**#465277**

Check whether the following are quadratic equations :

(i) $(x+1)^2 = 2(x-3)$

(ii) $x^2 - 2x = (-2)(3-x)$

(iii) $(x-2)(x+1) = (x-1)(x+3)$

(iv) $(x-3)(2x+1) = x(x+5)$

(v) $(2x-1)(x-3) = (x+5)(x-1)$

(vi) $x^2 + 3x + 1 = (x-2)^2$

(vii) $(x+2)^3 = 2x\left(x^2-1\right)$

(viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$

**Solution**

i)

$(x+1)^2 = 2(x-3)$

$x^2 + 2x + 1 = 2x - 6$

$x^2 + 7 = 0$ → Quadratic

ii)

$x^2 - 2x = -2(3-x)$

$x^2 - 2x = -6 + 2x$

$x^2 + 4x + 6 = 0$ → Quadratic

iii)

$(x-2)(x+1) = (x-1)(x+3)$

$x^2 - x - 2 = x^2 + 2x - 3$

$3x = 1$ → Non Quadratic

iv)

$(x-3)(2x+1) = (x)(x+5)$

$2x^2 - 5x - 3 = x^2 + 5x$

$x^2 + 10x - 3 = 0$ → Quadratic

v)

$(x-3)(2x-1) = (x-1)(x+5)$

$2x^2 - 7x + 3 = x^2 + 4x - 5$

$x^2 - 11x + 8 = 0$ → Quadratic

vi)

$x^2 + 3x + 1 = (x-2)^2$

$x^2 + 3x + 1 = x^2 - 4x + 4$

$7x = 3$ → Non Quadratic

vii)

$(x+2)^3 = 2x(x^2-1)$

$x^3 + 8 + 6x(x+2) = 2x^3 - 2x$

$x^3 + 8 + 6x^2 + 12x = 2x^3 - 2x$

$-x^3 + 6x^2 + 14x + 8 = 0$ → Non Quadratic

viii)

$x^3 - 4x^2 - x + 1$

$= x^3 + 3x^2(-2) + 3x(-2)^2 + (-2)^3$

$x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 12x - 8$

$x^3 - 4x^2 - x + 1 - x^3 + 6x^2 - 12x + 8 = 0$

$2x^2 - 13x + 9 = 0$ → Non Quadratic

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**#465278**

Represent the following situations in the form of quadratic equations :

(i) The area of a rectangular plot is $528 m^2$. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohans mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohans present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

**Solution**

i)

Let the breadth be $x$ m and the length will be $2x + 1$ m.

Area $= l \times b$

Area $= x(2x + 1) = 528$

$2x^2 + x - 528 = 02x^2 + 33x - 32x - 528 = 0$

$\Rightarrow 2x(x - 16) + 33(x - 16) = 0$

$\Rightarrow (2x + 33)(x - 16) = 0$

$\Rightarrow x = 16, \frac{33}{2}$

Breadth $= 16$ m and length $= 33$ m

ii)

Let one number be $x$ then the next number will be $x + 1$

$x(x + 1) = 306$

$\Rightarrow x^2 + x - 306 = 0$

$\Rightarrow x^2 + 18x - 17x - 306 = 0$

$\Rightarrow x(x + 18) - 17(x + 18) = 0$

$\Rightarrow (x + 18)(x - 17) = 0$

$\Rightarrow x = 17, -18$

The numbers are $17$ and $18$.

iii)

Let Rohan's present age $= x$ yrs.

Then his mother's present age $= x + 26$ yrs

After $3$ yrs

Rohan's age $= x + 3$ yrs

His mother's age $= x + 29$ yrs

$(x + 3)(x + 29) = 360$

$\Rightarrow x^2 + 32x + 87 - 360 = 0$

$\Rightarrow x^2 + 32x - 273 = 0$

$\Rightarrow x^2 + 39x - 7x - 273 = 0$

$\Rightarrow x(x + 39) - 7(x + 39) = 0$

$\Rightarrow (x + 39)(x - 7) = 0$

$\Rightarrow x = 7, -39$

So, Rohan's present age $= 7$ yrs.

iv)

Let the speed of the train $= x$ km/hr

$\frac{480}{(x - 8)} - \frac{480}{x} = 3$

$\Rightarrow 480x - 480x + 3840 = 3x(x - 8)$

$\Rightarrow 3x^2 - 24x - 3840 = 0$

$\Rightarrow x^2 - 8x - 1280 = 0$

$\Rightarrow x^2 - 40x + 32x - 1280 = 0$

$\Rightarrow x(x - 40) + 32(x - 40) = 0$

$\Rightarrow (x + 32)(x - 40) = 0$

$\Rightarrow x = 40, -32$

The speed of the train is $40$ km/hr.

