#422402

Topic: Arithmetic Progression

In an A.P.,the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20^{th} term is -112.

Solution

First term $= 2$.

Let d be the common difference of the A.P.

Therefore, the A.P. is $2, 2 + d, 2 + 2d, 2 + 3d, \ldots$

Sum of first five terms $= 10 + 10d$

Sum of next five terms $= 10 + 35d$

According to the given condition ,

$10 + 10d = \frac{1}{4}(10 + 35d) \Rightarrow 40 + 40d = 10 + 35d \Rightarrow 30 = -5d \Rightarrow d = -6 \therefore a_{20} = a + (20 - 1)d = 2 + (19)(-6) = 2 - 114 = -112$

Thus, the 20^{th} term of the A.P. is -112.

#423193

Topic: Arithmetic Progression

The p^{th}, q^{th} and r^{th} terms of an A.P. are a, b, c respectively. Show that $(q - r)a + (r - p)b + (p - q)c = 0$

Solution

Let t and d be the first term and the common difference of the A.P. respectively,

The n^{th} term of an A.P. is given by $a_n = t + (n - 1)d$ Therefore,

$a_p = t + (p - 1)d = a \quad \ldots(1) a_q = t + (q - 1)d = b \quad \ldots(2) a_r = t + (r - 1)d = c \quad \ldots(3)$

Subtracting equation (2) from (1), we obtain

$(p - 1 - q + 1)d = a - b \Rightarrow (p - q)d = a - b \therefore d = \frac{a - b}{p - q} \quad \ldots(4)$

Subtracting equation (4) from (3), we obtain

$(q - 1 - r + 1)d = b - c \Rightarrow (q - r)d = b - c \Rightarrow d = \frac{b - c}{q - r} \quad \ldots(5)$

Equating both the values of d obtained in (4) and (5), we obtain

$\frac{a - b}{p - q} = \frac{b - c}{q - r} \Rightarrow (a - b)(q - r) = (b - c)(p - q) \Rightarrow aq - bq - ar + br = bp - bq - cp + cq \Rightarrow (-aq + ar) + (bp - br) + (-cp + cq) = 0 \quad \Rightarrow -a(q - r) - b(r - p) - c(p - q) = 0 \Rightarrow a(q -$

Thus, the given result is proved.

#466081

Topic: Arithmetic Progression

In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is Rs.15 for the first km and Rs. 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs. 150 for the first metre and rises by Rs. 50 for each subsequent metre.

(iv) The amount of money in the account every year, when Rs. 10000 is deposited at compound interest at 8 % per annum.

Solution

(i) Fare for first km $=$ Rs. 15

Fare for second km $=$ Rs. $15 + 8 =$ Rs 23

Fare for third km $=$ Rs. $23 + 8 = 31$

Here, each subsequent term is obtained by adding a fixed number (8) to the previous term.

Hence, it is in A.P.

(ii) Let us assume, initial quantity of air $= 1$ .....1)

Therefore, quantity removed in first step $= \frac{1}{4}$

Remaining quantity after first step

$1 - \frac{1}{4} = \frac{3}{4}$ ....2)

Quantity removed in second step

$= \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$

Remaining quantity after second step

$= \frac{3}{4} - \frac{3}{16} = \frac{9}{16}$ ....3)

Here, each subsequent term is not obtained by adding a fixed number to the previous term.

Hence, it is not an AP.

(iii) Cost of digging of 1^{st} meter $= 150$

Cost of digging of 2^{nd} meter $= 150 + 50 = 200$

Cost of digging of 3^{rd} meter $= 200 + 50 = 250$

Here, each subsequent term is obtained by adding a fixed number (50) to the previous term.

Hence, it is an AP.

(iv) Amount in the beginning $=$ Rs. 10000

Interest at the end of 1^{st} year @ $8\% = 10000 \times 8$

Thus, amount at the end of 1^{st} year $= 10000 + 800 = 10800$

Interest at the end of 2^{nd} year @ $8\% = 10800 \times 8$

Thus, amount at the end of 2^{nd} year $= 10800 + 864 = 11664$

Since, each subsequent term is not obtained by adding a fixed number to the previous term; hence, it is not an AP.

#466082

Topic: Arithmetic Progression

Write first four terms of the AP, when the first term a and the common difference d are given as follows:

(i) $a = 10, d = 10$

(ii) $a = -2, d = 0$

(iii) $a = 4, d = -3$

(iv) $a = -1, d = \frac{1}{2}$

(v) $a = -1.25, d = -0.25$

Solution

An arithmetic progression is given by $a, (a+d), (a+2d), (a+3d), .$

where $a =$ the first term, $d =$ the common difference

(i) $10, 10+10, 10+2(10)$ and $10+3(10) = 10, 20, 30$ and 40

(ii) $-2, -2+(0), -2+2(0)$ and $-2+3(0) = -2, -2, -2$ and -2 (this is not an A.P)

(iii) $4, 4+(-3), 4+2(-3)$ and $4+3(-3) = 4, 4-3, 4-6$ and $4-9 = 4, 1, -2$ and -5

(iv)$-1, -1+\left(\frac{1}{2}\right), -1+2\left(\frac{1}{2}\right)$ and $-1+3\left(\frac{1}{2}\right) = -1, -\frac{1}{2}, 0$ and $\frac{1}{2}$

(v)$-1.25, -1.25+(-0.25), -1.25+2(-0.25)$ and $-1.25+3(-0.25) = -1.25, -1.50, -1.75$ and -2.0

#466083

Topic: Arithmetic Progression

For the following APs, write the first term and the common difference:

(i) $3, 1, -1, -3, \ldots.$

(ii) $-5, -1, 3, 7, \ldots..$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots.$

(iv) $0.6, 1.7, 2.8, 3.9, \ldots$

Solution

(i) $3, 1, -1, -3...$

Here, first term, $a = 3$

Common difference, $d =$ Second term $-$ First term

$d = 1-3 = -2$

(ii) $-5, -1, 3, 7...$

Here, first term, $a = -5$

Common difference, $d =$ Second term $-$ First term

$d = (-1)-(-5) = -1+5 = 4$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}$

Here, first term, $a = \frac{1}{3}$

Common difference, $d =$ Second term $-$ First term

$d = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) $0.6, 1.7, 2.8, 3.9...$

Here, first term, $a = 0.6$

Common difference, $d =$ Second term $-$ First term

$d = 1.7-0.6$

$= 1.1$

#466084

Topic: Arithmetic Progression

Which of the following are APs ? If they form an AP, find the common difference d and write three more terms.

(i) $2, 4, 8, 16, \dots.$

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}, \dots$

(iii) $-1.2, -3.2, -5.2, -7.2, \dots$

(iv) $-10, -6, -2, 2, \dots$

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}$

(vi) $0.2, 0.22, 0.222, 0.2222, \dots.$

(vii) $0, -4, -8, -12, \dots$

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \dots$

(ix) $1, 3, 9, 27$

(x) $a, 2a, 3a, 4a, \dots$

(xi) $a, a^2, a^3, a^4, \dots$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \dots$

(xiv) $1^2, 3^2, 5^2, 7^2, ..$

(xv) $1^2, 5^2, 7^2, 73, ..$

Solution

a, b, c are said to be in AP if the common difference between any two consecutive number of the series is same ie $b-a=c-b \Rightarrow 2b=a+c$

(i) It is not in AP, as the difference between consecutive terms is different.

