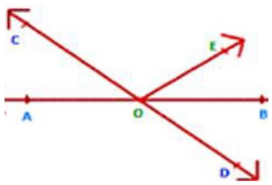
# NCERT SOLUTIONS
# CLASS-IX MATHS
# CHAPTER-6 LINES AND ANGLES

***Q1**: Lines AB and CD intersect at 0. If $\angle AOC + \angle BOE = 70^\circ$ and $\angle BOD = 40^\circ$, find $\angle BOE$ and reflex $\angle COE$.*




**SOL:**

Lines $AB and CD intersect at O$.

$\therefore \angle AOC = \angle BOD$ (Vertically Opposite Angles)

But $\angle BOD = 40^\circ$ . . (i)

$\therefore \angle AOC = 40^\circ$ (ii)

Now, $\angle AOC + \angle BOE = 70^\circ$
$\Rightarrow 40^\circ + \angle BOE = 70^\circ$

$\Rightarrow \angle BOE = 30^\circ$

Reflex $\angle COE = \angle COD + \angle BOD + \angle BOE$

$=\angle COD - 40^\circ + 30^\circ$ (iii) (using angle (i) and (ii)

$\angle COD = 180^\circ$ (as it is a straight line)

Thus ,$\angle COE = 180^\circ + 40^\circ + 30^\circ = 270^\circ$

***Q2**: In the figure, line $XY$ and $MN$ intersect at $O$.*

*If $\angle POY = 90^\circ$ and $a : b = 2 : 3$, find c.*

**Sol:**

Ray $OP$ stands on line $XY$

$\therefore \angle POX + \angle POY = 180^\circ$ (as is linear pair axiom)

Since $\angle POY = 90^\circ$ (given)

$\Rightarrow \angle POX = 90^\circ \angle POM + \angle XOM = 90^\circ$

$\Rightarrow a + b = 90^\circ$ (i)

$a : b = 2 : 3 \Rightarrow \frac{a}{b} = \frac{2}{3} \Rightarrow a = 2k, b = 3k$
$or\ \frac{a}{2} = \frac{b}{3} = k$

Substituting the values of a and b in (i), we get

$2k + 3k = 90^\circ \Rightarrow 5k = 90^\circ$
$\Rightarrow k = 18^\circ$

$\therefore a = 2k = 2(18^\circ) = 36^\circ$ …… (ii)
$b = 3k = 3(18^\circ) = 54^\circ$

Ray $OX$ stands on line MN (Linear pair axiom)

$\Rightarrow b + c = 180^\circ \Rightarrow 54^\circ + c = 180^\circ$

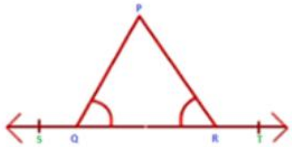
$\Rightarrow c = 180^\circ - 54^\circ$ (using equation (ii)

$\Rightarrow c = 126^\circ$

***Q3**: Prove that $\angle PQS = \angle PRT$ in the given isosceles triangle.*

*Sol:*

Sol:




Ray $QP$ stands on line $ST$.

$\therefore \angle PQS + \angle PQR = 180^\circ$ ............(i)

(as linear pair axiom ray $QP$ stands on line $ST$)

$\therefore \angle PQS + \angle PQR = 180^\circ$ .............(ii)

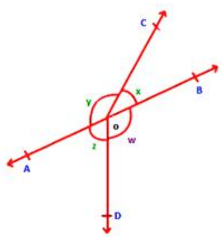
From (i) and (ii), we get

$\angle PQS + \angle PQR = \angle PRQ + \angle PRT \Rightarrow \angle PQS = \angle PRT$

(since $\angle PQR = \angle PRQ$)

***Q4: In the given figure if $x + y = w + z$ then prove that $AOB$ is a line.***

**Sol:**




$x + y = w + z$ ................(i)

We know, the sum of all the angles round a point is equal to $360^\circ$

So, $x + y + w + z = 360^\circ$

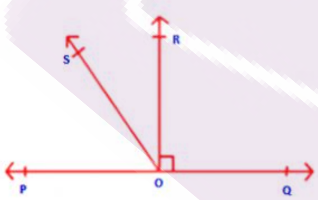
Putting the value of w+z in above equation we have,

$x + y + x + y = 360^\circ \Rightarrow 2(x + y) = 360^\circ \Rightarrow (x + y) = 180^\circ$

$\therefore AOB$ is a line.

