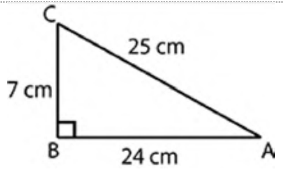
#466273



In ΔABC, right-angled at B, $AB = 24$ cm, $BC = 7$ cm. Determine:

(i) $\sin A$, $\cos A$

(ii) $\sin C$, $\cos C$

Solution

$\angle B = 90^o$

$AC^2 = AB^2 + BC^2$ ...(Pythagoras theorem)

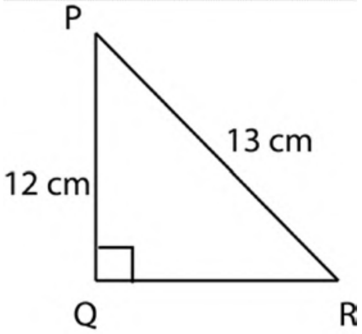
$AC^2 = (24)^2 + (7)^2$

$AC = 25$

(i) $\sin A = \frac{BC}{AC} = \frac{7}{25}$, $\cos A = \frac{AB}{AC} = \frac{24}{25}$

(ii) $\sin C = \frac{AB}{AC} = \frac{24}{25}$, $\cos C = \frac{BC}{AC} = \frac{7}{25}$

#466275



In Fig., find $\tan P - \cot R$

Solution

In $\triangle PQR$

$PR^2 = PQ^2 + QR^2$ ....(Pythagoras theorem)

$13^2 = 12^2 + QR^2$

$QR^2 = \sqrt{169 - 144} = \sqrt{25} = 5$cm

$\tan P = \frac{QR}{PQ} = \frac{5}{12}$, $\cot R = \frac{QR}{PQ} = \frac{5}{12}$

$\therefore \tan P - \cot R = \frac{5}{12} - \frac{5}{12} = 0$

#466278

If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.

Solution

Let ABC is right any $\triangle$, right angled at B.

$\sin A = \frac{3}{4} = \frac{BC}{AC}$.

Let $BC = 3x$ and $AC = 4x$ ...(x +ve direction)

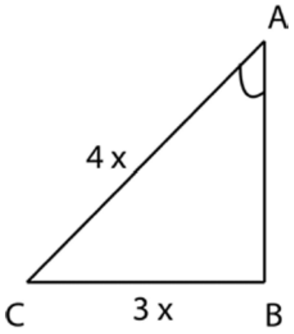
$AC^2 = AB^2 + BC^2$

$\Rightarrow AB^2 = 4x^2 - 3x^2 = 7x^2$

$\Rightarrow AB = \sqrt{7}x$

$\Rightarrow \cot B = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$

$\Rightarrow \tan A = \frac{BC}{AB} = \frac{3}{\sqrt{7}}$



#466279

Given $15\cot A = 8$, find $\sin A$ and $\sec A$.

Solution

Let ABC be a right angled $\triangle$ right angled at B.

$\cot A = \frac{8}{15}$

Let $AB = 8x$ and BC will be $15x$.

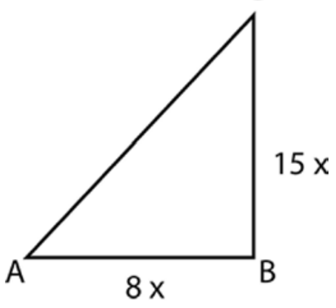
$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 64x^2 + 225x^2$

$\Rightarrow AC = \sqrt{25x^2} = 17x$

$\Rightarrow \sin A = \frac{BC}{AC} = \frac{15x}{17x} = \frac{15}{17}$

$\Rightarrow \sec A = \frac{AC}{AB} = \frac{17x}{8x} = \frac{17}{8}$



#466285

Given $\sec\theta = \frac{13}{12}$, calculate all other trignometric ratios.

