

CLASS
X
Unit-8

CBSE-*i*

TRIGONOMETRY

& ITS APPLICATIONS

(CORE)



CENTRAL BOARD OF SECONDARY EDUCATION

Shiksha Kendra, 2, Community Centre, Preet Vihar, Delhi-110 092 India



MATHEMATICS
TRIGONOMETRY
& ITS
APPLICATIONS
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The background of the page is a soft-focus image of a ruler and a pencil. The ruler is orange and white, with black markings and numbers. The pencil is light-colored with a pink eraser. The overall color palette is warm and educational.

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PREFACE

The Curriculum initiated by Central Board of Secondary Education -International (CBSE-i) is a progressive step in making the educational content and methodology more sensitive and responsive to the global needs. It signifies the emergence of a fresh thought process in imparting a curriculum which would restore the independence of the learner to pursue the learning process in harmony with the existing personal, social and cultural ethos.

The Central Board of Secondary Education has been providing support to the academic needs of the learners worldwide. It has about 11500 schools affiliated to it and over 158 schools situated in more than 23 countries. The Board has always been conscious of the varying needs of the learners in countries abroad and has been working towards contextualizing certain elements of the learning process to the physical, geographical, social and cultural environment in which they are engaged. The International Curriculum being designed by CBSE-i, has been visualized and developed with these requirements in view.

The nucleus of the entire process of constructing the curricular structure is the learner. The objective of the curriculum is to nurture the independence of the learner, given the fact that every learner is unique. The learner has to understand, appreciate, protect and build on values, beliefs and traditional wisdom, make the necessary modifications, improvisations and additions wherever and whenever necessary.

The recent scientific and technological advances have thrown open the gateways of knowledge at an astonishing pace. The speed and methods of assimilating knowledge have put forth many challenges to the educators, forcing them to rethink their approaches for knowledge processing by their learners. In this context, it has become imperative for them to incorporate those skills which will enable the young learners to become 'life long learners'. The ability to stay current, to upgrade skills with emerging technologies, to understand the nuances involved in change management and the relevant life skills have to be a part of the learning domains of the global learners. The CBSE-i curriculum has taken cognizance of these requirements.

The CBSE-i aims to carry forward the basic strength of the Indian system of education while promoting critical and creative thinking skills, effective communication skills, interpersonal and collaborative skills along with information and media skills. There is an inbuilt flexibility in the curriculum, as it provides a foundation and an extension curriculum, in all subject areas to cater to the different pace of learners.

The CBSE has introduced the CBSE-i curriculum in schools affiliated to CBSE at the international level in 2010 and is now introducing it to other affiliated schools who meet the requirements for introducing this curriculum. The focus of CBSE-i is to ensure that the learner is stress-free and committed to active learning. The learner would be evaluated on a continuous and comprehensive basis consequent to the mutual interactions between the teacher and the learner. There are some non-evaluative components in the curriculum which would be commented upon by the teachers and the school. The objective of this part or the core of the curriculum is to scaffold the learning experiences and to relate tacit knowledge with formal knowledge. This would involve trans-disciplinary linkages that would form the core of the learning process. Perspectives, SEWA (Social Empowerment through Work and Action), Life Skills and Research would be the constituents of this 'Core'. The Core skills are the most significant aspects of a learner's holistic growth and learning curve.

The International Curriculum has been designed keeping in view the foundations of the National Curricular Framework (NCF 2005) NCERT and the experience gathered by the Board over the last seven decades in imparting effective learning to millions of learners, many of whom are now global citizens.

The Board does not interpret this development as an alternative to other curricula existing at the international level, but as an exercise in providing the much needed Indian leadership for global education at the school level. The International Curriculum would evolve on its own, building on learning experiences inside the classroom over a period of time. The Board while addressing the issues of empowerment with the help of the schools' administering this system strongly recommends that practicing teachers become skillful learners on their own and also transfer their learning experiences to their peers through the interactive platforms provided by the Board.

I profusely thank Shri G. Balasubramanian, former Director (Academics), CBSE, Ms. Abha Adams and her team and Dr. Sadhana Parashar, Head (Innovations and Research) CBSE along with other Education Officers involved in the development and implementation of this material.

The CBSE-i website has already started enabling all stakeholders to participate in this initiative through the discussion forums provided on the portal. Any further suggestions are welcome.

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Syllabus –UNIT 8

Trigonometry and its applications (Core)

Revision of trigonometric facts	All T- ratios, values of T-ratios at $0^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}, 90^{\circ}$ Trigonometric Ratios and complementary angles
Trigonometric identities	$\sin^2\theta + \cos^2\theta = 1,$ $\sec^2\theta - \tan^2\theta = 1,$ $\operatorname{cosec}^2\theta - \cot^2\theta = 1,$ Problems based on trigonometric identities.
Angle of elevation and angle of depression	Describing angle of elevation and angle of depression for a given point.
Application problems	Problems involving angle measure of $30^{\circ}, 45^{\circ}, 60^{\circ}.$

Scope Document

Concepts

1. Trigonometric Identities
2. Line of sight
3. Angle of Elevation
4. Angle of Depression

Learning Objective

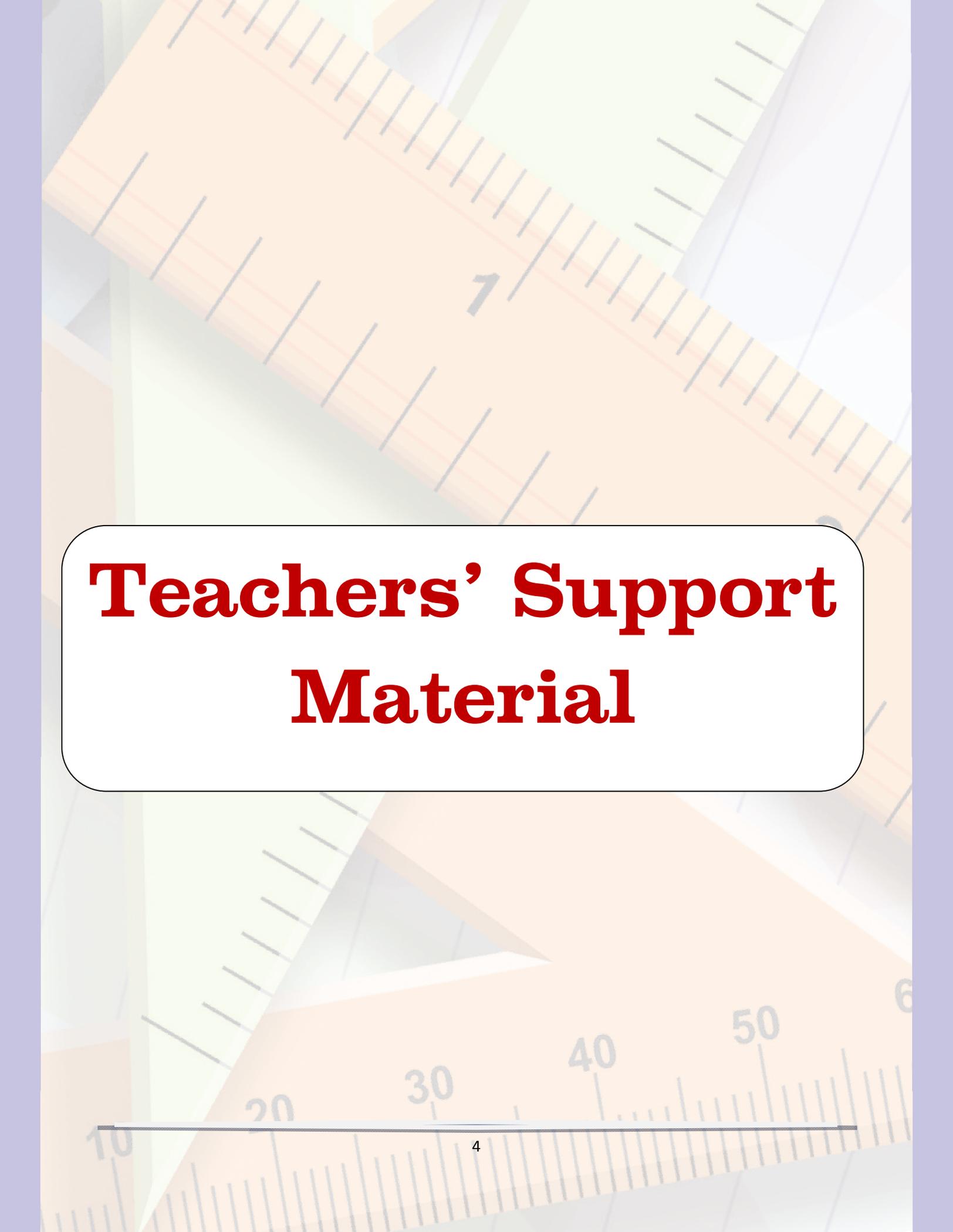
1. To prove that for any right angle triangle ABC, $\sin^2 A + \cos^2 A = 1$, using Pythagoras Theorem
2. To prove that for any right angle triangle ABC, $1 + \tan^2 A = \sec^2 A$, using above identity
3. To prove that for any right angle triangle ABC, $1 + \cot^2 A = \operatorname{cosec}^2 A$, using above identity
4. To prove other trigonometric identities using above identities.
 - a. To deduce the value of trigonometric expressions for given angles using relation between trigonometric ratios and trigonometric identities.
 - To describe the angle of elevation or angle of depression of a point w.r.t. the position of observer's eye
 - To draw a right angle triangle for a given situation and to mark required angle of depression/angle of elevation.
 - To apply trigonometry in solving simple problems (single triangle) from real life situation.

Extension activities:

1. To prove that in any right angle triangle
 - a) $\sin(A+B) \neq \sin(A) + \sin(B)$
 - b) $\cos(A+B) \neq \cos(A) + \cos(B)$
 - c) $\tan(A+B) \neq \tan(A) + \tan(B)$
2. Interesting quiz on trigonometry on <http://www.funtrivia.com/submitquiz.cfm?passthrough=qid:250308>
3. To construct a clinometer and to measure the height of school building or the other objects at height
4. To prove that in any right angle triangle $\sin(90^\circ - \theta) = \cos\theta$,
 $\cos(90^\circ - \theta) = \sin\theta$, $\tan(90^\circ - \theta) = \cot\theta$

Cross-curricular activities:

1. Trigonometry is used in architecture to make the buildings safe. How?
2. A wheel chair ramp can be designed using trigonometry. Steps and stairs pose a major obstacle to people confined to a wheelchair. Creating or designing the ramp means you need to evaluate the various dimensions of the structure and the needs of the person. Trigonometry offers simple ways to determine the length and width of the ramp, angle of incline, and the total area of landings. Design a wheel chair ramp.

The background features a collage of educational tools: several rulers in various colors (orange, yellow, blue) and orientations, and a pencil. The rulers have numerical markings and are slightly blurred, creating a sense of depth. The pencil is positioned diagonally across the center.

Teachers' Support Material

TEACHER'S NOTE

The teaching of Mathematics should enhance the child's resources to think and reason, to visualise and handle abstractions, to formulate and solve problems. As per NCF 2005, the vision for school Mathematics include :

1. Children learn to enjoy mathematics rather than fear it.
2. Children see mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on.
3. Children pose and solve meaningful problems.
4. Children use abstractions to perceive relation-ships, to see structures, to reason out things, to argue the truth or falsity of statements.
5. Children understand the basic structure of Mathematics: Arithmetic, algebra, geometry and trigonometry, the basic content areas of school Mathematics, all offer a methodology for abstraction, structuration and generalisation.
6. Teachers engage every child in class with the conviction that everyone can learn mathematics.

Students should be encouraged to solve problems through different methods like abstraction, quantification, analogy, case analysis, reduction to simpler situations, even guess-and-verify exercises during different stages of school. This will enrich the students and help them to understand that a problem can be approached by a variety of methods for solving it. School mathematics should also play an important role in developing the useful skill of estimation of quantities and approximating solutions. Development of visualisation and representations skills should be integral to Mathematics teaching. There is also a need to make connections between Mathematics and other subjects of study. When children learn to draw a graph, they should be encouraged to perceive the importance of graph in the teaching of Science, Social Science and other areas of study. Mathematics should help in developing the reasoning

skills of students. Proof is a process which encourages systematic way of argumentation. The aim should be to develop arguments, to evaluate arguments, to make conjunctures and understand that there are various methods of reasoning. Students should be made to understand that mathematical communication is precise, employs unambiguous use of language and rigour in formulation. Children should be encouraged to appreciate its significance.

At the secondary stage students begin to perceive the structure of Mathematics as a discipline. By this stage they should become familiar with the characteristics of Mathematical communications, various terms and concepts, the use of symbols, precision of language and systematic arguments in proving the proposition. At this stage a student should be able to integrate the many concepts and skills that he/she has learnt in solving problems.

The present unit on “Trigonometry and its Applications” focuses on lots of Problem solving activities and making of model in order to meet out the following learning objectives:

- To prove that for any right angled triangle ABC
 $\sin^2 A + \cos^2 A = 1$, using Pythagoras Theorem
- To prove that for any right angled triangle ABC
 $1 + \tan^2 A = \sec^2 A$, using above identity
- To prove that for any right angled triangle ABC
 $1 + \cot^2 A = \operatorname{cosec}^2 A$, using above identity
- To prove other trigonometric identities using above identities.
- To understand the difference between Pythagorean identities and other trigonometric identities.
- To deduce the value of trigonometric expressions for given angles using relation between trigonometric ratios and trigonometric identities.
- To describe the angle of elevation or angle of depression of a point w.r.t. the position of observer’s eye.

- To draw a right angled triangle for a given situation and to mark required angle of depression/ angle of elevation.
- To apply knowledge of trigonometric ratios and trigonometric identities in solving problems from real life situation involving single triangle or double triangles.

All the tasks are designed to take up the chapter keeping in mind the following pedagogical issues:

- To create supportive classroom environment in which learners can think together, learn together, participate in the discussions and can take intellectual decisions.
- To provide enough opportunities to each learner for expression so that teacher can have insight into the knowledge acquired, knowledge required, refinement required in the knowledge gained and the thinking process of the learner.
- Emphasis on creating a good communicative environment in the class.
- To cater to various learning styles.

Trigonometry and its applications is the extension unit of Introduction to Trigonometry. So, the teacher can start the chapter by taking lots of exercises related to the previously learnt concepts. Moreover, this unit is application of trigonometry, so lots of problem solving skills are required.

As a first **warm up (W1) activity** students can be motivated to speak about all the terms related to right angled triangle. Terms can be displayed on the screen or teacher can rotate a basket with paper slips in the class. Students can pick up a slip and speak about the word written over it. Students can give definition, their understanding of the term in their own words or any other information beyond the textbook knowledge. Each child should be encouraged by the teachers to speak without commenting right or wrong on his statements. If there is anything wrong in the child's answers, teacher can initiate a discussion or can illustrate with the help of drawing/picture.

Further **warm up activities** like a cartoon strip with a statement in the bubble will allow the students to perceive the situation in their own way. By writing their views on it and thinking loudly they can reach to the conclusion that trigonometric ratios are independent of the length of the sides and dependent on the angles between the two sides.

Some more **pre content activities** can be conducted in the class to reinforce the knowledge of trigonometric ratios, value of T-ratios at prescribed angles i.e. 0° , 30° , 45° , 60° , 90° . These tasks will help the students in following ways: -

1. To refresh the knowledge and the memory based concepts.

2. To acquire the skills in using them in problem solving.
3. To gear them for learning further concepts such as trigonometric identities and height and distance problems requiring knowledge of T-ratios and identities.

For this particular chapter all warm up and **pre-content activities** should focus on brushing up of previous knowledge and gradually going to the new terms required in this unit. This will give **comfortable feeling to the students** and will gear them for learning new concepts. New words to be used in this unit like angle of elevation, angle of depression, line of sight etc. can be introduced by asking the learners to find the meaning of the terms elevation, depression, sight from the dictionary. Students can also be asked to quote examples where they use these words. They can be encouraged to give examples from different subjects like geography, physics, English, history, etc.

