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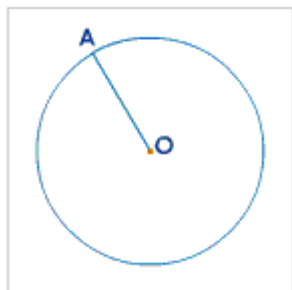
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# Circles

## Introduction

### Circle

A circle is defined as the locus of the points at a given distance from a certain fixed point.

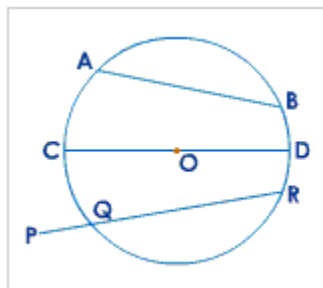


### Chord

The straight line joining any 2 points on the circle is called a chord.

AB is a chord.

The longest chord is called the diameter if it passes through the centre of the circle.

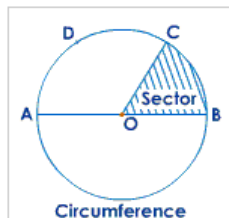


A diameter is twice the length of the radius.  $CD$  is a diameter.

A secant is a line cutting a circle into two parts.  $PQR$  is a secant.

### Circumference

The set of all the points on a circle constitute the circumference of the circle. In simple language we can say that the boundary curve of the circle (or perimeter) is its circumference.



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## Arc

Any part of the circumference is called an arc.

A diameter cuts a circle into 2 equal parts. An arc less than a semicircle is called a minor arc. An arc more than a semicircle is called a major arc.

$\widehat{ADC}$  is a minor arc and  $\widehat{ABC}$  is a major arc.

## Sector

A portion cut off by two radii is called a sector.

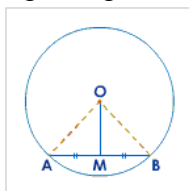
Segment: a portion of a circle cut off by a chord is called a segment.

## Concentric circles

Circles having the same centre are called concentric circles.

## Theorem 1

A straight line drawn from the centre of a circle to bisect a chord which is not a diameter, is at right angles to the chord.



### Data:

AB is a chord of a circle with centre O.

M is the mid-point of AB. OM is joined

### To Prove:

$$\angle AMO = \angle BMO = 90^\circ$$

### Construction:

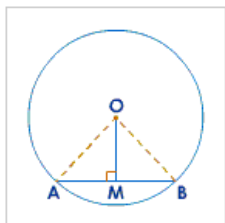
Join AO and BO.

### Proof:

Statement	Reason
In $\Delta^s$ AOM and BOM	
1. $AO = BO$	radii
2. $AM = BM$	data
3. $OM = OM$	common
4. $\Delta AOM \cong \Delta BOM$	(S.S.S.)
5. $\therefore \angle AMO = \angle BMO$	statement (4)
6. But $\angle AMO + \angle BMO = 180^\circ$	linear pair
7. $\therefore \angle AMO = \angle BMO = 90^\circ$	statements (5) and (6)

### Theorem 2: (Converse of theorem 1)

The perpendicular to a chord from the centre of a circle bisects the chord.



#### Data:

AB is a chord of a circle with centre O,  
 $OM \perp AB$ .

#### To Prove:

$AM = BM$ .

#### Construction:

Join AO and BO.

#### Proof:

Statement	Reason
In $\Delta AOM$ and $BOM$	
1. $\angle AMO = \angle BMO$	each $90^\circ$ (data)
2. $AO = BO$	radii
3. $OM = OM$	Common
4. $\Delta AOM \cong \Delta BOM$	(R.H.S.)
5. $AM = BM$	Statement (4)

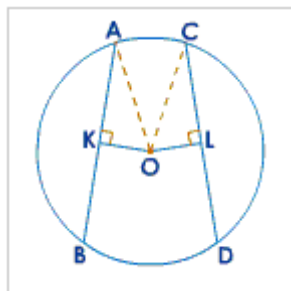
Converse of a theorem is the transposition of a statement consisting of 'data' and 'to prove'.

We elaborate it from the example of previous two theorems:

Theorem	Converse of theorem
1. Data: M is the mid-point of AB	To prove: M is the mid-point of AB.
2. To prove: $OM \perp AB$	Data: $OM \perp AB$

### Theorem 3

Equal chords of a circle are equidistant from the centre.



