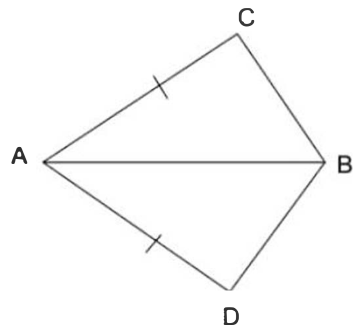
NCERT SOLUTIONS
CLASS-IX MATHS
CHAPTER-7 TRIANGLES

Exercise 7.1

Question 1:

In quadrilateral PQRS, PR = PQ and PQ bisects $\angle P$ (look at the fig.). Show that the $\triangle PQR = \triangle PQS$. What can you say about QR and QS?



Solution:

In $\triangle PQR$ and $\triangle PQS$, we have

PR = PS

$\angle RPQ = \angle SPQ$ (PQ bisects $\angle P$)

PQ = PQ (common)

$\triangle PQR = \triangle PQS$ (By SAS congruence)

Hence Proved.

Therefore, QR = QS (CPCT)

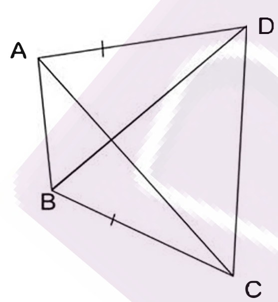
Question 2:

ABCD is a quadrilateral in which AD = BC and $\angle DAB$= $\angle CBA$ (see Fig.). Prove that

(i) $\triangle ABD = \triangle BAC$

(ii) BD = AC

(iii) $\angle ABD = \angle BAC$



Solution:

In the given figure, ABCD is a quadrilateral in which AD = BC and $\angle DAB$= $\angle CBA$.

In $\triangle DAB \, and \, \triangle BAC$, we have

AD = BC [Given]

$\angle DAB = \angle CBA$ [Given]

AB = AB [Common]

$\triangle ABD = \triangle BAC$ [By SAS congruence]

BD = AC [CPCT]

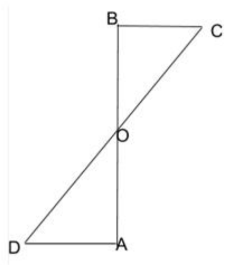
and $\angle ABD = \angle BAC$ [CPCT]

Hence Proved

Hence Proved.

Question 3:

AD and BC are equal perpendiculars to a line segment AB (see Fig.). Show that CD bisects AB.



Solution:

In $\triangle AOD = \triangle BOC$, we have,

$\angle AOD = \angle BOC$ [Vertically opposite angles)

$\angle CBO = \angle DAO (Each = 90^o)$

AD = BC [Given]

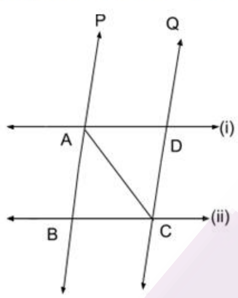
$\triangle AOD = \triangle BOC$ [By AAS congruence]

Also, AO = BO [CPCT]

Hence, CD bisects AB Proved.

Question 4:

L and m are two para;;e; lines intersected by another pair of parallel lines p and q (see fig.). Show that $\triangle ABC + \triangle CDA$.



Solution:

In the given figure, ABCD is a parallelogram in which AC is a diagonal, i.e., AB || DC and BC || AD.

$\triangle ABC = \triangle CDA$, we have,

$\angle BAC = \angle DCA$ (Alternate angles)

$\angle BCA = \angle DAC$ (Alternate angles)

AC = AC (Common)

$\triangle ABC = \triangle CDA$ (By SAS congruence)

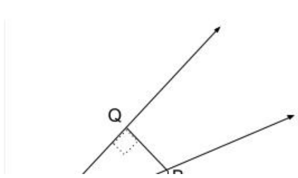
Hence Proved.