(ii) It is in AP with common difference $d=\frac{5}{2}-2=\frac{1}{2}$,

$t_n=a+(n-1)d$

$a=2$

$t_5=2+(5-1)\frac{1}{2}$

Next three terms are $4, \frac{9}{2}, 5$

(iii) It is in AP with common difference $d=-3.2+1.2=-2$,and $a=-1.2$

Next three terms are

$a+(5-1)d=-9.2,$

$a+(6-1)d=-11.2,$

$a+(7-1)d=-13.2$

(iv) It is in AP with common difference $d=-6+10=4$, and

$a=-10$

Next three terms are

$a+(5-1)d=6,$

$a+(6-1)d=10,$

$a+(7-1)d=14$

(v) It is in AP with common difference $d=3+\sqrt{2}-3=\sqrt{2}$, and

$a=3$

Next three terms are

$a+(5-1)d=3+4\sqrt{2},$

$a+(6-1)d=3+5\sqrt{2},$

$a+(7-1)d=3+6\sqrt{2}$

(vi) It is not in AP since $0.22-0.2 \neq 0.222-0.22$

(*vii*) It is in AP with common difference $d = -4 - 0 = -4$ and $a = 0$,

Next three terms are

$a + (5-1)d = -16$,

$a + (6-1)d = -20$,

$a + (7-1)d = -24$

(*viii*) It is in AP, with common difference 0, therefore next three terms will also be same as previous ones, i.e., $-\frac{1}{2}$

(*ix*) It is not in AP since $3 - 1 \neq 9 - 3$

(*x*) It is in AP with common difference $d = 2a - a = a$ and first term is a,

Next three terms are

$a + (5-1)d = 5a$,

$a + (6-1)d = 6a$,

$a + (7-1)d = 7a$

(*xi*) It is not in AP, as the difference is not constant.

(*xii*) It is in AP with common difference $d = \sqrt{2}$ and $a = \sqrt{2}$,

Next three terms are

$a + (5-1)d = 5\sqrt{2} = \sqrt{50}$,

$a + (6-1)d = \sqrt{72}$,

$a + (7-1)d = \sqrt{98}$

(*xiii*) It is not in AP as difference is not constant.

(*xiv*) It is not in AP as difference is not constant.

(*xv*) It is in AP with common difference $d = 5^2 - 1 = 24$ and $a = 1$,

Next three terms are

$a + (5-1)d = 97$,

$a + (6-1)d = 121$,

$a + (7-1)d = 145$

#466085

Topic: Arithmetic Progression

Sr. no.	a	d	n	a_n
(i)	7	3	8	----
(ii)	-18	----	10	0
(iii)	----	-3	18	-5
(iv)	-18.9	2.5	----	3.6
(v)	3.5	0	105	----

Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n is the n^{th} term of the AP:

Solution

(i) $a = 7, d = 3, n = 8, a_n = ?$

We know that,

For an A.P. $a_n = a + (n - 1)d$

$= 7 + (8 - 1)3$

$= 7 + (7)3$

$= 7 + 21 = 28$

Hence, $a_n = 28$

(ii) Given that

$a = -18, n = 10, a_n = 0, d = ?$

We know that,

$a_n = a + (n - 1)d$

$0 = -18 + (10 - 1)d$

$18 = 9d$

$d = \frac{18}{9} = 2$

Hence, common difference, $d = 2$

(iii) Given that

$d = -3, n = 18, an = -5$

We know that,

$a_n = a + (n - 1)d$

$-5 = a + (18 - 1)(-3)$

$-5 = a + (17)(-3)$

$-5 = a - 51$

$a = 51 - 5 = 46$

Hence, $a = 46$

(iv) $a = -18.9, d = 2.5, a_n = 3.6, n = ?$

We know that,

$a_n = a + (n - 1)d$

$3.6 = -18.9 + (n - 1)2.5$

$3.6 + 18.9 = (n - 1)2.5$

$22.5 = (n - 1)2.5$

$(n - 1) = \frac{22.5}{2.5} = 9$

$n - 1 = 9$

$n = 10$

Hence, $n = 10$

(v) $a = 3.5, d = 0, n = 105, a_n = ?$

We know that,

$a_n = a + (n - 1)d$

$a_n = 3.5 + (105 - 1)0$

$a_n = 3.5 + 104 \times 0$

$a_n = 3.5$

#466086

Topic: Arithmetic Progression

Choose the correct choice in the following and justify :

(i) 30^{th} term of the AP: $10, 7, 4, \dots$, is

(A) 97 (B) 77 (C) -77 (D) 87

(ii) 11^{th} term of the AP: $-3, -\dfrac{1}{2}, 2$,...., is

(A) 28 (B) 22 (C) 38 (D) $-48\dfrac{1}{2}$

Solution

(i) Given that

A.P. $10, 7, 4, \ldots$

First term, $a = 10$

Common difference, $d = a_2 = a_1 = 7 - 10 = -3$

We know that $a_n = a + (n-1)d$

$a_{30} = 10 + (30 - 1)(-3)$

$= 10 + (29)(-3)$

$= 10 - 87$

$= -77$

Hence, the correct answer is option C.

(ii) A.P. is $-3, \dfrac{-1}{2}, 2, \ldots\ldots$

First term $a = -3$

Common difference, $d = a_2 - a_1 = \dfrac{-1}{2} - (-3) = \dfrac{5}{2}$

We know that, $a_n = a + (n-1)d$

$a_{11} = -3 + (11-1)\dfrac{5}{2} = -3 + (10)\dfrac{5}{2} = -3 + 25 = 22$

Hence, the answer is option B.

#466087

Topic: Arithmetic Progression

(i) $2, \boxed{}, 26$

(ii) $\boxed{}, 13, \boxed{}, 3$

(iii) $5, \boxed{}, \boxed{}, 9\dfrac{1}{2}$

(iv) $-4, \boxed{}, \boxed{}, \boxed{}, \boxed{}, 6$

(v) $\boxed{}, 38, \boxed{}, \boxed{}, \boxed{}, -22$

In the following APs, find the missing terms in the boxes :

Solution

(i) For this A.P.,

$a = 2$

$a_3 = 26$

We know that, $a_n = a + (n-1)d$

$a_3 = 2 + (3-1)d$

$26 = 2 + 2d$

$24 = 2d$

$d = 12$

$a_2 = 2 + (2-1)12$

$= 14$

Therefore, 14 is the missing term.

(ii) For this A.P.,

$a_2 = 13$ and

$a_4 = 3$

We know that, $a_n = a + (n - 1)d$

$a_2 = a + (2 - 1)d$

$13 = a + d$ ... (i)

$a_4 = a + (4 - 1)d$

$3 = a + 3d$... (ii)

On subtracting (i) from (ii), we get,

$-10 = 2d$

$d = -5$

From equation (i), we get,

$13 = a + (-5)$

$a = 18$

$a_3 = 18 + (3 - 1)(-5)$

$= 18 + 2(-5) = 18 - 10 = 8$

Therefore, the missing terms are 18 and 8 respectively.