Solution

Let $\triangle ABC$ be right angled triangle (right angled at B)

$\sec Q = \frac{AC}{AB} = \frac{13}{12}$

Let $AC = 13x$ and $AB = 12x$

$AC^2 = AB^2 + BC^2$

$BC^2 = AC^2 - AB^2$

$x^2 = 169 - 144$

$x^2 = 25$

$x = \sqrt{25} = 5$

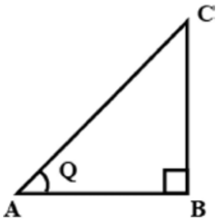
$\sin Q = \frac{BC}{AC} = \frac{5x}{13x} = \frac{5}{13}$

$\cos Q = \frac{AB}{AC} = \frac{12}{13}$

$\tan Q = \frac{BC}{AB} = \frac{5x}{12x} = \frac{5}{12}$

$\cot Q = \frac{1}{\tan Q} = \frac{1}{\frac{5}{12}} = \frac{12}{5}$

$\text{cosec } Q = \frac{AC}{BC} = \frac{13x}{5x} = \frac{13}{5}$



#466287

If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Solution

Let $\triangle ABC$ be right angled at C.

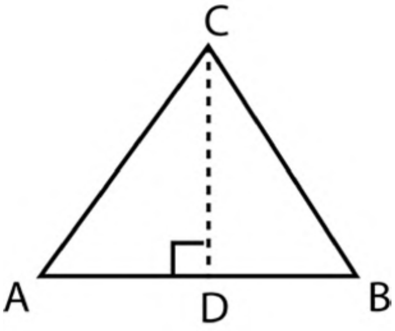
$\cos A = \cos B$ ...given

$\therefore \frac{AC}{AB} = \frac{BC}{AB}$

$\Rightarrow AC = BC$

$\therefore \triangle ABC$ is an isosceles right angled triangle.

$\Rightarrow \angle B = \angle A$



#466289

If $\cot A = \frac{7}{8}$, evaluate :

(i) $\frac{(1 + \sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(1 - \cos\theta)}$ (ii) $\cot^2\theta$

Solution

In $\triangle ABC$,

$\angle B = 90^o$ and $\angle C = Q$

$\cot Q = \frac{BC}{AB} = \frac{7}{8}$

Let $BC = 7x$, $AB = 8x$

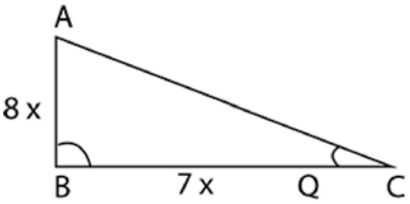
$AC^2 = AB^2 + BC^2$

$AC^2 - 5x^2 + 7x^2 = 113x^2$,

$AC = \sqrt{113}x$

(i) $\frac{(1 + \sin Q)(1 - \sin Q)}{(1 + \cos Q)(1 - \cos Q)} = \frac{1 - \sin^2 Q}{1 - \cos^2 Q} = \frac{1 - \left(\frac{8}{\sqrt{113}}\right)^2}{1 - \left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1 - \frac{64}{113}}{1 - \frac{49}{113}} = \frac{49}{64}$

(ii) $\cot^2 Q = \left(\frac{7}{8}\right) = \frac{49}{69}$



#466290

If $3\cot A = 4$, check whether $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Solution

$\angle B = 90^o$

$\cot A = \frac{A}{B} = \frac{4}{3}$

Let $AB = 4x$, $BC = 3x$.

$AC^2 = AB + BC^2$

$AC^2 = 16k^2 + 9x^2$

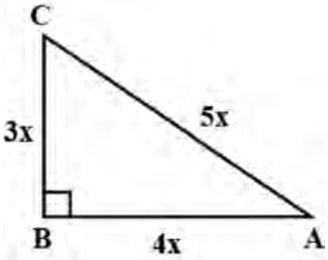
$AC = 5x$

Now, $\tan A = \frac{3}{4}$

LHS : $\frac{1 - \tan^2 A}{1 + \tan^2 A} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{7}{25}$

RHS : $\cos 2A - \sin 2A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

$LHS = RHS$



#466294

In triangle ABC, right-angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of:

(i) $\sin A \cos C + \cos A \sin C$

(ii) $\cos A \cos C - \sin A \sin C$

Solution

In $\triangle ABC$,

$\angle B = 90^o$, $\tan A = \frac{1}{\sqrt{3}} = \frac{BC}{AB}$

Let $BC = 1x$, $AB = \sqrt{3}x$

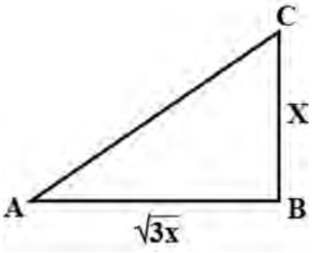
$AC^2 = AB^2 + BC^2$

$AC^2 = (\sqrt{3}x)^2 + (x)^2 = 4x^2$

$AC = 2x$

(i) $\sin A \cos C + \cos A \sin C = \frac{1}{2} \times \frac{1}{2} + \frac{B}{2} \times \frac{B}{2} = \frac{1}{4} + \frac{3}{4} = 1$