***Q5: In the figure $POQ$ is a line. Ray $QR$ is a perpendicular to line $PQ$. $OS$ is another ray lying between rays $OP and OR$. Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.***

**Sol:**




Given, ray $QR$ is perpendicular to line $PQ$.

$\therefore \angle QOR = \angle POR = 90^\circ$ ...............(i)

$\angle QOS = \angle QOR + \angle ROS$ ................(ii)

$\angle QOS = \angle POR - \angle ROS$ ..............(iii)

From equation (ii) and (iii), we have

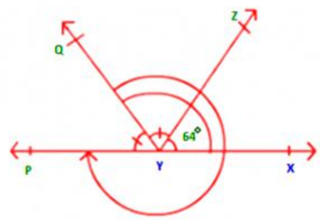
$\angle QOS - \angle POS = \angle QOR + \angle ROS - \angle POR + \angle ROS = (\angle QOR - \angle POR) + 2\angle ROS = 2\angle ROS$

(As we know $\angle QOR = \angle POR$ from equation (i) )

$\Rightarrow \angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

**Q6: It is given that $\angle XYZ = 64^\circ$ and $XY$ is produced to point $P$. Draw a figure from the given information.If ray $YQ$ bisect $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.**

**Sol:**




Given, ray $YZ$ stands on line $PX$.

$\therefore \angle XYZ + \angle ZYP = 180^\circ$

(As forms a linear pair axiom)

$\Rightarrow 64^\circ + \angle ZYP = 180^\circ$

$\Rightarrow \angle ZYP = 116^\circ$ ……………(i)

Given ray $YQ$ bisect $\angle ZYP$

$\therefore \angle PYQ = \angle ZYQ = \frac{1}{2}\angle ZYP = \frac{1}{2}(116^\circ) = 58^\circ$ ……………..(ii)

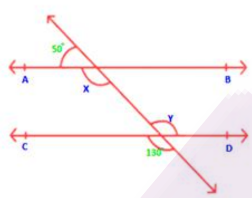
$\therefore reflex \angle QYP = 360^\circ - 58^\circ = 302^\circ$

Again ,$\angle XYQ = \angle XYZ + \angle ZYQ$

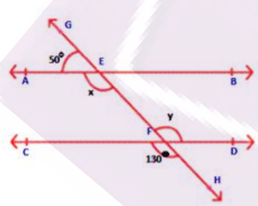
$= 64^\circ + 58^\circ$ (From eq (ii))

$= 122^\circ$

**Q7: In the given figure, find the values of $x andy$ and then show that $AB \parallel CD$.**




**Sol:**




Ray $AE$ stands on line $GH$

$\therefore \angle AEG + \angle AEH = 180^\circ$

(as forms Linear pair axioms)

$\Rightarrow 50^\circ + x = 180^\circ$ (as $\angle AEG$ is given)

$\Rightarrow x = 180^\circ - 50^\circ = 130^\circ$ …….(i)

$y = 130^\circ$ ……..(ii) (vertically opposite angles)

From (i) and (ii) we have,

$x = y$

But these angles are alternate interior angles and as they are equal.

So we can say that $AB \parallel CD$

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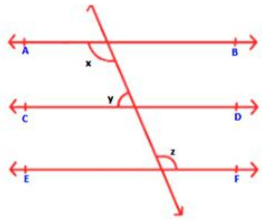
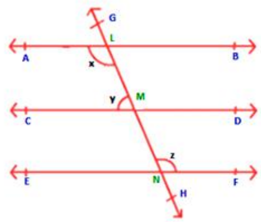
**Q8: In the figure, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, find $x$.**




**Sol:**




Given $AB \parallel CD$, $CD \parallel EF$

$AB \parallel EF$

( as lines parallel to the same line are parallel to each other)

$x = z$ ............(i) (alternate interior angles)

$x + y = 180^\circ$ ............(ii) (consecutive interior angles on the same side of the transversal $GH$ to parallel lines $AB$ and $CD$.