(iii) For this A.P.,

$a_1 = 5$ and

$a_4 = 9\frac{1}{2}$

We know that, $a_n = a + (n - 1)d$

$a_4 = 5 + (4 - 1)d$

$9\frac{1}{2} = 5 + 3d$

$d = \frac{3}{2}$

$a_2 = a + d$

$a_2 = 5 + \frac{3}{2}$

$a_2 = \frac{13}{2}$

$a_3 = a_2 + \frac{3}{2}$

$a_3 = 8$

Therefore, the missing terms are $6\frac{1}{2}$ and 8 respectively.

(iv) For this A.P.,

$a = -4$ and

$a_6 = 6$

We know that,

$a_n = a + (n - 1)d$

$a_6 = a + (6 - 1)d$

$6 = -4 + 5d$

$10 = 5d$

$d = 2$

$a_2 = a + d = -4 + 2 = -2$

$a_3 = a + 2d = -4 + 2(2) = 0$

$a_4 = a + 3d = -4 + 3(2) = 2$

$a_5 = a + 4d = -4 + 4(2) = 4$

Therefore, the missing terms are $-2, 0, 2,$ and 4 respectively.

(v) For this A.P.,

$a_2 = 38$

$a_6 = -22$

We know that

$a_n = a + (n-1)d$

$a_2 = a + (2-1)d$

$38 = a + d$... (i)

$a_6 = a + (6-1)d$

$-22 = a + 5d$ ... (ii)

On subtracting equation (i) from (ii), we get

$-22 - 38 = 4d$

$-60 = 4d$

$d = -15$

$a = a_2 - a = 38 - (-15) = 53$

$a_3 = a + 2d = 53 + 2(-15) = 23$

$a_4 = a + 3d = 53 + 3(-15) = 8$

$a_5 = a + 4d = 53 + 4(-15) = -7$

Therefore, the missing terms are $53, 23, 8$ and -7 respectively.

#466088

Topic: Arithmetic Progression

Which term of the AP : $3, 8, 13, 18, \ldots,$ is 78?

Solution

Given A.P. is $3, 8, 13, 18, ...$

For the above AP,

$a = 3$

$d = a_2 - a_1 = 8 - 3 = 5$

Let n^{th} term of this A.P. be $78.$

$a_n = a + (n-1)d$

$78 = 3 + (n-1)5$

$75 = (n-1)5$

$(n-1) = 15$

$n = 16$

Hence, 16^{th} term of this A.P. is $78.$

#466089

Topic: Arithmetic Progression

Find the number of terms in each of the following APs :

(i) $7, 13, 19, \ldots, 205$

(ii) $18, 15\frac{1}{2}, 13, \ldots, -47$

Solution

(i) $a = 7$

$d = a_2 - a_1 = 13 - 7 = 6$

Considering there are n terms in this A.P.

$a_n = 205$

We know that $a_n = a + (n-1)d$

$205 = 7 + (n-1)6$

$198 = (n-1)6$

$33 = (n-1)$

$n = 34$

The series has 34 terms.

(ii) $a = 18$

$d = a_2 - a_1 = 15\frac{1}{2} - 18$

$\Rightarrow d = \frac{31 - 36}{2} = \frac{-5}{2}$

Considering there are n terms in this A.P.

$a_n = -47$

We know, $a_n = a + (n-1)d$

$-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$

$\Rightarrow -47 - 18 = (n-1)\left(-\frac{5}{2}\right)$

$\Rightarrow -65 = (n-1)\left(-\frac{5}{2}\right)$

$\Rightarrow n - 1 = \frac{-130}{-5} = 26$

$\Rightarrow n = 27$

The series has 27 terms.

#466090

Topic: Arithmetic Progression

Check whether -150 is a term of the AP : $11, 8, 5, 2.....$

Solution

Given series is $11, 8, 5, 2...........$

$a = 11$ and $d = 8 - 11 = -3$

$T_n = -150$

$a + (n-1)d = -150$

$11 + (n-1)(-3) = -150$

$11 - 3n + 3 = -150$

$14 - 3n = -150$

$3n = -164$

$n = -\frac{164}{3} = -54.66$

n is not an integer.

So, it is not in A.P.

#466091

Topic: Arithmetic Progression

Find the 31^{st} term of an AP whose 11^{th} term is 38 and the 16^{th} term is 73.

Solution

Given that,

$a_{11} = 38$

$a_{16} = 73$

We know that,

$a_n = a + (n-1)d$

$a_{11} = a + (11-1)d$

$38 = a + 10d$ ... (i)

Similarly,

$a_{16} = a + (16-1)d$

$73 = a + 15d$ ... (ii)

On subtracting (i) from (ii), we get

$35 = 5d$

$d = 7$

From equation (i),

$38 = a + (10)(7)$

$38 - 70 = a$

$a = -32$

$\therefore a_{31} = a + (31-1)d$

$= -32 + 30(7)$

$= -32 + 210$

$= 178$

Hence, 31^{st} term is 178.

#466092

Topic: Arithmetic Progression

An AP consists of 50 terms of which 3^{rd} term is 12 and the last term is 106. Find the 29^{th} term.

Solution

Given, $a_3 = 12$, $a_{50} = 106$

$a_n = a + (n-1)d$

$a_3 = a + (3-1)d$

$12 = a + 2d$ ... (i)

$a_{50} = a + (50-1)d$

$106 = a + 49d$ ... (ii)

On subtracting (i) from (ii), we get

$94 = 47d$

$d = 2$

From equation (i), we get

$12 = a + 2(2)$

$a = 12 - 4 = 8$

$\therefore a_{29} = a + (29-1)d$

$= 8 + (28)2$

$= 8 + 56$

$= 64$

Therefore, 29^{th} term is 64.

#466093

Topic: Arithmetic Progression

If the 3^{rd} and the 9^{th} terms of an AP are 4 and -8 respectively, then which term of this AP is zero.

Solution

Given that, $a_3 = 4, a_9 = -8$

We know that,

$a_n = a + (n-1)d$

$a_3 = a + (3-1)d$

$4 = a + 2d \dots$ (i)

And

$a_9 = a + (9-1)d$

$-8 = a + 8d \dots$ (ii)

On subtracting equation (i) from (ii), we get,

$-12 = 6d$

$d = -2$

From equation (i), we get,

$4 = a + 2(-2)$

$\Rightarrow a = 8$

Let n^{th} term of this A.P. be zero.

$a_n = a + (n-1)d$

$0 = 8 + (n-1)(-2)$

$\Rightarrow n = 5$

Hence, 5th term of this A.P. is 0.

#466094

Topic: Arithmetic Progression

The 17^{th} term of an AP exceeds its 10^{th} term by 7. Find the common difference.

