NCERT SOLUTIONS
CLASS-IX MATHS
CHAPTER-8 QUADRILATERALS

Q1. The angles of quadrilateral are in the ratio $3:5:9:13$. Find the angles of the quadrilateral.

Solution:

Let the common ratio between the angles be x.
We know that the 'Sum of the interior angles of the quadrilateral' = $360°$
Now,

$3x + 5x + 9x + 13x = 360°$

$\Rightarrow 30x = 360°$

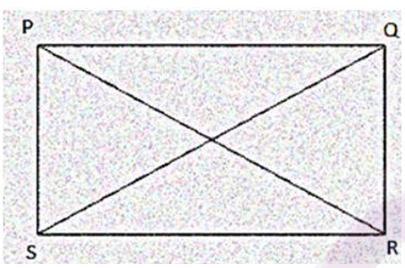
$\Rightarrow x = 12°$

Therefore the Angles of the quadrilateral are:
$3x = 3 \times 12 = 36°$

$5x = 5 \times 12 = 60°$ $9x = 9 \times 12 = 108°$ $13x = 13 \times 12 = 156°$

Q2. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Solution:

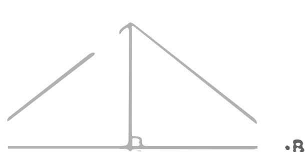


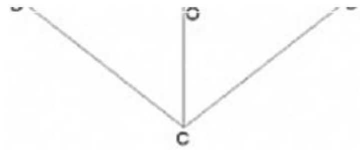
Given,
PQ = RS
To show,

PQRS is a rectangle we have to prove that one of its interior angle is right angled.
Proof,
In $\triangle PQR \ \ and \ \ \triangle QPS$,
$QR = QP$ (Common side)
$PR = PS$ (Opposite sides of a parallelogram are equal)
$PR = QS$ (Given)
Therefore, $\triangle PQR \cong \triangle QPS$by SSS congruence condition.
∠P = ∠Q (by CPCT)
also,
∠P + ∠Q = 180° (Sum of the angles on the same side of the transversal)
⇒ 2∠P = 180°
⇒ ∠P = 90°
Thus PQRS is a rectangle.

Q3. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Solution:





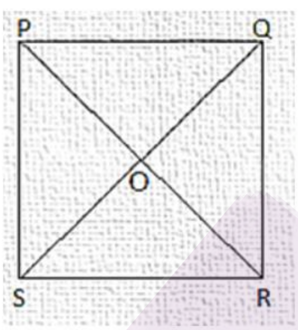
Let ABCD be a quadrilateral whose diagonals bisect each other at right angles.

Given,
$OA = OC, OB = OD \ \ and \ \ \angle AOB = \angle BOC = \angle OCD = \angle ODA = 90^\circ$

To show,
ABCD is parallelogram and $AB = BC = CD = DA$
Proof,
In $\triangle AOB \ \ and \ \ \triangle COB,$
$OA = OC$ (Given)
$\angle AOB = \angle COB$ (Opposite sides of a parallelogram are equal)
$OB = BO$ (Common)
Therefore, $\triangle AOB \cong bigtriangleupCOB$ (by SAS congruence condition).
Thus, $AB = BC$ (by CPCT-Corresponding parts of Congruent)
Similarly we can prove,
$AB = BC = CD = DA$
Opposites sides of a quadrilateral are equal hence $ABCD$ is a parallelogram.
Thus, $ABCD$ is rhombus as it is a parallelogram whose diagonals intersect at right angle.

Q4. Show that the diagonals of a square are equal and bisect each other at right angles.

Solution:



Let PQRS be a square and its diagonals PR and QS intersect each other at O.
To show,
$PR = QS, PO = OR \ \ and \ \ \angle POQ = 90^\circ$

Proof,
In $\triangle PQR \ \ and \ \ \triangle QRS,$
$QR = QP$ (Common)
$\angle PQR = \angle QPS = 90^\circ$
$PR = PS$ (Given)
Therefore, $\triangle PQR \cong \triangle QRS$ by SAS congruence condition.
Thus, $PR = PS$ (by CPCT).

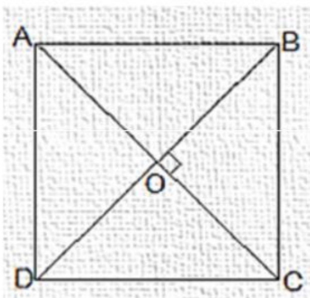
Therefore, diagonals are equal.
Now,
In $\triangle POQ \ \ and \ \ \triangle ROS,$
$\angle QPO = \angle SRO$ (Alternate interior angles)
$\angle POQ = \angle ROS$ (Vertically opposite)
$PQ = RS$ (Given)
Therefore, ΔAOB ≅ ΔCOD (by AAS congruence condition).
Thus, $PO = RO$ by CPCT. (Diagonal bisect each other.)
Now,
In $\triangle POQ \ \ and \ \ \triangle ROQ$

In $\triangle POQ$ and $\triangle ROQ$,
$OQ = QO$ (Given)
$PO = RO$ (diagonals are bisected)
$PQ = RQ$ (Sides of the square)
Therefore, $\triangle POQ \cong \triangle ROQ$ by SSS congruence condition.
also, $\angle POQ = \angle ROQ$

$\angle POQ + \angle ROQ = 180^\circ$ (Linear pair)
Thus, $\angle POQ = \angle ROQ = 90^\circ$ (Diagonals bisect each other at right angles)

Q5. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Solution:

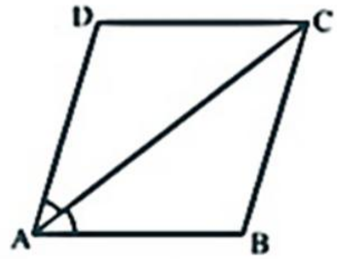


Given,
Let $ABCD$ be a quadrilateral in which diagonals AC and BD bisect each other at right angle at O.
To prove,
Quadrilateral $ABCD$ is a square.
Proof,
In $\triangle AOB$ and $\triangle COD$,
$AO = CO$ (Diagonals bisect each other)
$\angle AOB = \angle COD$ (Vertically opposite)
$OB = OD$ (Diagonals bisect each other)
Therefore, $\triangle AOB \cong \triangle COD$by SAS congruence condition.
Thus, $AB = CD$ by CPCT. …………… (i)
also,
$\angle OAB = \angle OCD$ (Alternate interior angles)
⇒ $\Rightarrow AB \parallel CD$

Now,
In $\triangle AOD$ and $\triangle COD$,
$AO = CO$ (Diagonals bisect each other)
$\angle AOD = \angle COD$ (Vertically opposite)
$OD = OD$ (Common)
Therefore, $\triangle AOD \cong \triangle COD$ (by SAS congruence condition).
Thus, $AD = CD$ (by CPCT). ………………… (ii)
also,
$AD = BC$ and $AD = CD$
$\Rightarrow AD = BC = CD = AB$ ……………….(iii)
also, $\angle ADC = \angle BCD$ (by CPCT).
and $\angle ADC + \angle BCD = 180^\circ$ (co-interior angles)
$\Rightarrow 2\angle ADC = 180^\circ$
$\Rightarrow \angle ADC = 90^\circ$ ……………. (iv)
One of the interior angle is right angle.
Thus, from (i), (ii) , (iii) and (iv) the given quadrilateral ABCD is a square.

Q6. Diagonal AC of a parallelogram $ABCD$ bisects $\angle A$. Show that
(i) it bisects $\angle C$ also,
(ii) $ABCD$ is a rhombus.

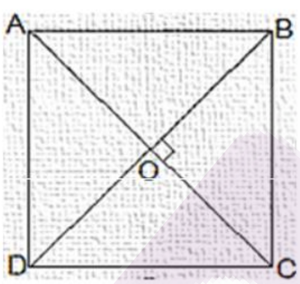
Solution:



(i)In $\triangle ADC \text{ and } \triangle CBA$,
$AD = CB$ (Opposite sides of a parallelogram)
$DC = BA$ (Opposite sides of a parallelogram)
$AC = CA$ (Common)
Therefore, $\triangle ADC \cong \triangle CBA$ by SSS congruence condition.
Thus,
$\angle ACD = \angle CAB$ (by CPCT)
and $\angle CAB = \angle CAD$ (Given)
$\Rightarrow \angle ACD = \angle BCA$
Thus, AC bisects $\angle C$ also.

(ii) $\angle ACD = \angle CAD$ (Proved)
⇒ $AD = CD$ (Opposite sides of equal angles of a triangle are equal)
Also, $AB = BC = CD = DA$ (Opposite sides of a parallelogram)
Thus, $ABCD$ is a rhombus.

***Q7.** $ABCD$ **is a rhombus. Show that diagonal** AC **bisects** $\angle A$ **as well as** $\angle C$ **and diagonal** BD **bisects** $\angle B$ **as well as** $\angle D$.*
Solution:

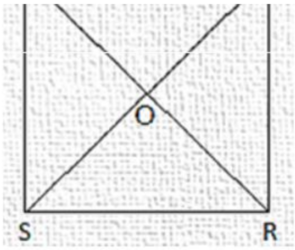


Let $ABCD$ is a rhombus and $AC \text{ and } BD$ are its diagonals.
Proof,
$AD = CD$ (Sides of a rhombus)
$\angle DAC = \angle DCA$ (Angles opposite of equal sides of a triangle are equal.)
also, $AB \parallel CD$
$\Rightarrow \angle DAC = \angle BCA$ (Alternate interior angles)
$\Rightarrow \angle DCA = \angle BCA$
Therefore, $AC \text{ bisect } \angle C$.
Similarly, we can prove that diagonal $AC \text{ bisect } \angle A$.

Also, by preceding above method we can prove that diagonal $BD \text{ bisect } \angle B \text{ as well as} \angle D$.

Q8.** $PQRS$ **is a rectangle in which diagonal** PR **bisects** $\angle P$ **as well as** $\angle R$**. Show that:
(i)** $PQRS$ **is a square
***(ii) diagonal** QS **bisects** $\angle Q$ **as well as** $\angle S$.*

Solution:

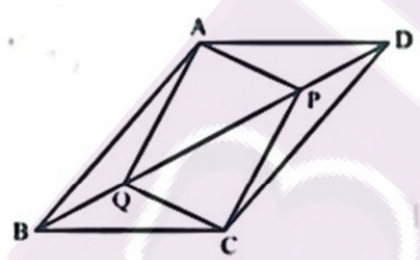


(i) $\angle SPR = \angle SRP$ (PR bisects $\angle P$ as well as $\angle R$)
$\Rightarrow PS = RS$ (Sides opposite to equal angles of a triangle are equal)
also, $RS = PQ$ (Opposite sides of a rectangle)
Therefore, $PQ = QR = RS = SP$
Thus, $PQRS$ is a square.

(ii) In $\triangle QRS$,
$QR = RS$
$\Rightarrow \angle RSQ = \angle RQS$ (Angles opposite to equal sides are equal)
also, $\angle RSQ = \angle PQS$ (Alternate interior angles)
$\Rightarrow \angle RQS = \angle PQS$
Thus, QS bisects $\angle Q$.
Now,
$\angle RQS = \angle PSQ$
$\Rightarrow \angle RSQ = \angle PSQ$
Thus, BD bisects $\angle D$

Q9. In parallelogram $ABCD$, two points P and Q are taken on diagonal BD such that $DP = BQ$. Show that:
(i) $\triangle APD \cong \triangle CQB$
(ii) $AP = CQ$
(iii) $\triangle AQB \cong \triangle CPD$
(iv) $AQ = CP$
(v) $APCQ$ is a parallelogram

Solution:



(i) In $\triangle APD$ and $\triangle CQB$,
$DP = BQ$ (Given)
$\angle ADP = \angle CBQ$ (Alternate interior angles)
$AD = BC$ (Opposite sides of a ||gm)
Thus, $\triangle APD \cong \triangle CQB$ (by SAS congruence condition).

(ii) $AP = CQ$ by CPCT as $\triangle APD \cong \triangle CQB$.

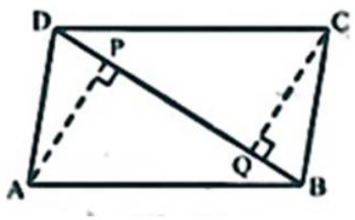
(iii) In $\triangle AQB$ and $\triangle CPD$,
$BQ = DP$ (Given)
$\angle ABQ = \angle CDP$ (Alternate interior angles)
$AB = BC = CD$ (Opposite sides of a parallelogram)
Thus, $\triangle AQB \cong \triangle CPD$ (by SAS congruence condition).

(iv) $AQ = CP$ by CPCT as $\triangle AQB \cong \triangle CPD$.

(v) From (ii) and (iv), it is clear that APCQ has equal opposite sides also it has equal opposite angles. Thus, $APCQ$ is a parallelogram.

10. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig. 8.21). Show that
(i) $\triangle APB \cong \triangle CQD$
(ii) AP = CQ

Solution:

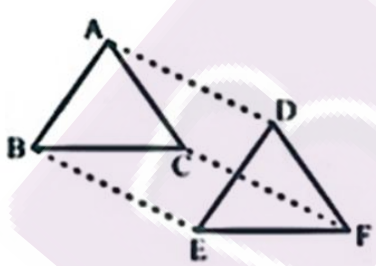


(i) In $\triangle APB\ \ and\ \ \triangle CQD$,
$\angle ABP = \angle CDQ$ (Alternate interior angles)
$\angle APB = \angle CQD$ (equal to right angles as AP and CQ are perpendiculars)
AB = CD (ABCD is a parallelogram)
Thus, $\triangle APB \cong \triangle CQD$ (by AAS congruence condition).

(ii) AP = CQ by CPCT as $\triangle APB \cong \triangle CQD$.

Q11. In $\triangle ABC\ \ and\ \ \triangle DEF$***,*** $AB = DE,\ AB \parallel DE,\ BC = EF\ and\ BC \parallel EF$ ***. Vertices A, B and C are joined to vertices D, E and F respectively.***
Show that
(i) quadrilateral ABED is a parallelogram
(ii) quadrilateral BEFC is a parallelogram
(iii) $AD \parallel CF$ ***and AD = CF***
(iv) quadrilateral ACFD is a parallelogram
(v) $AC = DF$

(vi) $\triangle ABC \cong \triangle DEF$
Solution:



(i) $AB = DE\ \ and\ \ AB \parallel DE$ (Given)

Thus, quadrilateral ABED is a parallelogram because two opposite sides of a quadrilateral are equal and parallel to each other.

(ii) Again $BC = EF\ \ and\ \ BC \parallel EF.$

Thus, quadrilateral BEFC is a parallelogram.

(iii) Since ABED and BEFC are parallelograms.
$\Rightarrow AD = BE\ and\ BE = CF$ (Opposite sides of a parallelogram are equal)

Thus, $AD = CF.$

Also, $AD \parallel BE\ \ and\ \ BE \parallel CF$ (Opposite sides of a parallelogram are parallel)

Thus, $AD \parallel CF.$

(iv) AD and CF are opposite sides of quadrilateral ACFD which are equal and parallel to each other. Thus, it is a parallelogram.
(v) $AC \parallel DF \ \ and \ \ AC = DF$ because ACFD is a parallelogram.

(vi) $\triangle ABC \ \ and \ \ \triangle DEF$,

AB = DE (Given)

BC = EF (Given)

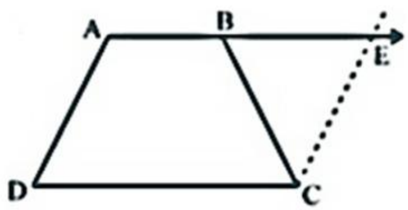
AC = DF (Opposite sides of a parallelogram)

Thus, $\triangle ABC \cong \triangle DEF$ (by SSS congruence condition).

Q12. ABCD is a trapezium in which $AB \parallel CD$ and $AD = BC$ .Show that
(i) $\angle A = \angle B$

(ii) $\angle C = \angle D$
(iii) $\triangle ABC \cong \triangle BAD$
(iv) diagonal AC = diagonal BD

[Hint : Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]
Solution:



Construction: Draw a line through C parallel to DA intersecting AB produced at E.
(i) CE = AD (Opposite sides of a parallelogram)
AD = BC (Given)
Therefor, BC = CE
$\Rightarrow \angle CBE = \angle CEB$also,
$\angle A + \angle CBE = 180^\circ$ (Angles on the same side of transversal and $\angle CBE = \angle CEB$)
$\angle B + \angle CBE = 180^\circ$ (Linear pair)
$\Rightarrow \angle A = \angle B$

(ii) $\angle A + \angle D = \angle B + \angle C = 180^\circ$ (Angles on the same side of transversal)
$\Rightarrow \angle A + \angle D = \angle A + \angle C$ (as, $\angle A = \angle B$)

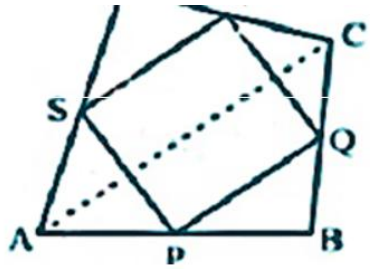
$\Rightarrow \angle D = \angle C$

(iii) In $\triangle ABC \ \ and \ \ \triangle BAD$,
AB = AB (Common)
$\angle DBA = \angle CBA$
AD = BC (Given)
Thus, $\triangle ABC \cong \triangle BAD$ (by SAS congruence condition).

(iv) Diagonal AC = diagonal BD (by CPCT as $\triangle ABC \cong \triangle BAD$.)

Exercise 2:

Q1. $ABCD$ is a quadrilateral in which P, Q, R and S are mid-points of the sides $AB, BC, CD \ and \ DA$. AC is a diagonal. Show that :
(i) $SR \parallel AC$ and $SR = \frac{1}{2}AC$
(ii) PQ = SR
(iii) PQRS is a parallelogram.



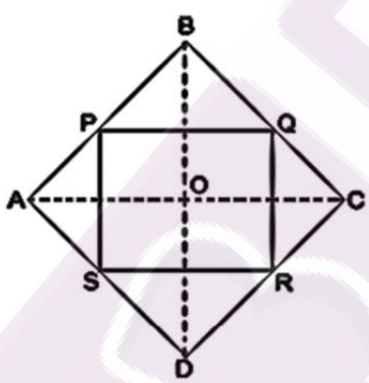
Solution:

(i) In $\triangle DAC$,
R is the mid point of DC and S is the mid point of DA.
Thus by mid point theorem, $SR \parallel AC$ and $SR = \frac{1}{2}AC$

(ii) In $\triangle BAC$,
P is the mid point of AB and Q is the mid point of BC.
Thus by mid point theorem, $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$
also, $SR = \frac{1}{2}AC$
Thus, PQ = SR

(iii) $SR \parallel AC$– from (i)
and, $PQ \parallel AC$– from (ii)
$\Rightarrow SR \parallel PQ$– from (i) and (ii)
also, PQ = SR
Thus, PQRS is a parallelogram.