From (i) and (ii), we have

$z + y = 180^\circ$ $y : z = 3 : 7$

Now solving the ratios, we get

Sum of ratios =3+7=10

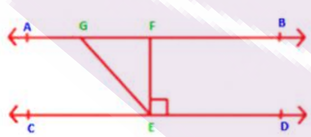
$\Rightarrow y = \frac{3}{10} \times 180^\circ = 54^\circ$ and

$z = \frac{7}{10} \times 180^\circ = 126^\circ$

As we know $x = z$

$\therefore x = z = 126^\circ$

**Q9: In the figure, if $AB \parallel CD$, $EF \perp CD$ and $\angle GED = 126^\circ$, find $\angle AGE, \angle GEF$ and $\angle FGE$.**




**Sol:**

(i) $\angle AGE = \angle GED = 126^\circ$ (as forms alternate interior angles)

(ii) $\angle GED = \angle GEF + \angle FED = 126^\circ$

$\Rightarrow \angle GEF + 90^\circ = 126^\circ$

(as given that $EF \perp CD$)

$\Rightarrow \angle GEF = 126^\circ - 90^\circ = 36^\circ$

(iii) $\angle CEG + \angle GED = 180^\circ$

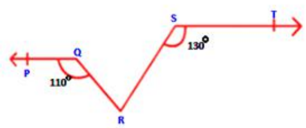
And its given that $\angle GED = 126^\circ$

$\Rightarrow \angle CEG + 126^\circ = 180^\circ \Rightarrow \angle CEG = 180^\circ - 126^\circ \Rightarrow \angle CEG = 54^\circ$

$\angle FGE = \angle CEG = 54^\circ$ (alternate angles)

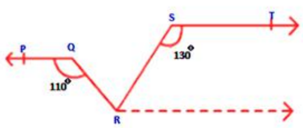
***Q10: In the figure, if $PQ \parallel ST$, $\angle PQR = 110^\circ$ and $\angle RST = 130^\circ$, find $\angle QRS$.***

***[Hint:Draw a line parallel to $ST$ through point $R$.***




**Sol:**

From the hint we draw a line $RU$ parallel to $ST$ through point $R$.




$\angle RST + \angle SRU = 180^\circ$

(Sum of the consecutive interior angles on the same side of the transversal is $180^\circ$)

$\Rightarrow 130^\circ + \angle SRU = 180^\circ$

$\Rightarrow \angle SRU = 180^\circ - 130^\circ = 50^\circ$ .........(i)

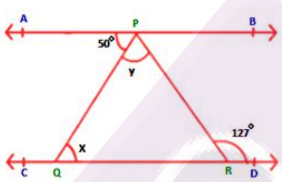
$\angle QRU = \angle PQR = 110^\circ$ (alternate interior angles)

$\Rightarrow \angle QRS + \angle SRU = 110^\circ$

$\Rightarrow \angle QRS + 50^\circ = 110^\circ$ (using (i))

$\Rightarrow \angle QRS = 110^\circ - 50^\circ = 60^\circ$

***Q11: In the figure, if $AB \parallel CD$, $\angle APQ = 50^\circ$ and $\angle PRD = 127^\circ$, find $x and y$.***




**Sol:**

From the figure we can see that

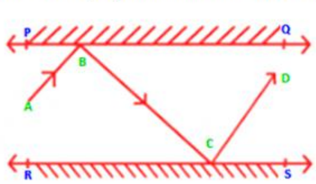
$\angle APQ = x = 50^\circ$ (as forms alternate interior angles)

$\angle PRD = x + y = 127^\circ$ (as we know that exterior angles of a triangle is equal to the sum of the two interior opposite angles )

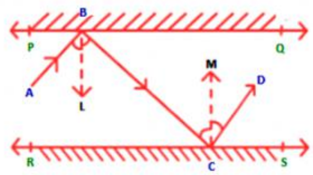
$\Rightarrow 50^\circ + y = 127^\circ \Rightarrow y = 127^\circ - 50^\circ = 77^\circ$

***Q12 : In the Fig, $PQ$ and $RS$ are two mirrors placed parallel to each other. An incident ray $AB$ strikes the mirror $PQ$ at $B$, the reflected ray moves along the path $BC$ and strikes the mirror $RS$ at $C$ and again reflects back along $CD$. Prove that $AB \parallel CD$.***

***(Hint: Draw perpendiculars at A and B to the two plane mirrors. Recall that the angle of the incidence is equal to angle of reflection.)***



**Sol:**




**Construction:** Draw ray $BL \perp PQ$ and ray $CM \perp RS$.

$BL \perp PQ$, $CM \perp RS$ and $PQ \parallel RS$.

$\therefore BL \parallel CM$

$\angle LBC = \angle MCB$ ......(i) (Alternate interior angles)

$\angle ABL = \angle LBC$ .........(ii) (angle of incidence= angle of reflection)

$\angle MCB = \angle MCD$ ..........(iii) (angle of incidence = angle of reflection).