(ii) $\cos A \cos C - \sin A \sin C = \frac{13}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{13}{2} = \frac{13}{4} - \frac{13}{4} = 0$



#466297

In ΔPQR, right-angled at Q, $PR + QR = 25$cm and $PQ = 5$cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

Solution

Given, $PR + QR = 25$, $PQ = 5$

Let $RP = x$

$\therefore QR = 25 - x$

$PR^2 = PD^2 + QR^2$ ...(Pythagoras theorem)

$x^2 = (5)^2 + (25 - x)^2$

$x^2 = 25 + 625 + x^2 - 50x$

$50x = 650$

$x = 13$

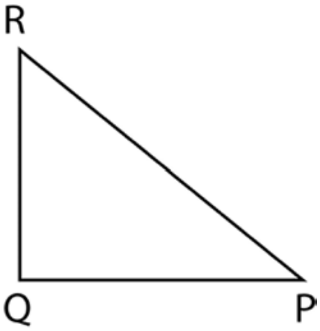
$\therefore PR = 13$

$QR = 25 - 13 = 12$

$\sin P = \frac{QR}{PR} = \frac{12}{13}$

$\cos P = \frac{PQ}{PR} = \frac{5}{13}$

$\tan P = \frac{QR}{PQ} = \frac{12}{5}$



#466299

State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A.

(iv) $\cot A$ is the product of $\cot$ and A.

(v) $\sin\theta = \frac{4}{3}$ for some angle θ.

Solution

(i) False

$\angle B = 90^o, AB = 3, BC = 4, AC = 5$

$\tan A = \frac{4}{3} > d. \quad ...(AC^2 = AB^2 + BC^2)$

(ii) True

$AC^2 = AB^2 + BC^2$

$(12x)^2 = (5x)^2 + BC^2,$

$BC = \sqrt{119x}$

(it is possible) Pythagoras theorem

(iii) False

$\cos A$ is cosine A, $\csc A$ is cosecant

(iv) False

$\cot A$ = cotangent of $\angle A$ not product of $\cot$ and A.

(v) False

$\sin Q = \frac{\text{perpendicular}}{\text{hypotenuse}}$

$\sin Q > 1; \frac{4}{3} > 1$ [$\because$ hypotenuse $>$ base $>$ perpendicular]

$\sin Q$ will always less than 1.

#466302

Evaluate the following:

(i) $\sin 60^\circ \cos 30^\circ + \sin 360^\circ \cos 60^\circ$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + cosec 30^\circ}$

(iv) $\frac{\sin 30^\circ + \tan 45^\circ - cosec 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Solution

(i) $\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{3}{4} + \frac{1}{4} = 1$

(ii) $2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 2 + \frac{3}{4} - \frac{3}{4} = 2$

(iii) $\frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\sqrt{3}}{2\sqrt{2} + 2\sqrt{6}} = \frac{2\sqrt{3}(\sqrt{6} - \sqrt{2})}{(2\sqrt{6})^2 - (2\sqrt{2})^2} = \frac{\sqrt{18} - \sqrt{2}}{8} = \frac{3\sqrt{2} - \sqrt{6}}{8}$

(iv) $\frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} = \frac{(3\sqrt{3} - 4)^2}{(3\sqrt{3})^2 - (4)^2} = \frac{43 - 24\sqrt{3}}{11}$

(v) $\frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{5\left(\frac{1}{4}\right) + \left(\frac{16}{3}\right) - 1}{\frac{1}{4} + \frac{3}{4}} = \frac{\frac{15 + 64 - 12}{12}}{\frac{4}{4}} = \frac{67}{12}$

#466306

Choose the correct option and justify your choice:

$\frac{2\tan 30^\circ}{1 + \tan^2 30^\circ} =$

[✓ A] $\sin 60^\circ$

B $\cos 60^\circ$

C $\tan 60^\circ$

D $\sin 30^\circ$

Solution

$\Rightarrow \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2} = \sin 60^o$