The first important sub-topic trigonometric identities can be introduced with discussion on simple algebraic identities and equations. Gradually, they can be shifted towards trigonometric identities with the help of right angle triangle, Pythagoras theorem and trigonometric ratios.

Once the identities are introduced with the help of Pythagoras theorem, it can be shared with the students that the following three basic identities are called Pythagorean identities.

$$\sin^2 A + \cos^2 A = 1$$

$$1 + \tan^2 A = \sec^2 A,$$

$$1 + \cot^2 A = \operatorname{cosec}^2 A$$

Rest of the identities can be proved with the help of these three identities.

While explaining the method of proving identities teacher must write some equality statements on the board which are not identities. For example,

$\sin\theta + \cos\theta = 1$. An open challenge should be thrown in front of the students to narrate various ways through which they can make out whether the given statement is identity or equation.

Before explaining the method of finding height and distance with the help of trigonometry teacher can provoke the students to think about various ways to find the distance of stars and the sun from the earth, height of the school building, height of the fan in the classroom, height of the trees in the school compound etc. Most of them may respond about use of Pythagoras Theorem, their response can be shifted to think about

the use of trigonometric ratios/identities in solving the height and distance problems. Terms like angle of elevation, angle of depression, line of sight etc. can be introduced with reference to the historical account. After explaining the height and distance problems the learning can be extended by throwing a discussion in the class referring to the following point: -

Which method is better, Pythagoras Theorem or use of T-ratios to find the distance and height. Why?

Teacher can also give the exposure of various instruments used to find the angle of elevation or depression like clinometers, theodolite and sextant and ask the students to make these instruments and to find the height/distance using all the three instruments. Further they can be asked to record their observations, share their experiences and to narrate which one is best according to them.

Trigonometry is used everywhere. It is used by the architects in making the building safe. Let the students explore how they are doing it. This will enhance the student's skills for disaster management.

Moreover, while preparing a stage with backdrop and wings, making light arrangements, green-rooms knowledge of trigonometry can be used very well. Students can be motivated to script a play on historical development of trigonometry and its presentation on stage using the knowledge acquired in this unit. This activity will help the students to acquire the life skill of being self dependent.

To help the disabled people wheelchair, special ramps etc. are designed to make them comfortable. Using the knowledge of trigonometry students can be inspired to design one such chair as a part of SEWA.

Post content activities (PC) may contain lots of tasks based on memory and vocabulary as well as problem solving assignments.

ACTIVITY - SKILL MATRIX

Type of Activity	Name of Activity	Skill to be developed
Warm UP(W1)		Application, calculation
Warm UP(W2)		Application, calculation
Warm UP(W3)		Computational skills
Pre-Content (P1)		Calculation skill
Pre-Content (P2)	Table of trig. Values	Understanding application and calculation skill
Content (CW 1)	Complimentary angle	Thinking skill
Content (CW 2)	Use of T-ratios	Thinking skill, computational skills
Content (CW 3)	Trigonometric identities	Thinking skill, creativity
Content (CW 4)	Angle of elevation and angle of depression	Problem understanding, drawing skill, thinking skill, computational skill
Content (CW 5)	Representation of situation through right angle triangle	Problem understanding, drawing skill, thinking skill, computational skill
Content (CW 6)	Application in trigonometry in daily life	Problem understanding, drawing skill, thinking skill, computational skill
Post - Content (PCW 1)		Understanding application and calculation skill
Post - Content (PCW 2)		Understanding application and calculation skill
Post - Content (PCW 3)	Clinometric activity	Problem understanding, drawing skill, thinking skill, computational skill Mathematical modelling observation and recording.

Activity 1 - warm up (W1)

Specific objective:

To recall the sides and angles in a right triangle

Description: In **earlier** classes the students have gained the knowledge of right triangle, its sides and angles. This is a starter activity through which the students will revise the learnt concept of finding the angles and sides of a right triangle using Pythagoras theorem and angle sum property of a triangle

Execution: Prepare some flash cards on which various right triangles are drawn. Some of the sides and angles are missing. Ask the students to find them.

Parameters for assessment:

- Able to use Pythagoras theorem
- Able to apply the angle sum property

Extra reading:

http://www.mathsisfun.com/right_angle_triangle.html

Pythagorean Theorem <http://www.grc.nasa.gov/WWW/K-12/airplane/pythag.html>

Activity 2 – warm up (W2)

Specific objective:

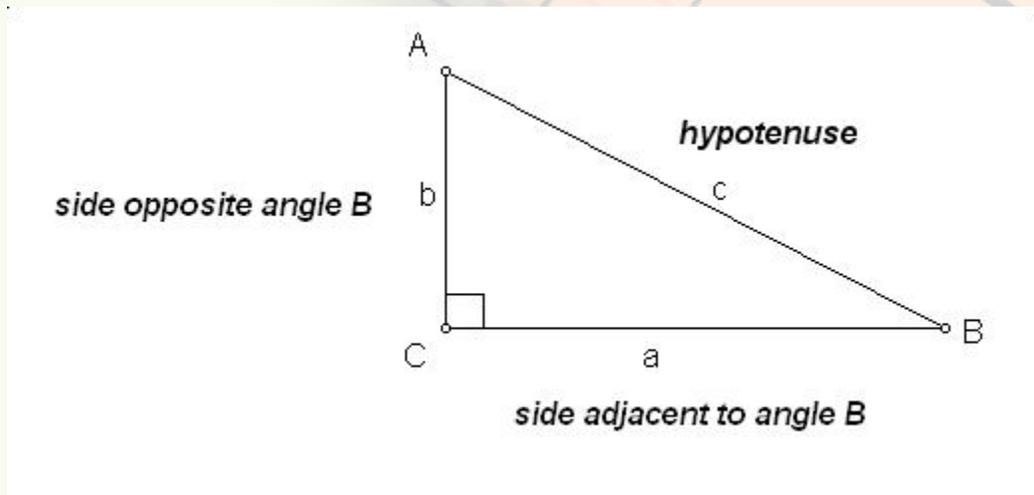
To recall the naming of sides in a right triangle according to marked angle

Description: In **earlier** classes the students have gained the knowledge of right triangle, its sides and angles. This is a starter activity through which the students will be motivated for learning to name the adjacent side and opposite side of a marked angle.

Execution: As a starter activity give two problems to students. Either draw them on board or distribute the worksheet (W2). Let students mark adjacent side and opposite side w.r.t the marked angle.

Parameters for assessment:

- Able to name correctly the adjacent side and opposite side of a marked angle.



Memory Aid: .

Video watch <http://mykhmsmathclass.blogspot.com/2009/06/video-naming-sides-of-right-triangle.html>

Activity 3 – warm up (W3)

Specific objective:

To motivate the students to write T-ratios in a given right triangle

Description: As the students have already done the unit-7, Introduction to Trigonometry, the essential pre-requisite for the present unit i.e. the knowledge of Trigonometric ratios can be revised to warm up the students to take up further concepts on Trigonometry. They will be given an interesting Memory Aid to remember all t-ratios

Execution: As a starter activity give two problems to students. Either draw them on board or distribute the worksheet (W3). Let students write all T- ratios for the given figure.

Parameters for assessment:

- Able to write T-ratios in a right triangle for a marked angle.

Memory Aid:

Soh --> Sine = opposite/hypotenuse

Cah --> Cosine = adjacent/hypotenuse

Toa --> Tangent = opposite/adjacent

Watch Video <http://mykhmsmathclass.blogspot.com/2009/06/video-trigonometric-ratios.html>

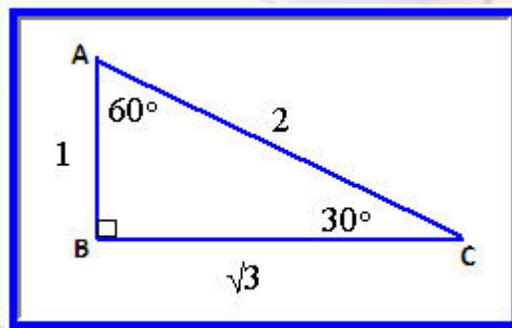
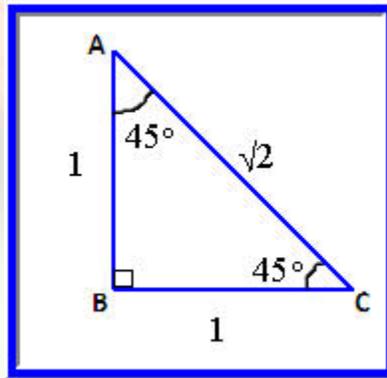
Activity 4 - Pre Content (P1)

Specific objective:

To recall and use values of all trigonometric ratios at 0° , 30° , 45° , 60° , 90°

Description: Before learning more about trigonometry in a right triangle, the previous knowledge of students may be tested for the value of T- ratios.

You may use the given pictures to ask oral questions.



Execution: You may assess the students orally or through a worksheet or a puzzle (jig saw). It can be a match the following activity also. Let students learn values of T ratios at specific angles.

Parameters for assessment:

Able to write and speak values of T- ratios at specific angles - 0° , 30° , 45° , 60° , 90°

Memory Aid:

- Write 0, 1, 2, 3, 4 over each column as shown below.

- Divide each number by 4 and take the square root. The values obtained are sine ratios.
- Write these ratios in reverse order and obtain the cosine ratios
- (sin A = cos B if A + B = 90°)
- Divide each sine ratio by cosine ratio and obtain the values of corresponding tangent ratios

$$\left(\because \frac{\sin A}{\cos A} = \tan A \right).$$

	$\sqrt{\frac{0}{4}}$	$\sqrt{\frac{1}{4}}$	$\sqrt{\frac{2}{4}}$	$\sqrt{\frac{3}{4}}$	$\sqrt{\frac{4}{4}}$
T-ratios	0	30°	45°	60°	90°
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined

Watch Video

Finding T ratios at 30 degrees and 60 degrees

<http://mykhsmathclass.blogspot.com/2009/06/video-finding-trigonometric-ratios-at.html>

Finding T ratios at 45 degrees <http://mykhsmathclass.blogspot.com/2009/06/video-finding-t-ratio-at-45-degrees.html>

Activity 5 - Pre Content (P2)

Specific objective:

To review and recall the knowledge of relation between T ratios at a given angle and T ratios for its complementary angle.

Description: P2 is designed for assessing the previous knowledge of students in terms of relation between T ratios and knowledge of complementary angles

Execution: Ask the students to recall T ratios and do the matching exercise given in P2.

Parameters for assessment:

- Able to tell the relation between T -ratios
- Able to tell T ratios of complementary angles

Memory Aid:

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Complimentary angles

$$\sin \theta = \cos (90^\circ - \theta)$$

$$\cos \theta = \sin (90^\circ - \theta)$$

$$\tan \theta = \cot (90^\circ - \theta)$$

$$\sec \theta = \operatorname{cosec} (90^\circ - \theta)$$

$$\operatorname{cosec} \theta = \sec (90^\circ - \theta)$$

$$\cot \theta = \tan (90^\circ - \theta)$$

Activity 6 - Content (CW1)

Specific objective: To explore T-ratios of complementary angles.

Description: This is an application based exercise in which the students will use the knowledge of T-ratios of complementary angles and solve problems.

Execution: Students will recall the T-ratios of complementary angles in a right triangle using the meaning of complementary angles. They will then solve the problems given in the worksheet.

Discuss the proof

If $\sin 3\theta = \cos(\theta - 6^\circ)$, where 3θ and $(\theta - 6^\circ)$ are acute angles, then find the value of θ .

Solution: $\sin 3\theta = \cos(\theta - 6^\circ)$

$$\text{So, } \sin 3\theta = \sin \{90^\circ - (\theta - 6^\circ)\}$$

$$\text{Or } \sin 3\theta = \sin (90^\circ - \theta + 6^\circ)$$

$$\text{Or } \sin 3\theta = \sin (96^\circ - \theta)$$

$$\text{So, } 3\theta = (96^\circ - \theta) \quad [\sin B = \sin \theta \text{ means } \angle B = \theta]$$

$$\text{Or } 3\theta + \theta = 96^\circ$$

$$\text{Or } 4\theta = 96^\circ$$

$$\text{Or } \theta = 24^\circ$$

Parameters for assessment:

- Knows the T-ratios of complementary angles
- Able to solve problems using T-ratios of complementary angles

Extra Reading

Useful Videos

<http://mykhmsmathclass.blogspot.com/2009/06/finding-day-of-given-date.html>

Activity 7 - Content (CW2)

Specific objective:

Using the knowledge of T- ratios in a right triangle and evaluating the given expression involving T-ratios

Description: This is quite an interesting activity through which students will get an opportunity to find the value of a given expression involving trigonometric functions by using their knowledge of trigonometry and Pythagoras theorem in a right triangle.

Execution: Firstly, students will learn a method to find all T-ratios when any of the one T-ratio is given. Proceeding further, the value of expressions involving T-ratios can be calculated.

Parameters for assessment:

- Applying Pythagoras theorem in a right triangle
- Finding T-ratios correctly

Activity 8 - Content (CW3)

Specific objective:

To solve problems based on trigonometric identities.

$$\sin^2\theta + \cos^2\theta = 1,$$

$$\sec^2\theta - \tan^2\theta = 1,$$

$$\operatorname{cosec}^2\theta - \cot^2\theta = 1$$

Description: This worksheet is designed for enhancing student's skills on proving and using trigonometric identities.

Execution: Let students explore right triangle and investigate the proofs of standard trigonometric identities using the Pythagoras theorem. Discuss some examples in the classroom. Ask students to prove the identities given in the worksheet CW3 using the standard identities.

Parameters for assessment:

- Able to prove trigonometric identities
- Able to use standard identities for proving other trigonometric identities

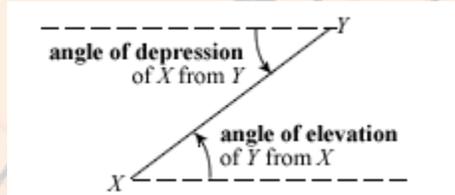
Watch Videos

<http://mykhmsmathclass.blogspot.com/2009/06/videos-problems-using-trigonometric.html>

Activity 9 - Content (CW4)

Specific objective:

To learn to label angle of elevation, angle of depression and line of sight in a given situation



Description: After learning to solve a right triangle using trigonometry, students will be provided an opportunity to explore its utility in daily life. For this they will first understand some basic terms like angle of depression, angle of elevation and line of sight.

Execution: On a worksheet, various situations will be depicted through pictures. Students will mark the angle of elevation, angle of depression and line of sight.

Parameters for assessment:

- Understanding on concept
- Correct marking of angle of depression/elevation and line of sight on the worksheet

Extra Reading

<http://mathcentral.uregina.ca/QQ/database/QQ.09.03/anjum1.html>

<http://www.thatquiz.org/tq/previewtest?F/C/Z/B/84221300663838>

Watch Video

<http://mykhmsmathclass.blogspot.com/2009/06/video-angle-of-elevation-and-depression.html>

Activity 10 – Content (CW5)

Specific objective:

To represent the given situation through right triangle diagrams.

Description: Trigonometry is widely used in many areas. Students are supposed to solve word problems using trigonometry. This worksheet has been prepared to enhance the skills of representing a given situation through a diagram.

Execution: Teacher will demonstrate some examples on the boards. Following it, CW5 will be distributed. Each student will draw diagrams for a given situation.

Parameters for assessment:

- Correct representation of the given situation through a right triangle diagram

Activity 11 – Content (CW6)

Specific objective:

To learn to apply the knowledge of trigonometry in solving daily life problems

Description: After learning to represent a given situation in a right triangle, have a discussion on application of trigonometry. Students will be then asked to solve the right triangle and find the unknown side using trigonometric ratios.

Execution: Distribute the worksheet. Explain the template of how to write and solve a problem. All the students will solve the problems followed by a discussion on using trigonometry in solving problems.