Solution

We know that,

For an A.P $a_n = a + (n-1)d$

$a_{17} = a + (17-1)d$

$a_{17} = a + 16d$

Similarly, $a_{10} = a + 9d$

It is given that

$a_{17} - a_{10} = 7$

$(a + 16d) - (a + 9d) = 7$

$7d = 7$

$\therefore d = 1$

Therefore, the common difference is 1.

#466095

Topic: Arithmetic Progression

Which term of the AP : $3, 15, 27, 39, \dots$ will be 132 more than its 54^{th} term.

Solution

Given A.P. is $3, 15, 27, 39, ...$

$a = 3$

$d = a_2 - a_1 = 15 - 3 = 12$

$a_{54} = a + (54 - 1)d$

$= 3 + (53)(12)$

$= 3 + 636 = 639$

Then, 132 more than its 54^{th} term is $132 + 639 = 771$

We have to find the term of this A.P. which is 771.

Let n^{th} term be 771.

$a_n = a + (n - 1)d$

$771 = 3 + (n - 1)12$

$768 = (n - 1)12$

$(n - 1) = 64$

$n = 65$

Therefore, 65^{th} term was 132 more than 54^{th} term.

#466096

Topic: Arithmetic Progression

Two APs have the same common difference. The difference between their 100^{th} terms is 100, what is the difference between their 1000^{th} terms.

Solution

Given that their 100th term's difference is 100

Let the first no. of first series be a_1 and second series be a_2.

then, $a_{100}(1) - a_{100}(2) = 100$ --- 1)

For 1st series ---- $a_{100} = a_1 + 99d$

2nd series ---- $a_{100} = a_2 + 99d$

Put these values in (1)

then,

$a_1 + 99d - (a_2 + 99d) = 100$

$a_1 + 99d - a_2 - 99d = 100$

therefore, $a_1 - a_2 = 100$ ---2)

Then, the difference between their 1000th terms is

for 1st series --- $a_{1000} = a_1 + 999d$

for 2nd series --- $a_{1000} = a_2 + 999d$

their 100th terms difference is

$a_{1000}(1) - a_{1000}(2)$

$a_1 + 999d - (a_2 + 999d)$

$a_1 + 999d - a_2 - 999d$

Therefore, we get the value $a_1 - a_2$

from (2), $a_1 - a_2 = 100$

Therefore, the difference between their 1000th terms is 100.

#466097

Topic: Arithmetic Progression

How many three-digit numbers are divisible by 7 ?

Solution

Three digit numbers which are divisible by 7 are $105, 112, 119, \ldots . 994$.

These numbers form an AP with $a = 105$ and $d = 7$.

Let number of three-digit numbers divisible by 7 be n

$a_n = 994$

$a_n = a + (n-1)d = 994$

$\Rightarrow 105 + (n-1)(7) = 994$

$\Rightarrow 7(n-1) = 994 - 105$

$\Rightarrow 7(n-1) = 889$

$\Rightarrow n - 1 = 127$

$\Rightarrow n = 128$

#466098

Topic: Arithmetic Progression

How many multiples of 4 lie between 10 and 250 ?

Solution

Since the first multiply by 4 between 10 to 250 is 12 and last term is 248.

Now a (first term) $= 12$ and last term $= 248$ and common diff is 4

Let n be total number of terms.

Since $a + (n-1)d =$ last term

Then, $248 = 12 + (n-1)4$

$236 = (n-1)4$

$n - 1 = 59$

$n = 60$

Therefore total number of term is 60.

#466099

Topic: Arithmetic Progression

For what value of n, are the n^{th} terms of two APs: $63, 65, 67, \ldots$ and $3, 10, 17, \ldots$ equal ?

Solution

First A.P. is $63, 65, 67, \ldots$

$\therefore a = 63$

$d = 65 - 63 = 2$

n^{th} term of this A.P. $= a_n = a + (n-1)d$

$a_n = 63 + (n-1)2 = 63 + 2n - 2$

$a_n = 61 + 2n$..................(1)

Second A.P is $3, 10, 17, \ldots$

$a = 3$

$d = 10 - 3 = 7$

n^{th} term of this A.P. $= 3 + (n-1)7$

$a_n = 3 + 7n - 7$

$a_n = 7n - 4$....................(2)

It is given that, n^{th} term of these A.P.s are equal to each other.

Equating equations (1) and (2),

$61 + 2n = 7n - 4$

$61 + 4 = 5n$

$5n = 65$

$n = 13$

Therefore, 13^{th} terms of both these A.P.s are equal to each other.

#466100

Topic: Arithmetic Progression

Determine the AP whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12 ?

Solution

As given $a + 2d = 16$.......2^{nd} term

And $a + 6d - (a + 4d) = 12$

Implies $d = 6$

$\therefore a + 2(6) = 16$, then $a = 4$

Then A.P is $a, a + d, a + 2d, a + 3d. \ldots \ldots \ldots \ldots$

So, $4, 4 + 6, 4 + 2(6), 4 + 3(6), 4 + 4(6). \ldots \ldots \ldots \ldots$

4, 10, 16, 22, 28, 34...........

#466101

Topic: Arithmetic Progression

Find the 20th term from the last term of the AP $: 3, 8, 13, \ldots, 253$.

Solution

3, 8, 13, ... , 253

$d = 8 - 3 = 5$.

Writing this A.P. in reverse order

253, 248, 243, ..., 13, 8, 5

Now, for the new A.P., we have

$a = 253$

$d = 248 - 253$

$d = -5$

We want to find 20th term, so $n = 20$

$a_{20} = 253 + (20 - 1)(-5)$

$\Rightarrow 1_{20} = 253 + (19)(-5)$

$\Rightarrow a_{20} = 253 - 95 = 158$

20^{th} term from the last term $= 158$.

#466102

Topic: Arithmetic Progression

The sum of the 4^{th} and 8^{th} terms of an AP is 24 and the sum of the 6^{th} and 10^{th} terms is 44. Find the first three terms of the AP.

Solution

We know that,

$a_n = a + (n - 1)d$

$a_4 = a + 3d$

Similarly,

$a_8 = a + 7d$

$a_6 = a + 5d$

$a_{10} = a + 9d$

Given that,

$a_4 + a_8 = 24$

$2a + 10d = 24$

$a + 5d = 12$ ....(i)

$a_6 + a_{10} = 44$

$a + 5d + a + 9d = 44$

$2a + 14d = 44$

$a + 7d = 22$....(ii)

On subtracting equation **(i)** from **(ii)**, we get,

$2d = 22 - 12$

$2d = 10$

$d = 5$

From equation **(i)**, we get

$a + 5d = 12$

$a + 5(5) = 12$

$a + 25 = 12$

$a = -13$

$a_2 = a + d$

$= -13 + 5 = -8$

$a_3 = a_2 + d = -8 + 5 = -3$

Therefore, the first three terms of this A.P. are $-13, -8$ and -3.

#466103

Topic: Arithmetic Progression

Subba Rao started work in 1995 at an annual salary of Rs. 5000 and received an increment of Rs. 200 each year. In which year did his income reach Rs. 7000?

Solution

It can be observed that the incomes that Subba Rao obtained in various years are in A.P. as every year, his salary is increased by Rs 200.

Therefore, the salaries of each year after 1995 are

5000, 5200, 5400, ...