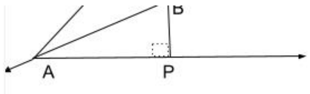
Question 5:

Line l is the bisector of an angle A and B is any point on l. BP and BQ are perpendicular from B to the arms of $\angle A$ ***(See fig.). Show that:***

(i) $\triangle APB \ and \ \triangle AQB$

(ii) BP = BQ or B is equidistant from the arms of $\angle A$





Solution:

In $\triangle APB \, and \, \triangle AQB$, we have

$\angle PAB = \angle AQB$ (l is the bisector of $\angle A$)

$\angle APB = \angle AQB \; (Each = 90^o)$

AB = AB

$\triangle APB = \triangle AQB$ (By AAS congruence)

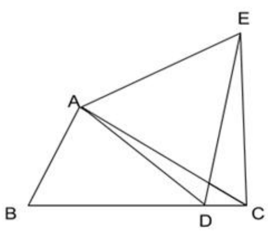
Also, BP = BQ (By CPCT)

i.e, B is equidistant from the arms of $\angle A$

Hence proved.

Question 6:

In the fig, AC = AE, AB = AD and $\triangle BAD = \triangle EAC$. Show that BC = DE.



Solution:

$\angle BAD = \triangle EAC$ (Given)

$\angle BAD + \angle DAC = \angle EAC + \angle DAC (Adding \angle DAC \; to \; both \; sides) \; \angle BAC = \angle EAC$

Now, in $\triangle ABC \, and \, \triangle ADE$, We have

AB = AD

AC = AE

$\angle BAC = \angle DAE$

$\triangle ABC = \triangle ADE$ (By SAS congruence)

BC = DE (CPCT)

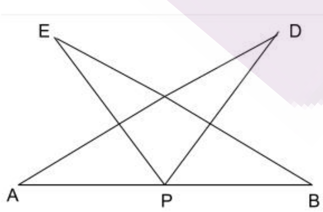
Hence Proved.

Question 7:

AB is a line segment and p is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE \; and \; \angle EPA = \angle DPB$ (see fig.). Show that

(i) $\triangle DAP = \triangle EBP$

(ii) AD = BE



Solution:

In $\triangle DAP \; and \; \triangle EBP$, we have

AP = BP (P is the midpoint of the line segment AB)

$\angle BAD = \angle ABE$ (Given)

$\angle EPB = \angle DPA \, (\angle EPA = \angle DPB \Rightarrow \angle EPA + \angle DPE = \angle DPB + \angle DPE) \, \triangle DPA = \triangle EPB$

$\angle EPB = \angle DPA (\angle EPA = \angle DPB \Rightarrow \angle EPA + \angle DPE = \angle DPB + \angle DPE) \triangle DPA = \triangle EPB$

AD = BE (ASA)

AD = BE

Hence Proved.

Question 8:

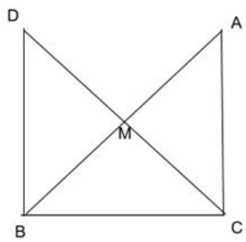
In right triangle ABC, right angles at C, M is the mid-point of hypotenuse AB, C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see fig.). Show that :

(i) $\triangle AMC = \triangle BMD$

(II) $\angle DBC$ ***is a right angle.***

(iii) $\triangle DBC = \triangle ACB$

(iv) $CM = \frac{1}{2}AB$



Solution:

In $\triangle BMB and \triangle DMC$, we have

(i) DM = CM (given)

BM = AM (M is the midpoint of AB)

$\angle DMB = \angle AMC$ (Vertically opposite angles)

$\triangle AMC = \triangle BMD$ (By SAS)

Hence Proved.

(ii) AC || BD ($\angle DBM \ and \ \angle CAM$ are .alternate angles)

$\Rightarrow \angle DBC + \angle ACB = 180^o$ (Sum of co-interior angles)

$\Rightarrow \angle DBC = 90^o$

Hence proved.

(iii) In $\triangle DBC \ amd \ \triangle ACB$, We have

DB = AC (CPCT)

BC = BC (Common)

$\angle DBC = \angle ACB (Each = 90^o)$

$\triangle DBC = \triangle ACB$ (By SAS)

Hence proved.

(iv) AB = CD

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$ (CPCT)

Hence, $\frac{1}{2}AB = CM$ Proved.