#### Data:

AB and CD are equal chords of a circle with centre O.  $OK \perp AB$  and  $OL \perp CD$ .

#### To Prove:

$OK = OL$

#### Construction:

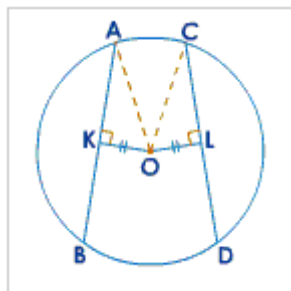
Join AO and CO.

#### Proof:

Statement	Reason
1. $AK = \frac{1}{2} AB$	$\perp$ from the centre bisects the chord.
2. $CL = \frac{1}{2} CD$	$\perp$ from the centre bisects the chord.
3. But $AB = CD$	data
4. $\therefore AK = CL$	statements (1), (2) and (3)
In $\triangle AOK$ and $\triangle COL$	
5. $\angle AKO = \angle CLO$	each $90^\circ$ (data)
6. $AO = CO$	radii
7. $AK = CL$	statement (4)
8. $\therefore \triangle AOK \cong \triangle COL$	(R.H.S.)
9. $\therefore OK = OL$	statement (8)

### Theorem 4 (Converse of 3)

Chords which are equidistant from the centre of a circle are equal.



#### Data:

AB, CD are chords of a circle with centre O.

$OK \perp AB$ ,  $OL \perp CD$  and  $OK = OL$ .

#### To Prove:

$AB = CD$ .

#### Construction:

Join AO and CO.

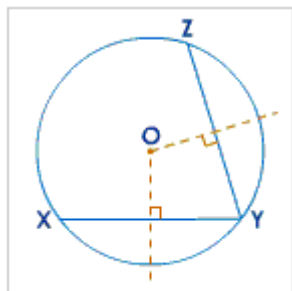
#### Proof:

Statement	Reason
In $\triangle AOK$ and $\triangle COL$	
1. $\angle AKO = \angle CLO$	each $90^\circ$ (data)
2. $AO = CO$	radii
3. $OK = OL$	data
4. $\triangle AOK \cong \triangle COL$	(R.H.S.)
5. $\therefore AK = CL$	statement (4)
6. But $AK = \frac{1}{2} AB$	$\perp$ from centre bisects the chord.
7. $CL = \frac{1}{2} CD$	$\perp$ from centre bisects the chord
8. $\therefore AB = CD$	statements (5), (6) and (7)



### Theorem 5

There is one circle, and only one, which passes through three given points not in a straight line.



#### Data:

X, Y and Z are three points not in a straight line.

#### To Prove:

A unique circle passes through X, Y and Z.

#### Construction:

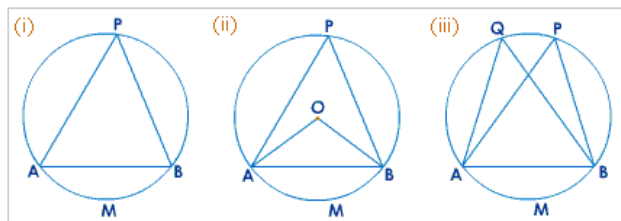
Join XY and YZ. Draw perpendicular bisectors of XY and YZ to meet at O.

#### Proof:

Statement	Reason
1. $OX = OY$	O lies on the $\perp$ bisector of XY.
2. $OY = OZ$	O lies on the $\perp$ bisector of YZ
3. $OX = OY = OZ$	statements (1) and (2)
4. O is the only point equidistant from X, Y and Z.	statement (3)
5. With O as centre and radius OX, a circle can be drawn to pass through X, Y and Z.	statement (4)
6. $\therefore$ the circle with centre O is a unique circle passing through X, Y and Z.	statement (5)

## Angle Properties (Angle, Cyclic Quadrilaterals and Arcs)

In fig.(i), the straight line AB subtends  $\angle APB$  on the circumference.



$\angle APB$  can be said to be subtended by arc AMB, on the remaining part of the circumference.