---

**#465282**

Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$

(ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

**Solution**

i)

$x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x - 5) + 2(x - 5) = 0$

$\Rightarrow (x + 2)(x - 5) = 0$

$\Rightarrow x = 5, -2$

ii)

$2x^2 + x - 6 = 0$

$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x + 2) - 3(x + 2) = 0$

$\Rightarrow (x + 2)(2x - 3) = 0$

$\Rightarrow x = \frac{3}{2}, -2$

iii)

$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0$

$\Rightarrow (x + \sqrt{2})(\sqrt{2}x + 5) = 0$

$\Rightarrow x = -\sqrt{2}, \frac{-5}{\sqrt{2}}$

iv)

$2x^2 - x + \frac{1}{8} = 0$

$\Rightarrow 16x^2 - 8x + 1 = 0$

$\Rightarrow 16x^2 - 4x - 4x + 1 = 0$

$\Rightarrow 16x\left(x - \frac{1}{4}\right) - 4\left(x - \frac{1}{4}\right) = 0$

$\Rightarrow \left(x - \frac{1}{4}\right)(16x - 4) = 0$

$\Rightarrow x = \frac{1}{4}, \frac{1}{4}$

v)

$100x^2 - 20x + 1 = 0$

$\Rightarrow 100x^2 - 10x - 10x + 1 = 0$

$\Rightarrow 10x(10x - 1) - 1(10x - 1) = 0$

$\Rightarrow (10x - 1)(10x - 1) = 0$

$\Rightarrow x = \frac{1}{10}, \frac{1}{10}$

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**#465283**

Find two numbers whose sum is $27$ and product is $182$.

**Solution**

Let the first number be $x$ then the second number will be $27 - x$.

$x(27 - x) = 182$

$\Rightarrow 27x - x^2 = 182$

$\Rightarrow x^2 - 27x + 182 = 0$

$\Rightarrow x^2 - 13x - 14x + 182 = 0$

$\Rightarrow x(x - 13) - 14(x - 13) = 0$

$\Rightarrow (x - 14)(x - 13) = 0$

$\Rightarrow x = 14, 13$

If the first number is $14,$ then the second number is $13$ and if the first number is $13,$ then the second number is $14$.

---

**#465284**

Find two consecutive positive integers, sum of whose squares is 365.

**Solution**

Let the first number is $x,$ then the next number will be $x + 1$.

$x^2 + (x + 1)^2 = 365$

$\Rightarrow x^2 + x^2 + 2x + 1 = 365$

$\Rightarrow 2x^2 + 2x - 364 = 0$

$\Rightarrow x^2 + x - 182 = 0$

$\Rightarrow x^2 + 14x - 13x - 182 = 0$

$\Rightarrow x(x + 14) - 13(x + 14) = 0$

$\Rightarrow (x - 13)(x + 14) = 0$

$\Rightarrow x = 13, -14$

Considering positive number, the numbers are $13$ and $14$.

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**#465287**

The altitude of a right triangle is $7$ cm less than its base. If the hypotenuse is $13$ cm, find the other two sides.

**Solution**

Let the base be $x$ cm. Then the altitude will be $x - 7$ cm.

By Pythagoras theroem,

$x^2 + (x - 7)^2 = 13^2$

$\Rightarrow x^2 + x^2 - 14x + 49 = 169$

$\Rightarrow 2x^2 - 14x - 120 = 0$

$\Rightarrow x^2 - 7x - 60 = 0$

$\Rightarrow x^2 - 12x + 5x - 60 = 0$

$\Rightarrow x(x - 12) + 5(x - 12) = 0$

$\Rightarrow (x + 5)(x - 12) = 0$

$\Rightarrow x = 12, -5$

Length cannot be negative, then the base is $12$ cm and the height is $5$ cm.