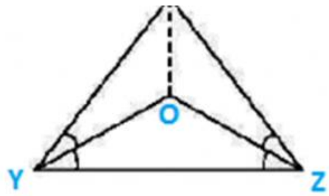
<u>Exercise 7.2:</u>

Question 1:

In an isosceles triangle XYZ with XY=XZ ,the bisector of $\angle B$ ***and*** $\angle C$

Intersect each other at O .join A at O .Show that:

(1)OY=OZ (2)XO bisects $\angle A$



Solution:

$\frac{1}{2}\angle XYZ = \frac{1}{2}\angle XZY \; \angle ZYO = \angle XZY$

[OY and OZ are bisector of

$\angle Y$ and $\angle Z$ respectively]

OBY=OZ [sides opposite to equal angles are equal]

Again, $\frac{1}{2}\angle XYZ = \frac{1}{2}\angle XZY$

$\angle XYO = \angle XYO$ [$\therefore$OY and OZ are bisector of $\angle Y$ and $\angle Z$ respectively]

In $\Delta XYO = \Delta XYO$, we have

XY=XZ [Given]

OY=OZ [proved XYove]

$\angle XYO = \angle XZO$ [proved above]

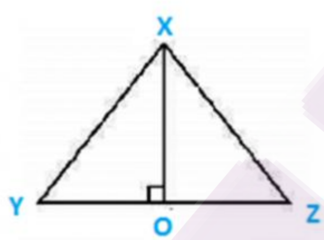
$\Delta XYO = \Delta XZO$ [SAS congruence]

ΔXYO= $\angle ZXO$ [CPCT]

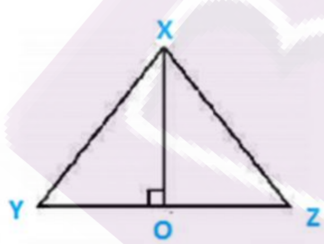
XO bisects $\angle X$ **proved**

Question 2:

In ΔXYZ ,XO is the perpendicular bisector of YZ .Prove that ΔXYZ is an isosceles triangle in which XY=XZ



Solution:



In $\Delta XYO = \Delta XZO$, we have

$\angle XOZ = \angle XOZ$ [] each =90°]

YO=ZO [XO bisects YZ]

XO=XO [common]

$\therefore \Delta XYO = \Delta XZO$ [SAS]

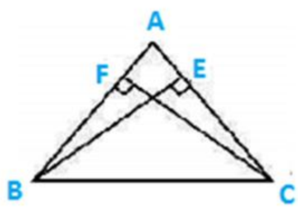
$\therefore$ XY=XZ [CPCT]

Hence, ΔXYZ is an isosceles triangle .**Proved**

Question 3:

ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and

AB respectively (see figure).Show that these altitudes are equal.

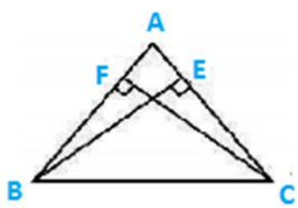


Solution:

In $\Delta ABC,$

AB=AC [Given]

$\angle B=\angle C$ [angles opposite to equal sides of a triangle are equal]



Now in right triangles BFC and CEB ,

$\angle BFC=\angle CEB$ [Each =90°]

$\angle FBC=\angle ECB$ [proved above]

BC=BC

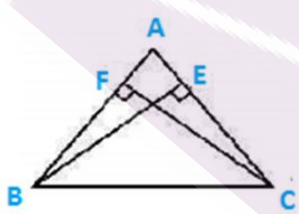
$\therefore \Delta BFC=\Delta CEB$ [AAS]

Hence, BE=CF [CPCT] **proved**

Question 4:

ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see.fig) show that

(1) $\Delta ABE\cong\Delta ACF$



(2) AB=AC i,e ABC is an isosceles triangle.

Solution(1) in ΔABE and ACF , we have

BE=CF [Given]

$\angle BAE=\angle CAF$ [common]

$\angle BEA=\angle CFA$ [Each =90°]

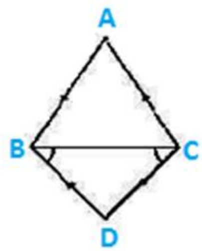
So, $\Delta ABE=\angle ACF$ [AAS]proved

(2) also, AB=AC [CPCT]

i,e., ABC is an isosceles triangle **Proved.**

Question 5:

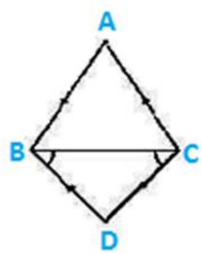
ABC and DBC are two isosceles triangle on the same base BC .show that