Here, $a = 5000$

$d = 200$

Let after n^{th} year, his salary be Rs 7000.

Therefore, $a_n = a + (n-1)d$

$7000 = 5000 + (n-1)200$

$200(n-1) = 2000$

$(n-1) = 10$

$n = 11$

Therefore, in 11^{th} year, his salary will be Rs 7000.

#466104

Topic: Arithmetic Progression

Ramkali saved Rs. 5 in the first week of a year and then increased her weekly savings by Rs. 1.75. If in the n^{th} week, her weekly savings become Rs. 20.75. Find n.

Solution

According to the question, $a = 5$, $d = 1.75$, $a_n = 20.75$

$a_n = a + (n-1)d$

Subsituting the values in above equation,

$20.75 = 5 + (n-1) \times 1.75$

$15.75 = (n-1)(1.75)$

$\Rightarrow n - 1 = \dfrac{15.75}{1.75}$

$\Rightarrow n - 1 = 9$

$\Rightarrow n = 10$

$n = 10$.

#466105

Topic: Arithmetic Progression

Find the sum of the following APs:

(i) $2, 7, 12, \ldots,$ to 10 terms.

(ii) $37, 33, 29, \ldots,$ to 12 terms.

(iii) $0.6, 1.7, 2.8, \ldots,$ to 100 terms.

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots,$ to 11 terms.

Solution

(i) 2, 7, 12...... to 10^{th} term

Here $a = 2, n = 10$

And $d = 7 - 2 = 5$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2(2) + (10-1)5] = 5(4 + 9 \times 5) = 245$$

(ii) $37, 33, 29, ...$ to 12^{th} term

Here $a = 37, n = 12$

And $d = 33 - 37 = -4$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}[2(37) + (12-1)(-4)] = 6(74 - 11 \times 4) = -180$$

(iii) 0.6, 1.7, 2.8........... to 100 term

Here $a = 0.6, n = 100$

And $d = 1.7 - 0.6 = 1.1$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{100} = \frac{100}{2}[2(0.6) + (100-1)(1.1)] = 50(1.2 - 99 \times 1.1) = 5505$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}.......$ to 11^{th} term

$$a = \frac{1}{15}, n = 11$$

$$d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$$

$$S_{11} = \frac{11}{2}\left[2(\frac{1}{15} + (11-1)\frac{1}{60})\right]$$

$$= \frac{11}{2}\left(\frac{2}{15} + \frac{10}{60}\right)$$

$$= \frac{11}{2} \times \frac{9}{30} = \frac{33}{20}$$

#466106

Topic: Arithmetic Progression

Find the sums given below :

(i) $7 + 10\frac{1}{2} + 14 + ... + 84$

(ii) $34 + 32 + 30 + ... + 10$

(iii) $-5 + (-8) + (-11) + ... + (-230)$

Solution

(i) $7, 10\frac{1}{2}, 14........84$

Here, first $a = 7$ and $l = 84$

$d = 10\frac{1}{2} - 7 = 3\frac{1}{2} = \frac{7}{2}$

$a_n = a + (n-1)d$

$84 = 7 + (n-1)\frac{7}{2} \Rightarrow 84 - 7 = \frac{7}{2}(n-1) \Rightarrow n - 1 = 22 \Rightarrow n = 23$

and last term $l = 84$

We know that $S_n = \frac{n}{2}(a + l)$

$S_n = \frac{23}{2}(7 + 84)$

$= \frac{23 \times 91}{2}$

$= \frac{2093}{2}$

$= 1046\frac{1}{2}$

(ii) $34, 32, 30....................10$

Here $a = 34, l = 10$

$d = 32 - 34 = -2$

$a_n = a + (n-1)d$

$10 = 34 + (n-1)(-2) \Rightarrow 10 - 34 = (-2)(n-1) \Rightarrow n - 1 = 12 \Rightarrow n = 13$

and last term $l = 10$

We know that $S_n = \frac{n}{2}(a + l)$

$S_n = \frac{13}{2}(34 + 10)$

$= \frac{13 \times 44}{2}$

$= 286$

(iii) $-5 + (-8) + (-11) + ... + (-230)$

Here $a = -5, l = -230$

$d = (-8 - (-5)) = -3$

$a_n = a + (n-1)d$

$230 = -5 + (n-1)(-3) \Rightarrow -230 + 5 = (-3)(n-1) \Rightarrow n - 1 = 75 \Rightarrow n = 76$

and last term $l = -230$

We know that $S_n = \frac{n}{2}(a + l)$

$S_n = \frac{76}{2}(-5 + (-230))$

$= -8930$

#466107

Topic: Arithmetic Progression

In an AP:

(i) Given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n.

(ii) Given $a = 7$, $a_{13} = 35$, find d and S_{13}.

(iii) Given $a_{12} = 37$, $d = 3$, find a and S_{12}.

(iv) Given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.

(v) Given $d = 5$, $S_9 = 75$, find a and a_9.

(vi) Given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n.

(vii) Given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d.

(viii) Given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a.

(ix) Given $a = 3$, $n = 8$, $S = 192$, find d.

(x) Given $l = 28$, $S = 144$, and there are total 9 terms. Find a.

Solution

We know that in an AP

If first term is $a_1 = a$, common difference is d then the n^{th} term is

$a_n = a + (n-1)d$

Sum of first n terms of this AP is

$S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}(2a + (n-1)d)$

Now,

(i)

$a_1 = a = 5$

$d = 3$

$a_n = 50 = a + (n-1)d$

$\Rightarrow 50 = 5 + (n-1)3$

$\Rightarrow n = 16$

$S_n = \frac{n}{2}(a_1 + a_n)$

$\Rightarrow S_n = \frac{16}{2}(5 + 50)$

$\Rightarrow S_n = 8 \times 55 = 440$

(ii)

$a_1 = 7$

$a_{13} = 35$

$\Rightarrow n = 13$

$a_{13} = a_1 + (n-1)d$

$\Rightarrow 35 = 7 + (13-1)d$

$\Rightarrow d = \frac{7}{3}$

$S_{13} = \frac{13}{2}(a_1 + a_{13})$

$\Rightarrow S_{13} = \frac{13}{2}(7 + 35)$

$\Rightarrow S_{13} = 13 \times 21 = 273$

(iii)

$a_{12} = 37$

$d = 3$

$\Rightarrow n = 12$

$a_{12} = a_1 + (n-1)d = a + (12-1)3$

$\Rightarrow 37 = a + 33$

$\Rightarrow a = 4$

$S_{12} = \dfrac{n}{2}(a_1 + a_{12})$

$\Rightarrow s_{12} = \dfrac{12}{2}(4 + 37) = 6 \times 41 = 256$

(iv)

$a_3 = 15$

$\Rightarrow a_3 = a + (3-1)d \Rightarrow 15 = a + 2d \quad ...(1)$

$S_{10} = 125$

$\dfrac{10}{2}(2a + 9d) = 125$

$\Rightarrow 2a + 9d = 25 \quad ....(2)$

From (1) and (2), we get

$d = -1, a = 17$

$a_{10} = 17 + (9)(-1) = 8$

(v)

