IX Math
Ch 2: Polynomials
Chapter Notes

Top Definitions

1. A polynomial p(x) in one variable x is an algebraic expression in x of the form
$p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_2x^2 + a_1x + a_0$, where
(i) $a_0, a_1, a_2......a_n$ are constants
(ii) $x_0, x_1, x_2......x_n$ are variables
(iii) $a_0, a_1, a_2......a_n$ are respectively the coefficients of $x_0, x_1, x_2......x_n$.
(iv) Each of $a_n x^n + a_{n-1}x^{n-1}, a_{n-2}x^{n-2}, ... a_2x^2, a_1x, a_0$, with $a_n \neq 0$, is called a term of a polynomial.

2. A leading term is the term of highest degree.

3. Degree of a polynomial is the degree of the leading term.

4. A polynomial with one term is called a monomial.

5. A polynomial with two terms is called a binomial.

6. A polynomial with three terms is called a trinomial.

7. A polynomial of degree 1 is called a linear polynomial. It is of the form ax+b. For example: x-2, 4y+89, 3x-z.

8. A polynomial of degree 2 is called a quadratic polynomial. It is of the form $ax^2 + bx + c$. where a, b, c are real numbers and $a \neq 0$ For example: $x^2 - 2x + 5$ etc.

9. A polynomial of degree 3 is called a cubic polynomial and has the general form $ax^3 + bx^2 + cx + d$. For example: $x^3 + 2x^2 - 2x + 5$ etc.

10. A bi-quadratic polynomial p(x) is a polynomial of degree 4 which can be reduced to quadratic polynomial in the variable $z = x^2$ by substitution.

11. The zero polynomial is a polynomial in which the coefficients of all the terms of the variable are zero.

12. Remainder theorem: Let p(x) be any polynomial of degree greater than or equal to one and let a be any real number. If p(x) is divided by the linear polynomial x – a, then remainder is p(a).

13. Factor Theorem: If p(x) is a polynomial of degree $n \geq 1$and a is any real number then (x-a) is a factor of p(x), if p(a) =0.

14. Converse of Factor Theorem: If p(x) is a polynomial of degree $n \geq 1$and a is any real number then p(a) =0 if (x-a) is a factor of p(x).

15. An algebraic identity is an algebraic equation which is true for all values of the variables occurring in it.

Top Concepts

1. The degree of non-zero constant polynomial is zero.
2. A real number 'a' is a zero/ root of a polynomial p(x) if p (a) = 0.
3. The number of real zeroes of a polynomial is less than or equal to the degree of polynomial.
4. Degree of zero polynomial is not defined.
5. A non zero constant polynomial has no zero.
6. Every real number is a zero of a zero polynomial.
7. Division algorithm: If p(x) and g(x) are the two polynomials such that degree of p(x) $\geq$ degree of g(x) and g(x)$\neq$ 0, then we can find polynomials q(x) and r(x) such that:
   p (x) = g(x) q(x) + r(x)
   where, r(x) =0 or degree of r(x) < degree of g(x).
8. If the polynomial p(x) is divided by (x+a), the remainder is given by the value of p (-a).
9. If the polynomial p(x) is divided by (x-a), the remainder is given by the value of p (a).

10. If p (x) is divided by ax + b = 0; a ≠ 0, the remainder is given by $p\left(\frac{-b}{a}\right)$; a ≠ 0.

11. If p (x) is divided by ax - b = 0 , a ≠ 0 , the remainder is given by $p\left(\frac{b}{a}\right)$; a ≠ 0.

12. A quadratic polynomial $ax^2 + bx + c$ is factorised by splitting the middle term bx as px +qx so that pq =ac.

13. The quadratic polynomial $ax^2 + bx + c$ will have real roots if and only if $b^2 - 4ac \geq 0$.

14. For applying factor theorem the divisor should be either a linear polynomial of the form x-a or it should be reducible to a linear polynomial.

Top Formulae

1. Quadratic identities:
   a. $(x+y)^2 = x^2 + 2xy + y^2$
   b. $(x-y)^2 = x^2 - 2xy + y^2$
   c. $(x-y)(x+y) = x^2 - y^2$
   d. $(x+a)(x+b) = x^2 + (a+b)x + ab$
   e. $(x+y+z)^2 = x^2 + y^2 + z^2 + +2xy + 2yz + 2zx$

   Here x, y, z are variables and a, b are constants

2. Cubic identities:
   a. $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$
   b. $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$
   c. $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
   d. $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$
   e. $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$
   f. If $x + y + z = 0$ then $x^3 + y^3 + z^3 = 3xyz$

   Here, x, y & z are variables.