-2x}{4-3x}$$

$\Rightarrow 4x - 3x^2 = 3 - 2x$

$\Rightarrow 3x^2 - 6x + 3 = 0$

$\Rightarrow (x-1)^2 = 0$

$x = 1, 1.$

2. By the method of completion of squares show that the equation $4x^2+3x+5=0$ has no real roots.

**Ans:** $4x^2+3x+5=0$

$$\Rightarrow x^2 + \frac{3}{4}x + \frac{5}{4} = 0$$

$$\Rightarrow x^2 + \frac{3}{4}x + \left(\frac{3}{8}\right)^2 = \frac{-5}{4} + \frac{9}{64}$$

$$\Rightarrow \left(x + \frac{3}{8}\right)^2 = \frac{-71}{64}$$

$$\Rightarrow \left(x + \frac{3}{8}\right)^2 = \frac{-71}{64}$$

$$\Rightarrow x + \frac{3}{8} = \sqrt{\frac{-71}{64}}$$ not a real no.

Hence QE has no real roots.

3. The sum of areas of two squares is $468 m^2$ If the difference of their perimeters is 24cm, find the sides of the two squares.

**Ans:** Let the side of the larger square be $x$.
Let the side of the smaller square be $y$.

APQ $x^2+y^2 = 468$

Cond. II $\quad 4x-4y = 24$

$\Rightarrow x - y = 6$

$\Rightarrow x = 6 + y$
$x^2 + y^2 = 468$
$\Rightarrow (6+y)^2 + y^2 = 468$
on solving we get $y = 12$
$\Rightarrow x = (12+6) = 18$ m
$\therefore$ sides are 18m & 12m.

4. A dealer sells a toy for Rs.24 and gains as much percent as the cost price of the toy. Find the cost price of the toy.

**Ans:** Let the C.P be $x$
$\therefore$ Gain = $x$%

$\Rightarrow$ Gain = $x.\dfrac{x}{100}$

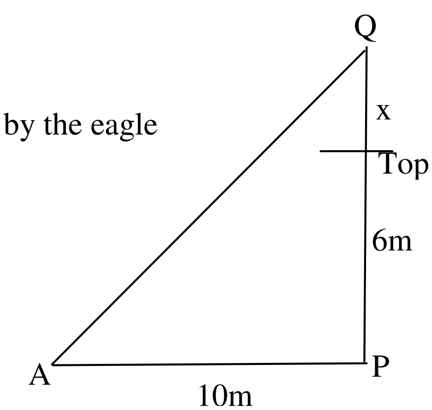
S.P = C.P +Gain

SP = 24

$$\Rightarrow x + \frac{x^2}{100} = 24$$

On solving $x$=20 or -120 (rej)
$\therefore$C.P of toy = Rs.20

5. A fox and an eagle lived at the top of a cliff of height 6m, whose base was at a distance of 10m from a point A on the ground. The fox descends the cliff and went straight to the point A. The eagle flew vertically up to a height $x$ metres and then flew in a straight line to a point A, the distance traveled by each being the same. Find the value of x.




**Ans:** Distance traveled by the fox = distance traveled by the eagle
$(6+x)^2 + (10)^2 = (16 - x)^2$
on solving we get
$x = 2.72$m.

**6.** A lotus is 2m above the water in a pond. Due to wind the lotus slides on the side and only the stem completely submerges in the water at a distance of 10m from the original position. Find the depth of water in the pond.

**Ans:** $(x+2)^2 = x^2 + 10^2$

$x^2 + 4x + 4 = x^2 + 100$

$\Rightarrow 4x + 4 = 100$

$\Rightarrow x = 24$

Depth of the pond = 24m

7 Solve x = $\sqrt{6+\sqrt{6+\sqrt{6}}}$.........

**Ans:** $x = \sqrt{6+\sqrt{6+\sqrt{6+}}}$

$\Rightarrow x = \sqrt{6+x}$

$\Rightarrow x^2 = 6 + x$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow (x - 3)(x + 2) = 0$

$\Rightarrow x = 3$

8. The hypotenuse of a right triangle is 20m. If the difference between the length of the other sides is 4m. Find the sides.