#466308

Choose the correct option and justify your choice:

$\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$

A $\tan 90^\circ$

B 1

C $\sin 45^\circ$

[✓ D] 0

Solution

The value of $\frac{1 - \tan^2 45^o}{1 + \tan^2 45^o}$

$= \frac{1 - (1)^2}{1 - (1)^2}$

$= \frac{1 - 1}{1 + 1} = \frac{0}{2} = 0$

#466311

Choose the correct option and justify your choice:

$\sin 2A = 2\sin A$ is true when $A =$

[✓ A] 0°

B 30°

C 45°

D 60°

Solution

$\sin 2A = 2\sin A$ ....given

If $A = 0^o$,

then L.H.S. $= \sin 2A = \sin 0^o = 0$

and R.H.S $= 2\sin A = 2\sin 0^o = 0$

So, L.H.S = R.H.S

If $A = 30^o$,

then L.H.S. $= \sin 2A = \sin 60^o = \dfrac{\sqrt{3}}{2}$

and R.H.S $= 2\sin A = 2\sin 30^o = 1$

So, L.H.S $\neq$ R.H.S

If $A = 45^o$,

then L.H.S. $= \sin 2A = \sin 90^o = 1$

and R.H.S $= 2\sin A = 2\sin 45^o = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

So, L.H.S $\neq$ R.H.S

If $A = 60^o$,

then L.H.S. $= \sin 2A = \sin 120^o = \dfrac{\sqrt{3}}{2}$

and R.H.S $= 2\sin A = 2\sin 60^o = \dfrac{2 \times \sqrt{3}}{2} = \sqrt{3}$

So, L.H.S $\neq$ R.H.S

#466314

Choose the correct option and justify your choice :

$\dfrac{2\tan 30^\circ}{1 - \tan^2 30^\circ} =$

[✓ A] $\sqrt{3}$

B $\dfrac{1}{\sqrt{3}}$

C 2

D $\sqrt{2}$

Solution

We know that $\tan 30^0 = \frac{1}{\sqrt{3}}$

The value of $\frac{2\tan 30^o}{1 - \tan^2 30^o}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3}$$

#466317

If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}; 0° < A + B \leq 90°; A > B$, find A and B.

Solution

$\tan(A + B) = \sqrt{3}, \tan(A + B) = \tan 60^o$

$\Rightarrow A + B = 60^o$ .... (i)

$\tan(A + B) = \frac{1}{\sqrt{3}}$

$\tan(A - B) = \tan 30^o$

$\Rightarrow A - B = 30^o$ ....(ii)

From (i) and (ii), we get

$B + 45 = 60^o$

$B = 15^o$

$\angle A = 45^o, \angle B = 15^o$

#466319

State whether the following are true or false. Justify your answer.

(i) $\sin(A + B) = \sin A + \sin B$

(ii) The value of $\sin\theta$ increases as θ increases.

(iii) The value of $\cos\theta$ increases as increases.

(iv) $\sin\theta = \cos\theta$ for all values of θ.

(v) $\cot A$ is not defined for $A = 0°$.

Solution

(i) False

$\sin(A + B) = \sin A + \sin B$

$A = 45^o, B = 45^o$

LHS: $\sin(45^o + 45^o) = \sin 90^o = 1$

RHS : $\sin A + \sin B = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

$LHS \neq RHS$

(ii) True

$\sin 0^o = 0 \sin 30^o = \frac{1}{2} = \sin 45^o = \frac{1}{\sqrt{2}}, \sin 60^o = \frac{\sqrt{3}}{2}$

$\sin 90^o = 1$

$\therefore$ θ increase ans value of $\sin\theta$ also increase.

