#463573

Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$

(ii) $y^2 + \sqrt{2}$

(iii) $3\sqrt{t} + t\sqrt{2}$

(iv) $y + \frac{2}{y}$

(v) $x^{10} + y^3 + t^{50}$

Solution

(i) In $4x^2 - 3x + 7$. all the indies of x are whole numbers. So, it is a polynomial in one variable x.

(ii) In $y^2 + \sqrt{2}$ the index of y is a whole number. So, it is a polynomial in one variable y.

(iii) In $3\sqrt{t} + t\sqrt{2} = 3t^{\frac{1}{2}} + \sqrt{2}\ t$, here the exponent of first term is $\frac{1}{2}$, which is not whole number therefore is it not a polynomial.

(iv) In $y + \frac{2}{y} = y + 2y^{-1}$, here the exponent of first term is -1, which is not whole number, therefore, is it not a polynomial.

(v) In $x^{10} + y^3 + t^{50}$ is not a polynomial in one variable as three variable x, y and t occur in it.

#463574

Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$ (ii) $2 - x^2 + x^3$ (iii) $\frac{\pi}{2}x^2 + x$ (iv) $\sqrt{2}x - 1$

Solution

Coefficient of x^2 in $ax^2 + bx + c = 0$ is a.

Hence, coefficients of x^2 in

(i) $2 + x^2 + x$ is 1

(ii) $2 - x^2 + x^3$ is -1

(iii) $\frac{\pi}{2}x^2 + x$ is $\frac{\pi}{2}$

(iv) $\sqrt{2}x - 1$ is 0

#463575

Give one example each of a binomial of degree 35 and of a monomial of degree 100.

Solution

Binomial of degree 35 may be taken as $x^{35} + 4x$

And monomial of degree 100 may be taken as $5x^{100}$

Note: This question might have different answers.

#463576

Write the degree of each of the following polynomials:

(i) $5x^3 + 4x^2 + 7x$

(ii) $4 - y^2$

(iii) $5t - \sqrt{7}$

(iv) 3

Solution

(i) The power of term is $5x^3$ and exponent is 3. So degree is 3.

(ii) The power of term is y^2 and exponent is 2. So degree is 2.

(iii) The power of term is $5t$ and exponent is 1. So degree is 1.

(iv) The only term here is 3 which can be written as $3x^0$ and so exponent is 0 or degree is 0.

#463577

Classify the following as linear, quadratic and cubic polynomials:

(i) $x^2 + x$

(ii) $x - x^3$

(iii) $y + y^2 + 4$

(iv) $1 + x$

(v) $3t$

(vi) r^2

(vii) $7x^3$

Solution

(i) The highest degree of $x^2 + x$ is 2, so it is a quadratic polynomials.

(ii) The highest degree of $x - x^3$ is 3, so it is a cubic polynomials.

(iii)The highest degree of $y + y^2 + 4$ is 2, so it is a quadratic polynomials.

(iv) The highest degree of x in $(1 + x)$ is 1, so it is a linear polynomials.

(v)The highest degree of t in $3t$ is 1, so it is a linear polynomials.

(vi)The highest degree of r^2 is 2, so it is a quadratic polynomials.

(vii)The highest degree of x in $7x^3$ is 3, so it is a cubic polynomials.

#463838

Classify the following polynomials as monomials, binomials, and trinomials. Which polynomial do not fit in any of these three categories?

$x + y$, 1000, $x + x^2 + x^3 + x^4$, $7 + y + 5x$, $2y - 3y^2$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$, $4z - 15z^2$, $ab + bc + cd + da$, pqr, $p^2q + pq^2$, $2p + 2q$

Solution

Monomials :

1000, pqr

Binomials :

