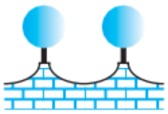
#464251

Topic: Combination of Solids



The front compound wall of a house is decorated by wooden spheres of a diameter 21cm, placed on small supports as shown in the figure. Eight such spheres are used for this purpose and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2

Solution

$r = \frac{21}{2} = 10.5$cm, SA $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times (10.5)^2$

$= 1386$cm^2

Cylindrical support,

r_1 of circular end $= 1.5$cm, $h = 7$cm

Curved Surface Area $= 2\pi rh = 2 \times \frac{22}{7} \times 1.5 \times 7 = 66$cm^2

Area of circular end $= \pi r^2 = \frac{22}{7} \times (1.5)^2 = 7.07$ cm^2

Area to be painted silver $= 8 \times (1386 - 7.07)$cm^2

$= 8 \times 1378.9 = 11031.44$cm^2

Cost of silver color $=$ Rs.0.25×11031.4

$=$ Rs.2757.8

Area to be painted black $= 8 \times 66 = 578$cm^2

Cost of painting black color $= 1528 \times 0.05$

$=$ Rs.26.40

Total cost $=$ Rs.$2757.8 + 26.4$

$=$ Rs.2784.2

#465129

Topic: Combination of Solids

2 cubes each of volume $64cm^3$ are joined end to end. Find the surface area of the resulting cuboid.

Solution

Volume of cubes = 64 cm^3

$(Edge)^3 = 64$

Edge $= 4$ cm

If cubes are joined end to end, the dimensions of the resulting cuboid will be

4 cm, 4 cm, 8 cm.

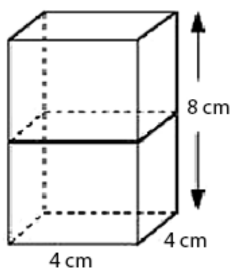
Surface area of cuboids $= 2(lb + bh + lh)$

$= 2(4 \times 4 + 4 \times 8 + 4 \times 8)$

$= 2(16 + 32 + 32)$

$= 2(16 + 64)$

$= 2(80) = 160\ cm^2$



#465130

Topic: Combination of Solids

A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.

Answer: 572

Solution

The radius(r) of the cylindrical part and the hemispherical part is the same (i.e., 7 cm)

Height of hemispherical part = Radius $= 7$ cm

Height of cylindrical part (h) $= 13 - 7 = 6$ cm

Inner surface are of the vessel = CSA of cylindrical part + CSA of hemispherical part

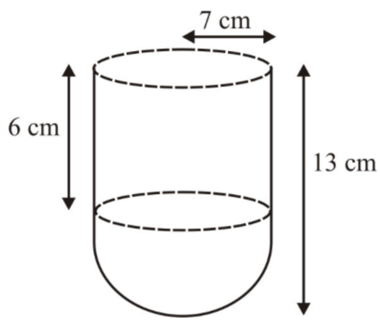
$= 2\pi rh + 2\pi r^2$

Inner surface area of vessel = $2 \times \frac{22}{7} \times 7 \times 6 + 2 \times \frac{22}{7} \times 7 \times 7$

$= 44(6 + 7)$

$= 44 \times 13$

$= 572 cm^2$



#465131

Topic: Combination of Solids

A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy rounded off to the nearest integer.

Answer: 215

Solution

As per the question we can draw the diagram,

Given:

The radius of the conical part and the hemispherical part is same (i.e.,3.5 cm)

Height of the hemispherical part, r $= 3.5 = \frac{7}{2}$

Height of conical part, $h = 15.5 - 3.5 = 12$ cm

Slant height(l) of conical part = $\sqrt{r^2 + h^2}$

$= \sqrt{\left(\frac{7}{2}\right)^2 + 12^2} = \sqrt{\frac{49}{4} + 144} = \sqrt{\frac{49 + 576}{4}}$

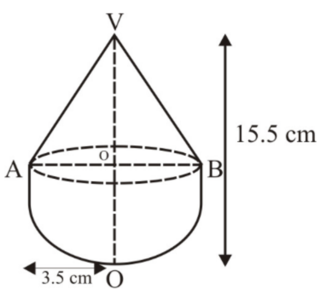
$= \sqrt{\frac{625}{4}} = \frac{25}{2}$

Total surface area of toy = CSA of conical part + CSA of hemispherical part

$= \pi rl + 2\pi r^2$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{25}{2} + 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 214.7$



#465132
Topic: Combination of Solids

A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Solution

As per the question we can draw the diagram,

The diagram shows that the greatest diameter possible for such hemisphere is equal to the cubes edge, i.e., 7cm.

Radius (r) of hemispherical part = $\frac{7}{2}$ = 3.5cm

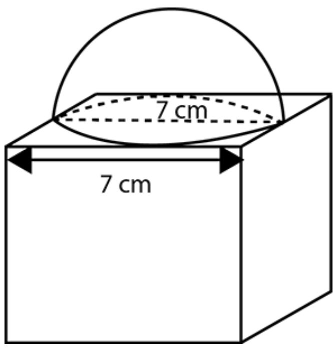
Total surface area of solid = Surface area of cubical part + CSA of hemispherical part Area of base of hemispherical part

$=6(Edge)^2 + 2\pi r^2 - \pi r^2 = 6(Edge)^2 + \pi r^2$

$= 6(7)^2 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 294 + 38.5$

$= 332.5\ cm^2$



#465133
Topic: Combination of Solids

A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Solution

Given:

Diameter of hemisphere = Edge of the cube $= l$

Radius of hemisphere = $\frac{l}{2}$

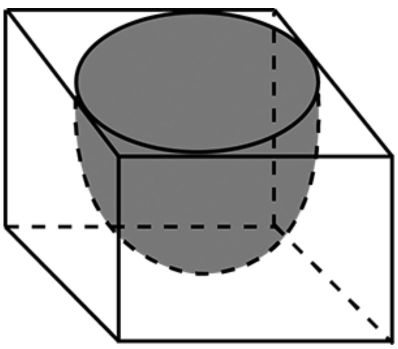
Total surface area of solid = Surface area of cubical part + CSA of hemisphere part - Area of base of hemispherical part

$= 6(Edge)^2 + 2\pi r^2 - \pi r^2 = 6(Edge)^2 + \pi r^2$

Total surface are of solid = $6l^2 + \pi \times \left(\frac{l}{2}\right)^2$

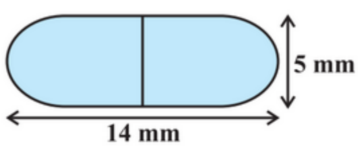
$= 6l^2 + \frac{\pi l^2}{4}$

$= \frac{1}{4}[24 + \pi]l^2\ unit^2$



#465134

Topic: Combination of Solids



A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends .The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area. (Use $\pi = \frac{22}{7}$).