Q2. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.
Solution:

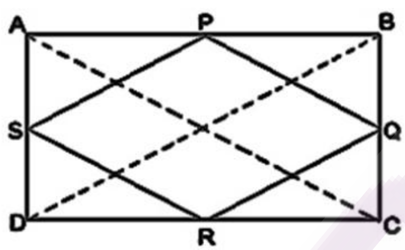


Given,
ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively.
To Prove,
PQRS is a rectangle.
Construction,
Join AC and BD

Proof,
In $\triangle DRS \ \ and \ \ \triangle BPQ$,
DS = BQ (Halves of the opposite sides of the rhombus)
$\angle SDR = \angle QBP$ (Opposite angles of the rhombus)
DR = BP (Halves of the opposite sides of the rhombus)
Thus, $\triangle DRS \cong \triangle BPQ$ (by SAS congruence condition).
RS = PQ (by CPCT) ………… (i)
In $\triangle QCR \ \ and \ \ \triangle SAP$,
RC = PA (Halves of the opposite sides of the rhombus)

$\angle RCQ = \angle PAS$ (Opposite angles of the rhombus)
CQ = AS (Halves of the opposite sides of the rhombus)
Thus, $\triangle QCR \cong \triangle SAP$ (by SAS congruence condition).
RQ = SP (by CPCT) ……(ii)
Now,
In $\triangle CBD$,
R and Q are the mid points of CD and BC respectively.
$\Rightarrow QR \parallel BD$
also,
P and S are the mid points of AD and AB respectively.
$\Rightarrow PS \parallel BD$
$\Rightarrow QR \parallel PS$
Thus, PQRS is a parallelogram.
also, $\angle PQR = 90^{\circ}$
Now,
In PQRS,
RS = PQ and RQ = SP from (i) and (ii)
$\angle Q = 90^{\circ}$
Thus, PQRS is a rectangle.

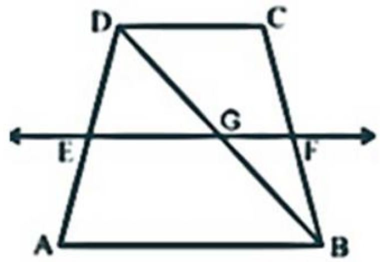
Q3. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.



Solution:

Given,
ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively.
Construction,
AC and BD are joined.
To Prove,
PQRS is a rhombus.
Proof,
In $\triangle ABC$
P and Q are the mid-points of AB and BC respectively
Thus, $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ (Mid point theorem) ……….(i)
In $\triangle ADC$,
$SR \parallel AC$ and $SR = \frac{1}{2}AC$ (Mid point theorem) ……(ii)
So, $PQ \parallel SR$ and $PQ = SR$
As in quadrilateral PQRS one pair of opposite sides is equal and parallel to each other, so, it is a parallelogram.
$PS \parallel QR$ and $PS = QR$ (Opposite sides of parallelogram) ….(iii)
Now,
In $\triangle BCD$,
Q and R are mid points of side BC and CD respectively.
Thus, $QR \parallel BD \ and \ QR = \frac{1}{2}BD$ (Mid point theorem) ……(iv)
AC = BD (Diagonals of a rectangle are equal) ……… (v)
From equations (i), (ii), (iii), (iv) and (v),
PQ = QR = SR = PS
So, PQRS is a rhombus.

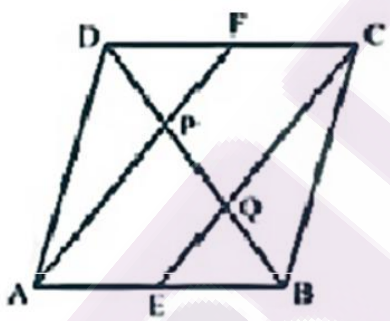
Q4. ABCD is a trapezium in which $AB \parallel DC$, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F. Show that F is the mid-point of BC.



Solution:

Given,
ABCD is a trapezium in which $AB \parallel DC$, BD is a diagonal and E is the mid-point of AD.
To prove,
F is the mid-point of BC.
Proof,
BD intersected EF at G.
In $\triangle BAD$,
E is the mid point of AD and also $EG \parallel AB$.
Thus, G is the mid point of BD (Converse of mid point theorem)
Now,
In $\triangle BDC$,
G is the mid point of BD and also $GF \parallel AB \parallel DC$.
Thus, F is the mid point of BC (Converse of mid point theorem)

Q5. In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively. Show that the line segments AF and EC trisect the diagonal BD.



Solution:

Given
ABCD is a parallelogram. E and F are the mid-points of sides AB and CD respectively.
To show,
AF and EC trisect the diagonal BD.
Proof,

ABCD is a parallelogram
Therefore, $AB \parallel CD$

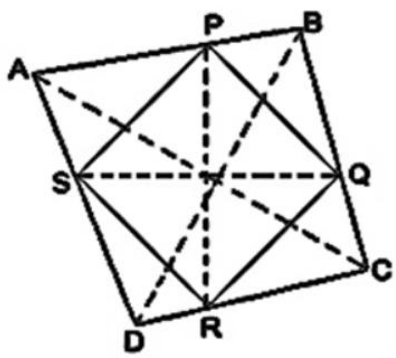
also, $AE \parallel FC$

Now,
$AB = CD$ (Opposite sides of parallelogram ABCD)
$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$
$\Rightarrow AE = FC$ (E and F are midpoints of side AB and CD)
AECF is a parallelogram (AE and CF are parallel and equal to each other)
$AF \parallel EC$ (Opposite sides of a parallelogram)
Now,
In $\triangle DQC$,

F is mid point of side DC and $FP \parallel CQ$ (as $AF \parallel EC$).
P is the mid-point of DQ (Converse of mid-point theorem)
$\Rightarrow DP = PQ$ ....(i)
Similarly,
In $\triangle APB$,
E is mid point of side AB and $EQ \parallel AP$ (as $AF \parallel EC$).
Q is the mid-point of PB (Converse of mid-point theorem)
$\Rightarrow PQ = QB$ ....(ii)
From equations (i) and (i),
DP = PQ = BQ
Hence, the line segments AF and EC trisect the diagonal BD.

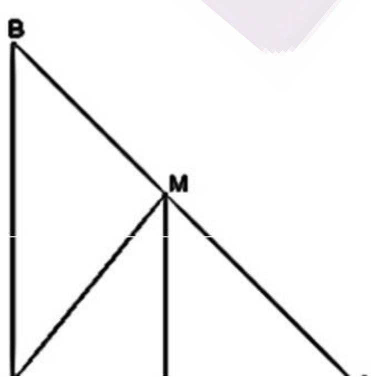
Q6. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

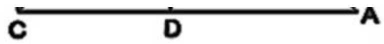
Solution:



Let ABCD be a quadrilateral and P, Q, R and S are the mid points of AB, BC, CD and DA respectively.
Now,
In $\triangle ACD$,
R and S are the mid points of CD and DA respectively.
Thus, $SR \parallel AC$.
Similarly we can show that,
$PQ \parallel AC$
$PS \parallel BD$
$QR \parallel BD$
Thus, PQRS is parallelogram.
PR and QS are the diagonals of the parallelogram PQRS. So, they will bisect each other.

Q7. ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that
(i) D is the mid-point of AC
(ii) $MD \perp AC$
(iii) $CM = MA = \frac{1}{2}AB$





Solution:

(i) In $\triangle ACB$,

M is the mid point of AB and $MD \parallel BC$

Thus, D is the mid point of AC (Converse of mid point theorem)

(ii) $\angle ACB = \angle ADM$ (Corresponding angles)

also, $\angle ACB = 90^\circ$

Thus, $\angle ADM = 90^\circ$ and $MD \perp AC$

(iii) In $\triangle AMD \ and \ \triangle CMD$,

AD = CD (D is the midpoint of side AC)

$\angle ADM = \angle CDM$ (Each 90°)

DM = DM (common)

Thus, $\triangle AMD \cong \triangle CMD$ (by SAS congruence condition).

AM = CM (by CPCT)

also, $AM = \frac{1}{2}AB$ (as M is mid point of AB)

Hence, $CM = AM = \frac{1}{2}AB$.