$x + y$, $2y - 3y^2$, $4z - 15z^2$, $p^2q + pq^2$, $2p + 2q$

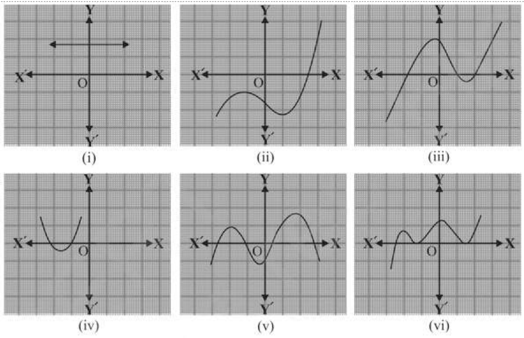
Trinomials :

$7 + y + 5x$, $2y - y^2 + 4y^3$, $5x - 4y + 3xy$

Polynomials that do not fit in any of these categories are :

$x + x^2 + x^3 + x^4$, $ab + bc + cd + da$

#464993



The graphs of $y = p(x)$ are given in the figure, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

Solution

(i) The graph does not intersect the x-axis, so there are no zeros in $p(x)$.

(ii) The graph intersects the x-axis at one place, so here there is one zero in $p(x)$.

(iii) The graph intersects the x-axis at three places, so that there are three zeros in $p(x)$.

(iv) The graph intersects the x-axis at two places, so here there are two zeros in $p(x)$.

(v) The graph intersects the x-axis at four places, so there are four zeros in $p(x)$.

(vi) The graph intersects the x-axis at three places, so that there are three zeros in $p(x)$.

#464994

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$

(ii) $4s^2 - 4s + 1$

(iii) $6x^2 - 3 - 7x$

(iv) $4u^2 + 8u$

(v) $t^2 - 15$

(vi) $3x^2 - x - 4$

Solution

(i) $x^2 - 2x - 8$

Factorize the equation, we get $(x + 2)(x - 4)$

So, the value of $x^2 - 2x - 8$ is zero when $x + 2 = 0, x - 4 = 0,$ i.e., when $x = -2$ or $x = 4$.

Therefore, the zeros of $x^2 - 2x - 8$ are -2 and 4.

Now,

Sum of zeroes $= -2 + 4 = 2 = -\frac{-2}{1} = -\frac{Coefficient\ \ of\ \ x}{Coefficient\ \ of\ \ x^2}$

Product of zeros $= (-2) \times (4) = -8 = \frac{-8}{1} = \frac{Constant\ \ term}{Coefficient\ \ of\ \ x^2}$

(ii) $4s^2 - 4s + 1$

Factorize the equation, we get $(2s - 1)(2s - 1)$

So, the value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0, 2s - 1 = 0,$ i.e., when $s = \frac{1}{2}$ or $s = \frac{1}{2}$.

Therefore, the zeros of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Now,

Sum of zeroes $= \frac{1}{2} + \frac{1}{2} = 1 = -\frac{-4}{4} = -\frac{Coefficient\ \ of\ \ s}{Coefficient\ \ of\ \ s^2}$

Product of zeros = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{1}{4} = \frac{Constant\ \ term}{Coefficient\ \ of\ \ s^2}$

(iii) $6x^2 - 3 - 7x$

Factorize the equation, we get $(3x+1)(2x-3)$

So, the value of $6x^2 - 3 - 7x$ is zero when $3x + 1 = 0, 2x - 3 = 0$, i.e., when $x = -\frac{1}{3}$ or $x = \frac{3}{2}$.

Therefore, the zeros of $6x^2 - 3 - 7x$ are $-\frac{1}{3}$ and $\frac{3}{2}$.

Now,

Sum of zeroes = $-\frac{1}{3} + \frac{3}{2} = \frac{7}{6} = -\frac{-7}{6} = -\frac{Coefficient\ \ of\ \ x}{Coefficient\ \ of\ \ x^2}$

Product of zeros = $-\frac{1}{3} \times \frac{3}{2} = -\frac{1}{2} = \frac{-3}{6} = \frac{-1}{2} = \frac{Constant\ \ term}{Coefficient\ \ of\ \ x^2}$

(iv) $4u^2 + 8u$

Factorize the equation, we get $4u(u+2)$

So, the value of $4u^2 + 8u$ is zero when $4u = 0, u + 2 = 0$, i.e., when $u = 0$ or $u = -2$.