$d = 5$

$S_9 = 75 = \dfrac{9}{2}(2a + 8d) = \dfrac{9}{2}(2a + 40) = 9(a + 20)$

$\Rightarrow a = -\dfrac{35}{3}$

$a_9 = a + 8d = -\dfrac{35}{3} + 8 \times 5 = 40 - \dfrac{35}{3}$

$a_9 = \dfrac{85}{3}$

(vi)

$a = 2$

$d = 8$

$S_n = 90$

$\Rightarrow \dfrac{n}{2}(2a + (n-1)d) = 90$

$\Rightarrow \dfrac{n}{2}(8n - 4) = 90$

$\Rightarrow n(4n - 2) = 90$

$\Rightarrow 2n^2 - n - 45 = 0$

$\Rightarrow n = 5 \quad (\because n > 0, \therefore$ we have neglected negative value of $n = -\dfrac{9}{2})$

$a_n = a_5 = a + (4)(d) = 2 + 4 \times 8 = 34$

(vii)

$a = 8$

$a_n = 62$

$S_n = 210 = \dfrac{n}{2}(a + a_n)$

$\Rightarrow 420 = n(8 + 62) = 70n$

$\Rightarrow n = 6$

$a_n = a + (n-1)d$

$62 = 8 + (5)d$

$\Rightarrow d = \dfrac{54}{5}$

(viii)

$a_n = 4$

$d = 2$

$a_n = a + (n-1)d$

$4 = a + (n-1)2$

$\Rightarrow = 6 = a + 2n$ ...(1)

$S_n = -14$

$S_n = \dfrac{n}{2}(a + a_n)$

$-14 = \dfrac{n}{2}(a + 4)$ ....(2)

From (1) and (2), we get

$n = 7$ or $n = -2,$ $(\because n > 0,$ we have to neglect negative value)

$n = 7,$ then $a = -8$

(ix)

$a = 3$

$n = 8$

$S = 192$

$S = 192 = \dfrac{n}{2}(2a + (n-1)d)$

$\Rightarrow 48 = 6 + 7d$

$\Rightarrow d = 6$

(x)

$l = a_n = 28$

$S_n = 144$

$n = 9$

$S_n = \dfrac{n}{2}(a + a_n)$

$144 = \dfrac{9}{2}(a + 28)$

$\Rightarrow a = 4$

#466108

Topic: Arithmetic Progression

How many terms of the AP : $9, 17, 25, \ldots$ must be taken to give a sum of 636 ?

Solution

Let there be n terms of this A.P.

For this A.P., $a = 9$

$d = 17 - 9 = 8$

As $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 636 = \frac{n}{2}[9 \times 2 + (n-1)8]$

$636 = n[9 + 4n - 4]$

$636 = n(4n + 5)$

$4n^2 + 5n - 636 = 0$

$4n^2 + 53n - 48n - 636 = 0$

$n(4n + 53) - 12(4n + 53) = 0$

$(4n + 53)(n - 12) = 0$

Either $4n + 53 = 0$ or $n - 12 = 0$

$n = \frac{53}{4}$ or $n = 12$

n cannot be $\frac{53}{4}$.(As the number of terms can neither be negative nor fractional.)

Therefore, $n = 12$ only.

#466109

Topic: Arithmetic Progression

The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Solution

According to the question,

$a = 5$, $a_n = 45$, $S_n = 400$

$S_n = \frac{n}{2}(a + a_n)$

Subsituting the values,

$400 = \frac{n}{2}[5 + 45]$

$400 = \frac{n}{2}[50]$

$n = 16$

$a_n = a + (n-1)d$

$45 = 5 + (16 - 1)d$

$40 = 15d$

$d = \frac{40}{15}$

$d = \frac{8}{3}$

#466110

Topic: Arithmetic Progression

The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution

According to the question, $a = 17$, $a_n = 350$, $d = 9$

To find n,

$a_n = a + (n - 1)d$

Subsituting the values

$350 = 17 + (n - 1)9$

$333 = (n - 1)9$

$(n - 1) = 37$

$n = 38$

$S_n = \dfrac{n}{2}(a + a_n)$

$S_{38} = \dfrac{13}{2}(17 + 350)$

$= 19 \times 367$

$= 6973$

Number of terms $= 38$

Sum of the terms $= 6973$.

#466111
Topic: Arithmetic Progression

Find the sum of first 22 terms of an AP in which $d = 7$ and 22^{nd} term is 149.

Solution

$d = 7$, $a_{22} = 149$

We want to find S_{22}.

$a_n = a + (n - 1)d$

$a_{22} = a + (22 - 1)7$

$149 = a + (21)(7)$

$149 = a + 147$

$a = 2$

$S_n = \dfrac{n}{2}(a + a_n)$

$S_{22} = \dfrac{22}{2}(2 + 149)$

$= 11 \times 151$

$= 1661$

#466112
Topic: Arithmetic Progression

Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Solution

$a_2 = 14$, $a_3 = 18$

$d = 18 - 14 = 4$

$a_2 = a + d$

$14 = a + 4$

$\Rightarrow a = 10$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{51} = \frac{51}{2}[2 \times 10 + (51 - 1)4] = \frac{51}{2}(20 + 200)$

$S_{51} = 51 \times 110 = 5610$

#466113

Topic: Arithmetic Progression

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Solution

Given

$S_7 = 49$ and $S_{17} = 289$

$S_7 = \frac{7}{2}[2a + (7-1)d] = 49$

$\Rightarrow 49 = \frac{7}{2}[2a + (7-1)d]$

$\Rightarrow 49 = \frac{7}{2}(2a + 6d)$

$\Rightarrow 7 = a + 3d$

$\Rightarrow a + 3d = 7$...................(i)

$S_{17} = \frac{17}{2}[2a + (17-1)d] = 289$

$\Rightarrow 289 = \frac{17}{2}[2a + (17-1)d]$

$\Rightarrow 289 = \frac{17}{2}(2a + 16d)$

$\Rightarrow 17 = a + 8d$

$\Rightarrow a + 8d = 17$......................(ii)

Substituting (i) from (ii), we get

$5d = 10$ or $d = 2$

From equation (i),

$a + 3(2) = 7$

$a + 6 = 7$ or $a = 1$

$S_n = \frac{n}{2}[2(1) + (n-1)2]$

$= \frac{n}{2}[2 + (n-1)2]$

$= \frac{n}{2}(2 + 2n - 2) = n^2$

#466114

Topic: Arithmetic Progression

Show that $a_1, a_2, \ldots, a_n$ form an AP where a_n is defined as below :