$\angle ABD = \angle ACD$

Solution. In isosceles ΔABC, We have

AB= AC



$\angle ABC = \angle ACB$ ———-(i)

[Angles opposite to equal sides are equal]

Now , in isosceles ΔDCB, We have

BD=CD

$\angle DBC = \angle DCB$ —————(ii)

[Angle opposite to equal sides are equal]

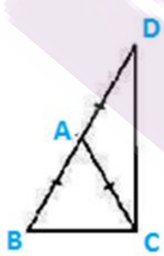
Adding (i) and (ii) , we have

$\angle ABC + \angle DBC = \angle ACB + \angle DCB$

$\angle ABD = \angle ACD$.**proved**

Question 6:

ΔABC is an isosceles triangle in which AB=AC side BA is produced to D such that AD =AB .show that $\angle BCD$ is a right angle.



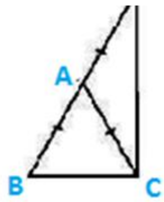
Solution:

AB=AC

$\angle ACB = \angle ABC$—————-(1)

[Angles opposite to equal sides are equal]

AB=AD [Given]



AD=AC [AB=AC]

$\therefore \angle ACD = \angle ADC$ ————————(ii)

[Angles opposite to equal sides are equal]

Adding (i) and (ii)

$\angle ACB + \angle ACD = \angle ABC + \angle ADC$ $\angle BCD = \angle ABC + \angle ADC$

Now in $\Delta BCD, We have$

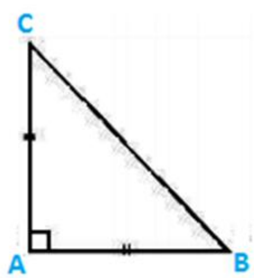
$\angle BCD + \angle DBC + \angle BDC ==$ 180 [Angle sum property of a triangle]

$\therefore \angle BCD + \angle BCD$=180°

$2\angle BCD = 180$ $2\angle BCD = 180$

Question 7:

ABC is a right angled triangle in which $\angle A$=90 and AB=AC.Find



$\angle B and \angle C$

Solution. In ΔABC, We have

$\angle A$=90

AB =AC

We know that angles opposite to equal sides of an isosceles triangle are equal

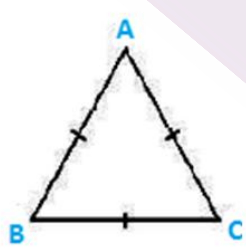
So, $\angle B = \angle C$

Since $\angle A$=90 , therefore sum of remaining two angles =90

Hence $\angle B = \angle C = 45$

Question 8:

Show that the angles of an equilateral triangle are 60 each.



Solution:

As ΔABC is an equilateral

So, AB=BC=AC

Now, AB=AC

$\angle ACB = \angle ABC$——-(ii)[angles sum property of a triangle]

Again BC=AC

$\angle BAC = \angle ABC$—-(ii) [same reason]

Now in ΔABC

$\angle ABC + \angle ACB + \angle BAC = 180$ [angle sum property of a triangle are equal]

$\angle ABC + \angle ACB + \angle BAC = 180$=180 [from (i) (ii)]

$3\angle ABC$=180

$\angle ABC$ =180/3=60°

Also from (i) and (ii)

$\angle ACB$=60 and $\angle BAC$

Hence each angle of an equilateral triangle is 60° **Proved**

Exercise 7.3

Question 1:

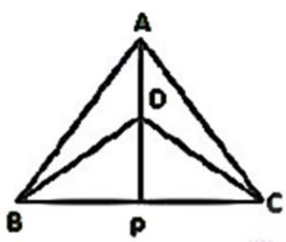
On the same base BC △XYZ and △DYZ are two isosceles triangles and vertices X and D are on the same side of YZ. If XD is extended to intersect YZ at P, show that

(i) △XYD ≈ △XZD

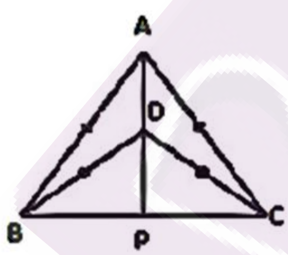
(ii) △XYP ≈ △XZP

(iii) XP bisects ∠X as well as ∠D.