---

**#465300**

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was $3$ more than twice the number of articles produced on that day. If the total cost of production on that day was Rs $90,$ find the number of articles produced and the cost of each article.

**Solution**

Let there be $x$ articles.

Then the production cost of each article $= 3 + 2x$

Total production cost $= x(3 + 2x) = 90$

$3x + 2x^2 = 902x^2 + 3x - 90 = 02x^2 + 15x - 12x - 90 = 02x(x - 6) + 15(x - 6) = 0(2x + 15)(x - 6) = 0x = 6, -\frac{15}{2}$

Number cannot be negative.

So, number of articles $= 6$ and the cost of each article $= 15$ Rs.

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**#465303**

Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

**Solution**

i)

$$2x^2 - 7x + 3 = 0 \Rightarrow x^2 - \frac{7x}{2} + \frac{3}{2} = 0 \Rightarrow x^2 - \frac{7x}{2} + \frac{3}{2} + \frac{49}{16} = \frac{49}{16} \Rightarrow x^2 - \frac{7x}{2} + \frac{49}{16} = \frac{49}{16} - \frac{3}{2} \Rightarrow \left(x - \frac{7}{4}\right)^2 = \frac{25}{16} \Rightarrow \left(x - \frac{7}{4}\right) = \pm\frac{5}{4} \Rightarrow x = \frac{7}{4} + \frac{5}{4} \quad \text{and} \quad x = \frac{7}{4} - \frac{5}{4} \Rightarrow x = 3, \frac{1}{2}$$

ii)

$$2x^2 + x - 4 = 0 \Rightarrow x^2 + \frac{x}{2} - 2 = 0 \Rightarrow x^2 + \frac{x}{2} + \frac{1}{16} = 2 + \frac{1}{16} \Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{33}{16} \Rightarrow x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4} \Rightarrow x = \frac{\sqrt{33} - 1}{4}, \frac{-\sqrt{33} - 1}{4}$$

iii)

$$4x^2 + 4\sqrt{3}x + 3 = 0 \Rightarrow x^2 + \sqrt{3}x + \frac{3}{4} = 0 \Rightarrow x^2 + \sqrt{3}x + \frac{3}{4} = -\frac{3}{4} + \frac{3}{4} \Rightarrow \left(x + \frac{\sqrt{3}}{2}\right)^2 = 0 \Rightarrow x + \frac{\sqrt{3}}{2} = 0 \Rightarrow x = -\frac{\sqrt{3}}{2}$$

**#465305**

Find the roots of the following equations:

(i) $x - \frac{1}{x} = 3, x \neq 0$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

**Solution**

i)

$$x - \frac{1}{x} = 3$$

$$\Rightarrow x^2 - 3x - 1 = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow a = 1, b = -3, c = -1$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9 + 4}}{2}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{13}}{2}$$

ii)

$$\frac{1}{(x+4)} - \frac{1}{(x-7)} = \frac{11}{30}$$

$$\Rightarrow \frac{(x - 7 - x - 4)}{(x+4)(x-7)} = \frac{11}{30}$$

$$\Rightarrow \frac{-11}{(x+4)(x-7)} = \frac{11}{30}$$

$$\Rightarrow -30 = x^2 - 3x - 28$$

$$\Rightarrow x^2 - 3x + 2 = 0$$

$$\Rightarrow x^2 - 2x - x + 2 = 0$$

$$\Rightarrow x(x-2) - 1(x-2) = 0$$

$$\Rightarrow (x-2)(x-1) = 0$$

$$\Rightarrow x = 1, 2$$

**#465306**

The sum of the reciprocals of Rehmans ages, (in years) 3 years ago and 5 years fromnow is $\frac{1}{3}$. Find his present age.

**Solution**

Let Rehman's present age $= x$ yrs.

3 yrs ago, Rehman's age $= x - 3$ yrs

5 yrs hence, his age will be $x + 5$ yrs

$\frac{1}{(x-3)} + \frac{1}{(x+5)} = \frac{1}{3}\frac{(x+5+x-3)}{(x+5)(x-3)} = \frac{1}{3}x^2 + 2x - 15 = 6x + 6x^2 - 4x - 21 = 0x^2 - 7x + 3x - 21 = 0x(x-7) + 3(x-7) = 0(x+3)(x-7) = 0x = -3, 7$

Rehman's present age is 7 yrs.

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**#465307**

In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

**Solution**

Let the marks in Math be $x$ and in English $(30 - x)$.

New marks in Math $= 2 + x$

New marks in English $= 27 - x$

$(2 + x)(27 - x) = 21054 - 2x + 27x - x^2 = 210x^2 - 25x + 156 = 0x^2 - 12x - 13x + 156 = 0x(x-12) - 13(x-12) = 0(x-12)(x-13) = 0x = 12, 13$

If her marks in Math is 12, then in English she got 18.

If math mark is 13, then she got 17 in English.

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**#465308**

The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

**Solution**

Let the shorter side be $x$ m.

Then diagonal will be $x + 60$ m.

The longer side will be $x + 30$ m.

$(x + 60)^2 = x^2 + (x + 30)^2$

On simplifying above equation, we get

$x^2 - 60x - 2700$

$= (x - 90)(x + 30)$

$x = 90, -30$

Neglecting negative value, we get

The shorter side is 90 m and the longer side is 120 m.

---

**#465309**

The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

**Solution**

Let the smaller number be $x$ and the larger number be $y$.

Given $x^2 - y^2 = 180$

Also, $x^2 = 8y$

Substituting $x^2 = 8y$ in the first equation, we get

$8y - y^2 = 180y^2 - 8y - 180 = 0y^2 - 18y + 10y - 180 = 0y(y-18) + 10(y-18) = 0(y+10)(y-18) = 0y = 18, -10$

Considering the positive root,

So, the larger number is 18.

$x^2 = 8y = 144x = 12$

The smaller number is 12 or $-12$.

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**#465310**

A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

**Solution**

Given distance = 360 km.

Let the speed of the train be $x$ km/hr.

Speed when increased by 5 km/hr $= (x + 5)$ km/hr

$$\frac{360}{x} - \frac{360}{(x+5)} = 1\frac{[360x + 1800 - 360x]}{x(x+5)} = 1x^2 + 5x - 1800 = 0x^2 + 45x - 40x - 1800 = 0x(x+45) - 40(x+45) = 0(x-40)(x+45) = 0x = 40, \ -45$$

The speed of the train is 40 km/hr.

---

**#465314**

Sum of the areas of two squares is $468_m2$. If the difference of their perimeters is 24 m, find the sides of the two squares.

**Solution**

Let the side of the first square be 'a'm and that of the second be $'A'$ m.

Area of the first square $= a^2$ sq m.

Area of the second square $= A^2$ sq m.

Their perimeters would be $4a$ and $4A$ respectively.

Given $4A - 4a = 24$

$A - a = 6$ --(1)

$A^2 + a^2 = 468$ --(2)

From (1), $A = a + 6$

Substituting for $A$ in (2), we get

$$(a+6)^2 + a^2 = 468a^2 + 12a + 36 + a^2 = 4682a^2 + 12a + 36 = 468a^2 + 6a + 18 = 234a^2 + 6a - 216 = 0a^2 + 18a - 12a - 216 = 0a(a+18) - 12(a+18) = 0(a-12)(a+18) = 0a = 12, \ -1$$

So, the side of the first square is 12 m. and the side of the second square is 18 m.

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**#465315**

Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

**Solution**

i)

$x^2 - 3x + 5 = 0 a = 2, b = -3, c = 5$

Discriminant, $D = b^2 - 4ac = 9 - 40 = -31$

Since $D < 0,$ the roots are imaginary for the given equation.

ii)

$3x^2 - 4\sqrt{3}x + 4 = 0 a = 3, b = -4\sqrt{3}, c = 4$

Discriminant, $D = b^2 - 4ac = 48 - 48 = 0$

Since $D = 0,$ the roots are real and equal for the given equation.

$3x^2 - 4\sqrt{3}x + 4 = 0 a = 3, b = -4\sqrt{3}, c = 4 x = \frac{[-b \pm \sqrt{b^2 - 4ac}]}{2a} x = \frac{4\sqrt{3}}{6} = \frac{2}{\sqrt{3}}$

iii)

$2x^2 - 6x + 3 = 0 a = 2, b = -6, c = 3$

Discriminant, $D = b^2 - 4ac = 36 - 24 = 12$

Since $D > 0,$ roots are real but not equal.

$x = \frac{[-b \pm \sqrt{b^2 - 4ac}]}{2a} x = \frac{6 \pm \sqrt{12}}{4} x = \frac{6 \pm 2\sqrt{3}}{4} x = \frac{3 \pm \sqrt{3}}{2}$

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**#465316**

Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

(ii) $kx(x - 2) + 6 = 0$

**Solution**

i)

If $2x^2 + kx + 3 = 0$ has equal roots, then $b^2 - 4ac = 0.$

Here, $a = 2, b = k, c = 3$

$x^2 + kx + 3 = 0 b^2 - 4ac = 0 k^2 - 4(2)(3) = 0 k^2 = 24 k = \pm\sqrt{24} = \pm 2\sqrt{6}$

ii)

If $kx^2 - 2kx + 6 = 0$ has equal roots, then $b^2 - 4ac = 0.$

Here, $a = k, b = -2k, c = 6$

$kx(x - 2) + 6 = 0 kx^2 - 2kx + 6 = 0 4k^2 - 4(k)(6) = 0 4k^2 - 24k = 0 4k(k - 6) = 0$

For $k = 0,$ equation is not quadratic.

$k = 6$

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**#465317**

Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is $800 m^2$ ? If so, find its length and breadth.

**Solution**

Let the breadth be $x$ m and the length will be $2x$ m.

The area = $2x^2$ sq m.

$2x^2 = 800 \Rightarrow x^2 = 400 \Rightarrow x = 20$

So, the breadth is $20$ m and the length $= 40$ m

**#465318**

Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48

**Solution**

Let their present ages be $x$ and $20 - x$ yrs respectively.

Four yrs ago, their ages would be (x-4) and (16-x) yrs respectively.

$(x - 4)(16 - x) = 4816x - x^2 - 64 + 4x = 48x^2 - 20x + 112 = 0b^2 - 4ac = -48$

Since the discriminant $< 0$, the roots would be imaginary.

Hence, such a situation is not possible.

**#465319**

Is it possible to design a rectangular park of perimeter 80 m and area $400 m^2$? If so, find its length and breadth.

**Solution**

Let the length be $l$ m and the breadth be $b$ m.

Then the area would be $lb = 400$

Perimeter would be $2(l + b) = 80$

$lb = 4002(l + b) = 80l + b = 40$

$b = 40 - l$ --(1)

Substituting (1) in Area, we get

$l(40 - l) = 40040l - l^2 = 400l^2 - 40l + 400 = 0(l - 20)(l - 20) = 0l = 20$

We now know that the length of the park is 20 m and the breadth of the park is also 20 m.

**#470302**

Solve:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a da
On a particular day, the total cost of production was $Rs.$ 750. We would like to find out the number of toys produced on that day.

**Solution**

i) Let John have $x$ marbles, so Jayanti will have $(45 - x)$ marbles.

After losing 5 marbles,

John will have $= (x - 5)$ marbles

Jivanti will have $= (45 - x - 5) = (40 - x)$ marbles

As per the given condition,

$(x - 5)(40 - x) = 124$

$\Rightarrow -(x - 5)(x - 40) = 124$

$\Rightarrow -x^2 + 45x - 200 = 124$

$\Rightarrow x^2 + 45x - 324 = 0$

$\Rightarrow x^2 - 36x - 9x - 324 = 0$

$\Rightarrow x(x - 36) - 9(x - 36) = 0$

$\Rightarrow (x - 36)(x - 9) = 0$

$\Rightarrow x - 36 = 0$ or $x - 9 = 0$

$\Rightarrow x = 36$ or $x = 9$

So, if John has 36 marbles, Jivanti will have 9 marbles.

If John has 9 marbles, Jivanti will have 36 marbles.

ii) Let the total number of toys produced in a day be $x$.

Cost of production of 1 toy $= Rs.\ (55 - x)$

$\therefore x(55 - x) = 750$

$\Rightarrow x^2 - 55x + 750 = 0$

$\Rightarrow x^2 - 25x - 30x + 750 = 0$

$\Rightarrow x(x - 25) - 30(x - 25) = 0$

$\Rightarrow (x - 25)(x - 30) = 0$

$\Rightarrow x - 25 = 0$ or $x - 30 = 0$

$\Rightarrow x = 25$ or $x = 30$

So, total number of toys produced in a day are 25 or 30.