(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Solution

(i) $a_n = 3 + 4n$

$a_1 = 3 + 4(1) = 7$

$a_2 = 3 + 4(2) = 11$

$a_2 = 3 + 4(3) = 15$

$d = a_3 - a_1 = 11 - 7 = 4$

Here $a = 7$, $d = 4$ and $n = 15$

$S_{15} = \frac{15}{2}[2 \times 7 + (15 - 1)4]$

$= \frac{15}{2}(14 + 56)$

$= \frac{15}{2} \times 70 = 525$

(ii) $a_n = 9 - 5n$

$a_1 = 9 - 5(1) = 9 - 5 = 4$

$a_2 = 9 - 5(2) = 9 - 10 = -1$

$a_3 = 9 - 5(3) = 9 - 15 = -6$

$d = a_2 - a_1 = -6 - (-1) = -5$

Here $a = 4$, $d = -5$ and $n = 15$

$S_{15} = \frac{15}{2}[2 \times 4 + (15 - 1)(-5)]$

$= \frac{15}{2}(8 - 70)$

$= \frac{15}{2} \times (-62) = -465$

#466115

Topic: Arithmetic Progression

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

Solution

If Sum of first $$n$ terms of an AP

$S_n = 4n - n^2$

Then, first term will be when $n = 1$ in above expression

$S_1 = a = 4(1) - (1)^2 = 3$

Sum of first two terms,

$n = 2, S_2 = 4(2) - 2^2 = 4$

So, second term will be

$a_2 = S_2 - S_1 = 4 - 3 = 1$

Similarly

$n = 3, S_3 = 4(3) - 3^2 = 3$

$a_3 = S_3 - S_2 = 3 - 4 = -1$

$n = 9, S_9 = 4(9) - 9^2 = -45$

$n = 10, s_{10} = 4(10) - (10)^2 = -60$

$a_{10} = S_{10} - S_9 = -60 - (-45) = -15$

$a_n = S_n - S_{(}n - 1)$

$a_n = 4n - n^2 - (4(n-1) - (n-1)^2)$

$\Rightarrow a_n = 5 - 2n$

#466116

Topic: Arithmetic Progression

Find the sum of the first 40 positive integers divisible by 6.

Solution

The first 40 positive integers that are divisible by 6 are $6, 12, 18, 24...$

$a = 6$ and $d = 6$.

We need to find S_{40}

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{40} = \frac{40}{2}[2(6) + (40 - 1)6]$

$= 20[12 + (39)6]$

$= 20(12 + 234)$

$= 20 \times 246$

$= 4920$

#466117

Topic: Arithmetic Progression

Find the sum of the first 15 multiples of 8.

Solution

The multiples of 8 are 8, 16, 24, 32...

These are in an A.P., having first term as 8 and common difference as 8.

Therefore, $a = 8$, $d = 8$, $S_{15} = ?$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}[2(8) + (15-1)8]$$

$$= \frac{15}{2}[6 + (14)(8)]$$

$$= \frac{15 \times 128}{2}$$

$= 960$

#466118

Topic: Arithmetic Progression

Find the sum of the odd numbers between 0 and 50.

Solution

The odd numbers between 0 and 50 are 1, 3, 5, 7, 9...49

Therefore, it can be observed that these odd numbers are in an A.P.

$a = 1$, $d = 2$, $l = 49$

$l = a + (n-1)d$

$49 = 1 + (n-1)2$

$48 = 2(n-1)$

$n - 1 = 24$

$n = 25$

$$S_n = \frac{n}{2}(a + l)$$

$$S_{25} = \frac{25}{2}(1 + 49)$$

$$= \frac{25 \times 50}{2} = 625$$

#466119

Topic: Arithmetic Progression

A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

Solution

It can be observed that these penalties are in an A.P. having first term as 200 and common difference as 50.

$a = 200$, $d = 50$

Penalty that has to be paid if he has delayed the work by 30 days $= S_{30}$

$$= S_{30} = \frac{30}{2}[2(200) + (30-1)50]$$

$= 15[400 + 1450]$

$= 15(1850)$

$= 27750$

Therefore, the contractor has to pay Rs 27750 as penalty.

#466120

Topic: Arithmetic Progression

A sum of Rs. 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs. 20 less than its preceding prize, find the value of each of the prizes.

Solution

Let the cost of 1^{st} prize be P.

Cost of 2^{nd} prize $= P - 20$

And cost of 3^{rd} prize $P - 40$

It can be observed that the cost of these prizes are in an A.P. having common difference as -20 and first term as P.

$a = P, d = -20$

Given that,

$S_7 = 700$

$\dfrac{7}{2}[2a + (7-1)(-20)]$

$\dfrac{2a + 6(-20)}{2} = 100$

$a + 3(-20) = 100$

$a - 60 = 100$

$a = 160$

Therefore, the value of each of the prizes was Rs $160,$ Rs $140,$ Rs $120,$ Rs $100,$ Rs $80,$ Rs 60 and Rs 40.

#466121

Topic: Arithmetic Progression

In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Solution

It can be observed that the number of trees planted by the students is in an AP.

1, 2, 3, 4, 5.................. .12

First term, $a = 1$

Common difference, $d = 2 - 1 = 1$

$S_n = \dfrac{n}{2}[2a + (n-1)d]$

$S_{12} = \dfrac{12}{2}[2(1) + (12-1)(1)]$

$= 6(2 + 11)$

$= 6(13)$

$= 78$

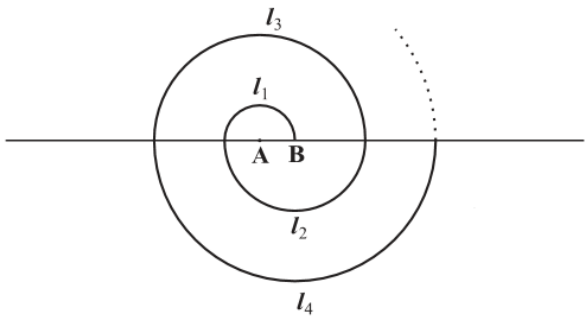
Therefore, number of trees planted by 1 section of the classes $= 78$

Number of trees planted by 3 sections of the classes $= 3 \times 78 = 234$

Therefore, 234 trees will be planted by the students.

#466122

Topic: Arithmetic Progression



A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, . . . as shown in Fig. What is the total length of such a spiral made up of thirteen consecutive semicircles?

Solution

Circumference of first semicircle $= \pi r = 0.5\pi$

Circumference of second semicircle $= \pi r = \pi$

Circumference of third semicircle $= \pi r = 1.5\pi$

It is clear that $a = 0.5\pi$, $d = 0.5\pi$ and $n = 13$

Hence, length of spiral can be calculated as follows:

$S = \frac{n}{2}[2a + (n-1)d]$

$= \frac{13}{2}(2 \times 0.5\pi + 12 \times 0.5\pi)$

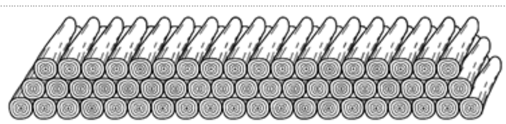
$= \frac{13}{2} \times 7\pi$

$= \frac{13}{2} \times 7 \times \frac{22}{7}$

$= 143$ cm

#466123

Topic: Arithmetic Progression



200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Fig.). In how many rows are the 200 logs placed and how many logs are in the top row.

Solution

It can be observed that the numbers of logs in rows are in an A.P.

20, 19, 18...

For this A.P.,

$a = 20,\ d = 19 - 20 = -1$

Let a total of 200 logs be placed in n rows.