(iv) XP is the perpendicular and bisector of YZ.



Solution:



(i) In △XYD and △XZD, we have

XY = XZ [Given]

YD = ZD [Given]

XD = XD [Common]

△XYD ≈ △XZD [SSS congruence].

(ii) In △XYP and △XZP, we have

XY = XZ

∠YXP = ∠ZXP

XP = XP

△XYP ≈ △XZP [SAS congruence].

(iii)△XYD ≈ △XDY

∠XDY = ∠XDZ

=> 180° — ∠XDY = 180° — ∠XDZ
Also, from part (ii), ∠YXPD = ∠ZXP

XP bisects DX as well as ∠D.

(iv) Now, YP = ZP and ∠YPX = ∠ZPX

But ∠YPX + ∠ZPX = 180°

So, 2∠BPA = 180°

Or, ∠YPX = 90°

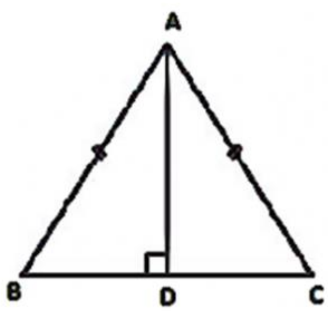
Since YP = ZP, therefore XP is perpendicular bisector of YZ.

Question 2:

Isosceles triangle XYZ has an altitude XD, in which XY = XZ. Show that

(i) XD bisects YZ

(ii) XD bisects ∠X



Solution:

(i) In △ABD and △ACD, we have ∠ADB = ∠ADC

XY = XZ

XD = XD

△XYD ≈ △XZD.

YD = ZD

Hence, XD bisects YC.

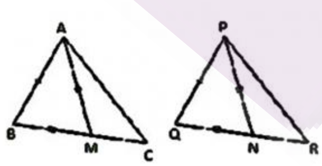
(ii) Also, ∠YXD = ∠ZXD

Hence XD bisects ∠X.

Question 3:

Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of △PQR. Show that:

(i) △ABM ≈ APQN (ii) △ABC ≈ △PQR



Solution:

(i) In △ABM and △PQN,

we have

BM = QN

$\frac{1}{2}BC = \frac{1}{2}QR$

AB = PQ

AM = PN

△ABM = △PQN [SSS Congruence]

∠ABM = ∠PQN.

(ii) Now, in △ABC and △PQR, we have

AB = PQ

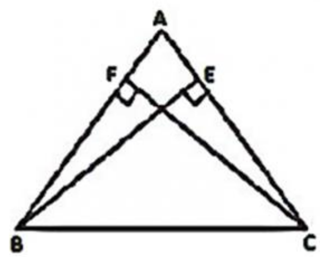
∠ABC = ∠PQR

BC = QR

△ABC ≊ △PQR [SAS congruence]

Question 4:

In triangle ABC, BE and CF are two equal altitudes. By Using RHS congruence rule, prove that the triangle ABC is isosceles.



Solution:

BE and CF are altitudes of a △ABC.

∠BEC = ∠CFB = 90°

Now, in right triangles BEB and CFB, we have

Hyp. BC = Hyp. BC

Side BE= Side CF

△BEC ≊ △CFB

∠BCE = ∠CBF

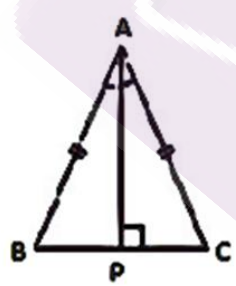
Now, in △ABC, ∠B = ∠C

AB = AC

Hence, △ABC is an isosceles triangle.

Question 5:

AB = AC in an isosceles triangle ABC. Draw AP perpendicular BC to show that ∠B= ∠C.



Solution:

Draw AP perpendicular BC. In △ABP and △ACP, we have

AB = AC

∠APB = ∠APC

AB = AP [Common]

△ABP = △ACP [By RHS congruence rule]

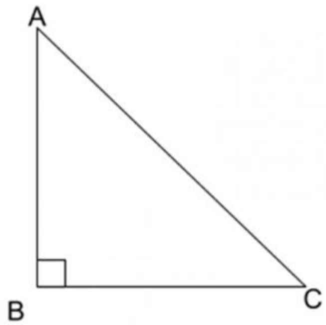
Also, ∠B = ∠C Proved. [CPCT]

Also, ∠B = ∠C Proved. [CPCT]

Exercise 7.4

Question 1:

Show that in a right-angled triangle, the hypotenuse is the longest side.