Answer: 220

Solution

Given:

Length of Capsule (l) = 14 mm

Diameter of Capsule = Diameter of Cylinder $= 5$ mm

Radius = $\frac{Diameter}{2}$

Therefore, Radius of each Hemisphere = Radius of Cylinder = r = $\frac{5}{2}$ = 2.5 mm

Length of Cylinder = AB = Total length of Capsule - Radius of left Hemisphere - Radius of Right Hemisphere

$= 14 - 2.5 - 2.5 = 9$ mm

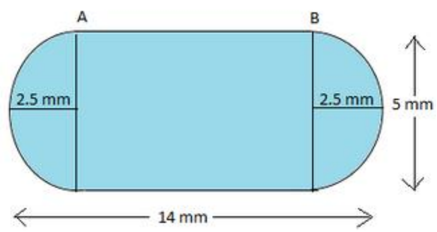
Surface Area of Capsule = Curved Surface Area of Cylinder + Surface Area of Left Hemisphere + Surface Area of Right Hemisphere

$= 2\pi rl + 2\pi r^2 + 2\pi r^2$

$= 2\pi rl + 4\pi r^2$

$= 2 \times \frac{22}{7} \times 2.5 \times 9 + 4 \times \frac{22}{7} \times 2.5^2$

$= \frac{22}{7}[45 + 25] = \frac{22}{7} \times 70 = 220$ mm^2



#465135

Topic: Combination of Solids

A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2. (Note that the base of the tent will not be covered with canvas.) (Use $\pi = \frac{22}{7}$).

Solution

Given:

Height (h) of the cylindrical part = 2.1 m

Diameter of the cylindrical part = 4 m

Radius of the cylindrical part = 2 m

Slant height (l) of conical part = 2.8 m

Area of canvas used = CSA of conical part + CSA of cylindrical part

$= \pi rl + 2\pi rh$

$= \pi \times 2 \times 2.8 + 2\pi \times 2 \times 2.1$

$= 2\pi[2.8 + 4.2]$

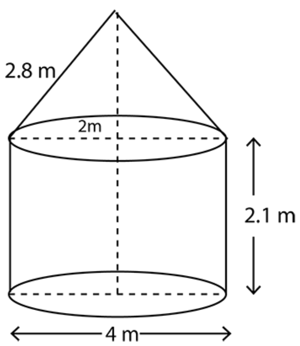
$= 2 \times \frac{22}{7} \times 7$

$= 44 m^2$

Cost of 1 m ^{2} canvas $= 500$ rupees

Cost of 44 m ^{2} canvas $= 44 \times 500 = 22000$ rupees.

Therefore, it will cost 22000 rupees for making such a tent.



#465136

Topic: Combination of Solids

From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2. (Use $\pi = \frac{22}{7}$).

Solution

Given:

Height (h) of the conical part = Height (h) of the cylindrical part $= 2.4$ cm

Diameter of the cylindrical part $= 1.4$ cm

Radius = $\frac{Diameter}{2}$

Radius(r) of the cylindrical part $= 0.7$ cm

Slant height (l) of conical part = $\sqrt{r^2 + h^2}$

$$= \sqrt{0.7^2 + 2.4^2} = \sqrt{0.49 + 5.76} = \sqrt{6.25} = 2.5$$

Total surface area of the remaining solid = CSA of cylindrical part + CSA of conical part + Area of cylindrical base

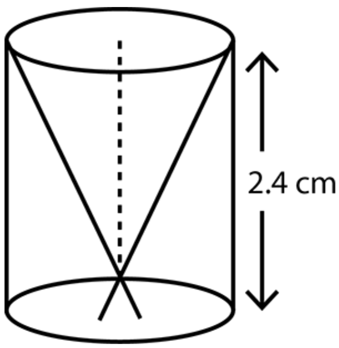
$$= 2\pi rh + \pi rl + \pi r^2$$

$$= 2 \times \frac{22}{7} \times 0.7 \times 2.4 + \frac{22}{7} \times 0.7 \times 2.5 + \frac{22}{7} \times 0.7 \times 0.7$$

$$= 4.4 \times 2.4 + 2.2 \times 2. + 2.2 \times 0.7$$

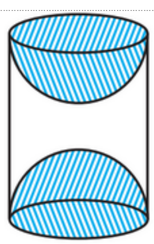
$$= 10.56 + 5.50 + 1.54 = 17.60 \text{ cm}^2$$

The total surface area of the remaining solid to the nearest cm^2 is 18 cm^2



#465137

Topic: Combination of Solids



A wooden article was made by scooping out a hemisphere from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface of the article.

Answer: 374

Solution

Given:

Height of Cylinder $= h = 10$ cm

Radius of Cylinder = Radius of Hemispheres $= r = 3.5$ cm

Surface area of article = CSA of cylindrical part + 2 x CSA of hemispherical part

$= 2\pi rh + 2 \times 2\pi r^2$

$= 2\pi \times 3.5 \times 10 + 2 \times 2\pi \times 3.5^2$

$= 70\pi + 49\pi$

$= 119\pi = 119 \times \dfrac{22}{7}$

$= 17 \times 22$

$= 374$ cm ^{2}

#465173

Topic: Combination of Solids

A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Solution

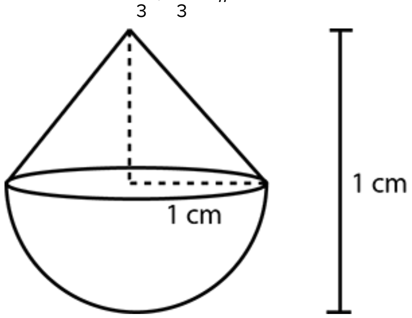
Given:

Height (h) of conical part = radius (r) of conical part $= 1$ cm

Radius (r) of hemispherical part = radius of conical part $= 1$ cm

Volume of solid = Volume of conical part + Volume of hemispherical part

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$
$$= \frac{1}{3}\pi \times 1^2 \times 1 + \frac{2}{3}\pi \times 1^3$$
$$= \frac{1}{3}\pi \times 1 + \frac{2}{3}\pi \times 1$$
$$= \frac{\pi}{3} + \frac{2\pi}{3} = \pi \text{ cm}^3$$