From (i), (ii) and (iii), we get $\angle ABL = \angle MCD$....(iv)

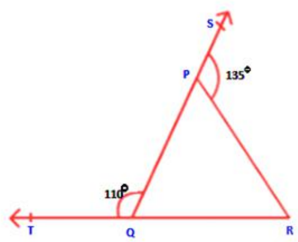
Adding (i) and (iv), we get

$\angle LBC + \angle ABL = \angle MCB + \angle MCD \Rightarrow \angle ABC = \angle BCD$

But these are alternate interior angles and they are equal.

So, $AB \parallel CD$

***Q13:** In the figure, sides $QP$ and $RQ$ are produced to point $S$ and $T$ respectively. If $\angle SRP = 135^\circ$ and $\angle PQT = 110^\circ$, find $\angle PRQ$.*




**Sol:**

TR is a line So, $\angle PQT + \angle PQR = 180^\circ$

$\Rightarrow 110^\circ + \angle PQR = 180^\circ$

$\Rightarrow \angle PQR = 180^\circ - 110^\circ = 70^\circ$ ......(i)

$QS$ is a line.

$\therefore \angle SPR + \angle QRP = 180^\circ \Rightarrow 135^\circ + \angle QRP = 180^\circ$

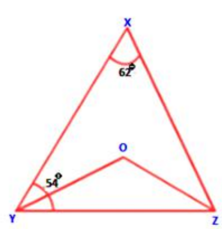
$\Rightarrow \angle QRP = 180^\circ - 135^\circ = 45^\circ$ .......(ii)

In $\triangle PQR, \angle PQR + \angle QPR + \angle PRQ = 180^\circ$ (sum of all the interior angles of a traingle is $180^\circ$.

$\Rightarrow 70^\circ + 45^\circ \angle + PRQ = 180^\circ$ (by equation (i) and (ii) )

$\Rightarrow 115^\circ + PRQ = 180^\circ \Rightarrow PRQ = 180^\circ - 115^\circ = 65^\circ$

***Q14 :** In the figure, $\angle X = 62^\circ$ $\angle XYZ = 54^\circ$. If $YO$ and $ZO$ of $\angle XYZ$ and $\angle XZY$ respectively of $\triangle XYZ$, find $\angle OZY$ and $\angle YOZ$.*




**Sol:**

In $\triangle XYZ$, $\angle XYZ + \angle YZX + \angle ZXY = 180^\circ$. (Sum of interior angles of a triangle is $180^\circ$)

$\Rightarrow 116^\circ + \angle YZX = 180^\circ$

$\Rightarrow \angle YZX = 180^\circ - 116^\circ = 64^\circ$ .. (i)

$YO$ is the bisector of $\angle XYZ$

$\Rightarrow \angle XYO = \angle OYZ = \frac{1}{2}\angle XYZ = \frac{1}{2}(54^\circ) = 27^\circ$ (ii)

$\Rightarrow ZO \text{ is the bisector of } \angle YZX$

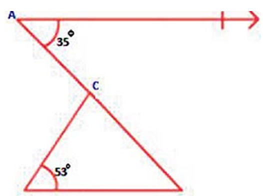
$\therefore \angle XZO = \angle OZY = \frac{1}{2}\angle YZX = \frac{1}{2}(64^\circ) = 32^\circ$ . (iii) (from equation (i))

In triangle $\triangle OYZ$, $\angle OYZ$, $\angle OZY$ $and \angle YOZ = 180^\circ$ (sum of interior angle of a triangle is 180°)

$\Rightarrow 27^\circ + 32^\circ + \angle YOZ = 180^\circ$ ( using equation (i) and (ii))

$\Rightarrow 59^\circ + \angle YOZ = 180^\circ \Rightarrow \angle YOZ = 180^\circ - 59^\circ = 121^\circ$

***Q15 : In the figure,*** $AB = \frac{1}{2}DE, \angle BAC = 35^\circ and \angle CDE = 53^\circ$***,find*** $\angle DCE$.




**Sol:**

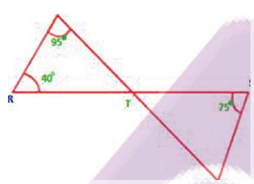
$\angle DEC = \angle BAC = 35^\circ$ . ..(i) (alternate interior angles)

$\angle CDE = 53^\circ$ .. .. . (ii) (given)

In $\triangle CDE, \angle CDE + \angle DEC + \angle DCE = 180^\circ$ (as sum of interior angles of a triangle is $180^\circ$.