Parameters for assessment:

- Drawing correct figure for a given problem
- Solving correctly and writing final answer with proper units

Extra Reading

<http://www.sylvum.com/cgi/online/serve.cgi/math/trigo/trigonometry3.html>

Activity 12 – Content (PCW1)

Based on assessing problems on T ratios for specific angles and proving trigonometric identities

Activity 13 – Content (PCW2)

Based on assessing problems on T ratios for complementary angles

Activity 14 – Content (PCW3)

Based on problems on applications of trigonometry

Activity 15 - Content (PCW4)

Instruction Sheet:

Objective

To make a clinometer and use it to measure the height of an object

Materials required: Stiff card, small pipe or drinking straw, thread, a weight (a metal washer is ideal).

Pre-requisite knowledge

Properties of right angled triangles.

Procedure

(A) To make clinometer:

1. Prepare a semi-circular protractor using any hard board and fix a viewing tube (straw or pipe) along the diameter.
2. Punch a hole (o) at the centre of the semicircle.
3. Suspend a weight {w} from a small nail fixed to the centre.
4. Ensure that the weight at the end of the string hangs below the protractor.
5. Mark degrees (in sexagesimal scale with 00 at the lowest and 10 to 90° proceeding both clockwise and anticlockwise). [Fig 1].

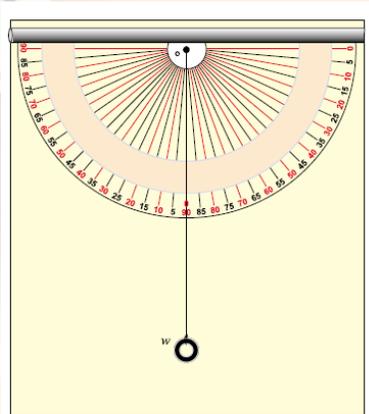


Fig 1

(B) To determine the height of an object:

6. First measure the distance of the object from you. Let the distance be d .
7. Look through the straw or pipe at the top of the object. Make sure you can clearly see the top of the object.
8. Hold the clinometer steady and let your partner record the angle the string makes on the scale of the clinometer. Let this angle be θ .

Observations

Using trigonometric ratio :

$$\tan \theta = \text{height} / \text{distance} = h/d$$

$$h = d \times \tan \theta$$

If, for example, $d = 100$ m and $\theta = 45^\circ$

$$h = 100 \times \tan 45^\circ = 100 \text{ m}$$

Remark

Students may be asked to change the distance of the object (by either moving the object or by changing their position) and note how the angle of elevation varies. They will notice that though d and θ will vary, the product $h = d \tan \theta$ will be constant (within measurement error).

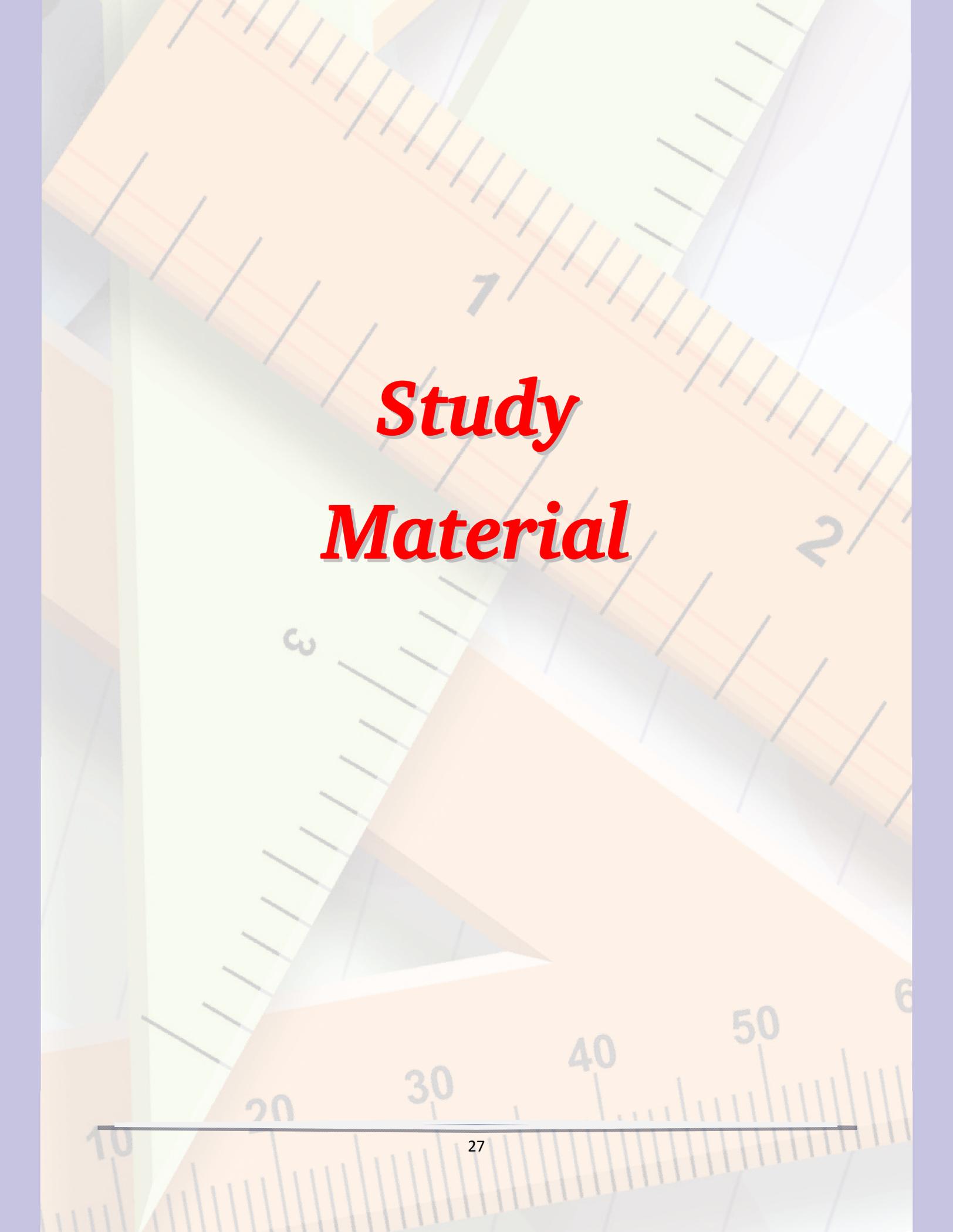
Extra reading <http://mykhmsmathclass.blogspot.com/2007/07/making-clinometer.html>

Assessment Plan

Rubric of Assessment

parameter	5	4	3	2	1
Trigonometric Identities	<ul style="list-style-type: none"> • Can prove the trigonometric identity $\sin^2 A + \cos^2 A = 1$ using Pythagoras Theorem • Can prove the trigonometric identity $1 + \tan^2 A = \sec^2 A$ using Pythagoras Theorem • Can prove the trigonometric identity $1 + \cot^2 A = \operatorname{cosec}^2 A$ using Pythagoras Theorem • can prove other trigonometric identities using above basic identities correctly • can find the value of trigonometric expressions using trigonometric identities correctly 				<ul style="list-style-type: none"> • Cannot prove the trigonometric identity $\sin^2 A + \cos^2 A = 1$ using Pythagoras Theorem • Cannot prove the trigonometric identity $1 + \tan^2 A = \sec^2 A$ using Pythagoras Theorem • Cannot prove the trigonometric identity $1 + \cot^2 A = \operatorname{cosec}^2 A$ using Pythagoras Theorem • cannot prove other trigonometric identities using above basic identities correctly • cannot find the value of trigonometric expressions using trigonometric identities correctly
Application problems	<ul style="list-style-type: none"> • Can define the terms: line of sight, angle of elevation, angle of depression correctly • Can recognize the line of sight, angle of elevation, angle of depression in a given figure correctly • Can represent the given word problem 				<ul style="list-style-type: none"> • Cannot define the terms: line of sight, angle of elevation, angle of depression correctly • Cannot recognize the line of sight, angle of elevation, angle of depression in a given figure correctly • Cannot represent the

	<p>through graph correctly</p> <ul style="list-style-type: none">• Can find the required length or height or angle accurately, for a given problem involving single triangle.• Can find the required length or height or angle accurately, for a given problem involving two triangles.				<p>given word problem through graph correctly</p> <ul style="list-style-type: none">• Cannot find the required length or height or angle accurately, for a given problem involving single triangle.• Cannot find the required length or height or angle accurately, for a given problem involving two triangles.
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***Study
Material***

Introduction

You have already been introduced to a branch of mathematics called trigonometry and the concepts of trigonometric ratios of an acute angle of a right triangle such as $\sin \theta$, $\cos \theta$, $\tan \theta$, $\operatorname{cosec} \theta$, $\sec \theta$ and $\cot \theta$. You have also obtained trigonometric ratios of some specific angles namely 30° , 45° , 60° and 0° , 90° geometrically. Now, in this chapter, we shall first recall these concepts briefly and extend this knowledge of trigonometry to establish some trigonometric identities and solve some daily life problem of heights and distances.

Trigonometric facts

- Trigonometric Ratios

Let ABC be a right triangle, right angled at B.(see figure (i))

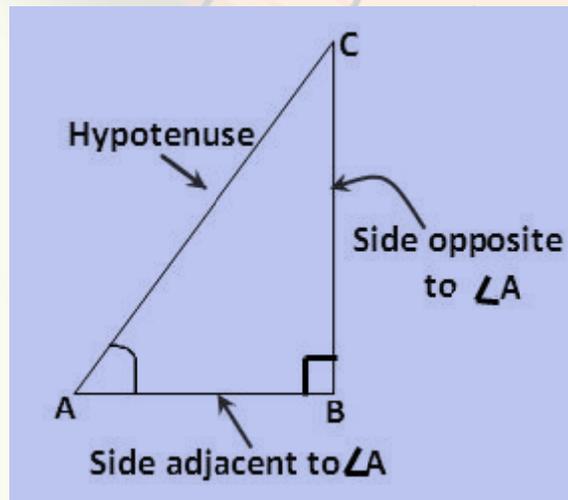


Figure (i)

$$\sin A = \frac{\text{side opposite to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$\tan A = \frac{BC}{AB}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{AC}{BC}$$

$$\sec A = \frac{1}{\cos A} = \frac{AC}{AB}$$

$$\cot A = \frac{1}{\tan A} = \frac{AB}{BC}$$

Similarly, in right ΔABC (Fig. ii)

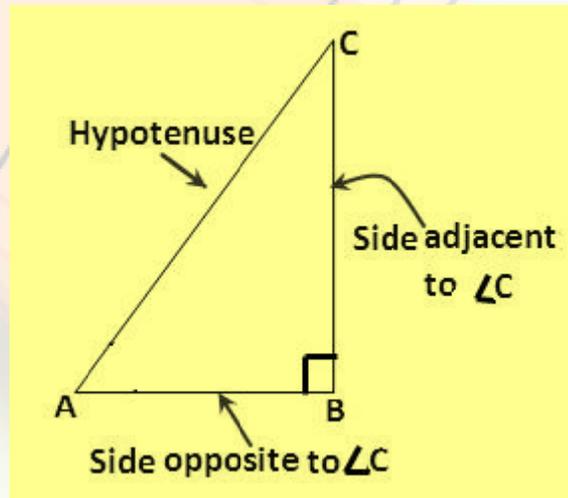


Figure (ii)

$$\sin C = \frac{AB}{AC}, \quad \cos C = \frac{BC}{AC}, \quad \tan C = \frac{AB}{BC},$$

$$\cos C = \frac{AC}{AB}, \quad \sec C = \frac{AC}{BC}, \quad \cot C = \frac{BC}{AB}$$

Recall that when one trigonometric ratio of an angle is known, we can determine the other trigonometric ratios also.

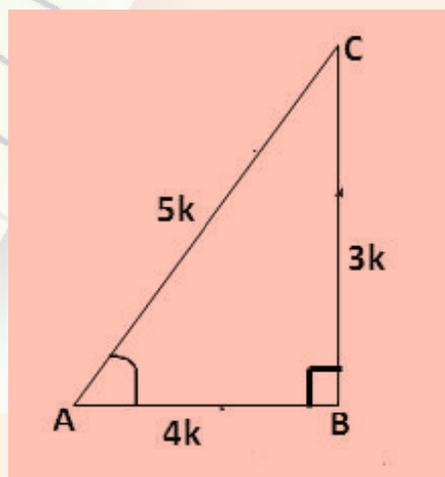


Figure (iii)

Example 1: Let $\sin A = \frac{3}{5}$, find other trigonometric ratios of A.

Solution: Consider right ΔABC (see figure (iii))

$$\text{Since } \sin A = \frac{3}{5} = \frac{BC}{AC},$$

Let $BC = 3k$, $AC = 5k$, where k is a positive constant.

$$AB = \sqrt{AC^2 - BC^2} \text{ (using Pythagoras Theorem)}$$

$$= \sqrt{25k^2 - 9k^2} = \sqrt{16k^2} = 4k$$

$$\text{So, } \cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}, \quad \tan A = \frac{3k}{4k} = \frac{3}{4}$$

$$\operatorname{cosec} A = \frac{5k}{3k} = \frac{5}{3}$$

$$\sec A = \frac{5k}{4k} = \frac{5}{4}$$

$$\cot A = \frac{4k}{3k} = \frac{4}{3}$$

Trigonometric Ratios of some specific angles

Recall the values of trigonometric ratios of 0° , 30° , 45° , 60° and 90° obtained geometrically. These values are given in the following table:

$\angle A$	0°	30°	45°	60°	90°
$\sin A$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos A$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec A	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot A	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

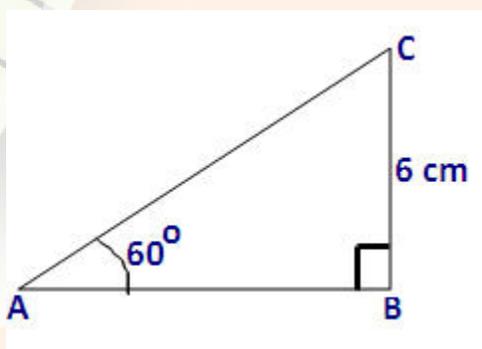
Note that as $\angle A$ increases from 0° to 90° , $\sin A$ increases from 0 to 1 and $\cos A$ decreases from 1 to 0. Maximum value of $\sin A$ and $\cos A$ is 1.

These values can be used to find other sides of a right triangle if one side and one acute angle are given and also to find value of some expressions involving these trigonometric ratios.

Example 2:

In a right triangle ABC, $BC = 6$ cm and $\angle A = 60^\circ$.

Determine AB and AC



Solution:

$$\text{As } \frac{AB}{BC} = \cot 60^\circ$$

$$\text{So, } AB = BC \cot 60^\circ = 6 \cot 60^\circ = 6 \times \frac{1}{\sqrt{3}} = \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3} \text{ cm}$$

$$\text{Also } \frac{AC}{BC} = \operatorname{cosec} 60^\circ = \frac{2}{\sqrt{3}}$$

$$\text{So } AC = 6 \times \frac{2}{\sqrt{3}} \text{ cm} = \frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3} \text{ cm}$$

Example 3: Evaluate:

$$\frac{5\sin^2 30^\circ + \cos^2 45^\circ + 4\tan^2 60^\circ}{2\sin 30^\circ \cos 45^\circ + \tan 45^\circ}$$

Solution: Given expression

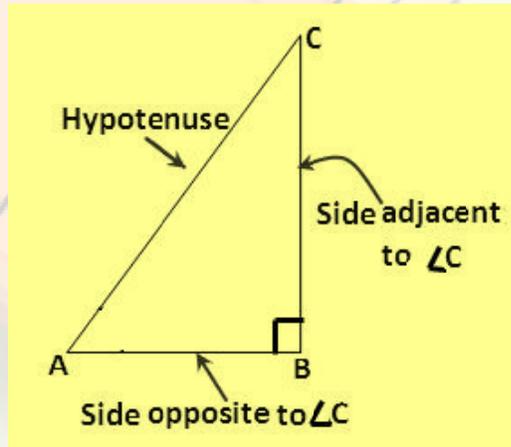
$$\begin{aligned} &= \frac{5\left(\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right) + 4\sqrt{3}}{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + 1} \\ &= \frac{\frac{5}{4} + \frac{1}{2} + 12}{\frac{1}{2} + 1} \\ &= \frac{5 + 2 + 48}{\frac{3}{2}} \\ &= \frac{55}{4} * \frac{2}{3} = \frac{55}{6} \end{aligned}$$

Trigonometric ratios of complementary angles

Recall that two angles are called complementary if their sum is 90° .