#465174

Topic: Combination of Solids

Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

Answer: 66

Solution

For the given statement first draw a diagram,

In this diagram, we can observe that

Height (h_1) of each conical part $= 2$ cm

Height (h_2) of cylindrical part $12 - 2 - 2 = 8$ cm

Radius (r) of cylindrical part = Radius of conical part = $\frac{3}{2}$ cm

Volume of air present in the model = Volume of cylinder $+ \ 2 \times$ Volume of a cone

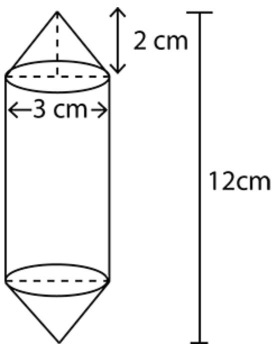
$= \pi r^2 h_2 + 2 \times \pi r^2 h_1$

$= \pi\left(\frac{3}{2}\right)^2 \times 8 + 2 \times \frac{1}{3}\pi\left(\frac{3}{2}\right)^2(2)$

$= \pi \times \frac{9}{4} \times 8 + \frac{2}{3}\pi \times \frac{9}{4} \times 2$

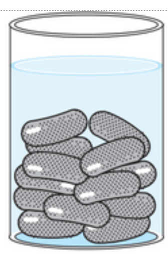
$= 18\pi + 3\pi = 21\pi$

$21 \times \frac{22}{7} = 66 \text{ cm}^2$



#465175

Topic: Combination of Solids



A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm .

Solution

Given:

Radius (r) of cylindrical part = Radius (r) of hemispherical part = $\frac{2.8}{2}$ = 1.4 cm

Length of each hemispherical part = Radius of hemispherical paart = 1.4 cm

Length (h) of cylindrical part = $5 - 2 \times$ length of hemispherical part

$= 5 - 2 \times 1.4 = 2.2$ cm

Volume of one gulab jamun = Volume of cylindrical part + 2 x Volume of hemispherical part

$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \pi r^2 h + \frac{4}{3}\pi r^3$

$= \pi \times (1.4)^2 \times 2.2 + \frac{4}{3}\pi(1.4)^3$

$= \frac{22}{7} \times 1.4 \times 1.4 \times 2.2 + \frac{4}{3} \times \frac{22}{7} \times 1.4 \times 1.4 \times 1.4$

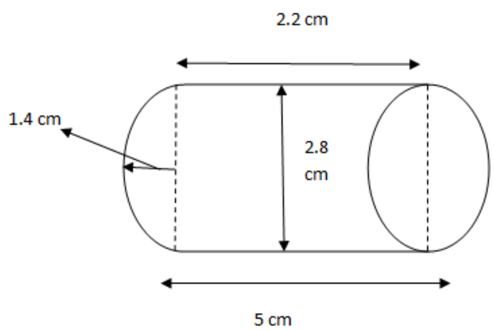
$= 13.552 + 11.498 = 25.05$ cm^3

So, Volume of 45 gulab jamuns $= 45 \times 25.05 = 1,127.25$cm^3

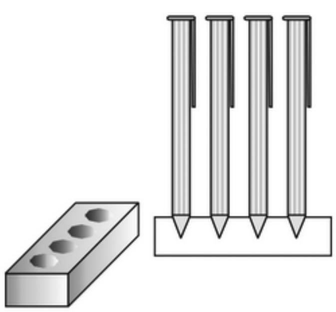
Volume of sugar syrup = 30% of volume

$= \frac{30}{100} \times 1,127.25$

$= 338.17$ cm $^3 \approx 338$ cm^3



#465176
Topic: Combination of Solids



A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand .

Solution

Depth (h) of each conical depression $= 1.4$ cm

Radius (r) of each conical depression $= 0.5$ cm

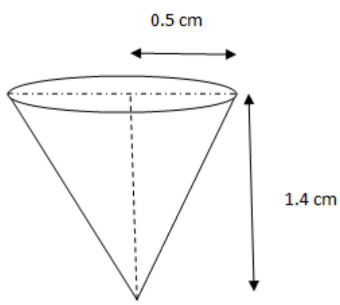
Volume of wood = Volume of cuboid - 4 x Volume of cones

$= l \times b \times h - 4 \times \frac{1}{3}\pi r^2 h$

$= 15 \times 10 \times 3.5 - 4 \times \frac{1}{3} \times \frac{22}{7} \times (\frac{1}{2})^2 \times 1.4$

$= 525 - 1.47$

$= 523.53 \ cm^3$



#465177
Topic: Combination of Solids

A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Solution

Given:

Height (h) of conical vessel $= 8$ cm

Radius (r_1) of conical vessel $= 5$ cm

Radius (r_2) of lead shots $= 0.5$ cm

Let x number of lead shots were dropped in the vessel.

Volume of the cone $= \frac{1}{3}\pi(r_1)^3$

Volume of water spilled = Volume of dropped lead shots

According to the question

$\frac{1}{4} \times \frac{1}{3}\pi r_1^2 h = x \times \frac{4}{3} \times \pi r_2^3$

$\Rightarrow r_1^2 h = x \times 16 r_2^3$

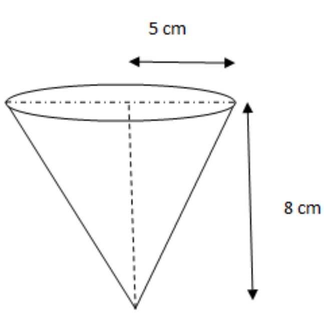
$\Rightarrow 5^2 \times 8 = x \times 16 \times (0.5)^3$

$\Rightarrow 200 = x \times 2$

$\Rightarrow x = \frac{200}{2}$

$x = 100.$

Therefore, the number of lead shots dropped in the vessel is 100.