(iii) False

$\cos 0 = 1, \cos 30 = \frac{13}{2}, \cos 45 = \frac{1}{\sqrt{2}}$

$\cos 60 = \frac{1}{2}, \cos 90 = 0$

Value of $\cos\theta$ does not increase from $0 < 0 < 90^o$

(iv) False

This is true for $\theta = 45^o$

Let $\theta = 90^o$

$\sin 90 = 1, \cos 90 = 0$

(v) True

$\cot A = \frac{\cos A}{\sin A}$

$\cot 0^o = \frac{\cos 0}{\sin 0} = \frac{1}{0} =$ not defined

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#466320

Evaluate:

(i) $\frac{\sin 18^\circ}{\cos 72^\circ}$

(ii) $\frac{\tan 26^\circ}{\cot 64^\circ}$

(iii) $\cos 48^\circ - \sin 42^\circ$

(iv) $cosec 31^\circ - \sec 59^\circ$

Solution

(i) $\frac{\sin(90 - 72)^o}{\cos 72} = \frac{\cos 72^o}{\cos 72^o} = 1$

(ii) $\frac{\tan(90 - 64)^o}{\cot 64^o} = \frac{\cot 64^o}{\cot 64^o} = 1$

(iii) $\cot 8^o - \sin 42^o = \cos(90 - 42)^o - \sin 42^o = \sin 42^o - \sin 42^o = 0$

(iv) $\cos 31^o - \sec 54^o = \cos(90 - 59)^o - \sec 59^o = \sec 59^o - \sec 59^o = 0$

#466322

Show that:

(i) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$

(ii) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

Solution

(i) $\tan(90-42)^o \tan(90-67)^o \tan 42^o \tan 67^o$

$= \cot 42^o \cot 67^o \tan 42^o \tan 67^o$

$= (1)(1) = 1$

(ii) $\cos(96-52)^o \cos(90-38)^o - \sin 38^o \sin 52^o$

$= \sin 52^o \sin 38^o - \sin 38^o \sin 52^o$

$= 0$

#466323

If $\tan 2A = \cot\left(A - 18^\circ\right)$, where $2A$ is an acute angle, find the value of A.

Solution

Given, $\tan 2A = \cot(a - 18^o)$

$\cot(90^o - 2A) = \cot(A - 18^o)$

$90 - 2A = A - 18$

$\therefore A = 36^o$

#466324

If $\tan A = \cot B$, prove that $A + B = 90^\circ$.

Solution

Given, $\tan A = \cot B$

$\tan A = \tan(90 - B)$

$A = 90^o - B,$

$\therefore A + B = 90^o$

#466325

If $\sec 4A = \operatorname{cosec}\left(A - 20^\circ\right)$, where $4A$ ia an acute angle , find the value of A.

Solution

Given, $\sec 4A = \operatorname{cosec}(A - 20)$

$\therefore \operatorname{cosec}(90 - 4A) = \operatorname{cosec}(A - 20)$

$\therefore 90 - 4A = A - 20$

$$A = \frac{110}{5}$$

$\therefore A = 22^o$

#466327

If A, B and C are interior angles of a triangle ABC, then show that

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}.$$

Solution

$\angle A + \angle B + \angle C = 180^o$ ...[Angle sum property of a triangle]

$\angle B + \angle C = 180 - \angle A$

$$\frac{\angle B + \angle C}{2} = 90^o - \frac{\angle A}{2}$$

$$\sin\left(\frac{B + C}{2}\right) = \sin\left(90 - \frac{A}{2}\right)$$

$$= \cos\frac{A}{2}$$

#466328

Express $\sin 67^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45°.

Solution

$\sin 67^o + \cos 75^o$

$= \sin(90 - 25)^o + \cos(90 - 15)^o$

$= \cos 23^o + \sin 15^o$

#466329

Express the trigonometric ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$.

Solution

We know,

$\text{cosec}^2 A = 1 + \cot^2 A$

$$\frac{1}{\text{cosec}^2 A} = \frac{1}{1 + \cot^2 A}$$

$$\sin^2 A = \frac{1}{1 + \cot^2 A}$$

$$\sin A = \pm\frac{1}{\sqrt{1 + \cot^2 A}}$$

$$\therefore \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

We know,

$\sec^2 A = 1 + \tan^2 A$

$$\sec^2 A = 1 + \frac{1}{\cot^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\therefore \sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

We know,

$$\tan A = \frac{1}{\cot A}$$

#466330

Write all the other trigonometric ratios of $\angle A$ in terms of $\sec A$.