In $\triangle ABC$, right angled at B,

$\angle A$ and $\angle C$ are complementary. If $\angle A = \theta$, then $\angle C = 90^\circ - \theta$.



Or if $\angle C = \theta$, then $\angle A = 90^\circ - \theta$.

From the above triangle ABC, you can see that

$$\sin A = \frac{BC}{AC} \text{ or } \sin \theta = \frac{BC}{AC}$$

$$\text{Also } \cos C = \frac{BC}{AC} \cos(90^\circ - \theta) = \frac{BC}{AC}$$

$$\text{So, } \sin A = \cos C \text{ or } \sin \theta = \cos(90^\circ - \theta)$$

$$\text{Similarly, } \cos \theta = \sin(90^\circ - \theta)$$

$$\tan \theta = \cot(90^\circ - \theta)$$

$$\operatorname{cosec} \theta = \sec(90^\circ - \theta)$$

$$\sec \theta = \operatorname{cosec}(90^\circ - \theta)$$

Using these results, we can simplify certain expressions as shown in the following example.

Example 4:

Evaluate:

$$\frac{\sin 17^\circ}{\cos 73^\circ}$$

Solution:

$$\sin 17^\circ = \cos(90^\circ - 17^\circ) = \cos 73^\circ$$

$$\text{So, } \frac{\sin 17^\circ}{\cos 73^\circ} = 1$$

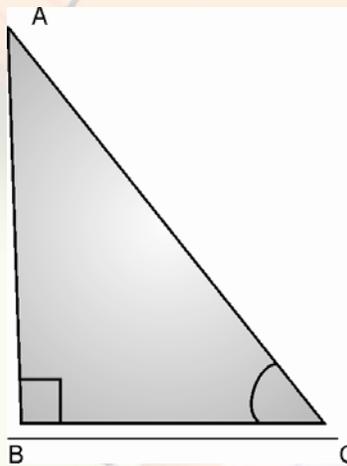
$$= 1$$

Example 5: Express $\sin 81^\circ + \tan 71^\circ$, in terms of trigonometric ratios of angles between 0° and 45°

Solution:
$$\begin{aligned} \sin 81^\circ + \tan 71^\circ &= \cos(90^\circ - 81^\circ) + \cot(90^\circ - 71^\circ) \\ &= \cos 9^\circ + \cot 19^\circ \end{aligned}$$

Trigonometric Identities

Consider ΔABC , right angled at B. let $\angle C = \theta$



We have, $AB^2 + BC^2 = AC^2$ (Pythagoras Theorem) (1)

Dividing both sides of (1) by AC^2 , we get

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

i.e. $\left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = 1$

or,

$$\boxed{\sin^2\theta + \cos^2\theta = 1} \quad (A)$$

[As $\frac{AB}{AC} = \sin\theta$ and $\frac{BC}{AC} = \cos\theta$]

Verify the equation (A) for $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$

You will find that (A) is true for all these values of θ . In fact, (A) is true for all the value of θ .

Here $0^\circ \leq \theta \leq 90^\circ$. So, (A) is called **trigonometric identity**.

Let us divide (1) by BC^2 , we get

$$\frac{AB^2}{BC^2} + \frac{BC^2}{BC^2} = \frac{AC^2}{BC^2}$$

$$\text{i.e. } \left(\frac{AB}{BC}\right)^2 + 1 = \left(\frac{AC}{BC}\right)^2 \quad \boxed{\tan^2\theta + 1 = \sec^2\theta}$$

Or,

(B)

[Recall $\frac{AB}{BC} = \tan\theta$ and $\frac{AC}{BC} = \sec\theta$]

This is also true for all the values of θ except $\theta = 90^\circ$, I.e. $0^\circ \leq \theta \leq 90^\circ$.

Thus (B) is also a **trigonometric identity**.

Let us divide (1) by AB^2 , we get

$$\frac{AB^2}{AB^2} + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

$$\text{i.e. } \left(\frac{BC}{AB}\right)^2 + 1 = \left(\frac{AC}{AB}\right)^2$$

or,

$$\boxed{\cot^2\theta + 1 = \operatorname{cosec}^2\theta}$$

(C)

[Recall $\frac{BC}{AB} = \cot\theta$ and $\frac{AC}{AB} = \operatorname{cosec}\theta$]

Type equation here.

This is true for all the values of θ such that $0^\circ \leq \theta \leq 90^\circ$.

Thus (C) is again a **trigonometric identity**.

The identities (A), (B) and (C) are basic trigonometric identities. These identities are useful in proving other trigonometric identities and also in expressing one trigonometric ratio in terms of other trigonometric ratios.

We illustrate the use of these trigonometric identities through some examples.

Example 7: Express the ratios

$\tan A$ and $\sin A$ in terms of $\cos A$.

Solution: $\tan A = \frac{\sin A}{\cos A} = \frac{\sqrt{1 - \cos^2 A}}{\cos A}$ [Using identity (A)]

$\sin A = \sqrt{1 - \cos^2 A}$ [Using identity (A)]

Example 8: Let $\cos A = \frac{\sqrt{3}}{2}$. Find the value of

- (1) $\sin A$ (2) $\tan A$

Solution:

(1) we know that

$\sin^2 A + \cos^2 A = 1$ (Identity A)

so, $\sin^2 A = 1 - \cos^2 A$

or $\sin A = \sqrt{1 - \cos^2 A}$

or $\sin A = \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{1 - \frac{3}{4}}$

$= \sqrt{\frac{1}{4}}$

Or $\sin A = \frac{1}{2}$

(Why $\sin A$ takes positive value only ?)

(Hint : $\sin A$ is ratio of length of sides of a triangle)

(2) we know that

$1 + \tan^2 A = \sec^2 A$

(identity (B))

$\tan^2 A = \sec^2 A - 1$

$\tan A = \sqrt{\sec^2 A - 1}$

$= \sqrt{\frac{1}{\cos^2 A} - 1}$

$= \sqrt{\frac{4}{3} - 1}$

$= \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$ (Why $\tan A$ takes positive value only ?)

Example 9: Prove the following identity

$$(1) (\sec\theta + \tan\theta)(1 - \sin\theta) = \cos\theta$$

$$(2) \tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$$

Solution: (1) R.H.S. is already simplified. So, we start with L.H.S.

$$\begin{aligned} \text{L.H.S.} &= (\sec\theta + \tan\theta)(1 - \sin\theta) \\ &= \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)(1 - \sin\theta) \\ &= \left(\frac{1 + \sin\theta}{\cos\theta}\right)(1 - \sin\theta) \\ &= \frac{1 - \sin^2\theta}{\cos\theta} = \frac{\cos^2\theta}{\cos\theta} && \text{[Using identity (A)]} \\ &= \cos\theta \\ &= \text{R.H.S.} \end{aligned}$$

Since L.H.S. = R.H.S., so the identity is true

$$\begin{aligned} (2) \text{ L.H.S.} &= \tan^2\theta - \sin^2\theta \\ &= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta \\ &= \frac{\sin^2\theta - \sin^2\theta \cos^2\theta}{\cos^2\theta} \\ &= \frac{\sin^2\theta(1 - \cos^2\theta)}{\cos^2\theta} \\ &= \left(\frac{\sin^2\theta}{\cos^2\theta}\right)(1 - \cos^2\theta) \\ &= \tan^2\theta \sec^2\theta && \text{[Using identity (A)]} \end{aligned}$$

Example 10: Prove the identity

$$\sec^4\theta(1 - \sin^4\theta) - 2\tan^2\theta = 1$$

$$\begin{aligned} \text{Solution: L.H.S.} &= \sec^4\theta(1 - \sin^4\theta) - 2\tan^2\theta \\ &= \sec^4\theta - \sec^4\theta\sin^4\theta - 2\tan^2\theta \\ &= \sec^4\theta - \frac{\sin^4\theta}{\cos^4\theta} - 2\tan^2\theta \\ &= \sec^4\theta - \tan^4\theta - 2\tan^2\theta \\ &= (\sec^2\theta + \tan^2\theta)(\sec^2\theta - \tan^2\theta) - 2\tan^2\theta \\ & \quad [a^2 - b^2 = (a + b)(a - b)] && \text{[Using identity (B)]} \\ &= (\sec^2\theta + \tan^2\theta)(1) - 2\tan^2\theta \\ &= \sec^2\theta - \tan^2\theta \\ &= 1 = \text{R.H.S.} && \text{[Using identity (B) again]} \end{aligned}$$

Example 11: Prove that

$$\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \operatorname{cosec} \theta - \cot \theta$$

Solution: L.H.S. = $\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \sqrt{\frac{(1 - \cos \theta)(1 - \cos \theta)}{(1 + \cos \theta)(1 - \cos \theta)}}$

$$= \sqrt{\frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}}$$

$$= \frac{1 - \cos \theta}{\sin \theta} \quad \text{[Using identity (A)]}$$

$$= \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta}$$

$$= \operatorname{cosec} \theta - \cot \theta$$

$$= \text{R.H.S.}$$

Example 12: Prove that

$$\tan^2 \theta + \cot^2 \theta + 2 = \sec^2 \theta \operatorname{cosec}^2 \theta$$

Solution: L.H.S. = $\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} + 2$

$$= \frac{\sin^4 \theta \cos^4 \theta}{\cos^2 \theta \sin^2 \theta} + 2$$

$$= \frac{(\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta}{\cos^2 \theta \sin^2 \theta} + 2$$

$$= \frac{1}{\cos^2 \theta \sin^2 \theta} - \frac{2 \sin^2 \theta \cos^2 \theta}{\cos^2 \theta \sin^2 \theta} + 2 \quad \text{[Using identity (A)]}$$

$$= \frac{1}{\cos^2 \theta \sin^2 \theta} - 2 + 2$$

$$= \sec^2 \theta \operatorname{cosec}^2 \theta$$

Alternatively

$$(1) \text{ L.H.S.} = (\tan^2 \theta + 1) + \cot^2 \theta + 1$$

$$= \sec^2 \theta + \operatorname{cosec}^2 \theta \quad \text{[Using identity (B) and (C)]}$$

$$= \frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}$$

$$\begin{aligned}
 &= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \sin^2\theta} \\
 &= \frac{1}{\cos^2\theta \sin^2\theta} \quad \text{[Using identity (A)]}
 \end{aligned}$$

Alternatively

$$\begin{aligned}
 (2) \quad \text{L.H.S} &= \sec^2\theta \operatorname{cosec}^2\theta \\
 &= (\tan\theta + \cot\theta)^2 \quad (\tan\theta \cdot \cot\theta = 1)^2 \\
 &= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2 \\
 &= \left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)^2 \\
 &= \frac{1}{\cos^2\theta\sin^2\theta} \quad \text{(using identity (A))} \\
 &= \sec^2\theta\operatorname{cosec}^2\theta
 \end{aligned}$$

Alternatively

$$\begin{aligned}
 (3) \quad \text{R.H.S.} &= \sec^2\theta \operatorname{cosec}^2\theta \\
 &= \frac{1}{\cos^2\theta\sin^2\theta} \\
 &= \left(\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}\right) \quad \text{(using identity (A))} \\
 &= \frac{\sin^2\theta}{\cos^2\theta\sin^2\theta^2} + \frac{\cos^2\theta}{\cos^2\theta\sin^2\theta^2} \\
 &= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} \\
 &= \sec^2\theta + \operatorname{cosec}^2\theta \\
 &= (1 + \tan^2\theta) + (1 + \cot^2\theta) \quad \text{(using identity (B) and (C))} \\
 &= \tan^2\theta + \cot^2\theta + 2
 \end{aligned}$$

Note : Four different ways are illustrated here to prove the identity Any one Strategy can be used.

Example 13: Prove that $\frac{\cos^2(45^\circ + \theta) + \cos^2(45^\circ - \theta)}{\tan(60^\circ + \theta) \tan(30^\circ - \theta)}$

$$\begin{aligned}
 \text{Solution: L.H.S.} &= \frac{\cos^2(45^\circ + \theta) + \cos^2(45^\circ - \theta)}{\tan(60^\circ + \theta) \tan(30^\circ - \theta)} \\
 &= \frac{\cos^2(45^\circ + \theta) + \sin^2\{90^\circ - (45^\circ - \theta)\}}{\tan(60^\circ + \theta) \cot\{90^\circ - (30^\circ - \theta)\}} \\
 &= \frac{\cos^2(45^\circ + \theta) + \sin^2(45^\circ + \theta)}{\tan(60^\circ + \theta) \cot(60^\circ + \theta)}
 \end{aligned}$$

$$= \frac{1}{\tan(60^\circ + \theta) * \frac{1}{\tan(60^\circ + \theta)}} \quad (\text{using } \sin^2\theta + \cos^2\theta = 1)$$

$$= \frac{1}{1} = 1 = \text{R.H.S.}$$

Example 14: Given $A = 30^\circ$, verify the identities

(1) $\sin 3A = 3 \sin A - 4 \sin^3 A$

(2) $\sin A = \sqrt{\frac{1 - \cos 2A}{2}}$

Solution: (1) L.H.S. = $\sin 3A = \sin (3 * 30)^\circ = \sin 90^\circ = 1$

R.H.S. = $3 \sin A - 4 \sin^3 A = 3 \sin 30^\circ - 4 \sin^3 30^\circ$

$$= 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$$

$$= \frac{3}{2} - \frac{1}{2} = 1$$

So, L.H.S. = R.H.S. and the identity is verified.

(3) L.H.S. = $\sin A = \sin 30^\circ = \frac{1}{2}$

R.H.S. = $\sqrt{\frac{1 - \cos 2A}{2}}$

$$= \sqrt{\frac{1 - \cos 60^\circ}{2}}$$

$$= \sqrt{\frac{1 - \frac{1}{2}}{2}}$$

$$= \sqrt{\frac{1}{4}} = \frac{1}{2}$$

As, L.H.S. = R.H.S. and hence the identity is verified

Example 15: Prove the following identity

$$\frac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1 + \sin\theta}{\cos\theta}$$

Solution: L.H.S. = $\frac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{\tan\theta + (\sec\theta - 1)}{\tan\theta - (\sec\theta - 1)}$

$$= \frac{[\tan\theta + (\sec\theta - 1)] [\tan\theta + (\sec\theta - 1)]}{[\tan\theta - (\sec\theta - 1)] [\tan\theta + (\sec\theta - 1)]}$$

$$= \frac{(\tan\theta + \sec\theta - 1)^2}{\tan^2\theta - (\sec\theta - 1)^2}$$

$$\begin{aligned}
&= \frac{\tan^2\theta + \sec^2\theta + 1 + 2\tan\theta\sec\theta - 2\sec\theta - 2\tan\theta}{\tan^2\theta - (\sec^2\theta + 1 - 2\sec\theta)} \\
&= \frac{2\sec^2\theta + 2\tan\theta\sec\theta - 2\sec\theta - 2\tan\theta}{\tan^2\theta - (1 + \tan^2\theta + 1 - 2\sec\theta)} \quad \text{(Using identity (B))} \\
&= \frac{\sec\theta(\sec\theta - 1) + \tan\theta(\sec\theta - 1)}{\sec\theta - 1} \\
&= \frac{(\sec\theta - 1)(\sec\theta + \tan\theta)}{(\sec\theta - 1)} \\
&= \sec\theta + \tan\theta \\
&= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} \\
&= \frac{1 + \sin\theta}{\cos\theta} = \text{R.H.S.}
\end{aligned}$$

Example 16: If $\sin\theta + \sin^2\theta = 1$, prove that $\cos^2\theta + \cos^4\theta = 1$

Solution: $\sin\theta + \sin^2\theta = 1$

$$\text{Or } \sin\theta = 1 - \sin^2\theta = \cos^2\theta \quad \text{(Using identity (A))}$$

$$\text{Or } \sin^2\theta = \cos^4\theta$$

$$\text{Or } 1 - \cos^2\theta = \cos^4\theta \quad \text{(using identity (A))}$$

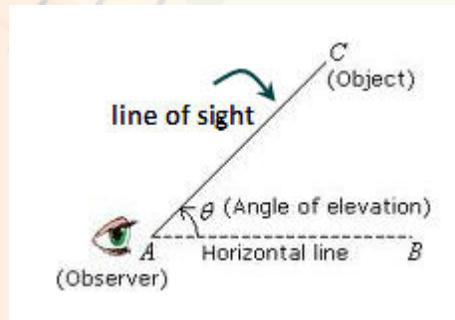
$$\text{So, } \cos^2\theta + \cos^4\theta = 1$$

Applications of Trigonometry

Trigonometry has a number of applications in day today life. For example, the height of a tower, mountain, building or tree, distance of a ship from a light house, width of a river, etc. can be determined by using knowledge of trigonometry. We would explain these applications through some examples.

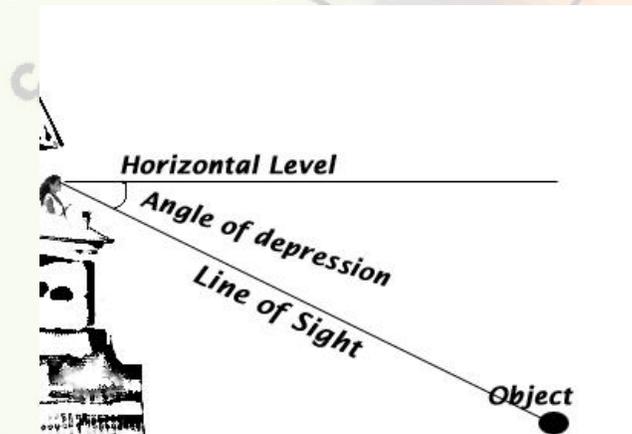
Before we proceed to solve such problems it is necessary to understand some basic terms which we explain below:

Line of sight: Suppose we are viewing an object clearly, the line of sight (or line of vision) to the object is the line from eye to the object, we are viewing (See figure below)



Angle of elevation: If the object is above the horizontal level of the eyes (i.e. if it above the eye-level), we have to turn our head upwards to view the object. In this process, our eyes move through an angle called the angle of elevation of the object from our eyes (See figure above)

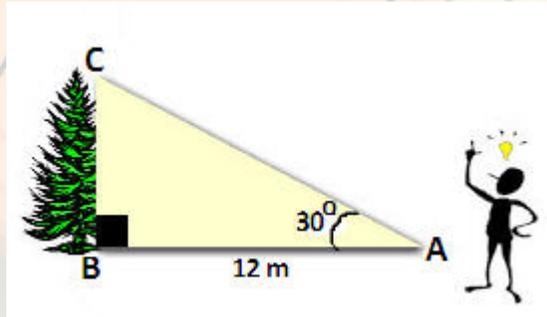
Angle of depression: Suppose a girl, sitting on the balcony of a house, observes an object (ball) lying on the ground at some distance from the building. In this case, she has to move her head downwards to view it. In this process, her eyes again move through an angle. Such an angle is called the angle of depression of the object. (see figure below)



Now, let us consider some problems involving heights and distances.

Example 17: A person standing on the ground is observing the top of a tree at a distance 12m from the foot of the tree. If an angle of elevation of the top of the tree is 30° , find the height of the tree.

Solution: Look at the figure:



In the right angle triangle ABC, here

$$AB = 12\text{m},$$

$$\angle CAB = 30^\circ.$$

We have to find BC, in order to find height of the tree.

Let $BC = h$ metre.

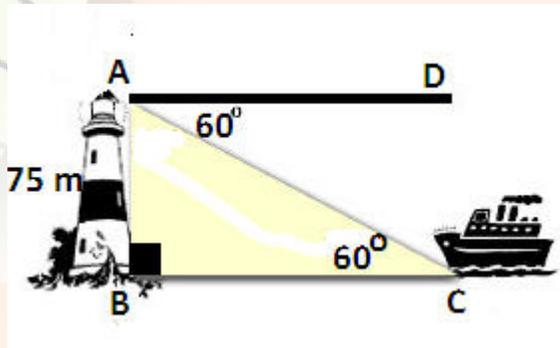
$$\text{We } \frac{BC}{BA} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\text{So, } BC = (BA) \frac{1}{\sqrt{3}} = 12 * \frac{1}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

Hence, the height of the tree is $4\sqrt{3}$ metres.

Example 18: As observed from the top of a 75m high light house from the sea-level, the angle of depression is 60° . Find the distance of the ship from the base of light house.

Solution: Let AB be the light house and the ship is at C.



Here, $\angle DAC =$ angle of depression. $= 60^\circ$

So, $\angle ACB = 60^\circ$ (Alternate angles)

$$AB = 75\text{m}$$

Let $BC = x$ metres.

$$\text{Now } \frac{AB}{BC} = \tan 60^\circ = \sqrt{3}$$

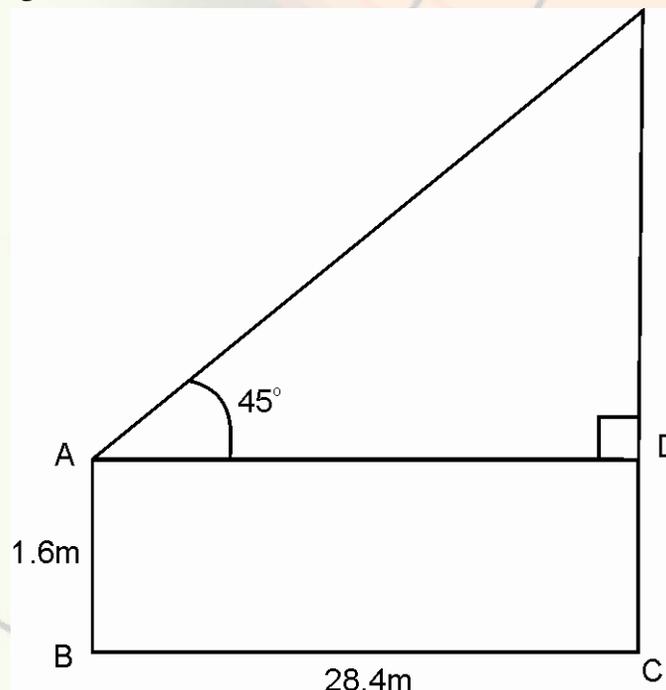
$$\text{Or } \frac{75}{x} = \sqrt{3}$$

$$\text{Or } x = \frac{75}{\sqrt{3}} = \frac{75\sqrt{3}}{3} = 25\sqrt{3}$$

Hence, the distance of the ship from the base of light house = $25\sqrt{3}$ m.

Example 19: An observer 1.6 m tall is standing on the ground and is 28.4 m away from a tower. The angle of elevation of the top of the tower from his eyes is 45° . What is the height of the tower?

Solution: See the figure.



Here, EC is the tower, AB is the observer.

We have to find EC and for this we first find ED .

In right triangle ADE

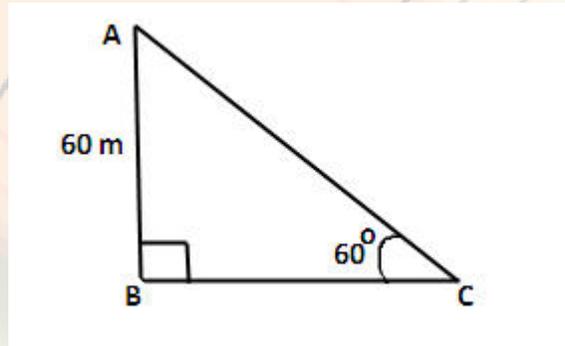
$$\frac{ED}{AD} = \tan 45^\circ = 1$$

So, $ED = AD = BC = 28.4$ m

Now height of tower = $EC = ED + DC = (28.4 + 1.6)$ m = 30m.

Example 20: A kite is flying at a height of 60m from the level ground, attached to a string inclined at 60° to the horizontal. Find the length of the string.

Solution: See the figure:



Here, AB is the height of the kite from the ground.

AC is the length of string and the angle of elevation i.e $\angle ACB = 60^\circ$

We have to find the length of string AC.

Let AC = x metres.

$$\text{Now, in } \triangle ABC, \frac{AB}{x} = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\text{i.e. } \frac{60}{x} = \frac{\sqrt{3}}{2}$$

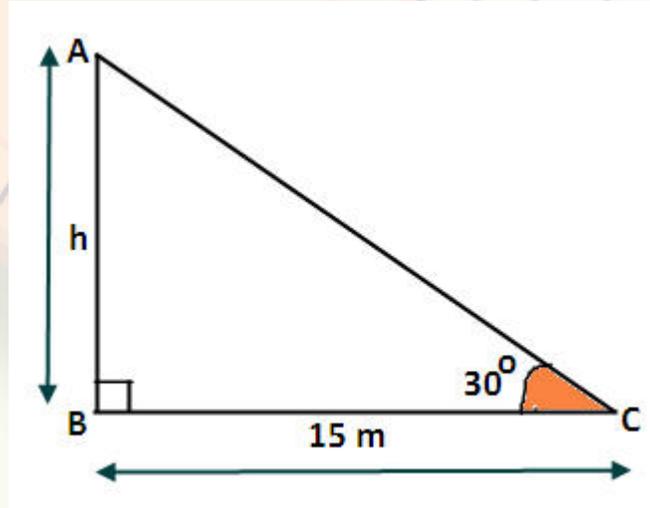
$$\text{or } \sqrt{3}x = 120$$

$$\text{or } x = \frac{120}{\sqrt{3}} = \frac{120\sqrt{3}}{3} = 40\sqrt{3}.$$

Hence, the length of string is $40\sqrt{3}$ metres.

Example 21: The shadow of a pole standing on the level ground is 15 metres when sun's altitude is 30° . Find the height of the pole.

Solution: See the figure.



Here, BC is the shadow of the pole when sun's altitude is 30° .

So, $BC = 15\text{m}$

Let AB be the height of the pole and let $AB = h$ metres.

From $\triangle ABC$, $\frac{AB}{BC} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

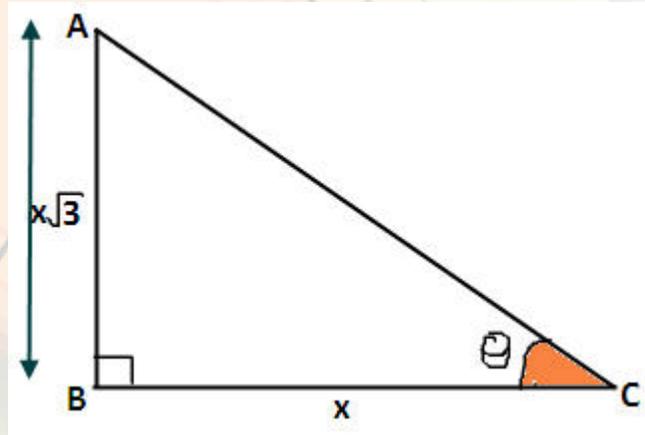
$$\frac{h}{15} = \frac{1}{\sqrt{3}}$$

$$\text{or } h = \frac{15}{\sqrt{3}} = \frac{15\sqrt{3}}{3} = 5\sqrt{3}.$$

Hence, the height of the pole is $5\sqrt{3}$.

Example 22: Find the angle of elevation of the sun when the length of the tree is $\sqrt{3}$ times its shadow. Find the sun's elevation?

Solution: See the figure;



Here, AB is height of the tree.

And BC is its shadow.

Let angle of elevation be θ .

So, $\angle ACB = \theta$.

Let BC = x metres.

So, AB = $x\sqrt{3}$

Now, from $\triangle ABC$,

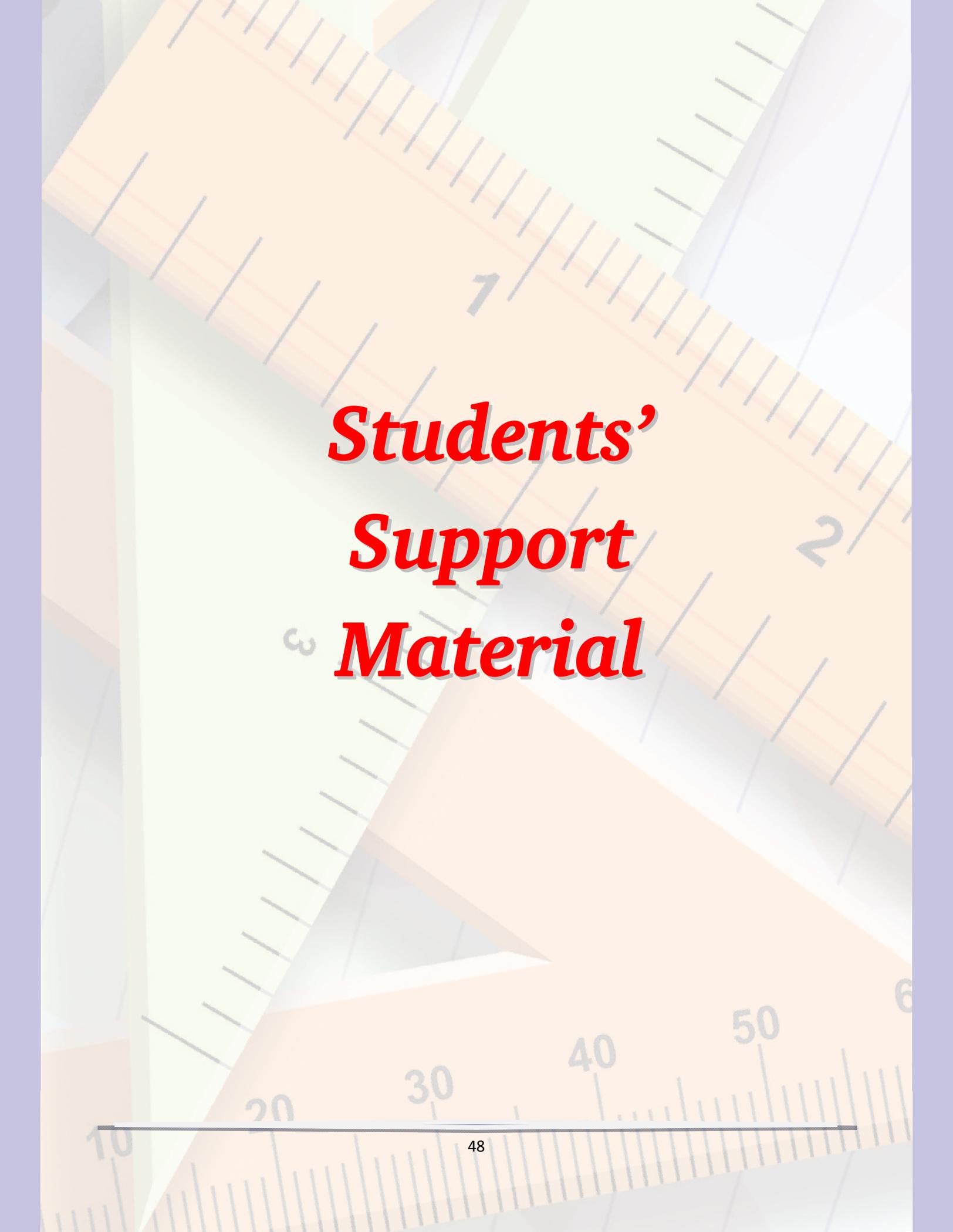
$$\frac{AB}{BC} = \tan\theta.$$

$$\frac{x\sqrt{3}}{x} = \tan\theta.$$

$$\sqrt{3} = \tan\theta.$$

So, $\theta = 60^\circ$

Hence, the sun's elevation = 60° .

The background of the page features several overlapping rulers and a protractor. The rulers are in various colors, including orange, yellow, and light blue, and are positioned at different angles. The protractor is a light blue semi-circle. The rulers have markings in millimeters and centimeters, with some larger numbers like 1, 2, 3, 20, 30, 40, 50, and 60 visible. The overall effect is a sense of precision and measurement.

***Students'
Support
Material***

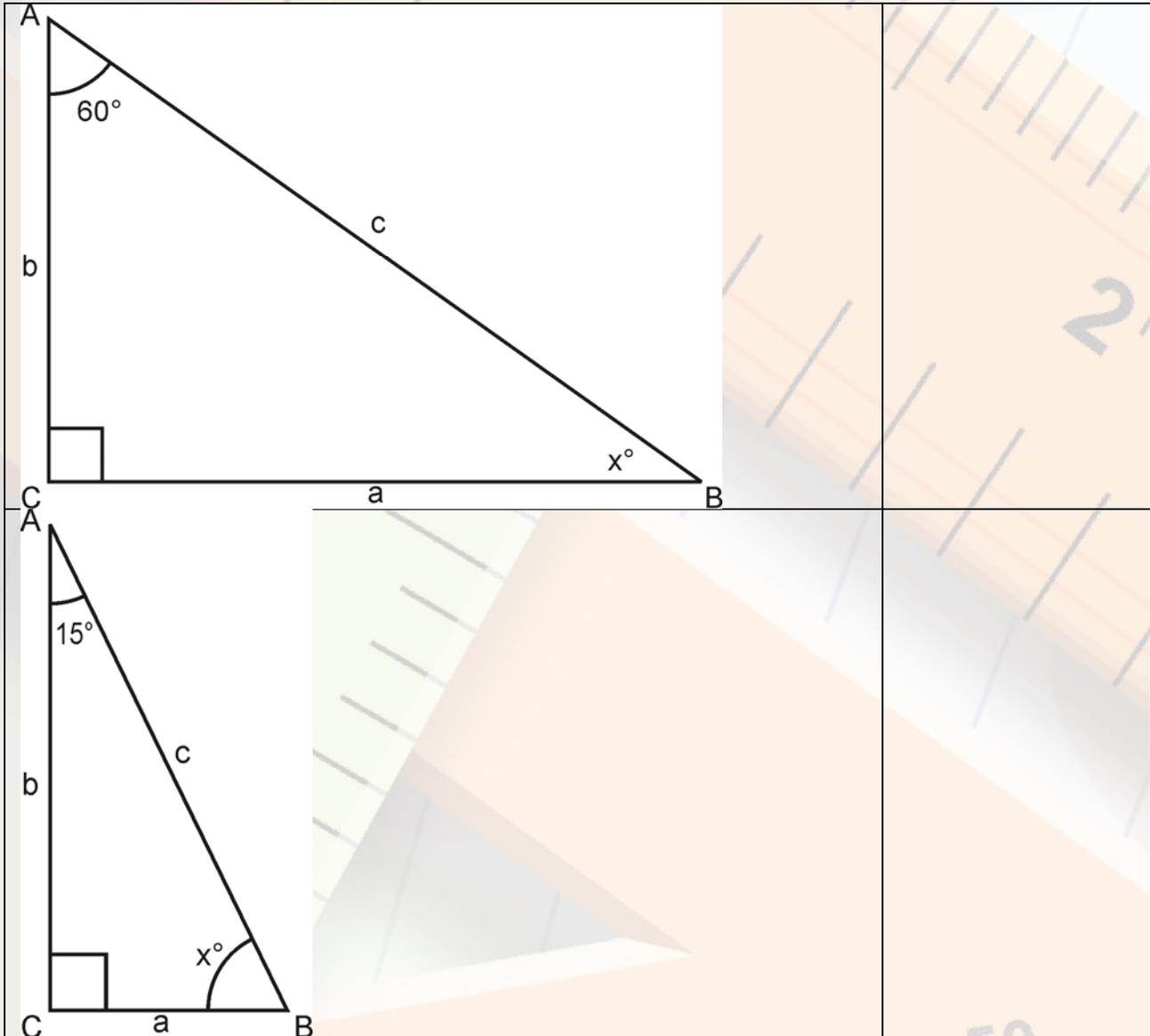
Student's Worksheet 1

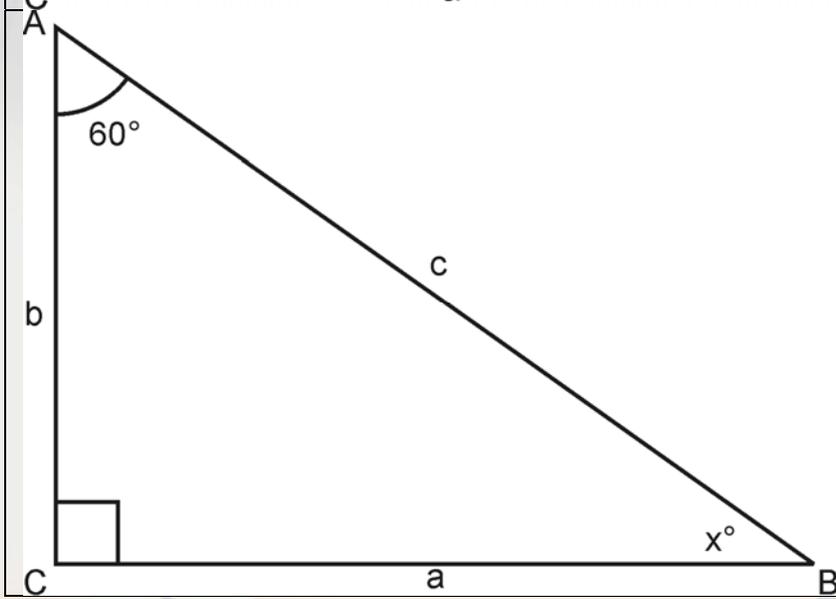
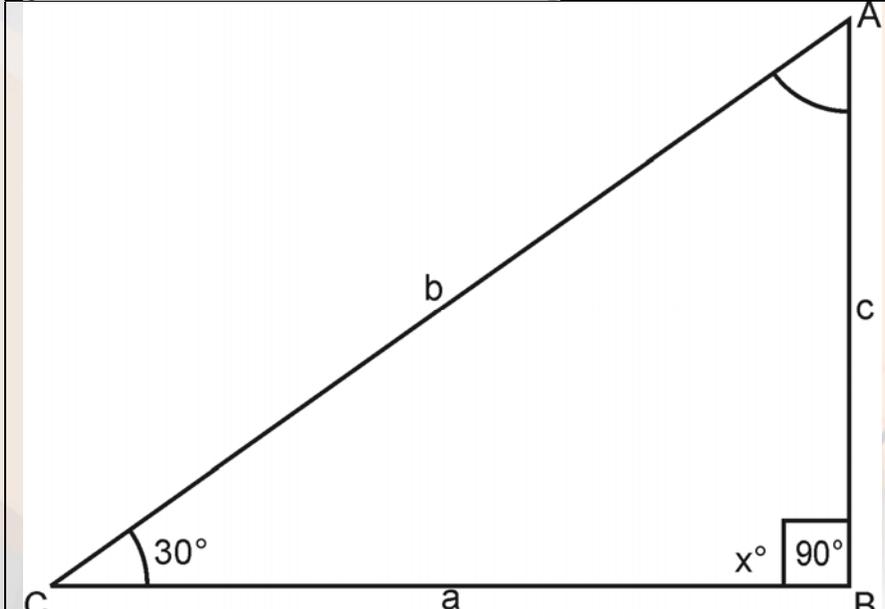
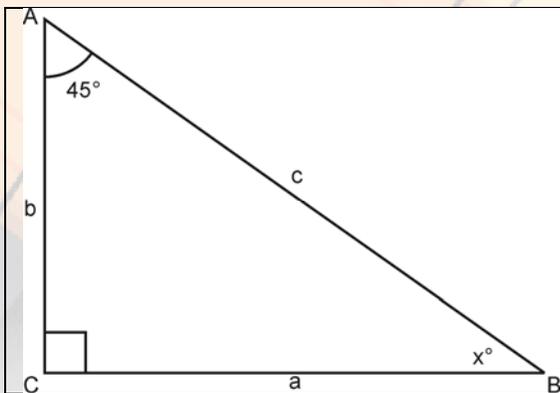
Warm up (W1)

Name of Student _____

Date _____

Find the value of x and write the relation between a , b and c .





Self Assessment Rubric 1 – Warm up (W1)

<input type="checkbox"/>	No Understanding
<input style="width: 15px; height: 15px;" type="checkbox"/>	Understanding of concept but not able to apply
<input style="width: 15px; height: 15px;" type="checkbox"/>	Understanding of concept, can apply but commit errors in calculation
<input style="width: 15px; height: 15px;" type="checkbox"/>	Understanding of concept, can apply accurately

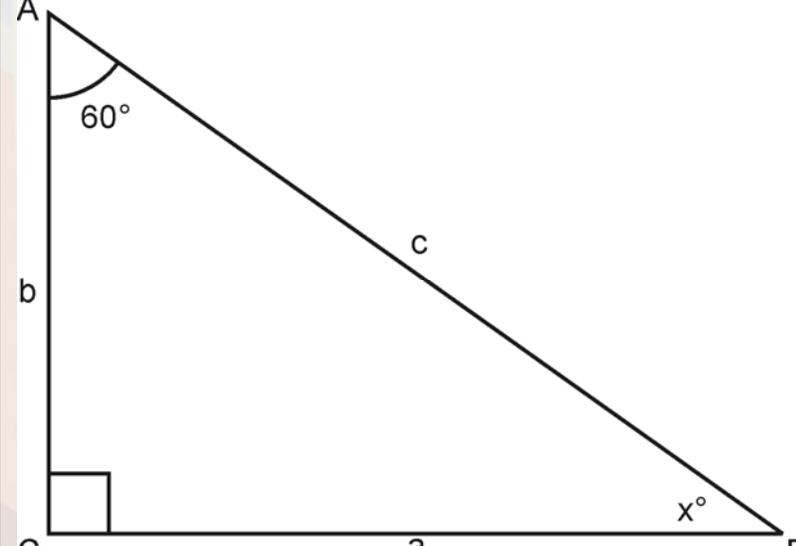
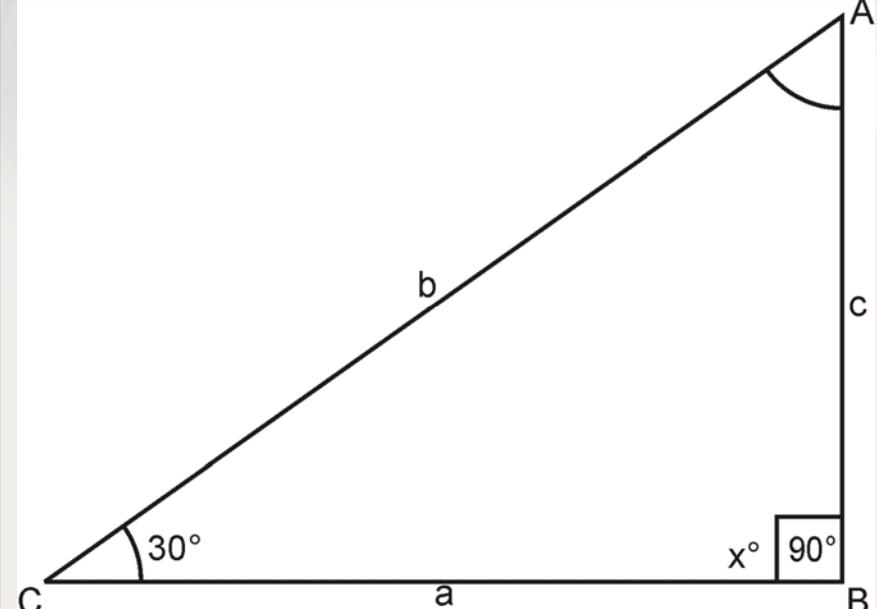
Parameter	<input style="width: 20px; height: 20px;" type="checkbox"/>	<input style="width: 20px; height: 20px;" type="checkbox"/>	<input style="width: 20px; height: 20px;" type="checkbox"/>	<input type="checkbox"/>
Has knowledge of Pythagoras theorem	<input style="width: 20px; height: 20px;" type="checkbox"/>	<input style="width: 20px; height: 20px;" type="checkbox"/>	<input style="width: 20px; height: 20px;" type="checkbox"/>	<input type="checkbox"/>
Can find angles in a triangle using angle sum property	<input style="width: 20px; height: 20px;" type="checkbox"/>	<input style="width: 20px; height: 20px;" type="checkbox"/>	<input style="width: 20px; height: 20px;" type="checkbox"/>	<input type="checkbox"/>

Student's Worksheet 2 Warm Up (W2)

Name of Student _____

Date _____

For the following right triangle fill in the blanks appropriately w.r.t to given angle

	<p>Adjacent side w.r.t $\angle B$ =</p> <p>Opposite side w.r.t $\angle B$ =</p> <p>Hypotenuse =</p>
	<p>Adjacent side w.r.t $\angle A$ =</p> <p>Opposite side w.r.t $\angle A$ =</p> <p>Hypotenuse =</p>

Self Assessment Rubric 2 – Warm up (W2)

<input type="checkbox"/>	No Understanding
<input style="background-color: green;" type="checkbox"/>	Understanding of concept but not able to apply
<input style="background-color: green; border: 1px solid black;" type="checkbox"/>	Understanding of concept, can apply but commit errors in calculation
<input style="background-color: green; border: 1px solid black;" type="checkbox"/>	Understanding of concept, can apply accurately

Parameter	<input style="background-color: green; border: 1px solid black;" type="checkbox"/>	<input style="background-color: green; border: 1px solid black;" type="checkbox"/>	<input style="background-color: green;" type="checkbox"/>	<input type="checkbox"/>
Has knowledge of terms adjacent side w.r.t an angle, opposite side w.r.t an angle and hypotenuse in a right triangle				

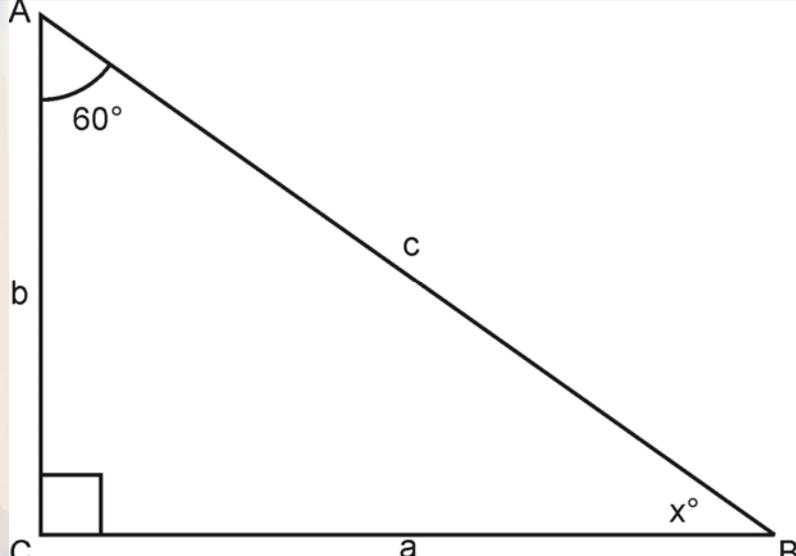
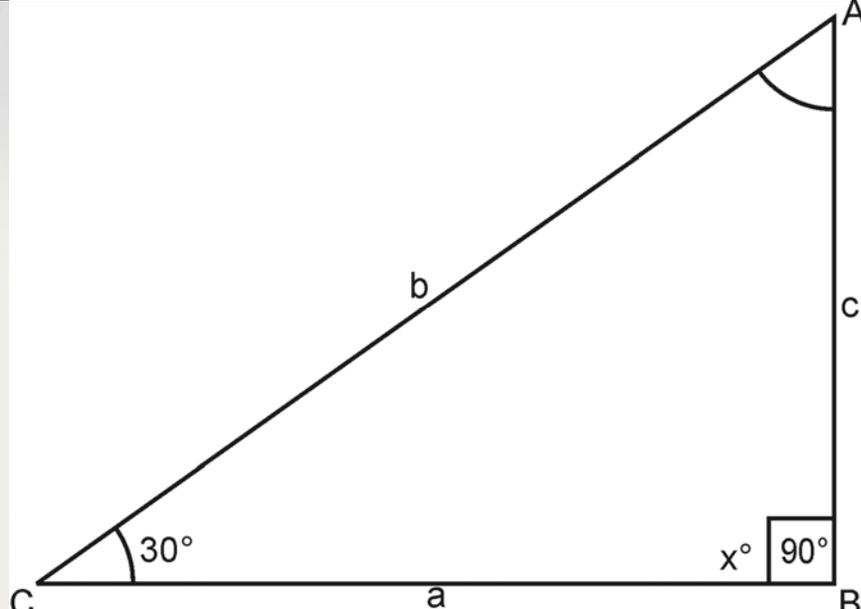
Student's Worksheet 3

Warm up (W3)

Name of Student _____

Date _____

Write the asked trigonometric ratios in the following figures.

	<p>$\sin x =$</p> <p>$\cos x =$</p> <p>$\tan x =$</p> <p>$\sec x =$</p> <p>$\operatorname{cosec} x =$</p> <p>$\cot x =$</p>
	<p>$\sin x =$</p> <p>$\cos x =$</p> <p>$\tan x =$</p> <p>$\sec x =$</p> <p>$\operatorname{cosec} x =$</p> <p>$\cot x =$</p>

Self Assessment Rubric 3 – Warm up (W3)

<input type="checkbox"/>	No Understanding
<input style="width: 20px; height: 20px;" type="checkbox"/>	Understanding of concept but not able to apply
<input style="width: 20px; height: 20px;" type="checkbox"/>	Understanding of concept, can apply but commit errors in calculation
<input style="width: 20px; height: 20px;" type="checkbox"/>	Understanding of concept, can apply accurately

Parameter	<input type="checkbox"/>	<input style="width: 20px; height: 20px;" type="checkbox"/>	<input style="width: 20px; height: 20px;" type="checkbox"/>	<input type="checkbox"/>
Can define T-ratios in a right triangle	<input type="checkbox"/>	<input style="width: 20px; height: 20px;" type="checkbox"/>	<input style="width: 20px; height: 20px;" type="checkbox"/>	<input type="checkbox"/>

Student's Worksheet 4 **Pre Content (P1)**

Name of Student _____

Date _____

Complete the following table

	0°	30°	45°	60°	90°
Sin					
Cos					
Tan					
Cot					
Sec					
Cosec					

Write True or False

1. $\sin 30^\circ = \cos 60^\circ$
2. $\sin 45^\circ = \cos 45^\circ$
3. $\sin 60^\circ = \cos 30^\circ$
4. $\sin 60^\circ = \cos 60^\circ$
5. $\sin 30^\circ = \cos 30^\circ$
6. $\tan 30^\circ = \cot 60^\circ$
7. $\tan 45^\circ = \cot 60^\circ$
8. $\tan 45^\circ = \cot 45^\circ$
9. $\sec 30^\circ = \cot 60^\circ$
10. $\operatorname{cosec} 30^\circ = \sec 60^\circ$

Fill in the blanks

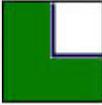
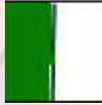
1. The angle θ between 0° and 90° at which the value of $\sin\theta$ and $\cos\theta$ coincides is -----
2. The angle θ between 0° and 90° at which the value of $\tan\theta$ and $\cot\theta$ coincides is -----
3. The angle θ between 0° and 90° at which the value of $\sec\theta$ and $\operatorname{cosec}\theta$ coincides is -----
4. $\sin 30^\circ \cdot \cos 30^\circ =$ -----
5. $\sin 45^\circ \cdot \cos 45^\circ =$ -----
6. $\tan 30^\circ \cdot \cot 30^\circ =$ -----
7. $\sin 60^\circ \cdot \cos 30^\circ =$ -----
8. $\sin^2 30^\circ \cdot \cos^2 60^\circ =$ -----
9. $\sin 60^\circ \cdot \cos 60^\circ =$ -----
10. $\tan^2 30^\circ \cdot \cot^2 60^\circ =$ -----

Without using tables, find the value of

$$\frac{\tan 30^\circ \cot 30^\circ + \sec 30^\circ \cdot \cos 30^\circ}{\sin 30^\circ \operatorname{cosec} 30^\circ}$$

Self Assessment Rubric 4 – Pre Content (P1)

	No Understanding
	Understanding of concept but not able to apply
	Understanding of concept, can apply but commit errors in calculation
	Understanding of concept, can apply accurately

Parameter				
Can write and tell values of T ratios at specific angles				
Can find relation between T ratios at specific angle				

Student's Worksheet 5

Pre Content (P2)

Name of Student _____

Date _____

1. Correctly Match the T ratios given in the first box with the expressions in the second box.

$\sin\theta$	$\cos\theta$	$\tan\theta$
$\sec\theta$	$\operatorname{cosec}\theta$	$\cot\theta$

$\frac{1}{\cos\theta}$	$\frac{1}{\cot\theta}$	$\frac{1}{\operatorname{cosec}\theta}$
$\frac{1}{\sin\theta}$	$\frac{\sin\theta}{\cos\theta}$	$\frac{1}{\sec\theta}$
$\frac{1}{\tan\theta}$	$\frac{\cos\theta}{\sin\theta}$	

2. Fill in the blanks

$$\sin(90 - \theta) =$$

$$\cos(90 - \theta) =$$

$$\tan(90 - \theta) =$$

$$\cot(90 - \theta) =$$

$$\sec(90 - \theta) =$$

$$\operatorname{cosec}(90 - \theta) =$$

3.
$$\frac{\sin^2 45^\circ + \cos^2 45^\circ}{\tan^2 60^\circ}$$

Self Assessment Rubric 5 – Pre Content (P2)

<input type="checkbox"/>	No Understanding
<input style="background-color: green;" type="checkbox"/>	Understanding of concept but not able to apply
<input style="background-color: green; border: 1px solid black;" type="checkbox"/>	Understanding of concept, can apply but commit errors in calculation
<input style="background-color: green; border: 1px solid black;" type="checkbox"/>	Understanding of concept, can apply accurately

Parameter	<input style="background-color: green; border: 1px solid black;" type="checkbox"/>	<input style="background-color: green; border: 1px solid black;" type="checkbox"/>	<input style="background-color: green;" type="checkbox"/>	<input type="checkbox"/>
Knows T- ratios				
Can find relation between T- ratios				

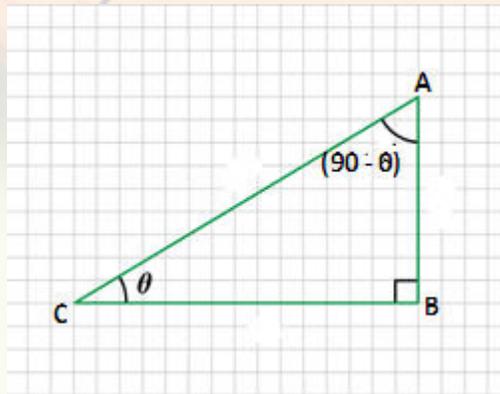
Student's Worksheet 6, Complimentary Angles Content (CW1)

Name of Student _____

Date _____

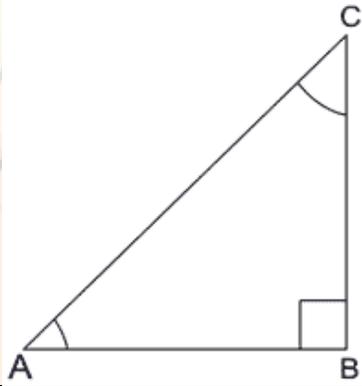
Solve the following questions and write reason wherever required.

- Using the given figure, complete the following table
-



$\sin \theta =$	$\sin (90^\circ - \theta) =$	
$\cos \theta =$	$\cos (90^\circ - \theta) =$	
$\tan \theta =$	$\tan (90^\circ - \theta) =$	
$\operatorname{cosec} \theta =$	$\operatorname{cosec} (90^\circ - \theta) =$	
$\sec \theta =$	$\sec (90^\circ - \theta) =$	
$\cot \theta =$	$\cot (90^\circ - \theta) =$	

3. Write reasons for the following statements

<p>In $\triangle ABC$, $\angle B = 90^\circ$</p> <p>$\angle C = 90^\circ - \angle A$ (Why?)</p>	
<p>$\sin A = BC / AC$ (Why?) $\operatorname{cosec} A = AC / BC$</p> <p>$\cos A = AB / AC$ (Why?) $\sec A = AC / AB$</p> <p>$\tan A = BC / AB$ (Why?) $\cot A = AB / BC$</p>	
<p>$\sin C = \sin (90^\circ - A)$ (Why?) $\operatorname{cosec} C = \operatorname{cosec} (90^\circ - A)$</p> <p>$\cos C = \cos (90^\circ - A)$ (Why?) $\sec C = \sec (90^\circ - A)$</p> <p>$\tan C = \tan (90^\circ - A)$ (Why?) $\cot C = \cot (90^\circ - A)$</p>	

4. Do you agree with the following results. Justify by taking suitable value for angle A.

$$\sin (90^\circ - A) = \cos A$$

$$\tan(90^\circ - A) = \cot A$$

$$\sec(90^\circ - A) = \operatorname{cosec} A$$

$$\cos (90^\circ - A) = \sin A$$

$$\cot(90^\circ - A) = \tan A$$

$$\operatorname{cosec} (90^\circ - A) = \sec A$$

5. Express $\sin 89^\circ + \tan 89^\circ$, in terms of trigonometric ratios of angles between 0° and 45° .

6. Simplify

a) $\frac{\sin 27^\circ}{\cos 63^\circ}$

b) $\frac{\sec 27^\circ}{\operatorname{cosec} 63^\circ}$

c) $\frac{\tan 27^\circ}{\cot 63^\circ}$

d) $\cos 48^\circ - \sin 42^\circ$

e) $\tan 48^\circ \cdot \tan 23^\circ \cdot \tan 42^\circ \cdot \tan 67^\circ$

Self Assessment Rubric 6 – Content (CW1)

<input type="checkbox"/>	No Understanding
<input style="width: 15px; height: 15px; background-color: green;" type="checkbox"/>	Understanding of concept but not able to apply
<input style="width: 15px; height: 15px; background-color: green; border: 1px solid black;" type="checkbox"/>	Understanding of concept, can apply but commit errors in calculation
<input style="width: 15px; height: 15px; background-color: green; border: 1px solid black;" type="checkbox"/>	Understanding of concept, can apply accurately

Parameter	<input style="width: 20px; height: 20px; background-color: green;" type="checkbox"/>	<input style="width: 20px; height: 20px; background-color: green; border: 1px solid black;" type="checkbox"/>	<input style="width: 20px; height: 20px; background-color: green; border: 1px solid black;" type="checkbox"/>	<input type="checkbox"/>
Knowledge of T-ratios of complementary angles				
Able to derive T-ratios of complementary angles				

Student's Worksheet 7, Use of T-ratios Content (CW2)

Name of Student _____

Date _____

Use the knowledge of T-Ratios and solve the following.

1. Given $\cot\theta = \frac{20}{21}$, determine $\cos\theta$ and $\operatorname{cosec}\theta$.

2. If $\sec\theta = \frac{25}{7}$, find the values of $\tan\theta$ and $\operatorname{cosec}\theta$.

In $\triangle ABC$, right angled at B, if $AB = 12\text{cm}$, $BC = 5\text{cm}$, find (i) $\sin A$
(ii) $\cot C$

3. If $\cos\theta = \frac{1}{3}$, evaluate $\frac{\sin\theta - \cot\theta}{2\tan\theta}$.

4. Given $\cos\theta = \frac{21}{29}$, determine the value of $\frac{\sec\theta}{\tan\theta - \sin\theta}$.

5. If $\tan A = \sqrt{2} - 1$, show that $\sin A \cdot \cos A = \frac{\sqrt{2}}{4}$.

6. If $\operatorname{cosec}\theta = 2$, find the value of $\cot A + \frac{\sin\theta}{1 + \cos\theta}$.

7. If $\sec\theta = \frac{5}{4}$, evaluate $\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$.

8. If $\sec\theta = \frac{13}{5}$, show that $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$.

9. If $\sin A = \frac{1}{3}$, evaluate $\cos A \operatorname{cosec} A + \tan A \sec A$.

10. In $\triangle ABC$, right angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of $\cos A \cos C - \sin A \sin C$

Self Assessment Rubric 7 – Content (CW2)

<input type="checkbox"/>	No Understanding
<input style="width: 100%; height: 100%;" type="checkbox"/>	Understanding of concept but not able to apply
<input style="width: 100%; height: 100%;" type="checkbox"/>	Understanding of concept, can apply but commit errors in calculation
<input style="width: 100%; height: 100%;" type="checkbox"/>	Understanding of concept, can apply accurately

Parameter	<input style="width: 100%; height: 100%;" type="checkbox"/>	<input style="width: 100%; height: 100%;" type="checkbox"/>	<input style="width: 100%; height: 100%;" type="checkbox"/>	<input type="checkbox"/>
Knowledge of T-ratios				
Able to solve right triangle				
Finding other T-ratios when any one is given				

Student's Worksheet 8, Trigonometry Identities Content (CW3)

Name of Student _____

Date _____

1. Use the trigonometric identities for proving the following. Write reasons at every step.

1. $(1 - \sin^2\theta) \sec\theta = 1$

2. $(1 - \cos\theta)(1 + \cos\theta)(1 + \cos^2\theta) = 1$

3. $(\sec^2\theta - 1)(\operatorname{cosec}^2\theta - 1) = 1$

4. $\sqrt{\operatorname{cosec}^2\theta - 1} = \cos\theta \operatorname{cosec}\theta$

5. $\sqrt{\frac{1 - \sin^2\theta}{1 - \cos^2\theta}} = \cot\theta$

2. Which of the following statements are identities?

(a) $\sin\theta + \cos\theta = 1$

(b) $\sin^2\theta + \cos^2\theta = 1$

What could be the possible number of ways to find out which statements are identities?

Self Assessment Rubric 8 – Content (CW3)

<input type="checkbox"/>	No Understanding
<input style="width: 100%; height: 100%;" type="checkbox"/>	Understanding of concept but not able to apply
<input style="width: 100%; height: 100%;" type="checkbox"/>	Understanding of concept, can apply but commit errors in calculation
<input style="width: 100%; height: 100%;" type="checkbox"/>	Understanding of concept, can apply accurately

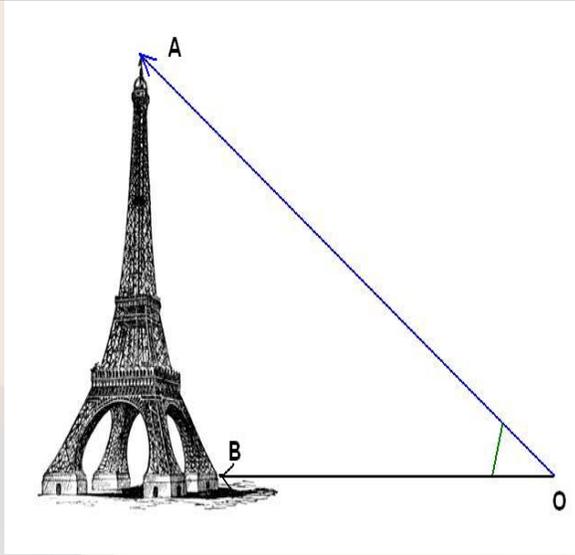
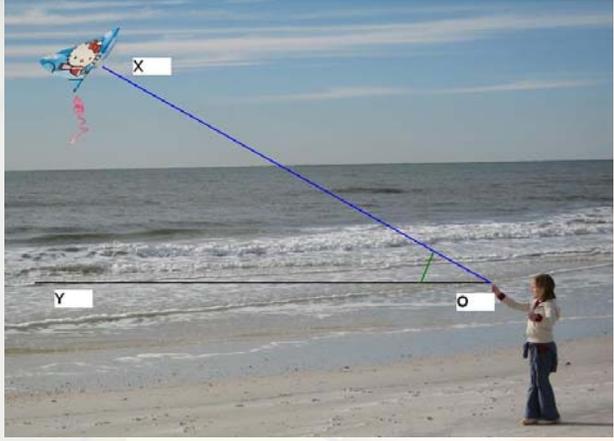
Parameter	<input style="width: 100%; height: 100%;" type="checkbox"/>	<input style="width: 100%; height: 100%;" type="checkbox"/>	<input style="width: 100%; height: 100%;" type="checkbox"/>	<input type="checkbox"/>
Can apply the standard identities to prove other trigonometric identities				
Can recognize the identities				

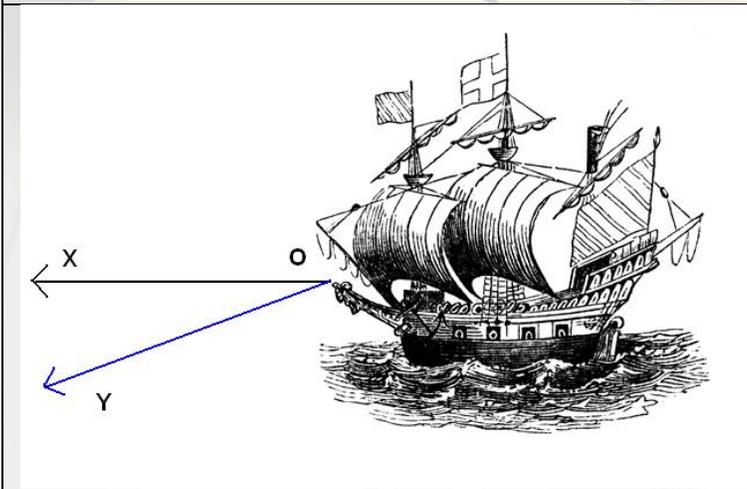
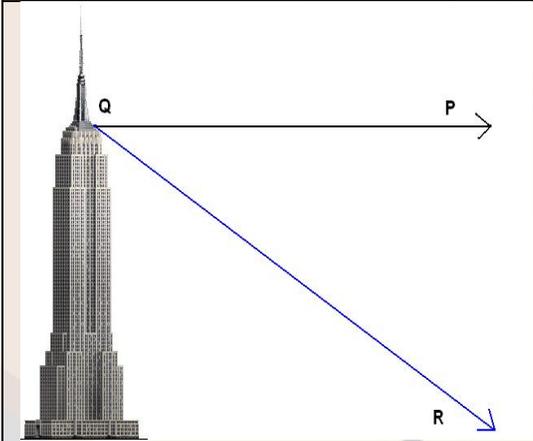
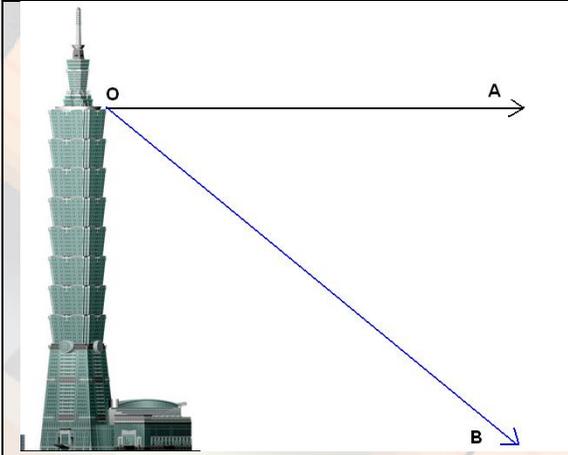
Student's Worksheet 9, Angle of Elevation/Depression Content (CW4)

Name of Student _____

Date _____

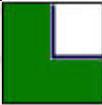
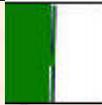
In the following figures label the angle of elevation / angle of depression and line of sight.

Figure	Angle of Elevation	Angle of depression	Line of Sight
			
			



Self Assessment Rubric 9 – Content (CW4)

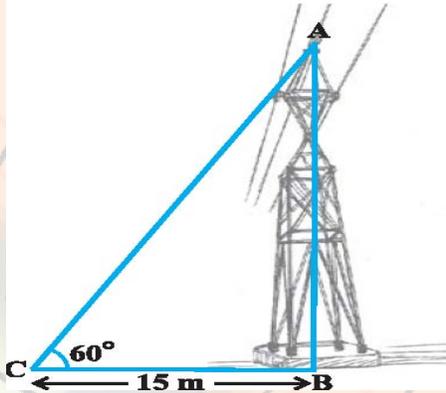
	No Understanding
	Understanding of concept but not able to apply
	Understanding of concept, can apply but commit errors in calculation
	Understanding of concept, can apply accurately

Parameter				
Labeling angle of elevation				
Labeling angle of depression				
Labeling line of sight				

Student's Worksheet 10, Representation of Situations through Right Angled Triangles

Content (CW5)

For each problem given below, draw a figure to illustrate the situation. The first one is done for you:

<p>A tower stands vertically on the ground. From a point on the ground, which is 15 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°.</p>	
<p>An observer 1.5 m tall is 28.5 m away from a chimney. The angle of elevation of the top of the chimney from her eyes is 45°.</p>	
<p>From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°.</p>	
<p>The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than</p>	

when it is 60° .	
The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45° , respectively.	
From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° , respectively. The bridge is at a height of 3 m from the banks.	
A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m.	
A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. The angle made by the rope with the ground level is 30° .	

Self Assessment Rubric 10 – Content (CW5)

<input type="checkbox"/>	No Understanding
<input type="checkbox"/>	Understanding of concept but not able to apply
<input type="checkbox"/>	Understanding of concept, can apply but commit errors in calculation
<input type="checkbox"/>	Understanding of concept, can apply accurately

Parameter	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Able to draw figure for the given word problem and explain	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Student's Worksheet 11, Application of Trigonometry in Daily Life

Content (CW6)

Solve the given problems using trigonometry.

Use the template:

Draw the diagram.	
What is given?	
What is to be determined?	
Formula used	
Solution	

1. A vertical stick 10 cm long casts a shadow 8 cm long. At the same time, a tower casts a shadow 30 m long. Determine the height of the tower.
2. A vertically straight tree, 15 m high is broken by the wind in such a way that its top just touches the ground and makes an angle of 60° with the ground, at what height from the ground did the tree break?
3. Solve all the problems of content worksheet 10 (CW5)

Self Assessment Rubric 11 – Content (CW6)

<input type="checkbox"/>	No Understanding
<input checked="" type="checkbox"/>	Understanding of concept but not able to apply
<input checked="" type="checkbox"/>	Understanding of concept, can apply but commit errors in calculation
<input checked="" type="checkbox"/>	Understanding of concept, can apply accurately

Parameter	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
Able to draw figure for the given word problem.				
Able to apply knowledge of trigonometric ratios and angles to solve the problems.				

Student's Worksheet 12

Post Content (PCW1)

Name of Student _____

Date _____

Section 1

- Find the values of
 - $\cos 90^\circ + \cos 245^\circ \cdot \sin 30^\circ \tan 45^\circ$
 - $\frac{\sin 30^\circ - \sin 90^\circ + 2\cos 0^\circ}{\tan 30^\circ \cdot \tan 60^\circ}$
 - $2\sqrt{2} \cos 45^\circ \cos 60^\circ + 2\sqrt{3} \sin 30^\circ \tan 60^\circ - \cos 0^\circ$
 - $\frac{\sin 60^\circ}{\cos^2 45^\circ} - 3\tan 30^\circ + 5\cos 90^\circ$
 - $\sqrt{\frac{1 - \cos^2 30^\circ}{1 - \sin^2 30^\circ}}$
- If $x = 30^\circ$, verify that $\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

Section 2

Prove the following identities from 1 to 6. Give reasons wherever required.

1.
$$\frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta}$$

2.
$$\frac{1}{\operatorname{cosec} \theta + \cot \theta} = \operatorname{cosec} \theta - \cot \theta$$

3.
$$\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta} = (\sec \theta + \tan \theta)^2 = \left(\frac{1 + \sin \theta}{\cos \theta}\right)^2$$

$$4. \quad \frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta - 1} + \frac{\operatorname{cosec} \theta}{\operatorname{cosec} \theta + 1} = 2 \sec^2 \theta$$

$$5. \quad \sec \theta (1 + \sin \theta) (\sec \theta - \tan \theta) = 1$$

$$6. \quad \frac{1 + \cos \theta}{1 - \cos \theta} = \frac{\tan^2 \theta}{(\sec \theta - 1)^2}$$

Student's Worksheet 13

Post Content (PCW2)

Name of Student _____

Date _____

Without using trigonometric tables, find the value of

$$i. \quad \frac{2 \sin 68^\circ}{\cos 22^\circ} - \frac{2 \cot 15^\circ}{5 \tan 75^\circ} = \frac{3 \tan 45^\circ \tan 20^\circ \tan 40^\circ \tan 50^\circ \tan 70^\circ}{5}$$

$$ii. \quad \sec^2 10^\circ - \cot^2 80^\circ + \frac{\sin 15^\circ \cos 75^\circ + \cos 15^\circ \sin 75^\circ}{\cos \theta \sin(90 - \theta) + \sin \theta \cos(90 - \theta)}$$

$$iii. \quad \frac{\sec^2(90 - \theta) - \cot^2 \theta}{2(\sin^2 25^\circ + \sin^2 65^\circ)} + \frac{2 \cos^2 60^\circ \tan^2 28^\circ \tan^2 62^\circ}{3(\sec^2 43^\circ - \cot^2 47^\circ)}$$

$$iv. \quad \frac{11 \sin 70^\circ}{7 \cos 20^\circ} - \frac{4 \cos 53^\circ \operatorname{cosec} 37^\circ}{7 \tan 15^\circ \tan 35^\circ \tan 55^\circ \tan 75^\circ}$$

Student's Worksheet 14

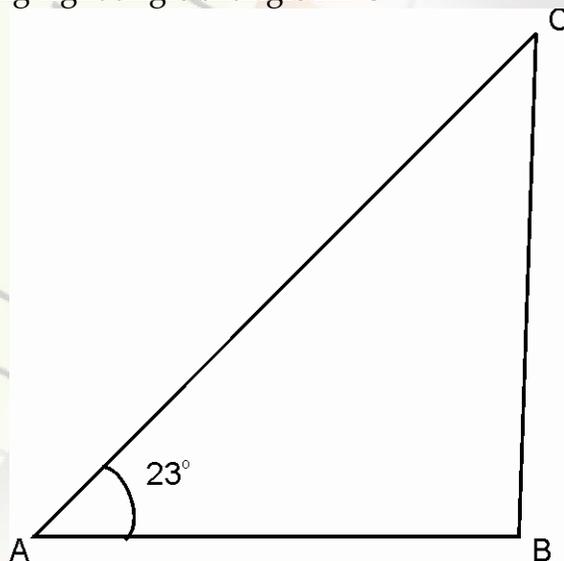
Post Content (PCW3)

Name of Student _____

Date _____

Use the knowledge of trigonometric ratios in right triangle and solve the following problems. Draw the diagrams, explain it and then solve.

1. The shadow of the flash staff is three times as long as the shadow of a flagstaff when the sun rays met the ground at an angle of 60° . Find the angle between the sun's rays and the ground at the time of longer shadow.
2. For the following right angle triangle ABC



frame a problem situation to be solved using trigonometric ratios.

Student's Worksheet 15

Post Content (PCW3)

Name of Student_____

Date_____

Make a clinometers using the instructions given below and use it to find the height of any taller object. Record your observations.

Instruction Sheet:

Objective

To make a clinometer and use it to measure the height of an object

Materials required: Stiff card, small pipe or drinking straw, thread, a weight (a metal washer is ideal).

Pre-requisite knowledge

Properties of right angled triangles.

Procedure

(A) To make clinometer:

1. Prepare a semi-circular protractor using any hard board and fix a viewing tube (straw or pipe) along the diameter.
2. Punch a hole (o) at the centre of the semicircle.
3. Suspend a weight {w} from a small nail fixed to the centre.
4. Ensure that the weight at the end of the string hangs below the protractor.

5. Mark degrees (in sexagesimal scale with 0° at the lowest and 10 to 90° proceeding both clockwise and anticlockwise). [Fig 1].

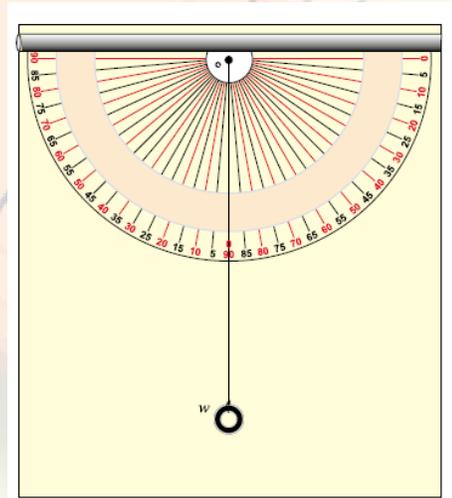


Fig 1

(B) To determine the height of an object:

6. First measure the distance of the object from you. Let the distance be d .
7. Look through the straw or pipe at the top of the object. Make sure you can clearly see the top of the object.
8. Hold the clinometer steady and let your partner record the angle the string makes on the scale of the clinometer. Let this angle be θ .

Observations

Using trigonometric ratio :

$$\tan \theta = \text{height} / \text{distance} = h/d$$

$$h = d \times \tan \theta$$

If, for example, $d = 100$ m and $\theta = 45^\circ$

$$h = 100 \times \tan 45^\circ = 100 \text{ m}$$

References

Extra Reading

Sides and angles of a right triangle

http://www.mathsisfun.com/right_angle_triangle.html

Naming sides of a right triangle

http://www.mathsteacher.com.au/year10/ch15_trigonometry/01_ratios/22sides.htm

Pythagorean Theorem <http://www.grc.nasa.gov/WWW/K-12/airplane/pythag.html>

Angle of elevation and depression

http://www.mathsteacher.com.au/year10/ch15_trigonometry/12_elevation_depression/23elevdep.htm

<http://mathcentral.uregina.ca/QQ/database/QQ.09.03/anjum1.html>

<http://www.thatquiz.org/tq/previewtest?F/C/Z/B/84221300663838>

Application of trigonometry

<http://www.syvum.com/cgi/online/serve.cgi/math/trigo/trigonometry3.html>

Making a clinometers <http://mykhmsmathclass.blogspot.com/2007/07/making-clinometer.html>

Useful Online Links

Naming sides of a right triangle

<http://mykhmsmathclass.blogspot.com/2009/06/video-naming-sides-of-right-triangle.html>

Trigonometric ratio <http://mykhmsmathclass.blogspot.com/2009/06/video-trigonometric-ratios.html>

Finding T ratios at 30 degrees and 60 degrees

<http://mykhmsmathclass.blogspot.com/2009/06/video-finding-trigonometric-ratios-at.html>

Finding T ratios at 45 degrees <http://mykhmsmathclass.blogspot.com/2009/06/video-finding-t-ratio-at-45-degrees.html>

T ratios of complementary angles (various types of problems)

<http://mykhmsmathclass.blogspot.com/2009/06/finding-day-of-given-date.html>

Problems using T identities <http://mykhmsmathclass.blogspot.com/2009/06/videos-problems-using-trigonometric.html>

Angle of elevation and depression

<http://mykhmsmathclass.blogspot.com/2009/06/video-angle-of-elevation-and-depression.html>