$\Rightarrow 53^\circ + 35^\circ + \angle DCE = 180^\circ \Rightarrow 88^\circ + \angle DCE = 180^\circ \Rightarrow \angle DCE = 180^\circ - 88^\circ = 92^\circ$

***Q16 . In the figure,if lines*** $PO$ $and$ $RS$ ***Intersect at point*** $T$***,such that*** $\angle PRT = 40^\circ \angle RPT = 95^\circ, \angle SQT = 75^\circ$, $find \angle SQT$.




**Sol:**

In In $\triangle PRT, \angle PTR + \angle PRT + \angle RPT = 180^\circ$ (as sum of interior angles of a triangle is $180^\circ$)

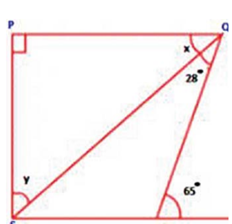
$\Rightarrow \angle PTR + 40^\circ + 95^\circ = 180^\circ \Rightarrow \angle PTR + 135^\circ = 180^\circ \Rightarrow \angle PTR = 180^\circ - 135^\circ \Rightarrow \angle PTR = 45^\circ$

$\Rightarrow \angle QTR = \angle PTR = 45^\circ$ (vertically opposite angles)

In $\triangle TSQ, \angle QTS + \angle TSQ + \angle SQT = 180^\circ$ (as sum of interior angles of a triangle is $180^\circ$

$\Rightarrow 45^\circ + 75^\circ + \angle SQT = 180^\circ \Rightarrow 120^\circ + \angle SQT = 180^\circ \Rightarrow \angle SQT = 180^\circ - 120^\circ \Rightarrow \angle SQT = 60^\circ$

***Q17 : In the figure, if*** $PQ \perp PS$, $\angle SQR = 28^\circ$ $and$ $\angle QRT = 65^\circ$***,then find the value of*** $x$ $and$ ***y.***




**Sol:**

$\angle QRT = \angle RQS + \angle QSR$ (exterior angle is equal to sum of the two opposite interior angles)

$\Rightarrow 65^\circ = 28^\circ + \angle QSR \Rightarrow \angle QSR = 65^\circ - 28^\circ = 37^\circ$

$PQ \perp SP$ (given)

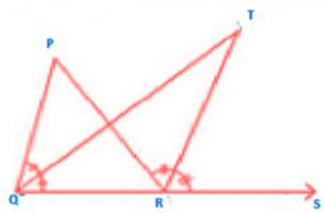
$\angle QPS + \angle PSR = 180^\circ$ (the sum of consecutive interior angles on the same side of the transversal in $180^\circ$ )

$\Rightarrow 90^\circ + \angle PSR = 180^\circ \Rightarrow \angle PSR = 180^\circ - 90^\circ = 90^\circ \Rightarrow \angle PSQ + \angle QSR = 90^\circ \Rightarrow y + 37^\circ = 90^\circ \Rightarrow Y = 90^\circ - 37^\circ = 53^\circ$

In $\triangle PSQ, \angle PSQ + \angle QSP + \angle QPS = 180^\circ$ (as sum of interior angles of a triangle is $180^\circ$.

$\Rightarrow x + y + 90\circ = 180^\circ \; x + 53^\circ + 90^\circ = 180^\circ \; x + 143^\circ = 180^\circ \; x = 180^\circ - 143^\circ = 37^\circ$

***Q18 : In the figure the side $QR$ $of \triangle PQR$ is produced to a point $S$. If the bisectors of $\angle PQR$ $and \angle PRS$ meet at point $T$, then prove that $\angle QTR = \frac{1}{2}\angle QPR.$***




**Sol:**

$\angle TRS$ is an exterior angle of $\triangle TQR$

$\angle TRS = \angle TQR + \angle QTR$ .....(i) (Since, the exterior angle is equal to the sum of the two interior opposite angles)

$\angle PRS$ is an exterior angle of $\triangle PQR$

$\angle PRS = \angle PQR + \angle QPR$ ......(ii) (since, the exterior angle is equal to the sum of the two interior opposite angles)

$\Rightarrow 2\angle TRS = 2\angle TQR + \angle QPR$ ($QT$ is the bisector of $\angle PQR \; and \; RT$ is the bisector of $\angle PRS$

$\Rightarrow \angle TRS$

$\Rightarrow 2(\angle TRS - \angle TQR) = \angle QPR$ ......(iii)

From (i), $\angle TRS - \angle TQR = \angle QTR$ ......(iv)

From (iii) and (iv) , we obtain

$2\angle QTR = \angle QPR \Rightarrow \angle QTR = \frac{1}{2}\angle QPR$