#465178

Topic: Combination of Solids

A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. (Use $\pi = 3.14$)

Solution

Height (h_1) of larger cylinder = 220 cm

Radius (r_1) of larger cylinder = $\dfrac{24}{2}$ = 12 cm

Height (h_2) of smaller cylinder = 60 cm

Radius (r_2) of smaller cylinder =8 cm

Total volume of pole = Volume of larger cylinder + Volume of smaller cylinder

$= \pi r_1^2 h_1 + \pi r_2^2 h_2$

$= (\pi(12)^2 \times 220) + (\pi(8)^2 \times 60)$

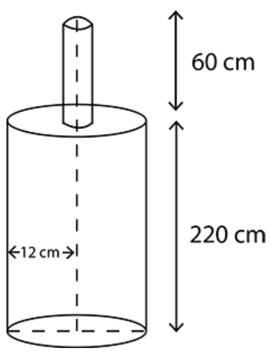
$\pi[144 \times 220 + 64 \times 60]$

$= 3.14[31,680 + 3,840]$

$= 3.14 \times 35520 = 111,532.8$ cm ^{3}

Mass of 1 cm^3 iron = 8 g

Mass of 111532.8 cm ^{3} iron = 11532.8 × 8 = 892262.4 g



#465179

Topic: Combination of Solids

A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Solution

Given:

Radius (r) of hemispherical part = Radius (r) of conical part $= 60$cm

Height (h_1) of conical part of solid $= 120$ cm

Height (h_1) of cylinder $= 180$ cm

Radius (r) of cylinder $= 60$ cm

Volume of water left = Volume of cylinder - Volume of soild

= Volume of cylinder - (Volume of cone + Volume of hemisphere)

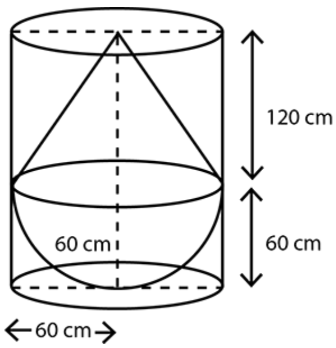
$= \pi r^2 h_1 - [\frac{1}{3}\pi r^2 h_2 + \frac{2}{3}\pi r^3]$

$= \pi \times 60^2 \times (180) - [\frac{1}{3}\pi \times 60^2 \times 120 + \frac{2}{3}\pi \times 60^3]$

$= \pi(60)^2[180 - (40 + 40)]$

$= \pi(3600)[100]$

$= 3,60,000 \times \frac{22}{7} = 1131428.57$ cm^3



#465180

Topic: Combination of Solids

A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and π = 3.14.

Solution

Given:

Height (h) of cylindrial part $= 8$ cm

Radius (r_2) of cylindrical part = $\frac{2}{2} = 1$ cm

Radius (r_1) of spherical part = $\frac{8.5}{2} = 4.25$ cm

Volume of vessel = Volume of sphere + Volume of cylinder

$$= \frac{4}{3}\pi r_1^3 + \pi r_2^2 h$$

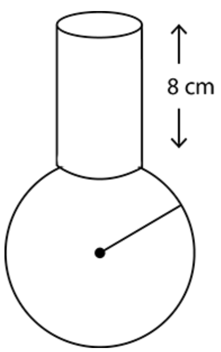
$$= \frac{4}{3}\pi\left(\frac{8.5}{2}\right)^3 + \pi(1)^2(8)$$

$$= \frac{4}{3} \times 3.14 \times 76.765625 + 8 \times 3.14$$

$$= 321.392 + 25.12$$

$$= 346.51 \text{ cm}^3$$

So, the child measurement is wrong.



#465181
Topic: Combination of Solids

A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.

Solution

Given:

Radius (r_1) of hemisphere $= 4.2$ cm

Radius (r_2) of cylinder $= 6$ cm

Height (h) = ?

The object fromed by recasting the hemisphere will be same in volume.

So, Volume of sphere = Volume of cylinder

$$\frac{4}{3}\pi r_1^3 = \pi r_2^2 h$$

$$\Rightarrow \frac{4}{3}\pi \times (4.2)^3 = \pi(6)^2 h$$

$$\Rightarrow \frac{4}{3} \times \frac{4.2 \times 4.2 \times 4.2}{36} = h$$

$h = (1.4)^3 = 2.74$ cm

Therefore, the height of cylinder so formed will be 2.74 cm.

#465182

Topic: Combination of Solids

Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Solution

Given:

Radius (r_1) of first sphere $= 6$ cm

Radius (r_2) of second sphere $= 8$ cm

Radius (r_3) of third sphere $= 10$ cm

Radius of the resulting sphere $= r$

The object formed by recasting these spheres will be same in volume as the sum of the voulmes of these spheres.

So, Volume of 3 spheres = Volume of resulting shape

$\frac{4}{3}\pi[r_1^3 + r_2^3 + r_3^3] = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{4}{3}\pi[6^3 + 8^3 + 10^3] = \frac{4}{3}\pi r^3$

$\Rightarrow 216 + 512 + 1000 = r^3$

$\Rightarrow 1728 = r^3$

$r = 12$ cm

Therefore, the radius of sphere so formed will be 12 cm.

#465183

Topic: Combination of Solids

A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Solution

The shape of the well will be cylindrical as shown in the figure below:

Given:

Depth (h) of well $= 20$ m

Radius (r) of circular end of well = $\frac{7}{2}m$

Area of platform = Length x Breadth $= 22 \times 14 m^2$

Assume height of the platform $= H$

Volume of soil dug from the well will be equal to the volume of soil scattered on the platform.

Volume of soil from well = Volume of soil used to make such platform

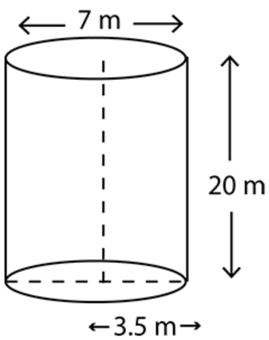
$\Rightarrow \pi r^2 h$ = Area of platform x Height of platform

$\Rightarrow \pi \times (\frac{7}{2})^2 \times 20 = 22 \times 14 \times H$

$H = \frac{22}{7} \times \frac{49}{4} \times \frac{20}{22 \times 14}$

$H = \frac{5}{2} = 2.5m$

Hence, the height of such platform will be 2.5 m.



#465184

Topic: Combination of Solids

A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Solution

The shape of the well will be cylindrical as shown in the figure below:

Given:

Depth (h_1) of well = 14 m

Radius (r_1) of the circular end of well = $\frac{3}{2}$m

Width of embankment = 4 m

From the figure, it can be observed that our embankment will be in a cylindrical shape having outer radius, $(r_2) = 4 + \frac{3}{2} = \frac{11}{2}$ m,

Let height of embankment be h_2.

Volume of soil dug from well = Volume of earth used to form embankment

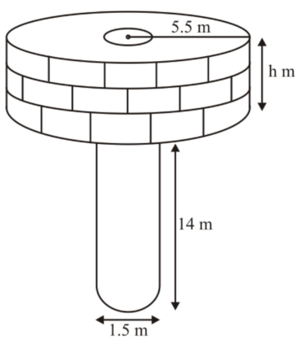
$\pi \times r_1^2 \times h_1 = \pi \times (r_2^2 - r_1^2) \times h_2$

$\Longrightarrow \pi \times \left(\frac{3}{2}\right)^2 \times 14 = \pi \times [\left(\frac{11}{2}\right)^2 - \left(\frac{3}{2}\right)^2 \times h_2]$

$\Longrightarrow \frac{9}{4} \times 14 = \frac{112}{4} \times h_2$

$h_2 = \frac{9}{8} = 1.125$ m

So, the height of the embankment will be 1.125 m.



#465185

Topic: Combination of Solids

A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Solution

Given:

Height (h_1) of cylindrical container = 15 cm

Radius = $\dfrac{Diameter}{2}$

Radius (r_1) of circular end of container = $\dfrac{12}{2} = 6\ cm$

Radius (r_2) of circular end of ice-cream cone = $\dfrac{6}{2} = 3\ cm$

Height (h_2) of conical part of ice-cream cone = 12 cm

Let n ice-cream cones be filled with ice-cream of the container.

Volume of ice-cream in cylinder = n (Volume of 1 ice-cream cone + Volume of hemispherical shape on the top)

$$\pi r_1^2 h_1 = n\left(\frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3}\pi r_2^3\right)$$

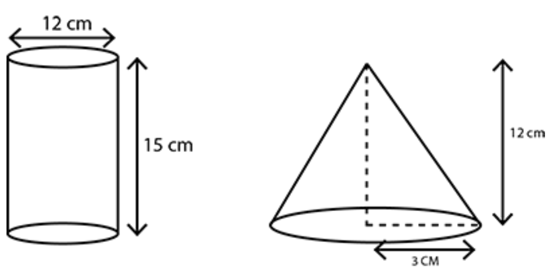
$$\Rightarrow \pi \times 6^2 \times 15 = n\left(\frac{1}{3}\pi 3^2 \times 12 + \frac{2}{3}\pi 3^3\right)$$

$$\Rightarrow n = \frac{30 \times 15}{\frac{1}{3} \times 9 \times 12 + \frac{2}{3} \times 27}$$

$$\Rightarrow n = \frac{36 \times 15 \times 3}{108 + 54}$$

$n = 10$

So, 10 ice-cream cones can be filled with the ice-cream in the container.



#465186

Topic: Combination of Solids

How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm?

Solution

Coins are cylindrical in shape as shown in the figure.

Given:

Height (h_1) of cylindrical coins $= 2$ mm $= 0.2$ cm

Radius (r) of circular end of coins = $\frac{1.75}{2} = 0.875$ cm

Let x coins be melted to form the required cuboids.

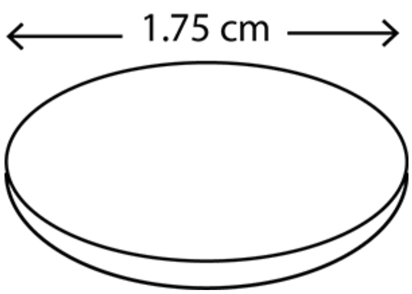
Volume of x coins = Volume of cuboids

$x \times \pi r^2 h_1 = l \times b \times h$

$\Rightarrow x \times \pi \times 0.875^2 \times 0.2 = 5.5 \times 10 \times 3.5$

$\Rightarrow x = \frac{5.5 \times 10 \times 3.5 \times 7}{0.875^2 \times 0.2 \times 22} = 400$

So, the number of coins melted to form such a cuboid is 400.



#465187
Topic: Combination of Solids

A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm. Find the radius and slant height of the heap.

Solution

Given:

Height (h_1) of cylindrical bucket = 32 cm

Radius (r_1) of circular end of bucket = 18 cm

Height (h_2) of conical heap = 24 cm

Let the radius of the circular end of conical heap be r_2.

The volume of sand in the cylindrical bucket will be equal to the volume of sand in the conical heap.

Volume of sand in the cylindrical bucket = Volume of sand in conical heap

$\pi \times r_1^2 \times \ = \frac{1}{3}\pi \times r_2^2 \times h_2$

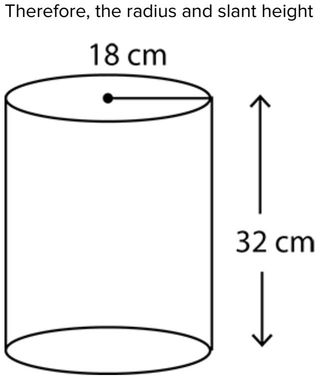
$\Rightarrow \pi \times 18^2 \times 32 = \frac{1}{3}\pi \times r_2^2 \times 24$

$\Rightarrow r_2^2 = \frac{3 \times 18^2 \times 32}{24} = 18^2 \times 4$

$\Rightarrow r_2 = 18 \times 2 = 36\ cm$

Slant height = $\sqrt{r_2^2 + h_2^2} = \sqrt{12^2 \times (3^2 + 2^2)} = 12\sqrt{13}\ cm$.

Therefore, the radius and slant height of the conical heap are 36 cm and $12\sqrt{13}$ cm respectively.



#465189

Topic: Combination of Solids

A farmer connects a pipe of internal diameter 20 cm form a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 kilometer per hour, in how much time will the tank be filled?

Solution

Radius (r_1) of circular end of pipe $= \dfrac{20}{200} = 0.1$ m

Area of cross-section $= \pi \times r_1^2 = \pi \times (0.1)^2 = 0.01\pi$ sq. m

Speed of water $= 3$ kilometer per hour $= \dfrac{3000}{60} = 50$ meter per minute.

Volume of water that flows in 1 minute from pipe $= 50 \times 0.01\pi = 0.5\pi$ cu. m

From figure 2, Volume of water that flows in t minutes from pipe $= t \times 0.5\pi$ cu. m

Radius (r_2) of circular end of cylindrical tank $= \dfrac{10}{2} = 5$ m

Depth (h_2) of cylindrical tank $= 2$ m

Let the tank be filled completely in t minutes.

Volume of water filled in tank in t minutes is equal to the volume of water flowed in t minutes from the pipe.

Volume of water that flows in t minutes from pipe $=$ Volume of water in tank

Therefore, $t \times 0.5\pi = \pi r_2^2 \times h_2$

$\Rightarrow t \times 0.5 = 5^2 \times 2$

$\Rightarrow t = \dfrac{25 \times 2}{0.5}$

$\Rightarrow t = 100$

Therefore, the cylindrical tank will be filled in 100 minutes.

#465193

Topic: Cone

A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Solution

Given:

Upper base diameter, D $= 4$ cm

Lower base diameter, d $= 2$cm

Height, h $= 14$ cm

So, R (upper base) $= 2$ cm,

r(lower $= 1$cm

Capacity of glass = Volume of frustum of cone

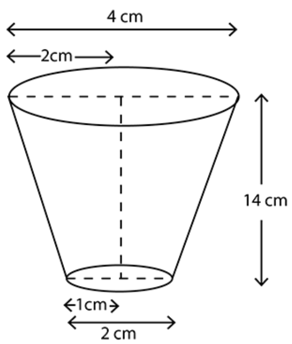
$= \frac{\pi h}{3}[R^2 + Rr + r^2]$

$= \frac{\pi \times 14}{3}[2^2 + 2 \times 1 + 1^2]$

$= \frac{22 \times 14[4 + 1 + 2]}{3 \times 7}$

$= \frac{308}{3} = 102\frac{2}{3}$ cm ^{3}

So, the capacity of the glass is $102\frac{2}{3}$ cm^3



#465194

Topic: Cone

The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Solution

$l = 4$ cm

Circumference of a circular end $= 18$ cm

$\Rightarrow 2\pi r_1 = 18$

$\Rightarrow \pi \times r_1 = \dfrac{18}{2} = 9$ ....(1)

Circumference of other circular end $= 6$ cm

$\Rightarrow 2\pi r_2 = 6$

$\Rightarrow \pi r_2 = \dfrac{6}{2} = 3$..(2)

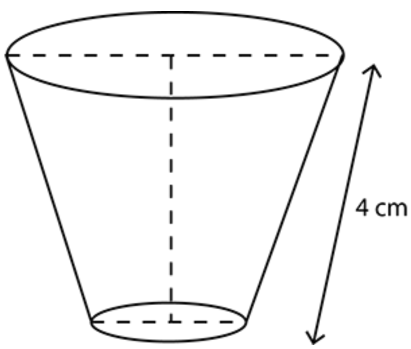
Adding (1) and (2)

Curved surface area

$= \pi(r_1 + r_2)l$

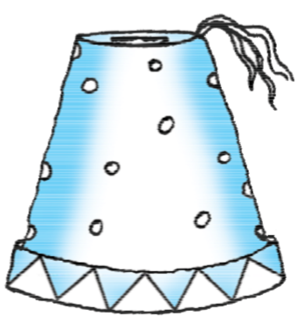
$= (9 + 3) \times 4$

$= 48 \text{ cm}^2$



#465195

Topic: Cone



A fez, the cap used by the Turks, is shaped like the frustum of a cone . If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.

Solution

Given:

Radius (r) of upper circular end $= 4$ cm

Radius (R) of lower circular end $= 10$ cm

Slant height (l) of frustum $= 15$ cm

Area of material used for making the fez = CSA of frustum + Area of upper circular end

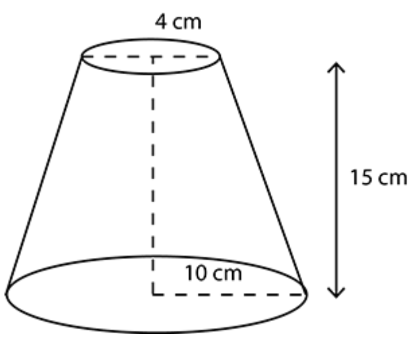
$= \pi(R+r)l + \pi r^2$

$= \pi(10+4) \times 15 + \pi \times 4^2$

$= 210\pi + 16\pi = \dfrac{226 \times 22}{7}$

$= 710\dfrac{2}{7}\ cm^2$

Therefore, the area of material used for making it is $710\dfrac{2}{7}\ cm^2$.



#465196

Topic: Cone

A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs. 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm^2. (Take $\pi = 3.14$).

Solution

Given:

Radius (R) of upper end of contianer $= 20$ cm

Radius (r) of lower end of container $= 8$ cm

Height (h) of container $= 16$ cm

Slant height (l) of frustum = $\sqrt{(R-r)^2+h^2}$

$= \sqrt{(20-8)^2+16^2} = \sqrt{12^2+16^2} = \sqrt{144+256} = \sqrt{400} = 20$ cm

Capacity of container = Volume of frustum

$= \frac{\pi h}{3}[R^2+r^2+Rr]$

$= \frac{3.14\times 16}{3}[20^2+8^2+20\times 8]$

$= \frac{3.14\times 16}{3}[400+64+160] = \frac{50.24}{3}[624]$

$= 10449.92$ cm^3

$= 10.45$ litres.

Cost of 1 litre milk = Rs.20

Cost of 10.45 litre milk $= 10.45\times 20 =$ Rs. 209

Area of metal sheet used to make the container = $\pi(R+r)l + \pi r^2$

$= \pi(20+8)20 + \pi\times 8^2 = 560\pi + 64\pi = 624\pi$ cm^2

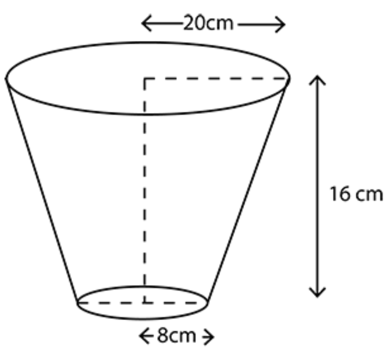
Cost of 100 cm^2 metal sheet = Rs. 8

Cost of 624π cm ^{2} metal sheet = $\frac{624\times 3.14\times 8}{100}$

$= 156.75$

Therefore, the cost of the milk which can completely fill the container is Rs. 209.

The cost of metal sheet used to make the container is Rs. 156.75.



#465210
Topic: Combination of Solids

A metallic right circular cone 20 cm high and whose vertical angle is 60^o is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{16}$ cm, find the length of the wire.

Solution

FRUSTUM:

Let r_1 and r_2 be the radii of the top and bottom surface, respectively of the frustum.

In ΔAOF,

$\tan 30^o = \frac{FO}{OA}$

$\frac{1}{\sqrt{3}} = \frac{FO}{10}$

$\Rightarrow FO = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$ cm

So, $r_1 = \dfrac{10\sqrt{3}}{3}$ cm

In ΔABD,

$\dfrac{BD}{AD} = \tan 30^o$

$\Rightarrow BD = \dfrac{20\sqrt{3}}{3}$ cm.

So, $r_2 = \dfrac{20\sqrt{3}}{3}$ cm

And height of frustum $(h) = 10$ cm.

So, Volume of frustum $= \dfrac{1}{3}\pi h(r_1{}^2 + r_2{}^2 + r_1 r_2)$

$$= \frac{1}{3}\pi \times 10\left[\left(\frac{10\sqrt{3}}{3}\right)^2 + \left(\frac{20\sqrt{3}}{3}\right)^2 + \frac{(10\sqrt{3})(20\sqrt{3})}{3 \times 3}\right]$$

$$= \frac{10}{3}\pi\left[\frac{100}{3} + \frac{400}{3} + \frac{200}{3}\right]$$

$$= \frac{10}{3} \times \frac{22}{7} \times \frac{700}{3} = \frac{22000}{9} cm^3$$

WIRE:

Let the radius of the wire r and l be the length of the wire.

So, $r = \dfrac{1}{32}$ cm

Volume of wire =Volume of cylinder

Volume of wire $= \pi r^2 l$

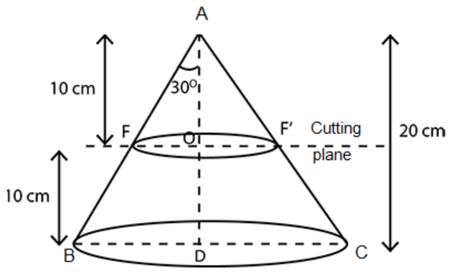
$$\frac{22000}{9} = \frac{22}{7} \times \left(\frac{1}{32}\right)^2 \times l$$

$$l = \frac{22000 \times 32 \times 32 \times 7}{22 \times 9}$$

$l = 7, 96, 444.44$ cm

$l = 7964.44$ m.

So, length of the wire is 7964.44 cm.



#465221

Topic: Combination of Solids

A copper wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm^3.

Solution

From the figure we can observed that 1 round of wire will cover 3 mm height of cylinder.

Number of rounds = $\dfrac{Height\ \ of\ \ cylinder}{Diameter\ \ of\ \ wire}$

$= \dfrac{12}{0.3} = 40$ rounds.

Length of wire required in 1 round = Circumference of base of cylinder

$= 2\pi r = 2\pi 5 = 10\pi$

Length of wire in 40 rounds $= 40 \times 10\pi$

$= \dfrac{400 \times 22}{7} = \dfrac{8800}{7} = 1257.14$ cm

$= 12.57$ m

Radius of wire = $\dfrac{0.3}{2} = 0.15$ cm.

Volume of wire = Area of cross-section of wire x Length of wire

$= \pi(0.15)^2 \times 1257.14$

$= 88.898\ \ cm^3$

Mass = Volume x Density

$= 88.898 \times 8.88$

$= 789.41$ gm

#465228

Topic: Combination of Solids

A right triangle whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate.)

Solution

The double cone so formed by revolving this right-angled triangle ABC about its hypotenuse is shown in the figure.

Hypotenuse, AC = $\sqrt{3^2 + 4^2}$

= $\sqrt{9 + 16}$ = $\sqrt{25}$ = 5 cm

Area od $\angle ABC = \frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times AC \times DB = \frac{1}{2} \times 4 \times 3$

$\frac{1}{2} \times 5 \times DB = 6$

So, DB = $\frac{12}{5}$ = 2.4 cm

Volume of double cone = Volume of cone 1 + Volume of cone 2

$= \frac{1}{3}\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$

$= \frac{1}{3}\pi r^2[h_1 + h_2] = \frac{1}{3}\pi r^2[DA + DC]$

$= \frac{1}{3} \times 3.14 \times 2.4^2 \times 5$

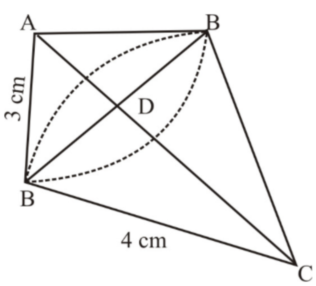
= 30.14 cm^3

Surface area of double cone = Surface area of cone 1 + Surface area of cone 2

$= \pi r l_1 + \pi r l_2$

$= \pi r[4 + 3] = 3.14 \times 2.4 \times 7$

= 52.75 cm^2



#465232

Topic: Combination of Solids

A cistern, internally measuring 150 cm x 120 cm x 110 cm, has 129600 cm ^{3} of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being 22.5 cm x 7.5cm x 6.5 cm?

Solution

Volume of cistern $= 150 \times 120 \times 110$

$= 1980000$ cm^3

Volume to be filled in cistern $= \dfrac{1980000}{129600}$

$= 1850400$ cm^3

Let x numbers of porous bricks were placed in the cistern.

Volume of x bricks $= x \times 22.5 \times 7.5 \times 6.5$

$= 1096.875x$

As each brick absorbs one-seventeenth of its volume, therefore, volume absorbed by these bricks

$= \dfrac{x}{17}(1096.875)$

$\Rightarrow 1850400 + \dfrac{x}{17}(1096.875) = (1096.875)x$

$\Rightarrow 1850400 = \dfrac{16x}{17}(1096.875)$

$x = 1792.41$

Therefore, 1792 bricks were placed in the cistern.

#465234
Topic: Combination of Solids

In one fortnight of a given month, there was a rainfall of 10 cm in a river valley. If the area of the valley is 7280 km^2, show that the total rainfall was approximately equivalent to the addition to the normal water of three rivers each 1072 km long, 75 m wide and 3 m deep.

Solution

Given:

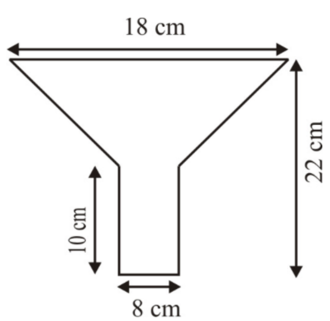
Area of the valley $= 7280$ km^2

Volume of rainfall in the area $= 7280 \times 10 \times 0.00001 = 0.728$ km^3

Volume of water in 3 rivers $= 3 \times 1 \times b \times h = 3 \times 1072 \times 75 \times 0.001 \times 3 \times 0.001 = 0.723$ km^3 (Converted m = km)

So, the 2 valleys are nearly similar.

#465326
Topic: Combination of Solids



An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel .

Solution

Given:

Radius (R) of upper circular end of frustum part = $\frac{18}{2}$ = 9 cm

Radius (r) of lower circular end of frustum part = Radius of circular end of cylindrical part = $\frac{8}{2}$ = 4 cm

Height (h_1) of frustum part = 22 − 10 = 12 cm

Height (h_2) of cylindrical part = 10 cm

Slant height (l) of frustum part = $\sqrt{(R-r)^2 + h_1^2} = \sqrt{(9-4)^2 + 12^2} = \sqrt{25+144}$

$= \sqrt{169} = 13$ cm

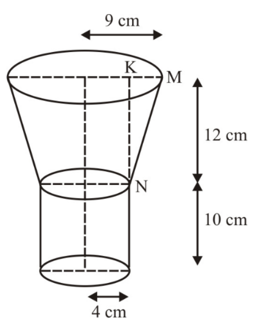
Area of tin sheet required = CSA of frustum part + CSA of cylindrical part

$= \pi(R+r)l + 2\pi r h_2$

$= \frac{22}{7} \times (9+4) \times 13 + 2 \times \frac{22}{7} \times 4 \times 10$

$= \frac{22}{7}[169+80] = \frac{22 \times 249}{7}$

$= 782\frac{4}{7}$ cm^2



#465355

Topic: Cone

Derive the formula for the curved surface area and total surface area of the frustum of a cone.

Solution

Let ABC be a cone. A frustum $DECB$ is cut by a plane parallel to its base.

Let r_1 and r_2 be the radii of the ends of the frustum of the cone and h be the height of the frustum of the cone.

In $\angle ABG$ and $\angle ADF$, $DF \parallel BG$

Therefore, $\angle ABG \sim \angle ADF$

$\frac{DF}{BG} = \frac{AF}{AG} = \frac{AD}{AB}$

$\frac{r_2}{r_1} = \frac{h_1 - h}{h_1} = \frac{l_1 - l}{1_1}$

$\frac{r_2}{r_1} = 1 - \frac{h}{h_1} = 1 - \frac{l}{l_1}$

$1 - \frac{l}{l_1} = \frac{r_2}{r_1}$

$1 - \frac{l}{l_1} = 1 - \frac{r_2}{r_1} = \frac{r_1 - r_2}{r_1}$

$\frac{l_1}{l} = \frac{r_1}{r_1 - r_2}$

$l_1 = \frac{r_1 l}{r_1 - r_2}$

CSA of frustum $DECB =$ CSA of cone $ABC -$ CSA cone ADE

$= \pi r_1 l_1 - \pi r_2(l_1 - l)$

$= \pi r_1\left(\frac{r_1}{r_1 - r_2}\right) - \pi r_2\left(\frac{r_1}{r_1 - r_2} - l\right)$

$\frac{\pi r_1^2 l}{r_1 - r_2} - \pi r_2\left(\frac{r_1 l - r_1 l + r_2 l}{r_1 - r_2}\right)$

$= \frac{\pi r_1^2 l}{r_1 - r_2} - \frac{\pi r_2^2 l}{r_1 - r_2}$

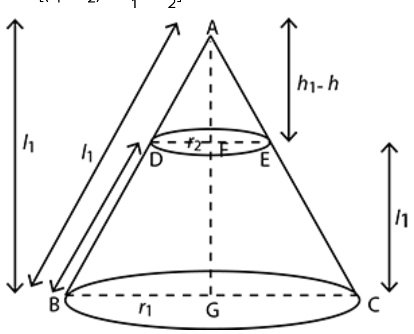
$= \pi l[\frac{r_1^2 - r_2^2}{r_1 - r_2}]$

CSA of frustum $= \pi(r_1 + r_2)l$.

Total surface area of frustum = CSA of frustum + Area of upper circular end + Area of lower circular end.

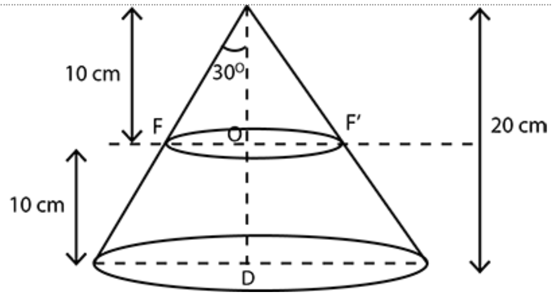
$= \pi(r_1 + r_2)l + \pi r_2^2 + \pi r_1^2$

$= \pi[(r_1 + r_2)l + r_1^2 + r_2^2]$



#465378

Topic: Cone



Derive the formula for the volume of the frustum of a cone.

Solution

Let ABC be a cone. A frustum $DECB$ is cut by a plane parallel to its base. Let r_1 and r_2 be the radii of the ends of the frustum of the cone and h be the height of the frustum of the cone.

In $\angle ABG$ and $\angle ADF$, $DF \parallel BG$

Therefore, $\angle ABG \sim \angle ADF$

$$\frac{DF}{BG} = \frac{AF}{AG} = \frac{AD}{AB}$$

$$\Rightarrow \frac{r_2}{r_1} = \frac{h_1 - h}{h_1} = \frac{l_1 - l}{1_1}$$

$$\Rightarrow \frac{r_2}{r_1} = 1 - \frac{h}{h_1} = 1 - \frac{l}{l_1}$$

$$\Rightarrow 1 - \frac{h}{h_1} = \frac{r_2}{r_1}$$

$$\Rightarrow \frac{h}{h_1} = 1 - \frac{r_2}{r_1} = \frac{r_1 - r_2}{r_1}$$

$$\Rightarrow \frac{h_1}{h} = \frac{r_1}{r_1 - r_2}$$

$$\Rightarrow h_1 = \frac{r_1 h}{r_1 - r_2}$$

Volume of frustum of cone = Volume of cone ABC- Volume of cone ADE

$$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 (h_1 - h)$$

$$= \frac{\pi}{3}[r_1^2 h_1 - r_2^2(h_1 - h)]$$

$$= \frac{\pi}{3}\left[r_1^2\left(\frac{r_1 h}{r_1 - r_2}\right) - r_2^2\left(\frac{r_1 h}{r_1 - r_2} - h\right)\right]$$

$$= \frac{pi}{3}\left[\left(\frac{r_1^3 h}{r_1 - r_2}\right) - r_2^2\left(hr_1 - hr_1 + hr_2\right)\right]$$

$$= \frac{\pi}{3}\left[\frac{r_1^3 h}{r_1 - r_2} - \frac{r_2^3 h}{r_1 - r_2}\right]$$

$$= \frac{\pi}{3}h\left[\frac{r_1^3 - r_2^3}{r_1 - r_2}\right]$$

$$= \frac{\pi}{3}h\left[\frac{(r_1 - r_2)(r_1^2) + r_2^2 + r_1 r_2}{r_1 - r_2}\right]$$

$$= \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$$