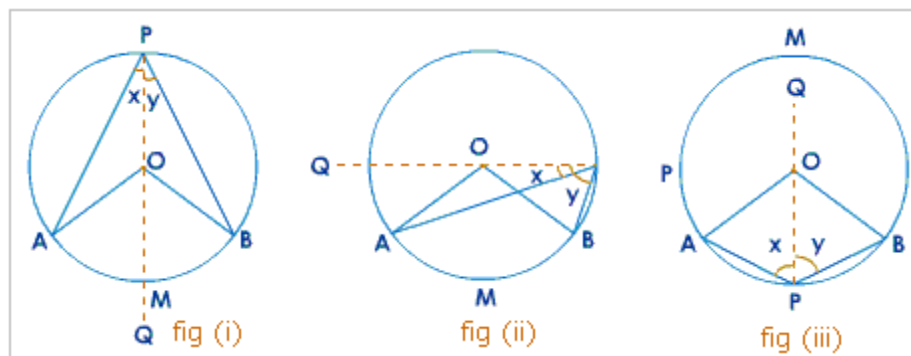
In fig.(ii), arc AMB subtends  $\angle APB$  on the circumference, and it subtends  $\angle AOB$  at the centre.

In fig. (iii),  $\angle APB$  and  $\angle AQB$  are in the same segment.

Let us study the theorems based on the angle properties of the circles.

### Theorem 6

The angle which an arc of a circle subtends at the centre is double the angle which it subtends at any point on the remaining part of the circumference.



### Data:

Arc AMB subtends  $\angle AOB$  at the centre O of the circle and  $\angle APB$  on the remaining part of the circumference.

### To Prove:

$$\angle AOB = 2 \angle APB$$

### Construction:

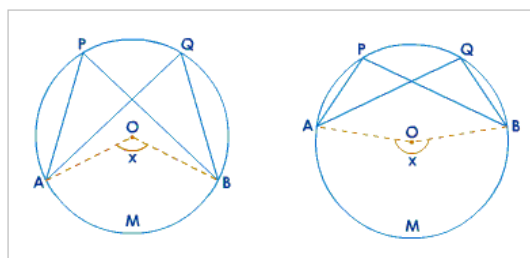
Join PO and produce it to Q. Let  $\angle APQ = x$  and  $\angle BPQ = y$ .

**Proof:**

Statement	Reason
1. $\angle AOQ = \angle x + \angle A$	ext. $\angle$ = sum of the int. opp. $\angle$ s
2. $\angle x = \angle A$	$\because OA = OP$ (radii)
3. $\therefore \angle AOQ = 2\angle x$	statements (1) and (2)
4. $\angle BOQ = 2\angle y$	same way as statement (3)
For fig.(i) and fig.(iii)	
5. $\angle AOQ + \angle BOQ = 2\angle x + 2\angle y$	statements (3) and (4)
6. $\Rightarrow \angle AOB = 2(\angle x + \angle y)$	statement (5)
7. For fig.(ii)	statements (3) and (4)
$\angle BOQ - \angle AOQ = 2\angle y - 2\angle x$	
8. $\angle AOB = 2(\angle y - \angle x)$	statement (8)
9. $\therefore \angle AOB = 2\angle APB$	statement (9)

**Theorem 7**

Angles in the same segment of a circle are equal.



**Data:**

$\angle APB$  and  $\angle AQB$  are in the same segment of a circle with centre O.

**To Prove:**

$$\angle APB = \angle AQB$$

**Construction:**

Join AO and BO.

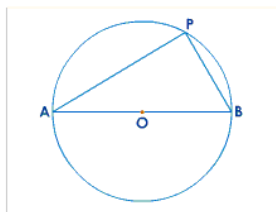
Let arc AMB subtend angle x at the centre O.

**Proof:**

Statement	Reason
1. $\angle x = 2\angle APB$	$\angle$ at centre = 2 x $\angle$ on the circumference
2. $\angle x = 2\angle AQB$	$\angle$ at centre = 2 x $\angle$ on the circumference
3. $\therefore \angle APB = \angle AQB$	statements (1) and (2)

### Theorem 8

The angle in a semicircle is a right angle.