Therefore, the zeros of $4u^2 + 8u$ are 0 and -2.

Now,

Sum of zeroes = $0 - 2 = -2 = -\frac{8}{4} = -2 = -\frac{Coefficient\ \ of\ \ u}{Coefficient\ \ of\ \ u^2}$

Product of zeros = $-0x - 2 = 0 = \frac{0}{4} = 0 = \frac{Constant\ \ term}{Coefficient\ \ of\ \ u^2}$

(v) $t^2 - 15$

Factorize the equation, we get $t = \pm\sqrt{15}$

So, the value of $t^2 - 15$ is zero when $t + \sqrt{15} = 0, t - \sqrt{15} = 0$, i.e., when $t = \sqrt{15}$ or $t = -\sqrt{15}$.

Therefore, the zeros of $t^2 - 15$ are $\pm\sqrt{15}$.

Now,

Sum of zeroes = $\sqrt{15} - \sqrt{15} = 0 = -\frac{0}{1} = 0 = -\frac{Coefficient\ \ of\ \ t}{Coefficient\ \ of\ \ t^2}$

Product of zeros = $\sqrt{15} \times \sqrt{-15} = -15 = \frac{-15}{1} = \frac{Constant\ \ term}{Coefficient\ \ of\ \ t^2}$

(vi) $3x^2 - x - 4$

Factorize the equation, we get$(x+1)(3x-4)$

So, the value of $3x^2 - x - 4$ is zero when x + 1 = 0, 3x - 4 = 0, i.e., when x = -1 or x = $\frac{4}{3}$.

Therefore, the zeros of $3x^2 - x - 4$ are -1 and $\frac{4}{3}$.

Now,

Sum of zeroes = $-1 + \frac{4}{3} = \frac{1}{3} = -\frac{-1}{3} = -\frac{Coefficient\ \ of\ \ x}{Coefficient\ \ of\ \ x^2}$

Product of zeros = $-1 \times \frac{4}{3} = -\frac{4}{3} = \frac{-4}{3} = \frac{Constant\ \ term}{Coefficient\ \ of\ \ x^2}$

#465007

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}, -1$ (ii) $\sqrt{2}, \frac{1}{3}$ (iii) $0, \sqrt{5}$