Solution

We know,

$\cos A = \dfrac{1}{\sec A}$

We know,

$\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$\sin A = \sqrt{1 - \left(\dfrac{1}{\sec A}\right)^2} = \sqrt{\dfrac{\sec^2 A - 1}{\sec^2 A}}$

$\therefore \sin A = \sqrt{\dfrac{\sec^2 A - 1}{\sec^2 A}}$

We know,

$\text{cosec } A = \dfrac{1}{\sin A}$

$\therefore \text{cosec } A = \dfrac{\sec A}{\sqrt{\sec^2 A - 1}}$

We know,

$\cot A = \dfrac{\cos A}{\sin A} = \dfrac{\cos A}{\sin A} = \dfrac{\dfrac{1}{\sec A}}{\dfrac{\sqrt{\sec^2 A - 1}}{\sec A}}$

$\therefore \cot A = \dfrac{1}{\sqrt{\sec^2 A - 1}}$

We know,

$\tan A = \dfrac{1}{\cot A}$

$\therefore \tan A = \sqrt{\sec^2 A - 1}$

#466332

Evaluate:

(i) $\dfrac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

Solution

(i)

We know,

$\sin 63^o = \sin(90-27)^o = \cos 27^o$

And,

$\cos 17^o = \cos(90-73)^o = \sin 73^o$

$$\therefore \frac{\sin^2 63^o + \sin^2 27^o}{\cos^2 17^o + \cos^2 73^o} = \frac{\cos^2 27^o + \sin^2 27^o}{\cos^2 17^o + \cos^2 73^o}$$

$= \frac{1}{1} = 1 \quad \cdots[\because \sin^{\theta} + \cos^2\theta = 1]$

(ii)

We know,

$\sin 25^o = \sin(90-65)^o = \cos 65^o$

$\cos 25^o = \cos(90-25)^o = \sin 65^o$

$\therefore \sin 25^o \cos 65^o + \cos 25^o \sin 65^o$

$= \cos 65^o \cos 65^o + \sin 25^o \sin 25^o$

$= \cos^2 65^o + \sin^2 65^o$

$= 1 \quad \cdots[\because \sin^{\theta} + \cos^2\theta = 1]$

#466334

Choose the correct option. Justify your choice.

$9\sec^2 A - 9\tan^2 A =$

A 1

[✓ **B**] 9

C 8

D 0

Solution

We know,

$1 + \tan^2 A = \sec^2 A$

$\therefore 9(\sec^2 A - \tan^2 A) = 9 \times 1 = 9$

#466336

Choose the correct option. Justify your choice

$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) =$

A 0

B 1

[✓ **C**] 2

D -1

Solution

The value of $(1+\tan\theta+\sec\theta)(1+\cot\theta-\text{cosec}\ \theta)$

$$=\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$=\frac{(\sin\theta+\cos\theta)^2-(1)^2}{\sin\theta\cos\theta}$$

$$=\frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}=2$$

#466337

Choose the correct option. Justify your choice

$(\sec A+\tan A)(1-\sin A)=$

A $\sec A$

B $\sin A$

C $cosecA$

[✓ D] $\cos A$

Solution

The value of $(\sec A+\tan A)(1-\sin A)$

$$\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$$

$$=\left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$$

$$=\frac{1-\sin^2 A}{\cos A}=\frac{\cos^2 A}{\cos A}=\cos A$$

#466340

Choose the correct option. Justify your choice.

$\dfrac{1+\tan^2 A}{1+\cot^2 A}=$

A $\sec^2 A$

B -1

C $\cot^2 A$

[✓ D] $\tan^2 A$

Solution

The value of $\dfrac{1+\tan^2 A}{1+\cot^2 A}$

$$=\frac{1+\dfrac{\sin^2 A}{\cos^2 A}}{1+\dfrac{\cos^2 A}{\sin^2 A}}$$

$$=\frac{\cos^2 A+\sin^2 A}{\sin^2 A+\cos^2 A}\times\frac{\sin^2 A}{\cos^2 A}$$

$$=\tan^2 A$$

#466355

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

(i) $(\text{cosec}\ \theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\text{cosec}\ \theta$

[Hint: Write the expression in terms of $\sin\theta$ and $\cos\theta$

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

[Hint: Simplify LHS and RHS separately].

