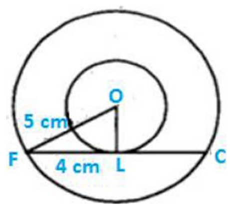
NCERT SOLUTIONS
CLASS-IX MATHS
CHAPTER-10 CIRCLES

Short Answer Questions

1. Out of the two concentric circle ,the radius of the outer circle is 5cm and the chord FC is of length 8cm is a tangent to the inner circle .Find the radius of the inner circle.

Sol. Let the chord FC of the larger circle touch the smaller circle at the point L.

Since FC is tangent at the point L to the smaller circle with the centre O.



$\therefore$ OL$\perp$FC

Since AC is chord of the bigger circle and OL$\perp$FC.

$\therefore$ OL bisects FC

FC=2FL

$\Rightarrow$ 8=2FL

$\Rightarrow$ FL=4cm

Now, consider right-angled ΔFLO,we obtain

$OL^2 = FO^2 - FL^2$

$= 5^2 - 4^2$

=25-16

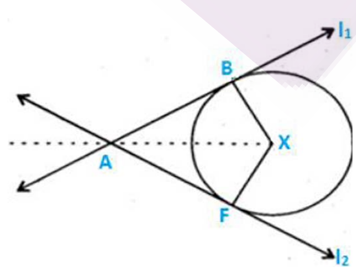
=9

OL $=\sqrt{9}$=3

Hence, the radius of the smaller or inner circle is 3cm

2. Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

Sol. Let l_1 and l_2,two intersecting lines, intersect at A, be the tangents from an external point A to a circle with centre X, at B and F respectively.



Join XB and XF

Now, in ΔABX and ΔARX, we have

AX=AX (common)

XB=XF (radii of same circle)

AB=AF

(tangents from an external point)

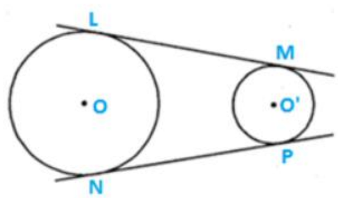
∴ ΔABX=ΔAFX

(by SSS congruence rule)

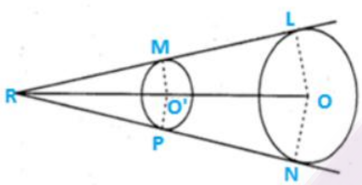
$\Rightarrow$ $\angle$XAB=$\angle$XAF

$\Rightarrow$ X lies on the bisector of the lines l_1 and l_2.

3.In the given figure,LM and NP are common tangents to two circles of unequal radii.Prove that LM=NP.



Sol. Let Chords LM and NP meet at the point R



Since RL and RN are tangents from an external point R to two Circles with centres O and O'.

∴ RL=RN .....(i)

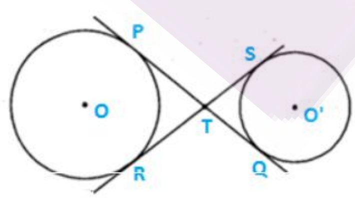
And RM=RP .....(ii)

Subtracting (ii) from (i),we have

RL-RM=RN-RP

∴ LM=NP

4.In the given figure, common tangents PQ and RS to two circles intersect at T. Prove that PQ=RS.



Sol. Clearly, TP and TR are two tangents from an external point T to the circle with centre O.

∴ TP=TR ......(i)

Also, TQ and TS are two tangents from an external point T to the circle with centre O'.

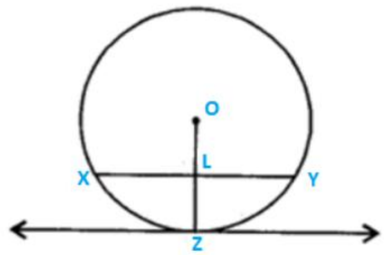
∴ TQ=TS .....(ii)

Adding (i) and (ii),we obtain

TP+TQ=TR+TS

$\Rightarrow$ PQ=RS

5.A chord XY of a circle is parallel to the tangent drawn at a point Z of the circle. Prove that Z bisects the arc XZY.



Sol. Since XY is parallel to the tangent drawn at the point Z and radius OZ is perpendicular to the tangent.