(iv) $1, 1$ (v) $\frac{-1}{4}, \frac{1}{4}$ (vi) $4, 1$

Solution

(i) $\frac{1}{4}$, -1

Using the quadratic equation formula,

$x^2 - (Sum\ \ of\ \ root)x + (Product\ \ of\ \ root) = 0$

Substitute the value in the formula, we get

$x^2 - \frac{1}{4}x - 1 = 0$

$4x^2 - x - 1 = 0$

(ii) $\sqrt{2}$, $\frac{1}{3}$

Using the quadratic equation formula,

$x^2 - (Sum\ \ of\ \ root)x + (Product\ \ of\ \ root) = 0$

Substitute the value in the formula, we get

$x^2 - \sqrt{2}x + \frac{1}{3} = 0$

Multiply by 3 to remove denominator,

$3x^2 - 3\sqrt{2}x + 1 = 0$

(iii) 0, $\sqrt{5}$

Using the quadratic equation formula,

$x^2 - (Sum\ \ of\ \ root)x + (Product\ \ of\ \ root) = 0$

Substitute the value in the formula, we get

$x^2 - 0x + \sqrt{5} = 0$

$x^2 + \sqrt{5} = 0$

(iv) 1, 1

Using the quadratic equation formula,

$x^2 - (Sum\ \ of\ \ root)x + (Product\ \ of\ \ root) = 0$

Substitute the value in the formula, we get

$x^2 - 1x + 1 = 0$

$x^2 - x + 1 = 0$

(v) $\frac{-1}{4}$, $\frac{1}{4}$

Using the quadratic equation formula,

$x^2 - (Sum\ \ of\ \ root)x + (Product\ \ of\ \ root) = 0$

Substitute the value in the formula, we get

$x^2 - \frac{-1}{4}x + \frac{1}{4} = 0$

Multiply by 4

$4x^2 + x + 1 = 0$

(vi) 4, 1

Using the quadratic equation formula,

$x^2 - (Sum\ \ of\ \ root)x + (Product\ \ of\ \ root) = 0$

Substitute the value in the formula, we get

$x^2 - 4x + 1 = 0$

#465008

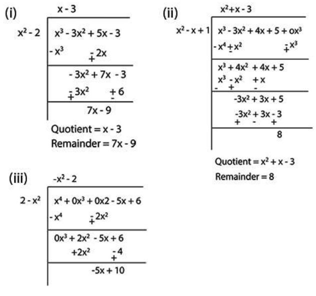
Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3,\ g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5,\ g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6,\ g(x) = 2 - x^2$

Solution



Quotient = x - 3
Remainder = 7x - 9

Quotient = x^2 + x - 3
Remainder = 8

Quotient = $-x^2$ - 2
Remainder = -5x + 10

#465009

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3,\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1,\ 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1,\ x^5 - 4x^3 + x^2 + 3x + 1$

Solution

(i) $t^2 - 3,\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$

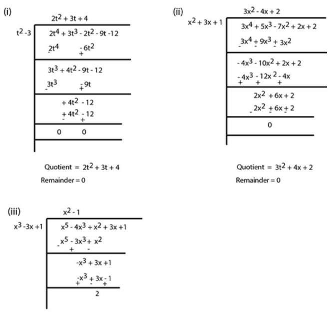
Remainder is 0, hence $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1,\ 3x^4 + 5x^3 - 7x^2 + 2x + 2$

Remainder is 0, hence $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1,\ x^5 - 4x^3 + x^2 + 3x + 1$

Remainder is 2, hence $x^3 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$



Quotient = $2t^2 + 3t + 4$
Remainder = 0

Quotient = $3t^2 + 4x + 2$
Remainder = 0

Quotient = $x^2 - 1$
Remainder = 2

#465010

Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Solution

Two zeroes are $\sqrt{\frac{1}{5}}$ and $-\sqrt{\frac{1}{5}}$

So we can write it as, x = $\sqrt{\frac{1}{5}}$ and x = $-\sqrt{\frac{1}{5}}$

we get $x - \sqrt{\frac{1}{5}} = 0$ and $x + \sqrt{\frac{1}{5}} = 0$

Multiply both the factors we get,

$x^2 - \frac{5}{3} = 0$

Multiply by 3 we get

$3x^2 - 5 = 0$ is the factor of $3x^4 + 6x^3 - 2x^2 - 10x - 5$

Now divide,

Quotient is $x^2 + 2x + 1 = 0$

Compare the equation with quadratic formula,

$x^2 - (Sum\ \ of\ \ root)x + (Product\ \ of\ \ root) = 0$

Sum of root $= 2$

Product of the root $= 1$

So, we get

$x^2 + x + x + 1 = 0$

$x(x + 1) + 1(x + 1) = 0$

$x + 1 = 0, x + 1 = 0$

$x = -1, x = -1$

So, our zeroes are -1, -1, $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

#465052

On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $(x - 2)$ and $(-2x + 4)$, respectively. Find $g(x)$.

Solution

We know the formula,

Dividend = Divisor x quotient + Remainder

$p(x) = g(x) \times q(x) + r(x)$

Substitute the values, we get

$x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) - 2x + 4$

Add $2x$ and subtract 4 both side,

$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x) \times (x - 2)$

Simplify and divide by $x - 2$

$$\frac{x^3 - 3x^2 + 3x - 2}{x - 2} = g(x)$$

So, $g(x) = x^2 - x + 1$

#465105

Give examples of polynomials $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) deg $p(x)$ = deg $q(x)$

(ii) deg $q(x)$ = deg $r(x)$

(iii) deg $r(x) = 0$

Solution

(i) deg p(x) = deg q(x)

We know the formula,

Dividend = Divisor x quotient + Remainder

$p(x) = g(x) \times q(x) + r(x)$

So here the degree of quotient will be equal to degree of dividend when the divisor is constant.

Let us assume the division of $4x^2$ by 2.

Here, $p(x) = 4x^2$

$g(x) = 2$

$q(x) = 2x^2$ and $r(x) = 0$

Degree of $p(x)$ and $q(x)$ is the same i.e., 2.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$4x^2 = 2(2x^2)$

Hence, the division algorithm is satisfied.

(ii) deg q(x) = deg r(x)

Let us assume the division of $x^3 + x$ by x^2,

Here, p(x) = $x^3 + x$, g(x) = x^2, q(x) = x and r(x) = x

Degree of q(x) and r(x) is the same i.e., 1.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + x = x^2 \times x + x$

$x^3 + x = x^3 + x$

Hence, the division algorithm is satisfied.

(iii) deg r(x) = 0

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of $x^4 + 1$ by x^3

Here, p(x) = $x^4 + 1$

g(x) = x^3

$q(x) = x$ and $r(x) = 1$

Degree of $r(x)$ is 0.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^4 + 1 = x^3 \times x + 1$

$x^4 + 1 = x^4 + 1$

Hence, the division algorithm is satisfied.

#465124

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

(ii) $x^3 - 4x^2 + 5x - 2; 2, 1, 1$

Solution

(i) $2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, -2$

$p(x) = 2x^3 + x^2 - 5x + 2$ .... (1)

Zeroes for this polynomial are $\frac{1}{2}, 1, -2$

Substitute the $x = \frac{1}{2}$ in equation (1)

$p\left(\frac{1}{2}\right) = 2(\frac{1}{2})^3 + (\frac{1}{2})^2 - 5(\frac{1}{2}) + 2$

$= \frac{1}{4} + \frac{1}{4} + \frac{5}{2} + 2$

$= 0$

Substitute the $x = 1$ in equation (1)

$p(1) = 2 \times 1^3 + 1^2 - 5 \times 1 + 2$

$= 2 + 1 - 5 + 2 = 0$

Substitute the $x = -2$ in equation (1)

$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$

$= -16 + 4 + 10 + 2 = 0$

Therefore, $\frac{1}{2}, 1, -2$ are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$ we obtain,

$a = 2, b = 1, c = -5, d = 2$

Let us assume $\alpha = \frac{1}{2}, \beta = 1, \gamma = -2$

Sum of the roots = $\alpha + \beta + \gamma = \frac{1}{2} + 1 = 2 = \frac{-1}{2} = \frac{-b}{a}$

$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$

Product of the roots = $\alpha\beta\gamma = \frac{1}{2} \times x \times (-2) = \frac{-2}{2} = \frac{d}{a}$

Therefore, the relationship between the zeroes and coefficient are verified.

(ii) $x^3 - 4x^2 + 5x - 2; 2, 1, 1$

$p(x) = x^3 - 4x^2 + 5x - 2$ .... (1)

Zeroes for this polynomial are $2, 1, 1$

Substitute $x = 2$ in equation (1)

$p(2) = 2^3 - 4 \times 2^2 + 5 \times 2 - 2$

$= 8 - 16 + 10 - 2 = 0$

Substitute $x = 1$ in equation (1)

$p(1) = x^3 - 4x^2 + 5x - 2$

$= 1^3 - 4(1)^2 + 5(1) - 2$

$= 1 - 4 + 5 - 2 = 0$

Therefore, $2, 1, 1$ are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$ we obtain,

$a = 1, b = -4, c = 5, d = -2$

Let us assume $\alpha = 2, \beta = 1, \gamma = 1$

Sum of the roots = $\alpha + \beta + \gamma = 2 + 1 + 1 = 4 = -\frac{-4}{1}\frac{-b}{a}$

Multiplication of two zeroes taking two at a time=$\alpha\beta + \beta\gamma + \alpha\gamma = (2)(1) + (1)(1) + (2)(1) = 5 = \frac{5}{1} = \frac{c}{a}$

Product of the roots = $\alpha\beta\gamma = 2 \times 1 \times 1 = 2 = -\frac{-2}{1} = \frac{d}{a}$

Therefore, the relationship between the zeroes and coefficient are verified.