$S_n = 200$

$S_n = \frac{n}{2}(a + (n - 1)d$

$200 = \frac{n}{2}[2(20) + (n - 1)(-1)]$

$400 = n(40 - n + 1)$

$400 = n(41 - n)$

$400 = 41n - n^2$

$n^2 - 16n - 25n + 400$

$n(n - 16) - 25(n - 16) = 0$

$(n - 16)(n - 25) = 0$

Either $(n - 16) = 0$ or $n - 25 = 0$

$n = 16$ or $n = 25$

$a_n = a + (n - 1)d$

$a_{16} = 20 + (16 - 1)(-1)$

$a_{16}\ 20 - 15$

$a_{16} = 5$

Similarly,

$a_{25} = 20 + (25 - 1)(-1)$

$a_{25} = 20 - 24$

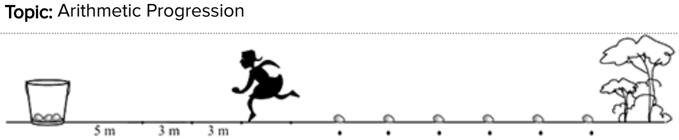
$= -4$

Clearly, the number of logs in 16^{th} row is 5. However, the number of logs in 25^{th} row is negative, which is not possible.

Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16^th row is 5.

#466124

Topic: Arithmetic Progression



In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see Fig.).

A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[Hint : To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$]

Solution

The distances of potatoes from the bucket are $5, 8, 11, 14...$

Distance run by the competitor for collecting these potatoes are two times of the distance at which the potatoes have been kept. Therefore, distances to be run are

10, 16, 22, 28, 34, ...

$a = 10$, $d = 16 - 10 = 16$

To find: S_{10}

$S_{10} = \frac{10}{2}[2(10) + (10 - 1)6]$

$= 5[20 + 54]$

$= 5(74)$

$= 370$

Therefore, the competitor will run a total distance of 370 m.

#466125
Topic: Arithmetic Progression

Which term of the AP : $121, 117, 113, \ldots,$ is its first negative term?

[Hint : Find n for $a_n < 0$]

Solution

Given A.P is $121, 117, 113..............$

$a = 121$ and $d = 117 - 121 = -4$

$a_n = a + (n - 1)d$

$= 121 + (n - 1)(-4)$

$= 121 - 4n + 4$

$= 125 - 4n$

We have find the first term of this A.P.

Therefore $a_n < 0$

$125 - 4n < 0$

$124 < 4n$

$n > \frac{125}{4}$

$n > 31.25$

$n = 32$

#466126
Topic: Arithmetic Progression

The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

Solution

We know that,

$a_n = a + (n - 1)d$

$a_3 = a + (3 - 1)d$

$a_3 = a + 2d$

Similarly, $a_7 = a + 6d$

Given that, $a_3 + a_7 = 6$

$(a + 2d) + (a + 6d) = 6$

$2a + 8d = 6$

$a + 4d = 3$

$a = 3 - 4d$ ....(i)

Also, it is given that $a_3 \times a_7 = 8$

$(a + 2d) \times (a + 6d) = 8$

From equation (i),

$(3 - 4d + 2d)(3 - 4d + 6d) = 8$

$\Rightarrow (3 - 2d)(3 + 2d) = 8$

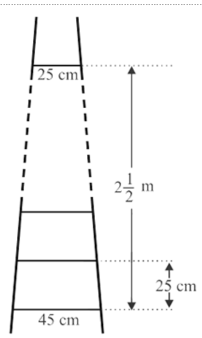
$\Rightarrow 9 - 4d^2 = 8$

$\Rightarrow 4d^2 = 1$

$\Rightarrow d = \pm\frac{1}{2}$

#466127

Topic: Arithmetic Progression



A ladder has rungs 25 cm apart. (see Fig.). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}m$ apart, what is the length of the wood required for the rungs?

Solution

It is given that the rungs are 25 cm apart and the top and bottom rungs are $2\frac{1}{2}$ m apart.

$\therefore$ total number of rungs.

$\frac{2\frac{1}{2} \times 100}{25} + 1$

$= \frac{250}{25} + 1 = 11$

Now, as the lengths of the rungs decrease uniformly, they will be in an A.P.

The length of the wood required for the rungs equals the sum of all the terms of this A.P.

First term, $a = 45$

Last term, $l = 25$

$n = 11$

$S_n = \frac{n}{2}(a + l)$

$S_{10} = \frac{11}{2}(45 + 25) = \frac{11}{2} \times 70 = 385$ cm

Therefore, length of wood is 385cm

#466128

Topic: Arithmetic Progression

The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.

Solution

Here $a = 1$, $d = 1$ and $a_{49} = 49$

As per question,

$S_{x-1} = S_{49} - S_x$............(i)$$

$S_x = \frac{x}{2}[2 + (x - 1) \times 1]$

$= \frac{x}{2}(x + 1) = \frac{x^2 + x}{2}$

Similarly,

$S_{x-1} = \frac{x-1}{2}[2 + (x - 1 - 1) \times 1]$

$= \frac{x-1}{2}(2 + x - 2) = \frac{(x-1)x}{x}\frac{x^2 - x}{2}$

Similarly,

$S_{49} = \frac{49}{2}[2 + 48 \times 1] = \frac{49}{2} \times 50 = 1225$

After substituting the values of S_{x-1}, S_{49} and S_x in equation (1), we get

$S_{x-1} == S_{49} - S_x$

$\frac{x^2 - x}{2} = 1225 - \frac{x^2 + x}{2}$

$\frac{x^2 - x}{2} + \frac{x^2 + x}{2} = 1225$

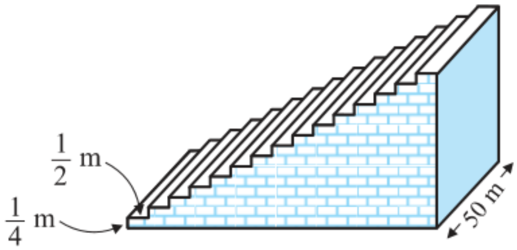
$\frac{x^2 - x + x^2 - x}{2} = 1225$

$x^2 = 1225$

$x = 35$

#466129

Topic: Arithmetic Progression



A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.

Each step has a rise of $\frac{1}{4}$m and a tread of $\frac{1}{2}$m. (see Fig.). Calculate the total volume of concrete required to build the terrace.

Solution

Dimensions in 1^{st} step $= 50 \times 0.25 \times 05$ m

Volume of first step $= 6.25$ cubic m

Dimensions of second step $= 50 \times 0.5 \times 05$m

Volume of 2^{nd} step $= 12.5$ cubic m

Dimensions of 3^{rd} step $= 50 \times 0.75 \times 05$ m

Volume of 3^{rd} step $= 18.75$ cubic m

Now, we have $a = 6.25$, $d = 6.25$ and $n = 15$

Sum of 15 terms can be calculated as follows:

$$S_{15} = \frac{15}{2}[2 \times 6.25 + 14 \times 6.25]$$
$$= \frac{15}{2} \times 100 = 750$$

Volume of concrete $= 750$ cubic m