$\therefore$ OR$\perp$XY

$\therefore$ OL bisects the chord XY.

$\therefore$ XL-LY

$\therefore$ arc XZ=arc ZY

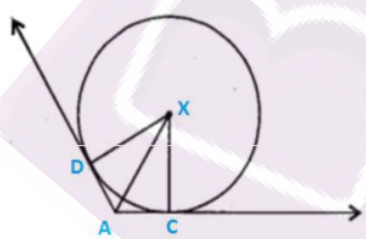
i.e., Z bisects arc XZY

Long answer questions

1.If from an external point <u>A</u> of a circle with centre X,two tangents AC and AD are drawn such that $\angle$DAC=120° , prove that AC+AD=AX i.e.,AX=2AC.

Sol. Join XC and XD.

since, XCAC



$\therefore$ [tangent to any circle is perpendicular to its radius at point of contact]

$\therefore$ $\angle$XCA=90°

In ΔXCB and ΔXDB,we have

CA=DA

[tangents from an external point]

XA=XA [common side]

XC=XD [radii of a circle]

∴ By SSS congruency , we have

$\Delta XCA \cong \Delta XDA$

$\Rightarrow\cong$ ∠XAC=∠XAD

=$\frac{1}{2}$∠CAD=$\frac{1}{2}\times 120°$=$60°$

In ΔXCA ,∠XCA=90°

$$\frac{AC}{AX} = \cos 60°$$

$$\frac{AC}{AX} = \overline{\frac{1}{2}}$$

$\Rightarrow$ AX=2AC

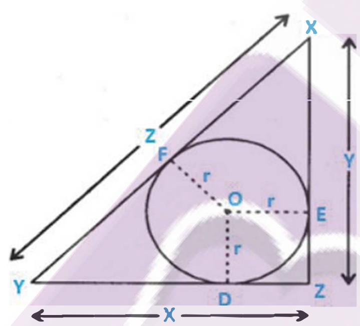
Also, AX=AC + AC

$\Rightarrow$ AX=AC + AD [Since, AC=AD]

2.If x,y,z are the sides of a right triangle where c is hypotenuse,prove that the radius r of the circle which touches the sides of the triangle is given by

$$r = \frac{x+y-z}{2}$$

Sol. Let the circle touches the sides YZ,ZX,XY of the right triangle XYZ at D,E,F respectively,where YZ=x,ZX=y and XY=z.Then XE = XF and YD = YF.



Also, ZE=ZD=r

I.e., y-r=XF,

x-r=yF

Or XY=z=XF+YF

$\Rightarrow$ Z=y-r+x-r

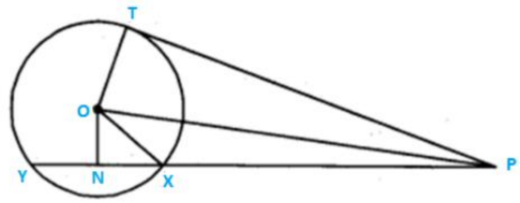
$\Rightarrow$ r = $\frac{x+y-z}{2}$

3.In the given figure,from an external poiny P,a tangent PT and a line segment PXY is drawn to a circle with the centre O.ON is perpendicular on the chord XY. Prove that:

(i)PX.PY= $PY^2 - XN^2$

(ii) $PN^2 - XN^2 = OP^2 - OT^2$

(iii) $PX.PY = PT^2$



Sol. (i) PX.PY=(PN-XN)(PN+YN)

=(PN-XN)(PN+XN)

since, XN=YN,as ON⊥Chord XY]

= $PN^2 - XN^2$

(ii) $PN^2 - XN^2$=($OP^2 ON^2$)- XN^2

Since,in rt. △ONP OP^2= ON^2 + PN^2]

=OP^2– ON^2– XN^2

=OP^2-(ON^2)+ XN^2)

=OP^2– OX^2

=OP^2– OT^2

Since,OX=OT=r]

(iii)From (i) and(ii),we have

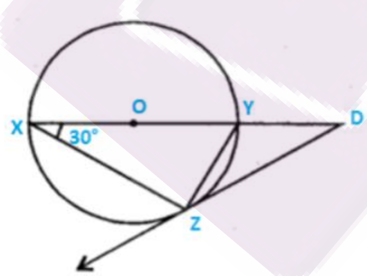
PX.PY= OP^2– OT^2

$PX.PY = PT^2$

∴ In rt. △OTP; OP^2= OT^2+ PT^2

4.XY is a diameter and XZ is a chord with centre O such that ∠YXZ= 30°.The tangent at Z intersects XY extended at a point D. Prove that YZ=YD.