#465125

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as $2, 7, 14$ respectively.

Solution

Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be α, β, γ.

Sum of the polynomial = $\alpha + \beta + \gamma = \dfrac{2}{1} = \dfrac{-b}{a}$

Sum of the product of its zeroes taken two at a time = $\alpha\beta + \beta\gamma + \alpha\gamma = \dfrac{-7}{1} = \dfrac{c}{a}$

Product of the root = $\alpha\beta\gamma = \dfrac{-14}{1} = \dfrac{-d}{a}$

If $a = 1, b = -2, c = -7, d = 14$.

Hence the polynomial is $x^3 - 2x^2 - 7x + 14$

#465126

If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$, find a and b.

Solution

p(x)= $x^3 - 3x^2 + x + 1$

zeroes are $a - b, a, a + b$

Comparing the given polynomial with $px^3 + qx^2 + rx + t$, we obtain

$p = 1, q = -3, r = 1, t = 1$

Sum of zeroes = $a - a + a + a + a = \dfrac{-q}{p} = 3a$

$\Rightarrow \dfrac{-q}{p} = 3a$

$\Rightarrow \dfrac{-(-3)}{1} = 3a$

$\Rightarrow 3 = 3a$

$\Rightarrow a = 1$

Then the zeroes becomes, $1 - b, 1, 1 + b$

Multiplication of zeroes $= (1 - b) \times 1 \times (1 + b) = \dfrac{-t}{p} = 1 - b^2$

$\Rightarrow \dfrac{-1}{1} = 1 - b^2$

$\Rightarrow b^2 = 2$

$b = \pm\sqrt{2}$

Therefore, $a = 1$ and $b = \pm\sqrt{2}$

#465127

If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Solution

The two zeroes of the polynomial is $2+\sqrt{3}, 2-\sqrt{3}$

Therefore, $(x-2+\sqrt{3})(x-2-\sqrt{3}) = x^2+4-4x-3$

$= x^2-4x+1$ is a factor of the given polynomial.

Using division algorithm, we get

$x^4-6x^3-26x^2+138x-35 = (x^2-4x+1)(x^2-2x-35)$

So, $(x^2-2x-35)$ is also a factor of the given polynomial.

$x^2-2x-35 = x^2-7x+5x-35$

$= x(x-7)+5(x-7)$

$= (x-7)(x+5)$

Hence, 7 and $-5$are the other zeros of this polynomial.

$$
\begin{array}{r l}
 & x^2 - 2x - 35 \\
x^2 - 4x + 1 \,\big) & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 - x^2 \\
 & - \quad + \quad - \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\hline
 & 0
\end{array}
$$

#465128

If the polynomial $x^4-6x^3+16x^2-26x+10-a$, is divided by another polynomial x^2-2x+k the remainder comes out to be $(x+a)$, find k and a.

Solution

Dividend = Divisor x Quotient + Remainder

∴ Dividend - Remainder = Divisor x Quotient

$x^4 - 6x^3 + 16x^2 - 26x + 10 - x - a$ is divisible by $x^2 - 2x + k$

Let us divide $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ by $x^2 - 2x + k$

Remainder $= 0,$

Therefore, $(-10 + 2k)x + (10 - a - 8k + k^2) = 0$

$-10 + 2k = 0, 10 - a - 8k + k^2 = 0$

For $-10 + 2k = 0$

$\Rightarrow 2k = 10$

$\Rightarrow k = 5$

For $10 - a - 8k + k^2 = 0$

$10 - a - 8 \times 5 + 25 = 0$

$-5 - a = 0$

$a = -5$

So, $k = 5, a = -5$