#### Data:

AB is a diameter of a circle with centre O. P is any point on the circle

#### To Prove:

$$\angle APB = 90^\circ$$

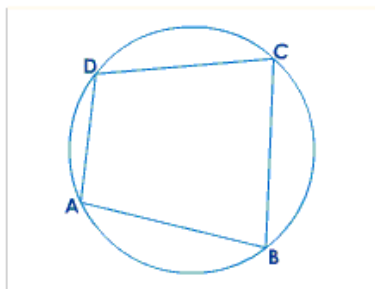
#### Proof:

Statement	Reason
1. $\angle APB = \frac{1}{2} \angle AOB$	$\angle$ at the centre = 2 $\times$ $\angle$ on the Oce.
2. $\angle AOB = 180^\circ$	AOB is a straight line
3. $\therefore \angle APB = \frac{1}{2} \times 180^\circ$	Statements (1) and (2)
4. $\therefore \angle APB = 90^\circ$	Statement (3)

### Cyclic Quadrilaterals

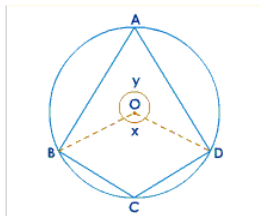
If the vertices of a quadrilateral lie on a circle, the quadrilateral is called a cyclic quadrilateral. The vertices are called concyclic points.

In the given figure, ABCD is a cyclic quadrilateral. The vertices A, B, C and D are concyclic points.



### Theorem 9

The opposite angles of a quadrilateral inscribed in a circle (cyclic) are supplementary.



#### Data:

ABCD is a cyclic quadrilateral; O is the centre of the circle.

#### To Prove:

- (i)  $\angle A + \angle C = 180^\circ$
- (ii)  $\angle B + \angle D = 180^\circ$

#### Construction:

Join BO and DO.

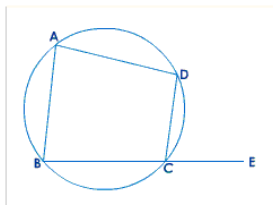
Let  $\angle BOD = x$  and reflex  $\angle BOD = y$

#### Proof:

Statement	Reason
1. $\angle A = \frac{1}{2} \angle x$	$\angle$ at the centre = $2(\angle$ on the circumference)
2. $\angle C = \frac{1}{2} \angle y$	$\angle$ at the centre = $2(\angle$ on the circumference)
3. $\angle A + \angle C = \frac{1}{2} \angle x + \frac{1}{2} \angle y$	Statements (1) and (2)
4. $\angle A + \angle C = \frac{1}{2} (\angle x + \angle y)$	Statement (3)
5. But $\angle x + \angle y = 360^\circ$	$\angle$ s at a point
6. $\therefore \angle A + \angle C = \frac{1}{2} \times 360^\circ$	statements (4) and (5)
7. $\therefore \angle A + \angle C = 180^\circ$	statement (6)
8. Also $\angle ABC + \angle ADC = 180^\circ$	same way as statement (7)

**Corollary:**

The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.



**Data:**

ABCD is a cyclic quadrilateral. BC is produced to E.

**To Prove:**

$$\angle DCE = \angle A$$

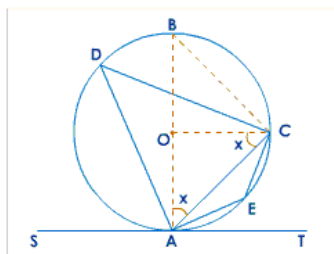
**Proof:**

Statement	Reason
1. $\angle A + \angle BCD = 180^\circ$	Opp. $\angle$ s of a cyclic quad.
2. $\angle BCD + \angle DCE = 180^\circ$	linear pair
3. $\therefore \angle BCD + \angle DCE = \angle A + \angle BCD$	Statements (1) and (2)
4. $\therefore \angle DCE = \angle A$	Statement (2)

**Alternate Segment Property**

**Theorem 10:**

The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment.



**Data:**

A straight line SAT touches a given circle with centre O at A. AC is a chord through the point of contact A.  $\angle ADC$  is an angle in the alternate segment to  $\angle CAT$  and  $\angle AEC$  is an angle in the alternate segment to  $\angle CAS$ .