Sol. Here, XOY is a diameter of the circle, such that ∠YXZ=30° and ZD be the tangent at Z.



∠XZY=90° ∠in a semi-circle]

Also, ∠ZYD=∠YXZ+∠XZY

[ext. ∠of a △ is equal to sum of interior opp.angles]

=30°+ 90°

=120°

∠YZD=∠YXZ=30°

In △,by angles sum property,we have

∠YDZ+∠ZYD+∠YZD=180°

∠YDZ+120°+30°=180°

∠YDZ=180°–120°–30°

=30°

\Rightarrow ∠YDZ=∠YDZ

=30°

Hence, YD=YZ

[sides opp.to equal angles]

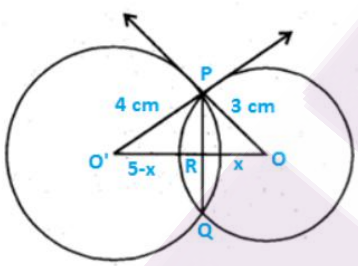
5.Two circles with centres O and O' of radii 3 cm and 4cm respectively intersect at two points P and Q such that O'P are tangents to the two circles.Find the length of the common chord PQ.

Sol. Clearly, ∠OPO'= 90°

OO'= $\sqrt{3^2+4^2}'$

=$\sqrt{9+16}=\sqrt{25}$

=5cm



Let RO=x

⇒ RO'=5-x

Now, PR^2= PO^2– RO^2

=9-x^2 ....(1)

Also, PR^2= PO^2– RO^2

=16-$(5-x)^2$..........(2)

From (1) and (2),we have

9-x^2=16-(25+x^2-10x)

9-x^2=16-25+x^2+10x)

10x=18

X=1.8cm

From (1),we have

PR^2=9-1.8^2

=9-3.24

=5.76

PR=$\sqrt{5.76}$

=2.4cm

Hence, the required length of the common chord is 2 x PR i.e., 2×2.4 i.e., 4.8cm.

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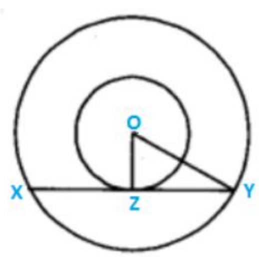
Exemplar problems

1. If V1 , V2 (V2 > V1) be the diameters of two concentric circles and Z be the length of a chord of a circle which is tangent to the other circle, prove that : $V_2{}^2 = Z^2 + V_1{}^2$.

Solution:-

Let XY be a chord of a circle which touches the other circle at Z. Then ΔOZY is a right triangle.



∴ By Pythagoras Theorem,

$OZ^2 + ZY^2 = OY^2$

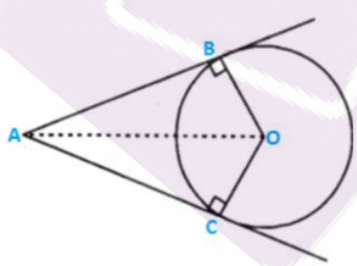
i.e., $\left(\frac{V1}{2}\right)^2 + \left(\frac{Z}{2}\right)^2 = \left(\frac{V2}{2}\right)^2$

$\Rightarrow \quad \frac{V_1{}^2}{4} + \frac{Z^2}{4} = \frac{V_2{}^2}{4}$

$\Rightarrow \quad V_2{}^2 + Z^2 = V_1{}^2$

2. Two tangents AB and AC are drawn from an external point to a circle with centre 0. Prove that BOCA is a cyclic quadrilateral.

Sol. We know that, tangents to a circle is perpendicular to its radius at the point of contact.



∴ OC⊥AC and OB⊥AB

∠OCA=∠OBA=90°

∠OCA+∠OBA=$90^\circ + 90^\circ$

=180°

Sum of opposite angles of quadrilateral BOCA is 180°

Hence, BOCA is a cyclic quadrilateral.