(v) $\frac{\cos A - \sin A - 1}{\cos A + \sin A + 1} = \text{cosec}\ A + \cot A$’

Using the identity $\text{cosec}^2 A = 1 + \cot^2 A$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

(viii) $(\sin A + \text{cosec}\ A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

(ix) $(\text{cosec}\ A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

[Hint: Simplify LHS and RHS separately]

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

Solution

(i)

LHS $= (\text{cosec}\ \theta - \cot\theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta}$

$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{1-\cos\theta}{1+\cos\theta}$

$= RHS$

(ii)

LHS : $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)(\cos A)}$

$= \frac{1+\sin^2 A + \cos^2 A + 2\sin A}{(1+\sin A)(\cos A)} = \frac{2+2\sin A}{(1+\sin A)(\cos A)}$

$= \frac{2(1+\sin A)}{(1+\sin A)(\cos A)} = \frac{2}{\cos A} = 2\sec A$

$= RHS$

(iii)

LHS : $\frac{\tan\theta}{1-\cos\theta} + \frac{\cot\theta}{1-\tan\theta}$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta - \cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta - \sin\theta}{\cos\theta}}$$

$$= \frac{1}{\sin\theta - \cos\theta}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$$

$$= \left[\frac{1}{\sin\theta - \cos\theta}\right]\left[\frac{\sin^3\theta - \cos^3\theta}{\sin\theta\cos\theta}\right]$$

$$= \frac{\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta}{\sin\theta\cos\theta} = \frac{1 + \sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$= \sec\theta\,\mathrm{cosec}\,\theta + 1$

= RHS

(iv)

$$\frac{1 + \sec A}{\sec A} = \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}} = \frac{\cos A + 1}{1}$$

$$= \frac{(\cos A + 1)(1 - \cos A)}{1 - \cos A}$$

$$= \frac{\sin^2 A}{1 - \cos A}$$

= RHS

(v) $\frac{\cos A - \sin A - 1}{\cos A + \sin A - 1} = \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} + \frac{1}{\sin A}}$

$$= \frac{\cot A - 1 + \mathrm{cosec}\,A}{\cos A + 1 - \mathrm{cosec}\,A}$$

$$\frac{(\cot A - 1 + \mathrm{cosec}\,A)^2}{(\cot A)^2 - (1 - \mathrm{cosec}\,A)^2}$$

$$= \frac{(2\mathrm{cosec}\,A - 2)(\mathrm{cosec}\,A + \cot A)}{(-1 - 1 + 2\cos A)}$$

$= \mathrm{cosec}\,A + \cot A$

$= RHS$

(vi)

$$\sqrt{\frac{(1 + \sin A)(1 + \sin A)}{(1 - \sin A)(1 + \sin A)}}$$

$$= \frac{1 + \sin A}{\cos A}$$

$= \sec A + \tan A$

$= RHS$

(vii)

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \tan\theta\left[\frac{(1 - 2\sin^2\theta)}{2 - 2\sin^2\theta - 1}\right]$$

$= \tan\theta =$ RHS

(viii)

$(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \text{cosec}^2 A + 2\sin A \cdot \text{cosec } A + \cos^2 A + \sec^2 A + 2\sec \cdot \cos A$

$= 1 + (1 + \cot^2 A + 1 + \tan^2 A) + 2 + 2$

$= 7 + \tan^2 A + \cot^2 A$

$= RHS$

(ix)

LHS : $(\text{cosec } A - \sin A)(\sec A - \cos A)$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right) = \frac{(\cos^2 A)(\sin^2 A)}{\sin A \cos A}$$

$= \sin A \cos A$

RHS : $\dfrac{1}{\tan A + \cot A} = \dfrac{\sin A\cos A}{\sin^2 + \cos^2 A} = \sin A \cos A$

$\therefore LHS = RHS$

(x)

LHS : $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} = \dfrac{1 + \dfrac{\sin^2 A}{\cos^2 A}}{1 + \dfrac{\cos^2 A}{\sin^2 A}} = \dfrac{\dfrac{1}{\cos^2 A}}{\dfrac{1}{\sin^2 A}} = \dfrac{\sin^2 A}{\cos^2 A} = \tan^2 A$

RHS : $\left(\dfrac{1 - \tan A}{1 - \cot A}\right)^2 = \dfrac{1 + \tan^2 - 2\tan A}{1 + \cot^2 A - 2\cot A}$

$$= \frac{\sec^2 A - 2\tan A}{\text{cosec }^2 A - 2\cot A}$$

$$= \frac{\dfrac{1 - 2\sin A\cos A}{\cos^2 A}}{\dfrac{1 - 2\sin A\cos A}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

$\therefore LHS = RHS$