**To Prove:**

- (i)  $\angle CAT = \angle ADC$
- (ii)  $\angle CAS = \angle AEC$

**Construction:**

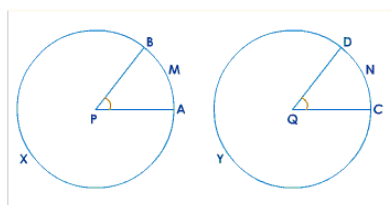
Draw AOB as diameter and join BC and OC.

**Proof:**

Statement	Reason
1. $\angle OAC = \angle OCA = x$	$\because OA = OC$ and supposition
2. $\angle CAT + \angle x = 90^\circ$	$\because$ tangent-radius property
3. $\angle AOC + \angle x + \angle x = 180^\circ$	sum of the angles of a $\Delta$
4. $\angle AOC = 180^\circ - 2\angle x$	statement (3)
5. Also $\angle AOC = 2\angle ADC$	$\angle$ at the centre = $2\angle$ on the Oce.
6. $\angle CAT = 90^\circ - x$	Statement (2)
7. $2\angle CAT = 180^\circ - 2x$	Statement (6)
8. $\therefore 2\angle CAT = 2\angle ADC$	Statement (4), (5) and (7)
9. $\angle CAT = \angle ADC$	Statement (8)
10. $\angle CAS + \angle CAT = 180^\circ$	Linear pair
11. $\angle ADC + \angle AEC = 180^\circ$	Opp. angles of a cyclic quad
12. $\angle CAS + \angle CAT = \angle ADC + \angle AEC$	Statements (10) and (11)
13. $\therefore \angle CAS = \angle AEC$	Statements (9) and (12)

**Theorem 11**

In equal circles (or in the same circle), if two arcs subtend equal angles at the centres, they are equal.



**Data:**

AXB and CYD are equal circles with centres P and Q; arcs AMB, CND subtend equal angles APB, CQD.

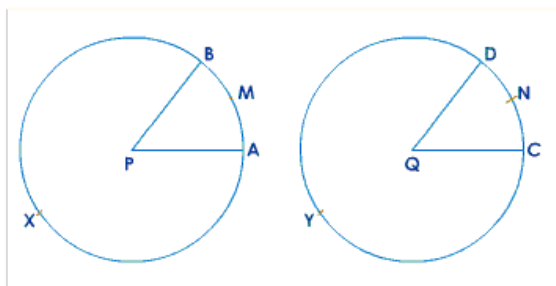
**To Prove:**

arc AMB = arc CND.

**Proof:**

Statement	Reason
1. Apply $\odot$ CYD to $\odot$ AXB so that centre Q falls on centre P and QC along PA and D on the same side as B.	$\because$ $\odot$ s are equal (data)
$\therefore$ Oce. CYD overlaps Oce. AXB.	
2. $\therefore$ C falls on A.	$\because$ PA = QC (data)
3. $\angle APB = \angle CQD$	data
4. $\therefore$ QD falls along PB	statements (1) and (3)
5. $\therefore$ D falls on B	$\because$ QD = PB (data)
6. $\therefore$ arc CND coincides with arc AMB.	statements (2) and (5)
7. arc AMB = arc CND	statement (6)

**Theorem 12 (Converse of 11)**



In equal circles (or in the same circle) if two arcs are equal, they subtend equal angles at the centres.

**Data:**

In equal circles AXB and CYD, equal arcs AMB and CND subtend  $\angle APB$  and  $\angle CQD$  at the centres P and Q respectively.

**To Prove:**

$$\angle APB = \angle CQD.$$



**Proof:**

Statement	Reason
1. Apply $\odot CYD$ to $AXB$ so that centre $Q$ falls on centre $P$ and $QC$ along $PA$ , and $D$ on the same side as $B$ .	$\odot s$ are equal (data)
$\therefore \odot ce.$ $CYD$ overlaps $\odot ce.$ $AXB$	$PA = QC$ (data)
2. $\therefore C$ falls on $A$	Data
3. arc $AMB =$ arc $CND$	Statements (1), (2) and (3)
4. $\therefore D$ falls on $B$ .	Statements (1), (2) and (4)
5. $\therefore QD$ coincides with $PB$ and $QC$ coincides with $PA$	Statement (5)
6. $\angle APB = \angle CQD$ .	