Solution:

ABC is a right angle triangle, right angled at B.

Now $\angle A + \angle C = 90^o$

Angles A and C are less than 90°

Now, $\angle B > \angle A$

AC > BC ….(i)

(Side opposite to greater angle is longer)

Again, $\angle B > \angle C$

AC > AB …(ii)

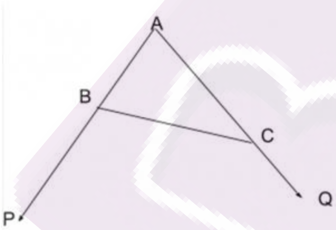
(Side opposite to greater angle are longer)

Hence, from (i) and (ii), we can say that AC (hypotenuse) is the longest side.

Hence proved.

Question 2:

In the figure, sides AB and AC of $\triangle ABC$ are extended to points P and Q respectively. Also, $\angle PBC < \angle QCB$. Show that AC > AB.



Solution:

$\angle ABC + \angle PBC = 180^o (Linear pair) \Rightarrow \angle ABC = 180^o - \angle PBC$

Similarly, $\angle ACB = 180^o - \angle QCB$

It is given that $\angle PBC < \angle QCB$

$180^o - \angle QCB < 180^o - \angle PBC$

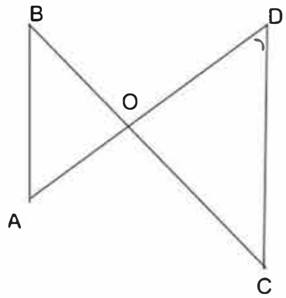
Or $\angle ACB < \angle ABC$ [From (i) and (ii)]

$\Rightarrow AB < AC \Rightarrow AC > AB$

Hence proved.

Question 3:

In the figure, $\angle B < \angle A$ and $\angle C < \angle D$. Show that AD < BC.



Solution:

$\angle B < \angle A$ (Given)

BO > AO ...(i)

(Side opposite to greater angle is longer)

$\angle C < \angle D$ (Given)

CO > DO ...(ii)

(Same reason)

Adding (i) and (ii)

BO + CO > AO + DO

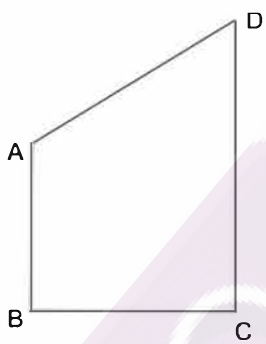
BC > AD

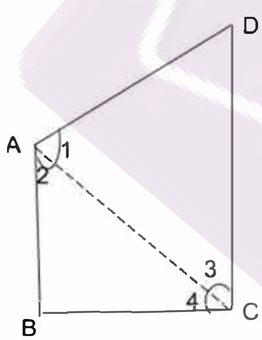
AD < BC

Hence Proved.

Question 4:

AB and CD are respectively the smallest and longest side of a quadrilateral ABCD (see fig.) Show that $\angle A > \angle C$





Solution:

Join AC.

Mark the angles as shown in the figure.

In $\triangle ABC$,

BC > AB (AB is the shortest side)

$\angle 2 > \angle 4$ ...(i)

[Angle opposite to longer side is greater]

In $\triangle ADC$,

CD > AD (CD is the longest side)

$\angle 1 > \angle 3$ ...(ii)

[Angle opposite to longer side is greater]

Adding (i) and (ii), we have

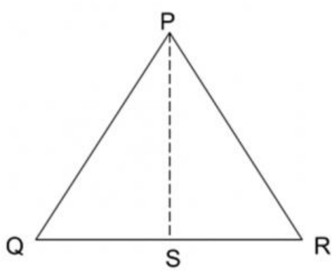
$\angle 2 + \angle 1 > \angle 4 + \angle 3 \Rightarrow \angle A > \angle C$

Similarly, by joining BD, we can prove that

$\angle B > \angle D$

Question 5:

In the figure, PR > PQ and PS bisects $\angle QPR$. Prove that $\angle PSR > \angle PSQ$



Solution:

PR > PQ

$\angle PQR > \angle PRQ$ ...(i)