**Ans:** APQ

$x^2 + y^2 = 20^2$

$x^2 + y^2 = 400$

also $x - y = 4$

$\Rightarrow x = 404 + y$

$(4 + y)^2 + y^2 = 400$

$\Rightarrow 2y^2 + 8y - 384 = 0$

$\Rightarrow (y + 16)(y - 12) = 0$

$\Rightarrow y = 12 \quad y = -16$ (N.P)

$\therefore$ sides are 12cm & 16cm

9. The positive value of k for which $x^2 + Kx + 64 = 0$ & $x^2 - 8x + k = 0$ will have real roots .

**Ans:** $x^2 + Kx + 64 = 0$

$\Rightarrow b^2 - 4ac \geq 0$

$K^2 - 256 \geq 0$

$K \geq 16$ or $K \leq -16$ ..............(1)

$x^2 - 8x + K = 0$

$64 - 4K \geq 0$

$\Rightarrow 4K \leq 64$

$K \leq 16$ ……………(2)

From (1) & (2) $K = 16$

10. A teacher on attempting to arrange the students for mass drill in the form of a solid square found that 24 students were left over. When he increased the size of the square by one student he found he was short of 25 students. Find the number of students.

**Ans:** Let the side of the square be $x$.

No. of students = $x^2 + 24$

New side = $x + 1$

No. of students = $(x + 1)^2 - 25$

APQ $\Rightarrow x^2 + 24 = (x + 1)^2 - 25$

$\Rightarrow x^2 + 24 = x^2 + 2x + 1 - 25$

$\Rightarrow 2x = 48$

$\Rightarrow x = 24$

$\therefore$ side of square = 24

No. of students = 576 + 24

= 600

11. A pole has to be erected at a point on the boundary of a circular park of diameter 13m in such a way that the differences of its distances from two diametrically opposite fixed gates A & B on the boundary in 7m. Is it possible to do so? If answer is yes at what distances from the two gates should the pole be erected.

**Ans:** AB = 13 m

BP = $x$

$\Rightarrow$ AP – BP = 7

$\Rightarrow$ AP = $x + 7$

APQ

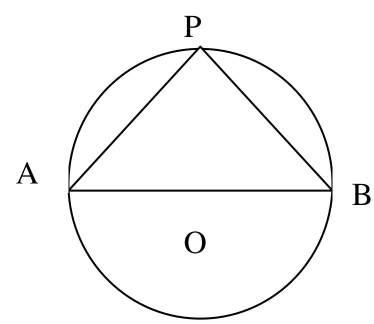
$\Rightarrow (13)^2 = (x + 7)^2 + x^2$

$\Rightarrow x^2 + 7x - 60 = 0$

$(x + 12)(x - 5) = 0$

$\Rightarrow x = -12$ N.P

$x = 5$




$\therefore$ Pole has to be erected at a distance of 5m from gate B & 12m from gate A.

12. If the roots of the equation $(a-b)x^2 + (b-c)x + (c - a) = 0$ are equal. Prove that 2a=b+c.

**Ans:** $(a-b)x^2 + (b-c)x + (c - a) = 0$

T.P $2a = b + c$

$B^2 - 4AC = 0$

$(b-c)^2 - [4(a-b)(c - a)] = 0$

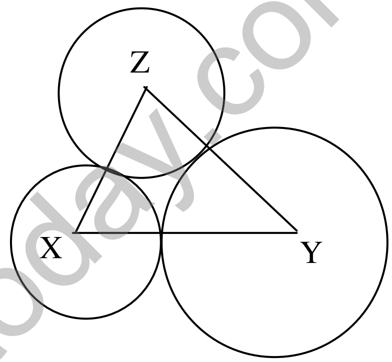
$b^2 - 2bc + c^2 - [4(ac - a^2 - bc + ab)] = 0$

$\Rightarrow b^2 - 2bc + c^2 - 4ac + 4a^2 + 4bc - 4ab = 0$
$\Rightarrow b^2 + 2bc + c^2 + 4a^2 - 4ac - 4ab = 0$
$\Rightarrow (b + c - 2a)^2 = 0$
$\Rightarrow b + c = 2a$

13. X and Y are centers of circles of radius 9cm and 2cm and XY = 17cm. Z is the centre of a circle of radius 4 cm, which touches the above circles externally. Given that $\angle XZY = 90^o$, write an equation in r and solve it for r.

**Ans:** Let r be the radius of the third circle

XY = 17cm $\Rightarrow$ XZ = 9 + r YZ = 2




APQ
$(r + 9)^2 + (r + 2)^2 = (1 + r)^2$
$\Rightarrow r^2 + 18r + 81 + r^2 + 4r + 4 = 289S$
$\Rightarrow r^2 + 11r - 10r = 0$
$(r + 17)(r - 6) = 0$
$\Rightarrow r = -17$ (N.P)
$r = 6$ cm
$\therefore$ radius = 6cm.