In case of the same circle:

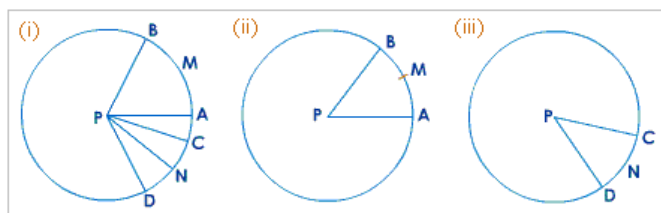
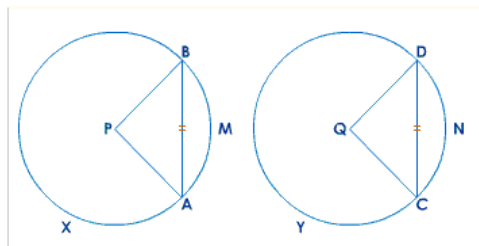


Fig.(ii) and fig.(iii) may be considered to be two equal circles obtained from fig.(i) and then the above proofs may be applied.

**Theorem 13**

In equal circles (or in the same circle), if two chords are equal, they cut off equal arcs.



**Data:**

In equal circles  $AXB$  and  $CYD$ , with centres  $P$  and  $Q$ , chord  $AB =$  chord  $CD$ .

**To Prove:**

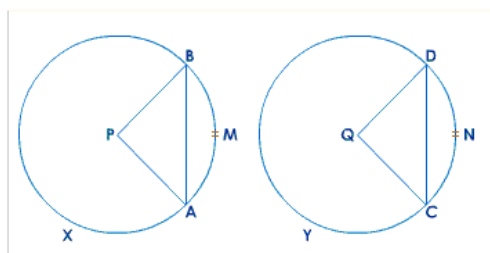
arc  $AMB =$  arc  $CND$ ; arc  $AXB =$  arc  $CYD$

**Proof:**

Statement	Reason
In $\Delta^s$ ABP and CDQ	
1. $AP = CQ$	radii of equal $\odot$ s.
2. $BP = DQ$	radii of equal $\odot$ s
3. $AB = CD$	data
4. $\Delta ABP \cong \Delta CDQ$	(S.S.S.)
5. $\therefore \angle APB = \angle CQD$	statement (4)
6. arc AMB = arc CND	statement (5)
7. $\odot AXB - \text{arc AMB} = \odot CYD - \text{arc CND}$	equal arcs [statement (6)]
8. $\therefore \text{arc AXB} = \text{arc CYD}$	statement (7)

**Theorem 14 (Converse of 13)**

In equal circles (or in the same circle) if two arcs are equal, the chords of the arcs are equal.



**Data:**

Equal circles AXB, CYD with centres P and Q have arc AMB = arc CND.

**To Prove:**

chord AB = chord CD

**Construction:**

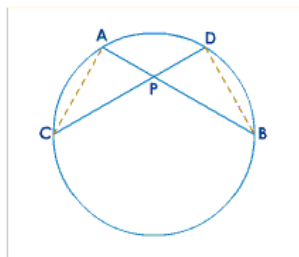
Join AP, BP, CQ and DQ.

**Proof:**

Statement	Reason
In $\Delta^s$ ABP and CDQ	
1. $AP = CQ$	radii of equal $\odot$ s
2. $BP = DQ$	radii of equal $\odot$ s
3. $\angle APB = \angle CQD$	$\because$ arc AMB = arc CND
4. $\therefore \Delta ABP \cong \Delta CDQ$	(S.A.S.)
5. $\therefore AB = CD$	statement (4)

### Theorem 15

If two chords of a circle intersect internally, then the product of the length of the segments are equal.



#### Data:

AB and CD are chords of a circle intersecting internally at P.

#### To Prove:

$$AP \times BP = CP \times DP.$$

#### Construction:

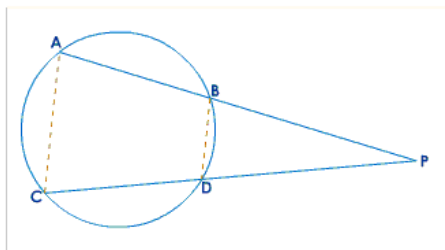
Join AC and BD.

#### Proof:

Statement	Reason
In $\triangle APC$ and $\triangle DPB$	
1. $\angle A = \angle D$	$\angle$ s in the same segment
2. $\angle C = \angle B$	$\angle$ s in the same segment
3. $\therefore \triangle APC \sim \triangle DPB$	AA similarity
4. $\therefore \frac{AP}{DP} = \frac{CP}{BP}$	Statement (3)
5. $\therefore AP \times BP = CP \times DP$	Statement (4)

### Theorem 16

If two chords of a circle intersect externally, then the product of the lengths of the segments are equal.



#### Data:

AB and CD are chords of a circle intersecting externally at P.

#### To Prove:

$$AP \times BP = CP \times DP.$$

#### Construction:

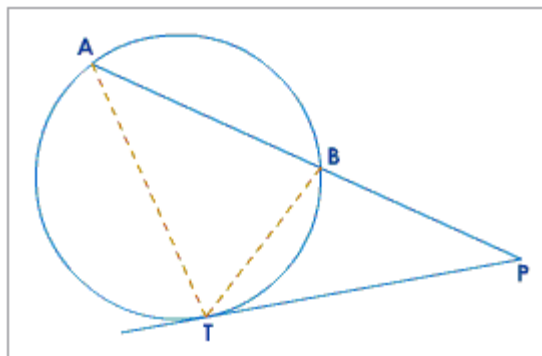
Join AC and BD.

#### Proof:

Statement	Reason
In $\triangle ACP$ and $\triangle BDP$	
1. $\angle A = \angle BDP$	ext. $\angle$ of a cyclic quad. = int. opp. $\angle$
2. $\angle C = \angle DBP$	ext. $\angle$ of a cyclic quad. = int. opp. $\angle$
3. $\therefore \triangle ACP \sim \triangle BDP$	AA similarity
4. $\therefore \frac{AP}{DP} = \frac{CP}{BP}$	Statement (3)
5. $\therefore AP \times BP = CP \times DP$	Statement (4)

### Theorem 17

If a chord and a tangent intersect externally, then the product of the lengths of the segments of the chord is equal to the square on the length of the tangent from the point of contact to the point of intersection.



#### Data:

A chord AB and a tangent TP at a point T on the circle intersect at P.

#### To Prove:

$$AP \times BP = PT^2$$

#### Construction:

Join AT and BT.

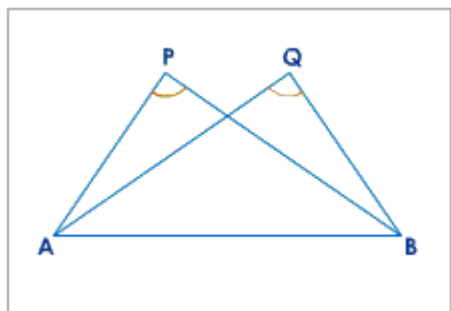
#### Proof:

Statement	Reason
In $\triangle APT$ and $\triangle BTP$	Angle in the alternate segment
1. $\angle A = \angle BTP$	
2. $\angle P = \angle P$	Common
3. $\therefore \triangle APT \sim \triangle BTP$	AA similarity
4. $\frac{AP}{PT} = \frac{PT}{BP}$	Statement (3)
5. $AP \times BP = PT^2$	Statement (4)

## Test for Concyclic Points

- (a) Converse of the statement, 'Angles in the same segment of a circle are equal', is one test for concyclic points. We state:

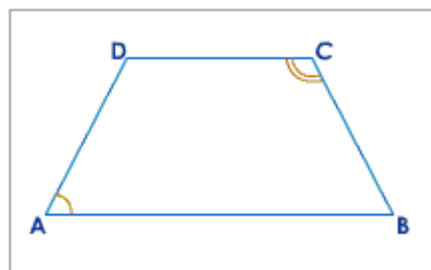
If two equal angles are on the same side of a line and are subtended by it, then the four points are concyclic. In the figure, if  $\angle P = \angle Q$  and the points P, Q are on the same side of AB, then the points A, B, Q and P are concyclic.



- (b) Converse of 'opposite angles of a cyclic quadrilateral are supplementary' is one more test for concyclic points.

We state:

If the opposite angles of a quadrilateral are supplementary, then its vertices are concyclic. In the figure, if  $\angle A + \angle C = 180^\circ$ , then A, B, C and D are concyclic points.



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