[Angle opposite to longer side is greater]

$\angle QPS > \angle RPS$ $(PS\ bisects\ \angle QPR)$ ...(ii)

In $\triangle PQS$, $\angle PQS + \angle QPS + \angle PSQ = 180^o$

$\Rightarrow \angle PSQ = 180^o - (\angle PQS + \angle QPS)$ ...(iii)

Similarly in $\triangle PRS$, $\triangle PSR = 180^o - (\angle PRS + \angle RPS)$

$\Rightarrow \angle PSR = 180^o - (\angle PSR + \angle QPS)$ [from (ii) ... (iv)

From (i), we know that $\angle PQS < \angle PSR$

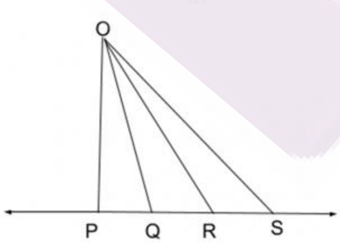
So from (iii) and (iv), $\angle PSQ < \angle PSR$

$\Rightarrow \angle PSR > \angle PSQ$

Hence proved.

Question 6:

Show that of all the segments drawn from a given point not on it, the perpendicular line segment is the shortest.



Solution:

We have a line l and O is the point not on l

$OP \perp i$

We have to prove that OP < OQ, OP < OR and OP < OS.

OP < OS

In $\triangle OPQ$, $\angle P = 90^o$

Therefore, $\angle Q$ is an acute angle (i.e, $\angle Q < 90^o$)

$\angle Q < \angle P$

Hence, OP < OQ (Side opposite to greater angle is longer)

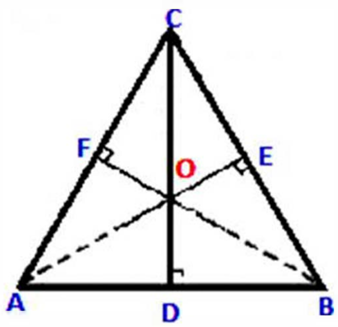
Similarly, we can prove that OP is shorter than OR, OS, etc.

Hence proved

Exercise 7.5

Question 1:

Find a point in the interior of $\triangle DEF$ which is at an equal distance or equidistant from all the vertices of $\triangle DEF$.



Solution:

Draw perpendicular bisectors of sides DE,

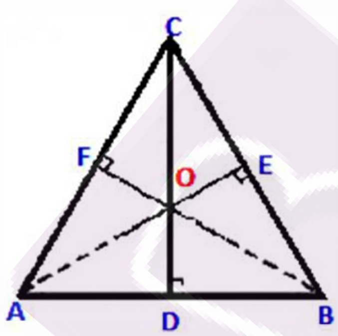
EF and FD, which meets at O.

Hence, O is the required point.

Question 2:

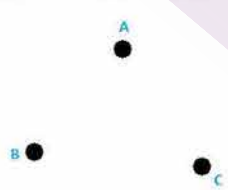
Find a point in the interior of a triangle such that it is at equal distances from all the sides of the triangle.

Solution:



Question 3:

People are concentrated at three points in a park namely A,B and C. (see Fig.).



A: is where swings and slides for children are present

B: is where a lake is present

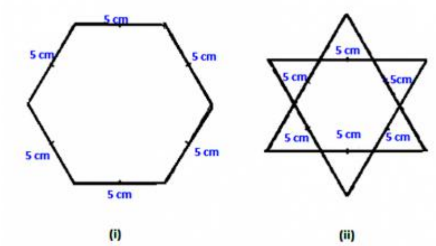
C: is where there is a large parking lot and exit

Where do you think an ice – cream parlor should be set up such that the maximum number of people can access it? Draw bisectors AB and BC Let these angle bisectors meet at O. O is the required point.

Solution: Join AB, BC and CA to get a triangle ABC. Draw the perpendicular bisector of AB and BC. Let them meet at O. Then O is equidistant from A, B and C. Hence, the parlor should be set up at O so that all the other points are equidistant from it.

Question 4:

Fill the star shaped and hexagonal rangolies [see fig.(i) and (ii)] by filling them with as many equilateral triangles as you can of side 1 cm. What is the number of triangles in both the cases? Which one has the most number of triangles?



Solution: