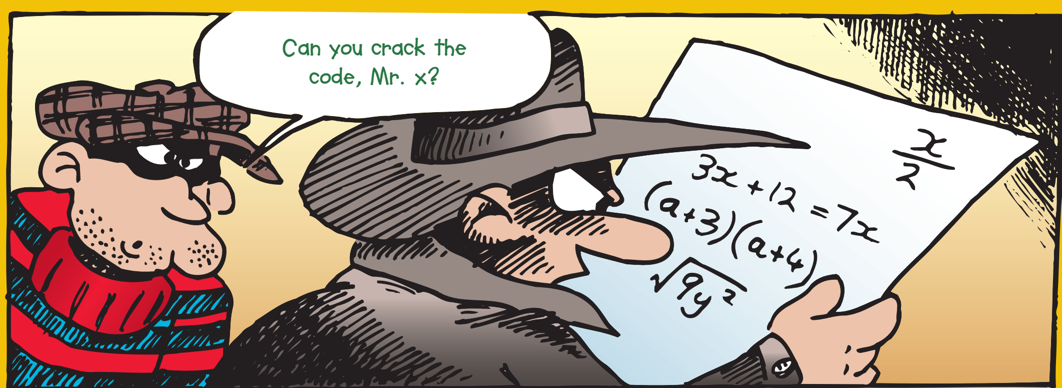
7

Factorising Algebraic Expressions



Chapter Contents

Learning Outcomes

Students will be able to:

- Factorise using common factors.
- Factorise by grouping in pairs.
- Factorise using the difference of two squares.
- Factorise quadratic trinomials.
- Simplify algebraic fractions by factorising.
- Perform operations with algebraic fractions.

Areas of Interaction

Approaches to Learning (Knowledge Acquisition, Problem Solving, Logical Thinking, Reflection), Human Ingenuity

In Chapter 4, Algebraic Expressions, you were shown how to expand various algebraic products that were written in a factorised form; that is, each product had to be rewritten without grouping symbols.

For example:

$$\begin{aligned} 3a(5-2a) &\rightarrow 15a - 6a^2 \\ (a-2)(a+7) &\rightarrow a^2 + 5a - 14 \\ (x+5)^2 &\rightarrow x^2 + 10x + 25 \\ (m+2)(m-2) &\rightarrow m^2 - 4 \end{aligned}$$

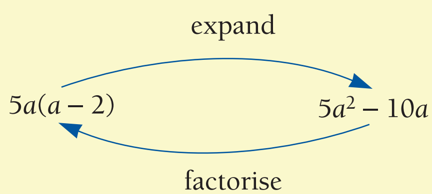
This chapter will show you how to reverse this process. You will learn how to factorise various algebraic expressions.

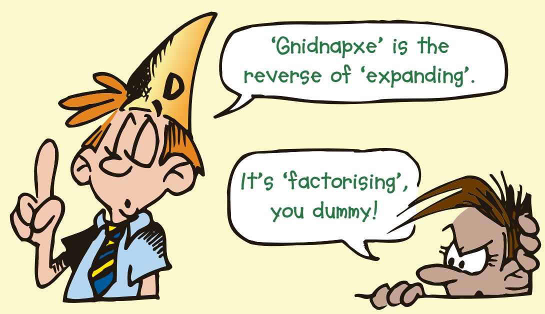
7:01 | Factorising Using Common Factors

To factorise an algebraic expression, we must determine the highest common factor (HCF) of the terms and insert grouping symbols, usually parentheses.

If we expand the expression $5a(a-2)$ we obtain $5a^2 - 10a$. To factorise $5a^2 - 10a$ we simply reverse the process. We notice that $5a$ is the HCF of $5a^2$ and 2, so $5a$ is written outside the brackets and the remainder is written inside the brackets: $5a^2 - 10a = 5a(a-2)$.

A factor of a given number is another number that will divide into the given number with no remainder. eg {1, 2, 3, 6, 9, 18} is the set of factors of 18.





worked examples

1 $5y + 15 = 5 \times y + 5 \times 3$ (HCF is 5)
$= 5(y + 3)$

2 $21x - 24y = 3 \times 7x - 3 \times 8y$ (HCF is 3)
$= 3(7x - 8y)$

3 $12ab + 18a = 6a \times b + 6a \times 3$ (HCF is $6a$)
$= 6a(b + 3)$

4 $5x^2 - 30x = 5x \times x - 5x \times 6$ (HCF is $5x$)
$= 5x(x - 6)$

5 $-12x^2 - 3x = -3x \times 4x - 3x \times 1$ (HCF is $-3x$)
$= -3x(4x + 1)$

6 $3a^2b - 9ab^2 + 15ab = 3ab \times a - 3ab \times b + 3ab \times 5$
$= 3ab(a - b + 5)$

$\boldsymbol{ab + ac = a(b + c)}$

Exercise 7:01

Foundation Worksheet 7:01

Common factors

1 Complete the following.
  a $6a + 12 = 6(\ldots + 2)$
2 Factorise.
  a $5x + 15$ b $a^2 - 3a$
3 Factorise.
  a $-6m - 15$ b $-2x^2 + 4x$

1 Factorise the following completely.

a $5x + 15y$	**b** $-3m - m^2$
c $6xy - 2x$	**d** $15p - 20q$
e $15pq - 20q$	**f** $12st^2 + 15st$
g $-18xy - 6x$	**h** $at - at^2$
i $7x^2y + xy$	**j** $a^2 + ab$

2 Factorise each of the following.

a $a^2 + ab + 3a$	**b** $xy - 3x^2 + 2x$
c $12st - 4t^3 + 8t$	**d** $36 - 12ab + 18b$
e $3ab - 9a^2b + 12ab^2 + a^2b^2$	**f** $4m - 8n - 12mn$
g $3 + 5m - 2n$	**h** $-3n - 5mn + 2n^2$
i $12x^2 + 8x - 4$	**j** $12x^2y^2 + 8xy^2 - 4y^2$

3 Examine this example
$x(x + 1) - 2(x + 1)$ has a common factor of $(x + 1)$ so it can be taken out as a common factor so $x(x + 1) - 2(x + 1) = (x + 1)(x - 2)$

Now factorise these similar types.

a $x(x + 4) + y(x + 4)$	**b** $4(a + 2) - b(a + 2)$
c $m(m - 1) - 3(m - 1)$	**d** $2(s - 3) + s(s - 3)$
e $2a(a - 1) - (a - 1)$	**f** $3m(9 - 2m) + 2(9 - 2m)$
g $x(x - 5) + 2(3x - 15)$	**h** $y(y + 5) + 2(-y - 5)$
i $x(3 - x) + 5(x - 3)$	**j** $ab(9 - a) - 2(a - 9)$

Note:
$(x + 1)(x - 2) = (x - 2)(x + 1)$

4 Factorise fully the following algebraic expressions.

a $9x + 6$	**b** $10 + 15a$	**c** $4m - 6n$
d $x^2 + 7x$	**e** $2a^2 - 3a$	**f** $12y - 6y^2$
g $ab - bx$	**h** $st - s$	**i** $4ab + 10bc$
j $-4m + 6n$	**k** $-x^2 - 3x$	**l** $-15a + 5ab$
m $3x + x^2 - ax$	**n** $ax + ay + az$	**o** $4m - 8n + 6p$
p $2(a + x) + b(a + x)$	**q** $x(3 + b) + 2(3 + b)$	**r** $y(x - 1) - 3(x - 1)$
s $5ab - 15ac + 10ad$	**t** $x^2 - 7x + xy$	**u** $a(a + 3) - (a + 3)$

7:02 | Factorising by Grouping in Pairs

prep quiz 7:02

Factorise these expressions.

1 $3a + 18$
2 $5x + ax$
3 $pq - px$
4 $3ax - 9bx$
5 $x^2 - 2x$
6 $a^3 + a^2$
7 $9 - 3a$
8 $-5m - 10$
9 $9(a + 1) + x(a + 1)$
10 $x(x + y) - 1(x + y)$

For some algebraic expressions, there may not be a factor common to every term. For example, there is no factor common to every term in the expression:

$3x + 3 + mx + m$

But the first two terms have a common factor of 3 and the remaining terms have a common factor of m. So:

$3x + 3 + mx + m = 3(x + 1) + m(x + 1)$

Now it can be seen that $(x + 1)$ is a common factor for each term.

$3(x + 1) + m(x + 1) = (x + 1)(3 + m)$

Therefore:

$3x + 3 + mx + m = (x + 1)(3 + m)$

The original expression has been factorised by grouping the terms in pairs.

worked examples

1 $$\begin{aligned} 2x + 2y + ax + ay &= 2(x + y) + a(x + y) \\ &= (x + y)(2 + a) \end{aligned}$$

2 $$\begin{aligned} a^2 + 3a + ax + 3x &= a(a + 3) + x(a + 3) \\ &= (a + 3)(a + x) \end{aligned}$$

3 $$\begin{aligned} ax - bx + am - bm &= x(a - b) + m(a - b) \\ &= (a - b)(x + m) \end{aligned}$$

4 $$\begin{aligned} ab + b^2 - a - b &= b(a + b) - 1(a + b) \\ &= (a + b)(b - 1) \end{aligned}$$

5 $$\begin{aligned} 5x + 2y + xy + 10 &= 5x + 10 + 2y + xy \\ &= 5(x + 2) + y(2 + x) \\ &= (x + 2)(5 + y) \end{aligned}$$

Note: Terms had to be rearranged to pair those with common factors.

$$\begin{aligned} \mathbf{ab + ac + bd + cd} &= \mathbf{a(b + c) + d(b + c)} \\ &= \mathbf{(b + c)(a + d)} \end{aligned}$$

Exercise 7:02

> **Foundation Worksheet 7:02**
> **Grouping in pairs**
> **1** Complete the factorising.
> **a** $3(x + 2) + a(x + 2)$
> **2** Factorise.
> **a** $am + 5a + 2m + 10$

1 Complete the factorisation of each expression below.

a $2(a + b) + x(a + b)$ **b** $a(x + 7) + p(x + 7)$
c $m(x - y) + n(x - y)$ **d** $x(m + n) - y(m + n)$
e $a^2(2 - x) + 7(2 - x)$ **f** $q(q - 2) - 2(q - 2)$
g $(x + y) + a(x + y)$ **h** $x(1 - 3y) - 2(1 - 3y)$

2 Factorise these expressions.

a $pa + pb + qa + qb$ **b** $3a + 3b + ax + bx$ **c** $mn + 3np + 5m + 15p$
d $a^2 + ab + ac + bc$ **e** $9x^2 - 12x + 3xy - 4y$ **f** $12p^2 - 16p + 3pq - 4q$
g $ab + 3c + 3a + bc$ **h** $xy + y + 4x + 4$ **i** $a^3 + a^2 + a + 1$
j $pq + 5r + 5p + qr$ **k** $xy - x + y - 1$ **l** $8a - 2 + 4ay - y$
m $mn + m + n + 1$ **n** $x^2 + my + xy + mx$ **o** $x^2 - xy + xw - yw$
p $x^2 + yz + xz + xy$ **q** $11a + 4c + 44 + ac$ **r** $a^3 - a^2 + a - 1$

3 Factorise the following.

a $xy + xz - wy - wz$ **b** $ab + bc - ad - cd$ **c** $5a + 15 - ab - 3b$
d $6x - 24 - xy + 4y$ **e** $11y + 22 - xy - 2x$ **f** $ax^2 - ax - x + 1$

• This is an exercise you can sink your teeth into!

7:03 | Factorising Using the Difference of Two Squares

prep quiz 7:03

Simplify: **1** $\sqrt{16}$ **2** $\sqrt{49}$ **3** $\sqrt{121}$

If x is positive, simplify: **4** $\sqrt{x^2}$ **5** $\sqrt{9x^2}$ **6** $\sqrt{64x^2}$

Expand and simplify: **7** $(x-2)(x+2)$ **8** $(x+5)(x-5)$ **9** $(7-a)(7+a)$ **10** $(3m+2n)(3m-2n)$

If the expression we want to factorise is the difference of two squares, we can simply reverse the procedure seen in section 3:07B.

$$(x-a)(x+a) \xrightarrow{\text{expand}} x^2 - a^2$$

$$x^2 - a^2 \xrightarrow{\text{factorise}} (x-a)(x+a)$$

worked examples

1 $x^2 - 9 = x^2 - 3^2$
$\quad = (x-3)(x+3)$

2 $25a^2 - b^2 = (5a)^2 - b^2$
$\quad = (5a-b)(5a+b)$

3 $a^4 - 64 = (a^2)^2 - 8^2$
$\quad = (a^2-8)(a^2+8)$

4 $36m^2 - 49n^2 = (6m)^2 - (7n)^2$
$\quad = (6m-7n)(6m+7n)$

$$\boldsymbol{x^2 - y^2 = (x-y)(x+y)}$$

Note: $(x-y)(x+y) = (x+y)(x-y)$

Exercise 7:03

1 Factorise each of these expressions.

a $x^2 - 4$ **b** $a^2 - 16$ **c** $m^2 - 25$ **d** $p^2 - 81$
e $y^2 - 100$ **f** $x^2 - 121$ **g** $9 - x^2$ **h** $1 - n^2$
i $49 - y^2$ **j** $a^2 - b^2$ **k** $x^2 - a^2$ **l** $y^2 - a^2$
m $9a^2 - 4$ **n** $16x^2 - 1$ **o** $25p^2 - 9$ **p** $49 - 4a^2$
q $25p^2 - a^2$ **r** $m^2 - 81n^2$ **s** $100a^2 - 9b^2$ **t** $81x^2 - 121y^2$

2 Factorise by first taking out a common factor.

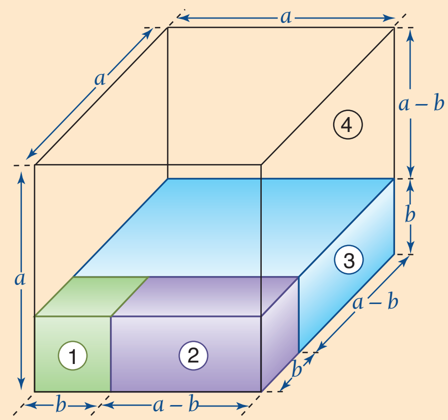
a $2x^2 - 32$ **b** $3x^2 - 108$
c $4a^2 - 100$ **d** $5y^2 - 20$
e $24a^2 - 6b^2$ **f** $3x^2 - 27y^2$
g $8y^2 - 128$ **h** $80p^2 - 5q^2$
i $4x^2 - 64$ **j** $3x^2 - 3$
k $72p^2 - 2$ **l** $2 - 18x^2$
m $8a^2 - 18m^2$ **n** $125 - 20a^2$
o $200x^2 - 18y^2$ **p** $98m^2 - 8n^2$

worked example

$18x^2 - 50 = 2(9x^2 - 25)$
$\quad = 2([3x]^2 - 5^2)$
$\quad = 2(3x-5)(3x+5)$

Challenge 7:03 | The difference of two cubes (Extension)

- The large cube has a volume of a^3 cubic units
  It is made up of four smaller parts
  (a cube and three rectangular prisms).
  Our aim is to find an expression for the difference of two cubes ($a^3 - b^3$).



1 Complete the table below.

2 Write an expression for the volume of the large cube (a^3) in terms of the volumes of the four smaller parts.
ie $a^3 = V_{①} + V_{②} + V_{③} + V_{④}$

3 Use your answer to question **2** to write an expression for $a^3 - b^3$.

Volume of part ①	Volume of part ②	Volume of part ③	Volume of part ④
$b \times b \times b$			

Express each volume as a product of its factors.

Note: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Applying this to algebraic expressions, we could factorise a difference of two cubes:

eg $x^3 - 8 = x^3 - 2^3$
$= (x - 2)(x^2 + 2x + 4)$

Exercises

Factorise these expressions using the formula above.

1 $m^3 - n^3$
2 $x^3 - y^3$
3 $a^3 - 8$
4 $m^3 - 27$
5 $x^3 - 1000$
6 $y^3 - 125$
7 $64 - n^3$
8 $27 - k^3$
9 $8m^3 - 27$
10 $64x^3 - 125y^3$
11 $125x^3 - 8y^3$
12 $27m^3 - 343n^3$

7:04 | Factorising Quadratic Trinomials

prep quiz 7:04

Expand:
1 $(x+2)(x+3)$ **2** $(a-1)(a+3)$ **3** $(m-7)(m-2)$
4 $(x+5)^2$ **5** $(a-2)^2$

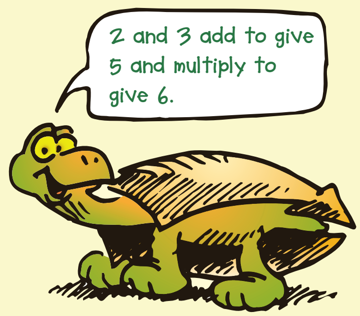
Find two numbers a and b where:
6 $a+b=5$ and $ab=6$ **7** $a+b=9$ and $ab=20$
8 $a+b=-2$ and $ab=-15$ **9** $a+b=3$ and $ab=-4$
10 $a+b=7$ and $ab=-18$

- An expression with three terms is called a *trinomial*.
- Expressions like x^2+3x-4 are called *quadratic trinomials*. The highest power of the variable is 2.
- Factorising is the reverse of expanding.
  $(x+a)(x+b) = x^2+ax+bx+ab$
  $= x^2+(a+b)x+ab$

  Using this result, to factorise x^2+5x+6 we look for two values a and b, where $a+b=5$ and $ab=6$.

  These numbers are 2 and 3, so:
  $x^2+5x+6=(x+2)(x+3)$

$(x+a)(x+b) \rightleftarrows x^2+(a+b)x+ab$



worked examples

Factorise:
1 $x^2+7x+10$ **2** m^2-6m+8 **3** y^2+y-12 **4** $x^2-9x-36$ **5** $3y^2+15y-72$

If $x^2+7x+10=(x+a)(x+b)$ then $a+b=7$ and $ab=10$.

Solutions

1
$2+5=7$
$2\times 5=10$
$\therefore x^2+7x+10$
$=(x+2)(x+5)$

2
$(-2)+(-4)=-6$
$(-2)\times(-4)=8$
$\therefore m^2-6m+8$
$=(m-2)(m-4)$

3
$(-3)+4=1$
$(-3)\times 4=-12$
$\therefore y^2+y-12$
$=(y-3)(y+4)$

4
$3+(-12)=-9$
$3\times(-12)=-36$
$\therefore x^2-9x-36$
$=(x+3)(x-12)$

5
$3y^2+15y-72$
$=3(y^2+5y-24)$
$(-3)+8=5$
$(-3)\times 8=-24$
$\therefore 3(y^2+5y-24)$
$=3(y-3)(y+8)$

Exercise 7:04

Foundation Worksheet 7:04

Factorising trinomials

1 Which two integers:
  a add to give 4?
  b multiply to give 5?

2 Factorise:
  a $m^2 + 8m + 9$
  b $n^2 - 3n + 2$

1 Factorise each of these trinomials.

a $x^2 + 4x + 3$	**b** $x^2 + 3x + 2$	
c $x^2 + 6x + 5$	**d** $x^2 + 7x + 6$	
e $x^2 + 9x + 20$	**f** $x^2 + 10x + 25$	
g $x^2 + 12x + 36$	**h** $x^2 + 10x + 21$	**i** $x^2 + 9x + 18$
j $x^2 + 14x + 40$	**k** $x^2 + 15x + 54$	**l** $x^2 + 13x + 36$
m $x^2 - 4x + 4$	**n** $x^2 - 12x + 36$	**o** $x^2 - 7x + 12$
p $x^2 - 9x + 20$	**q** $x^2 + 2x - 3$	**r** $x^2 + x - 12$
s $x^2 + 4x - 12$	**t** $x^2 + 7x - 30$	**u** $x^2 - x - 2$
v $x^2 - 10x - 24$	**w** $x^2 - 7x - 30$	**x** $x^2 - x - 56$

2 Factorise:

a $a^2 + 6a + 8$	**b** $m^2 + 9m + 18$	**c** $y^2 + 13y + 42$
d $p^2 + 7p + 12$	**e** $x^2 + 12x + 20$	**f** $n^2 + 17n + 42$
g $s^2 + 21s + 54$	**h** $a^2 + 18a + 56$	**i** $x^2 - 3x - 4$
j $a^2 - 2a - 8$	**k** $p^2 - 5p - 24$	**l** $y^2 + y - 6$
m $x^2 + 7x - 8$	**n** $q^2 + 5q - 24$	**o** $m^2 + 12m - 45$
p $a^2 + 18a - 63$	**q** $y^2 + 6y - 55$	**r** $x^2 - 2x + 1$
s $k^2 - 5k + 6$	**t** $x^2 - 13x + 36$	**u** $a^2 - 22a + 72$
v $p^2 + 22p + 96$	**w** $q^2 - 12q - 45$	**x** $m^2 - 4m - 77$

3 Factorise by first taking out a common factor (see example 5).

a $2x^2 + 6x + 4$	**b** $3x^2 - 6x - 9$	**c** $5x^2 - 10x - 40$
d $2x^2 + 16x + 32$	**e** $3x^2 - 30x - 33$	**f** $3x^2 + 21x + 36$
g $4a^2 - 12a - 40$	**h** $2n^2 + 8n + 6$	**i** $5x^2 - 30x + 40$
j $3x^2 - 21x + 36$	**k** $3a^2 - 15a - 108$	**l** $5x^2 + 15x - 350$

fun spot 7:04

Fun Spot 7:04 | How much logic do you have?

See if you can solve the three problems below.

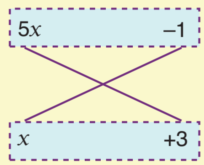
1 What is the next letter in this sequence?
O, T, T, F, F, S, S, ?

2 A man passing a beggar in the street exclaimed, 'I am that beggar's father!' But the beggar was not the man's son. How can this be?

3 Two guards are guarding two sacks. One guard always tells the truth, but the other guard always lies, but you do not know which guard is which. One of the sacks is full of gold; the other is full of peanuts. You are permitted to take one of the sacks but you are not sure which one contains the gold. You are also allowed to ask one of the guards just one question.
What question should you ask to ensure you get the sack of gold?

7:05 | Factorising Further Quadratic Trinomials

- In all quadratic trinomials factorised so far, the *coefficient* of x^2 has been 1. We will now consider cases where the *coefficient* of x^2 is not 1.
- To **expand** $(5x - 1)(x + 3)$ we can use a cross diagram.



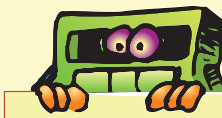
$5x^2$ is the product of the two left terms.
-3 is the product of the two right terms.
$14x$ is the sum of the products along the cross, ie $15x + (-x)$.

$\therefore\ (5x - 1)(x + 3) = 5x^2 + 14x - 3$

- One method used to factorise trinomials like $5x^2 + 14x - 3$ is called the cross method.

$3x^2 + 7x + 9$

coefficient of x^2



Remember

$5x^2 \qquad -3$

$(5x - 1)(x + 3)$

$-x$

$+15x$

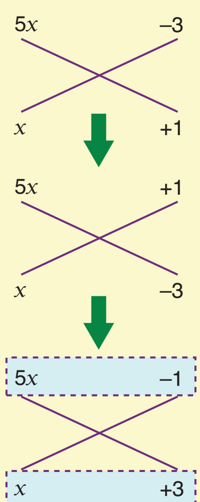
$= 5x^2 + 15x - x - 3$

$= 5x^2 + 14x - 3$

Cross method

To factorise $5x^2 + 14x - 3$, we need to reverse the expanding process.
We need to choose two factors of $5x^2$ and two factors of -3 to write on the cross.

Try: $\begin{cases} 5x \\ x \end{cases}$ and $\begin{cases} -3 \\ +1 \end{cases}$



- If $(5x - 3)$ and $(x + 1)$ are the factors of $5x^2 + 14x - 3$, then the products of numbers on the ends of each arm will have a sum of $+14x$.
- When we add the cross products here, we get:
  $(5x) + (-3x) = 2x$
  This does not give the correct middle term of $14x$, so $(5x - 3)$ and $(x + 1)$ are **not** factors.
- Vary the terms on the cross.

  Try: $\begin{cases} 5x \\ x \end{cases}$ and $\begin{cases} +1 \\ -3 \end{cases}$

  Cross product $= (-15x) + (x) = -14x$

- Try: $\begin{cases} 5x \\ x \end{cases}$ and $\begin{cases} -1 \\ +3 \end{cases}$

  Cross product $= (15x) + (-x) = 14x$
  $\therefore$ This must be the correct combination.
  $\therefore\ 5x^2 + 14x - 3 = (5x - 1)(x + 3)$

Examine the examples below. Make sure you understand the method.

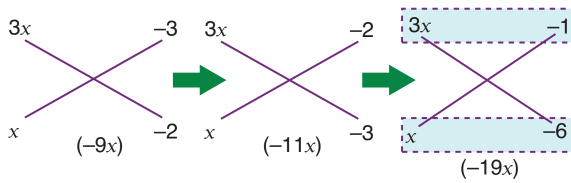
worked examples

Find the factors of:

1 $3x^2 - 19x + 6$ **2** $4x^2 - x - 3$ **3** $2x^2 + 25x + 12$

Solutions

1

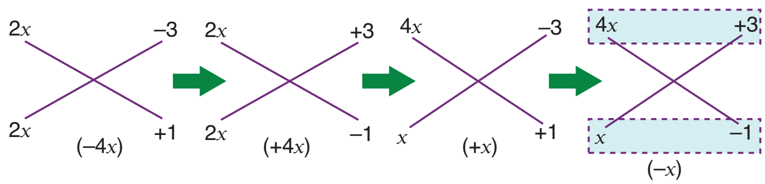


This cross product gives the correct middle term of $-19x$, so:

$3x^2 - 19x + 6 = (3x - 1)(x - 6)$

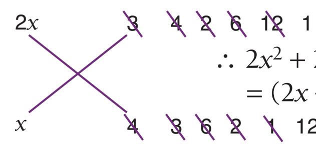
Note: The factors of +6 had to be both negative to give a negative middle term.

2



$\therefore\ 4x^2 - x - 3$
$= (4x + 3)(x - 1)$

3 In practice, we would not draw a separate cross for each new set of factors. We simply cross out the factors that don't work and try a new set.



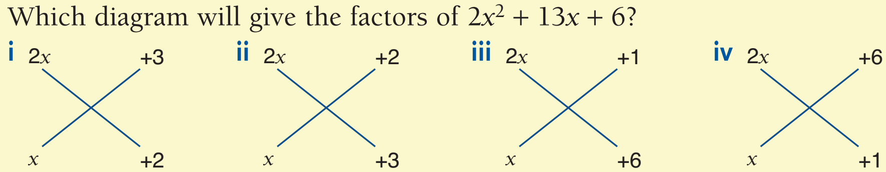
$\therefore\ 2x^2 + 25x + 12$
$= (2x + 1)(x + 12)$

■ To factorise a quadratic trinomial, $ax^2 + bx + c$, when a (the coefficient of x^2) is not 1, use the cross method.

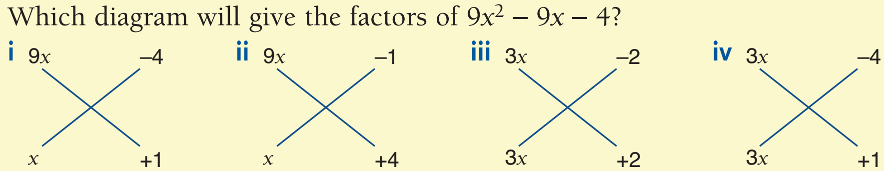
Note: Another method for factorising these trinomials is shown in Challenge 7:05 on page 187.

Exercise 7:05

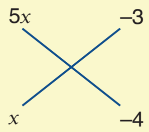
1 **a** Which diagram will give the factors of $2x^2 + 13x + 6$?

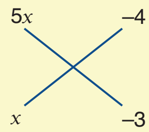


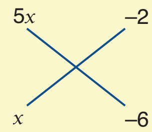
b Which diagram will give the factors of $9x^2 - 9x - 4$?

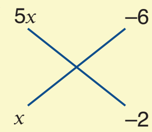


c Which diagram will give the factors of $5x^2 - 19x + 12$?

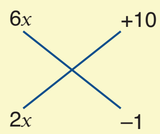
i 

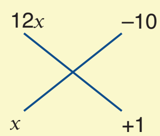
ii 

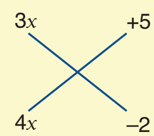
iii 

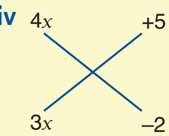
iv 

d Which diagram will give the factors of $12x^2 + 7x - 10$?

i 

ii 

iii 

iv 

2 Factorise these expressions.

a $2x^2 + 7x + 3$ **b** $3x^2 + 8x + 4$ **c** $2x^2 + 7x + 6$
d $2x^2 + 11x + 5$ **e** $3x^2 + 5x + 2$ **f** $2x^2 + 11x + 15$
g $4x^2 + 13x + 3$ **h** $5x^2 + 17x + 6$ **i** $2x^2 + 13x + 15$
j $2x^2 - 5x + 2$ **k** $3x^2 - 11x + 6$ **l** $5x^2 - 17x + 6$
m $4x^2 - 11x + 6$ **n** $10x^2 - 21x + 9$ **o** $5x^2 - 22x + 21$
p $2x^2 + x - 10$ **q** $3x^2 + 4x - 15$ **r** $4x^2 + 11x - 3$
s $2x^2 - x - 6$ **t** $2x^2 - 5x - 3$ **u** $3x^2 - x - 30$
v $6x^2 - 5x - 21$ **w** $2x^2 - 5x - 12$ **x** $4x^2 - x - 18$

3 Find the factors of the following:

a $12x^2 + 7x + 1$ **b** $6a^2 + 5a + 1$ **c** $6p^2 + 7p + 2$
d $10y^2 - 9y + 2$ **e** $12x^2 - 7x + 1$ **f** $9a^2 - 21a + 10$
g $8m^2 + 18m - 5$ **h** $6n^2 - 7n - 3$ **i** $21q^2 - 20q + 4$
j $20x^2 - x - 1$ **k** $8m^2 - 2m - 15$ **l** $18y^2 - 3y - 10$
m $6a^2 + 5a - 6$ **n** $15k^2 + 26k + 8$ **o** $8x^2 + 18x + 9$
p $4 - 3a - a^2$ **q** $2 + m - 10m^2$ **r** $6 + 7x - 3x^2$
s $6 - 7x - 3x^2$ **t** $15 - x - 28x^2$ **u** $2 + 9n - 35n^2$
v $3x^2 + 10xy + 8y^2$ **w** $2x^2 - 5xy + 2y^2$ **x** $5m^2 - 2mn - 7n^2$

4 Factorise by first taking out the common factor.

a $6x^2 + 10x - 4$ **b** $6a^2 - 2a - 4$ **c** $6a^2 + 9a - 27$
d $8x^2 + 12x - 36$ **e** $6x^2 + 28x + 16$ **f** $12p^2 + 12p - 9$
g $30q^2 + 55q - 35$ **h** $10m^2 - 46m + 24$ **i** $50a^2 + 15a - 5$
j $4 - 6x - 10x^2$ **k** $36 - 3t - 3t^2$ **l** $9 + 24x + 12x^2$

5 Complete each in as many ways as possible by writing positive whole numbers in the boxes and inserting operation signs.

a $(x \ldots \square)(x \ldots \square) = x^2 \ldots \square x \ldots 15$

b $(x \ldots \square)(x \ldots \square) = x^2 \ldots \square x - 12$

c $(x \ldots \square)(x \ldots \square) = x^2 \ldots 5x + \square$

d $(5x \ldots \square)(x \ldots \square) = 5x^2 \ldots \square x \ldots 2$

Challenge 7:05 | Another factorising method for harder trinomials

Example

Factorise $3x^2 + 14x - 5$.

$$\frac{(3x \quad)(3x \quad)}{3}$$

- Put '$3x$' in both sets of parentheses and divide by '3'.

$$\text{Now: } 3x^2 + 14x \overset{-15}{-5}$$

- Multiply $3 \times (-5)$ and write the answer above -5.

- Then find the two numbers that multiply to give -15 and add to give $+14$ (ie $+15$ and -1).
- Now place these numbers in the parentheses.

$$\frac{(3x + 15)(3x - 1)}{3}$$

- Then divide the '3' into the product above.

$$\frac{(\cancel{3}^1x + \cancel{15}^5)(3x - 1)}{\cancel{3}_1}$$

- The solution is: $(x + 5)(3x - 1)$

Note: the '10' in the denominator cancelled partly with each set of

Example

Factorise $10x^2 - 19x + 6$.

$$\frac{(10x \quad)(10x \quad)}{10}$$

$$\begin{bmatrix}(-15) \times (-4) = 60 \\ (-15) + (-4) = -19\end{bmatrix}$$

$$= \overset{60}{10x^2} - 19x + 6$$

$$= \frac{(10x - 15)(10x - 4)}{10}$$

$$= \frac{(\cancel{10}^2x - \cancel{15}^3)(\cancel{10}^5x - \cancel{4}^2)}{\cancel{10}\,\cancel{2}_1}$$

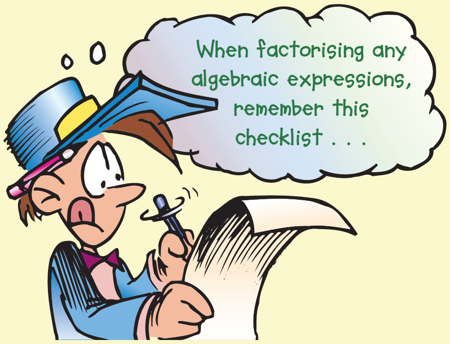
The solution is: $(2x - 3)(5x - 2)$.

Try the method with these trinomials.

1 $2x^2 + 7x + 6$ **2** $4x^2 - 19x - 5$ **3** $3x^2 - 13x + 12$

4 $6x^2 + 7x + 2$ **5** $5x^2 + 9x - 2$ **6** $12x^2 - 25x + 12$

7:06 | Factorising: Miscellaneous Types



First:
Always take out any common factor.
Then:
If there are two terms, is it a difference of two squares, $a^2 - b^2$?
If there are three terms, is it a quadratic trinomial, $ax^2 + bx + c$?
If there are four terms, can it be factorised by grouping the terms into pairs?

worked examples

1 $4x^2 - 36$
$= 4(x^2 - 9)$ common factor
$= 4(x - 3)(x + 3)$ diff. of 2 squares

2 $15x^2y - 20xy + 10xy^2$
$= 5xy(3x - 4 + 2y)$ common factor

3 $8x^2 - 40x + 32$
$= 8(x^2 - 5x + 4)$ common factor
$= 8(x - 4)(x - 1)$ quadratic trinomial

4 $12 - a - 6a^2$
$= (3 + 2a)(4 - 3a)$ quadratic trinomial

$3\ \cancel{6}$ $\cancel{-6a}$ $\cancel{a}$ $\cancel{-3a}$ $2a$

$4\ \cancel{2}$ $\cancel{a}$ $\cancel{-6a}$ $\cancel{2a}$ $-3a$

5 $ap - aq - 3p + 3q$
$= a(p - q) - 3(p - q)$ grouping terms into pairs
$= (p - q)(a - 3)$

Exercise 7:06

1 Factorise each of these expressions:

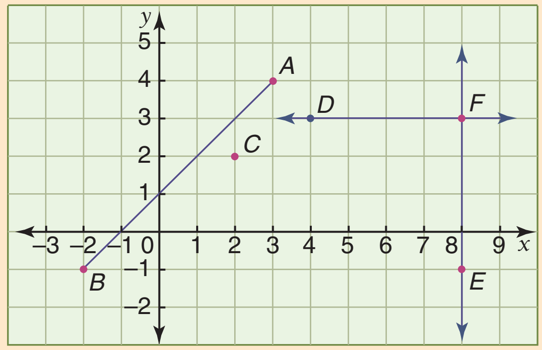
a $x^2 - 6x + 5$ **b** $x^2 - 9$ **c** $xy + 2y + 9x + 18$ **d** $a^2 - 9a$
e $a^2 - 6a + 9$ **f** $4x^2 - 1$ **g** $12x^2 - x - 35$ **h** $a^2 - 13a + 40$
i $5a^2b - 10ab^3$ **j** $p^2 - q^2$ **k** $pq - 3p + 10q - 30$ **l** $7x^2 + 11x - 6$
m $a^2 + 3a - ab$ **n** $16 - 25a^2$ **o** $1 - 2a - 24a^2$ **p** $4m + 4n - am - an$
q $5ay - 10y + 15xy$ **r** $15x^2 - x - 28$ **s** $x^2y^2 - 1$ **t** $x^2 - x - 56$
u $2mn + 3np + 4m + 6p$ **v** $100a^2 - 49x^2$
w $2 - 5x - 3x^2$ **x** $k^2 + 2k - 48$

2 Factorise completely:

a $2 - 8x^2$ **b** $5x^2 - 10x - 5xy + 10y$ **c** $2a^2 - 22a + 48$
d $3m^2 - 18m + 27$ **e** $x^4 - 1$ **f** $p^3 - 4p^2 - p + 4$
g $4x^2 - 36$ **h** $a^3 - a$ **i** $3a^2 - 39a + 120$
j $9 - 9p^2$ **k** $3k^2 + 3k - 18$ **l** $24a^2 - 42a + 9$
m $ax^2 + axy + 3ax + 3ay$ **n** $(x + y)^2 + 3(x + y)$ **o** $5xy^2 - 20xz^2$
p $6ax^2 + 5ax - 6a$ **q** $x^2 - y^2 + 5x - 5y$ **r** $3x^2 - 12x + 12$
s $63x^2 - 28y^2$ **t** $a^4 - 16$ **u** $(a - 2)^2 - 4$
v $1 + p + p^2 + p^3$ **w** $8t^2 - 28t - 60$ **x** $8 - 8x - 6x^2$

Fun Spot 7:06 | What did the caterpillar say when it saw the butterfly?

Answer each question and put the letter for that question in the box above the correct answer.



For the number plane on the right, find:

E the equation of the x-axis
E the distance BC
E the midpoint of AB
E the equation of AB
F the gradient of DF
H the intersection of DF and EF
I the distance of F from the origin
G the equation of the y-axis
E the distance AB
G the gradient of AB
E the y-intercept of DF
H the equation of DF
I the intersection of AB and BC

Simplify:

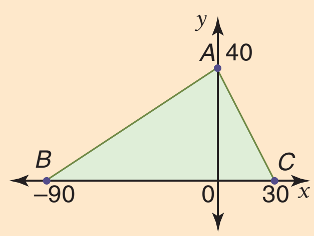
I $10x^2 + x^2$ **L** $10x^2 - x^2$ **L** $10x^2 \times x^2$
M $10x^2 \div x^2$ **N** $\frac{1}{4}$ of $8x^4$ **N** 5% of $40x$
N $(2x^2)^3$ **N** $\frac{x}{2} + \frac{x}{2}$

A playing card is chosen at random from a standard pack. Find the probability that it is:

O the Ace of spades **P** a heart
R a King **S** a picture card
T a red card greater than 3 but less than 9

Expand and simplify:

O $(x + 1)(x + 7)$ **O** $(a - b)(a + b)$
S $(a + b)^2$ **T** $(a - b)^2$



On this number plane, what is the length of:

T OC? **U** OA? **U** AC?
V BC? **W** OB? **Y** AB?
O What is the area of ΔABC?

$10\sqrt{97}$	$a^2 - b^2$	40	90	$\sqrt{73}$	$9x^2$	$10x^4$	$2x^4$	$5\sqrt{2}$	120	5	$\frac{1}{13}$	$x = 0$	$y = 0$	$\frac{5}{26}$	10	3	50	$\frac{1}{4}$

$11x^2$	$8x^6$	2400 units^2	$2x$	$(\frac{1}{2}, 1\frac{1}{2})$	$x^2 + 8x + 7$	0	$a^2 - 2ab + b^2$	$y = 3$	$\frac{1}{52}$	$a^2 + 2ab + b^2$	$y = x + 1$	30	$(8, 3)$	$(-2, -1)$	x	1	$\frac{3}{13}$

7:07 | Simplifying Algebraic Fractions: Multiplication and Division

prep quiz 7:07

Simplify the following:

1 $\dfrac{5a^2}{10a}$ **2** $\dfrac{12xy^2}{8x^2y}$ **3** $\dfrac{2x}{3} \times \dfrac{6}{x}$ **4** $\dfrac{2x}{3} \div \dfrac{4x}{9}$

Factorise:

5 $6x^2 + 9x$ **6** $x^2 + 7x + 12$ **7** $x^2 - 49$ **8** $3x^2 + 6x + 3$

9 $3x + 3y + ax + ay$ **10** $2x^2 + 9x - 5$

Just as numerical fractions can be simplified by cancelling common factors in the numerator and the denominator, so algebraic fractions can often be simplified in the same way after first factorising where possible. Look at the examples below.

worked examples

1
$$\frac{2x-2}{3x-3} = \frac{2\cancel{(x-1)}^1}{3\cancel{(x-1)}_1} = \frac{2}{3}$$

2
$$\frac{x^2+7x+12}{x^2-9} = \frac{(x+4)\cancel{(x+3)}^1}{(x-3)\cancel{(x+3)}^1} = \frac{x+4}{x-3}$$

3
$$\frac{6-6a^2}{3+3a+x+ax} = \frac{6(1-a)\cancel{(1+a)}^1}{(3+x)\cancel{(1+a)}^1} = \frac{6(1-a)}{3+x}$$

4
$$\frac{3x^2-9x}{3x^2-27} = \frac{{}^1\cancel{3}x\cancel{(x-3)}^1}{\cancel{3}_1\cancel{(x-3)}_1(x+3)} = \frac{x}{x+3}$$

Algebraic fractions should also be factorised before completing a multiplication or a division since the cancelling of common factors often simplifies these processes. Consider the following examples.

worked examples

1
$$\frac{5x+15}{x+1} \times \frac{2x+2}{5} = \frac{\cancel{5}^1(x+3)}{\cancel{(x+1)}_1} \times \frac{2\cancel{(x+1)}^1}{\cancel{5}_1} = 2(x+3)$$

2 $$\frac{x^2-9}{x^2+5x+6} \times \frac{3x+6}{x^2-2x-3} = \frac{\cancel{(x-3)}\cancel{(x+3)}}{\cancel{(x+2)}\cancel{(x+3)}} \times \frac{3\cancel{(x+2)}}{\cancel{(x-3)}(x+1)}$$
$$= \frac{3}{x+1}$$

3 $$\frac{6x-14}{3x-9} \div \frac{3x-7}{5x-15} = \frac{2\cancel{(3x-7)}}{3\cancel{(x-3)}} \times \frac{5\cancel{(x-3)}}{\cancel{(3x-7)}}$$
$$= \frac{10}{3}$$

4 $$\frac{a^2-16}{a^2-25} \div \frac{a^2-2a-8}{a^2+10a+25} = \frac{\cancel{(a-4)}(a+4)}{(a-5)\cancel{(a+5)}} \times \frac{\cancel{(a+5)}(a+5)}{\cancel{(a-4)}(a+2)}$$
$$= \frac{(a+4)(a+5)}{(a-5)(a+2)}$$

To simplify algebraic fractions, factorise both numerator and denominator, where possible, and then cancel.

Exercise 7:07

1 Factorise and simplify:

a $\dfrac{5x+10}{5}$

b $\dfrac{4}{2x+6}$

c $\dfrac{12}{3x-9}$

d $\dfrac{2x-10}{x-5}$

e $\dfrac{x+7}{3x+21}$

f $\dfrac{5a-5}{8a-8}$

g $\dfrac{3a+9}{6a+18}$

h $\dfrac{7m-28}{3m-12}$

i $\dfrac{x^2+x}{x^2-x}$

j $\dfrac{x^2-4}{x-2}$

k $\dfrac{a+1}{a^2-1}$

l $\dfrac{4y^2-9}{4y+6}$

m $\dfrac{a^2-4a}{3a-a^2}$

n $\dfrac{2x^2-2}{2x-2}$

o $\dfrac{x^2-36}{3x-18}$

p $\dfrac{a^2-3a-4}{a+1}$

q $\dfrac{x^2-6x+9}{x-3}$

r $\dfrac{x^2-4}{x^2+3x+2}$

s $\dfrac{x^2+3x+2}{x^2+5x+6}$

t $\dfrac{m^2+5m-24}{m^2-7m+12}$

u $\dfrac{t^2+7t+12}{t^2-9}$

v $\dfrac{a^2-x^2}{a^2+3a+ax+3x}$

w $\dfrac{2x^2-x-1}{4x^2-1}$

x $\dfrac{18a^2-8}{6a^2+a-2}$

2 Simplify the following:

a $\dfrac{2x+4}{3} \times \dfrac{6x}{x+2}$

b $\dfrac{5y-15}{2y+8} \times \dfrac{y+4}{10}$

c $\dfrac{2x-4}{3x-9} \times \dfrac{5x-15}{7x-14}$

d $\dfrac{5n+10}{n+3} \times \dfrac{6n+18}{4n+8}$

e $\dfrac{7y+28}{21} \times \dfrac{6}{6y+24}$

f $\dfrac{1+2a}{10+30a} \times \dfrac{6+18a}{1-2a}$

g $\dfrac{y^2+y}{2y+8} \times \dfrac{4y+6}{3y+3}$

h $\dfrac{x^2-3x}{x^2} \times \dfrac{2x^2+5x}{9x-27}$

i $\dfrac{x+3}{x^2-9} \times \dfrac{x-3}{x+1}$

j $\dfrac{3x+15}{x^2-25} \times \dfrac{x^2-49}{3x-21}$

k $\dfrac{a^2+5a+6}{a^2-4} \times \dfrac{a^2-a-2}{a^2-1}$

l $\dfrac{y^2+3y+2}{y^2+5y+6} \times \dfrac{y^2+7y+12}{y^2+5y+4}$

m $\dfrac{x^2+6x+5}{x^2+5x+4} \times \dfrac{x^2+7x+12}{x^2+12x+35}$

n $\dfrac{m^2-1}{m^2-6m+5} \times \dfrac{m^2-10m+25}{m^2-25}$

o $\dfrac{a^2-4}{a^2+3a-4} \times \dfrac{a^2-16}{a^2+2a-8}$

p $\dfrac{2x^2+4x+2}{x^2-1} \times \dfrac{x^2+3x-4}{4x+4}$

q $\dfrac{3x^2+5x+2}{x^2-x-2} \times \dfrac{x^2+x-6}{3x^2+11x+6}$

r $\dfrac{5a^2+16a+3}{25a^2-1} \times \dfrac{5a^2-a}{2a^2+5a-3}$

s $\dfrac{x^2-y^2+x-y}{x^2-2xy+y^2} \times \dfrac{10x-10y}{5x+5y+5}$

t $\dfrac{(a+b)^2-c^2}{a^2+ab+ac+bc} \times \dfrac{a^2+ab-ac-bc}{a+b+c}$

3 Simplify:

a $\dfrac{3a+6}{2} \div \dfrac{a+2}{4}$

b $\dfrac{x+2}{5x} \div \dfrac{7x+4}{10x}$

c $\dfrac{5m-10}{m+1} \div \dfrac{3m-6}{3m+3}$

d $\dfrac{6m+9}{2m-8} \div \dfrac{2m+3}{3m-12}$

e $\dfrac{3x}{5x+15} \div \dfrac{x^2+x}{x+3}$

f $\dfrac{24y-16}{4y+6} \div \dfrac{3y-2}{8y+12}$

g $\dfrac{5m-20}{4m+6} \div \dfrac{5m-20}{2m^2+3m}$

h $\dfrac{25k+15}{3k-3} \div \dfrac{5k+3}{3k}$

i $\dfrac{n^2-9}{2n+4} \div \dfrac{n+3}{2}$

j $\dfrac{y+7}{y-7} \div \dfrac{y^2-49}{y^2-7y}$

k $\dfrac{a^2+5a+4}{a^2-16} \div \dfrac{a^2-9}{a^2-a-12}$

l $\dfrac{x^2+6x+9}{x^2+8x+15} \div \dfrac{x^2+5x+6}{x^2+7x+10}$

m $\dfrac{x^2-4}{x^2-7x+10} \div \dfrac{x^2-x-6}{x^2-3x-10}$

n $\dfrac{p^2+7p+10}{p^2-2p-8} \div \dfrac{p^2+2p-15}{p^2+p-12}$

o $\dfrac{n^2-49}{n^2-9} \div \dfrac{n^2+14n+49}{n^2-6n+9}$

p $\dfrac{2x^2-8x-42}{x^2+6x+9} \div \dfrac{x^2-9x+14}{x^2+x-6}$

q $\dfrac{3x^2-48}{x^2-3x-4} \div \dfrac{x^2+4x}{x^3-x}$

r $\dfrac{2a^2-a-1}{a^2-1} \div \dfrac{6a^2+a-1}{3a^2+2a-1}$

s $\dfrac{x+y+x^2-y^2}{x^2+2xy+y^2} \div \dfrac{1+x-y}{2x+2y}$

t $\dfrac{p^2-(q+r)^2}{p^2+pq-pr-qr} \div \dfrac{p-q-r}{p^2-pq-pr+qr}$

7:08 | Addition and Subtraction of Algebraic Fractions

prep quiz 7:08

Simplify:

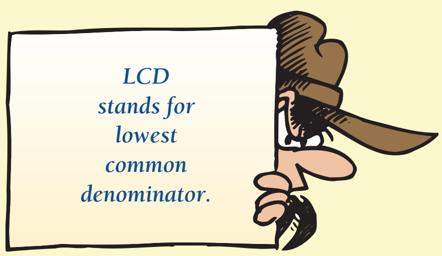
1 $\frac{1}{2}+\frac{3}{5}$ **2** $\frac{3}{4}+\frac{3}{8}$ **3** $\frac{9}{10}-\frac{3}{5}$ **4** $\frac{7}{15}-\frac{3}{20}$

5 $\frac{5}{x}+\frac{7}{x}$ **6** $\frac{2}{a}+\frac{1}{2a}$ **7** $\frac{2}{3a}+\frac{3}{2a}$ **8** $\frac{1}{x}-\frac{1}{4x}$

9 $\frac{a}{2x}+\frac{2a}{x}$ **10** $\frac{5m}{2n}-\frac{4m}{3n}$

The Prep Quiz above should have reminded you that, when adding or subtracting fractions, the lowest common denominator needs to be found. If the denominators involve two or more terms, factorising first may help in finding the lowest common denominator. For example:

$$\frac{2}{x^2-9}+\frac{5}{x^2+5x+6}=\frac{2}{(x-3)(x+3)}+\frac{5}{(x+3)(x+2)}$$



Here the LCD = $(x-3)(x+3)(x+2)$. Note that the factors of each denominator are present without repeating any factor common to both. Each numerator is then multiplied by each factor not present in its original denominator.

$$=\frac{2(x+2)+5(x-3)}{(x-3)(x+3)(x+2)}$$

$$=\frac{2x+4+5x-15}{(x-3)(x+3)(x+2)}$$

$$=\frac{7x-11}{(x-3)(x+3)(x+2)}$$

When adding or subtracting fractions:
- **factorise the denominator of each fraction**
- **find the lowest common denominator**
- **rewrite each fraction with this common denominator and simplify.**

worked examples

1 $\dfrac{2}{x+2}+\dfrac{1}{x+3}=\dfrac{2(x+3)+1(x+2)}{(x+2)(x+3)}$

$=\dfrac{2x+6+x+2}{(x+2)(x+3)}$

$=\dfrac{3x+8}{(x+2)(x+3)}$

2 $\dfrac{3}{2x+1}-\dfrac{4}{3x-1}=\dfrac{3(3x-1)-4(2x+1)}{(2x+1)(3x-1)}$

$=\dfrac{9x-3-8x-4}{(2x+1)(3x-1)}$

$=\dfrac{x-7}{(2x+1)(3x-1)}$

No factorising was needed in these first two examples.

3 $\dfrac{1}{x^2+5x+6}+\dfrac{2}{x+3}$

$=\dfrac{1}{(x+2)(x+3)}+\dfrac{2}{(x+3)}$

$=\dfrac{1+2(x+2)}{(x+2)(x+3)}$

$=\dfrac{1+2x+4}{(x+2)(x+3)}$

$=\dfrac{2x+5}{(x+2)(x+3)}$

4 $\dfrac{4}{x^2+x}-\dfrac{3}{x^2-1}$

$=\dfrac{4}{x(x+1)}-\dfrac{3}{(x-1)(x+1)}$

$=\dfrac{4(x-1)-3x}{x(x+1)(x-1)}$

$=\dfrac{4x-4-3x}{x(x+1)(x-1)}$

$=\dfrac{x-4}{x(x+1)(x-1)}$

5 $\dfrac{x+3}{x^2+2x+1}-\dfrac{x-1}{x^2-x-2}$

$=\dfrac{x+3}{(x+1)(x+1)}-\dfrac{x-1}{(x+1)(x-2)}$

$=\dfrac{(x+3)(x-2)-(x-1)(x+1)}{(x+1)(x+1)(x-2)}$

$=\dfrac{x^2+x-6-(x^2-1)}{(x+1)(x+1)(x-2)}$

$=\dfrac{x-5}{(x+1)^2(x-2)}$

Exercise 7:08

Foundation Worksheet 7:08
Addition and subtraction of algebraic fractions
1 Simplify:
a $\frac{2}{x}+\frac{5}{x}$ b $\frac{3}{x}-\frac{1}{x+1}$
2 Simplify:
a $\frac{6}{5a}-\frac{3}{2a}$ b $\frac{1}{x^2-1}+\frac{2}{x-1}$

1 Simplify each of the following. (*Note:* No factorising is needed.)

a $\dfrac{1}{x+1}+\dfrac{1}{x-1}$ b $\dfrac{1}{a+5}+\dfrac{1}{a+3}$ c $\dfrac{1}{y-7}-\dfrac{1}{y+1}$

d $\dfrac{2}{x+3}+\dfrac{3}{x+5}$ e $\dfrac{5}{m+1}-\dfrac{3}{m-2}$ f $\dfrac{6}{t+10}-\dfrac{3}{t+2}$

g $\dfrac{1}{2x-1}+\dfrac{3}{x-1}$ h $\dfrac{9}{3x+2}-\dfrac{7}{2x+5}$ i $\dfrac{8}{5x-1}+\dfrac{7}{3x+1}$

j $\dfrac{3}{2x}+\dfrac{5}{x+7}$ k $\dfrac{9}{2x+5}-\dfrac{5}{3x}$ l $\dfrac{1}{2a}-\dfrac{3}{2a+1}$

m $\dfrac{x}{x+3}+\dfrac{x}{x+1}$ n $\dfrac{a}{2a+1}-\dfrac{2a}{4a-1}$ o $\dfrac{x+1}{x+2}+\dfrac{x+2}{x+1}$

2 Simplify. (*Note*: The denominators are already factorised.)

a $\dfrac{1}{(x+1)(x+2)}+\dfrac{1}{x+1}$

b $\dfrac{1}{x(x+2)}+\dfrac{1}{x+2}$

c $\dfrac{1}{x+3}-\dfrac{1}{x(x+3)}$

d $\dfrac{1}{x-5}-\dfrac{1}{(x-5)(x+2)}$

e $\dfrac{3}{(x+2)(x+3)}+\dfrac{4}{x+2}$

f $\dfrac{5}{x+4}-\dfrac{3}{(x+1)(x+4)}$

g $\dfrac{1}{(x+1)(x+2)}+\dfrac{1}{(x+2)(x+3)}$

h $\dfrac{2}{(x-3)(x+3)}+\dfrac{4}{(x+3)(x+1)}$

i $\dfrac{3}{(x+7)(x-1)}+\dfrac{5}{(x+7)(x+1)}$

j $\dfrac{9}{(x+9)(x+3)}-\dfrac{7}{(x+3)(x-1)}$

k $\dfrac{1}{(2x+1)(x+5)}+\dfrac{3}{(x+5)(x+2)}$

l $\dfrac{5}{(2x-1)(3x+2)}-\dfrac{6}{x(2x-1)}$

m $\dfrac{x-1}{(x+3)(x+1)}+\dfrac{x+1}{(x+3)(x-1)}$

n $\dfrac{x+2}{x(x+3)}-\dfrac{x-1}{x(x+2)}$

3 Simplify, by first factorising each denominator where possible.

a $\dfrac{1}{x^2+x}+\dfrac{1}{x+1}$

b $\dfrac{1}{3x+9}-\dfrac{1}{x+3}$

c $\dfrac{2}{2x+3}+\dfrac{3}{4x+6}$

d $\dfrac{5}{x^2-1}+\dfrac{3}{x-1}$

e $\dfrac{1}{x^2-9}+\dfrac{1}{2x-6}$

f $\dfrac{1}{x^2+x}-\dfrac{1}{x^2-1}$

g $\dfrac{1}{x^2+2x+1}+\dfrac{1}{x^2-1}$

h $\dfrac{1}{x^2+7x+12}+\dfrac{1}{x^2+8x+16}$

i $\dfrac{2}{x^2+6x+8}+\dfrac{4}{x^2+5x+6}$

j $\dfrac{2}{x^2+7x+12}+\dfrac{4}{x^2+5x+4}$

k $\dfrac{3}{x^2-x-2}-\dfrac{4}{x^2-2x-3}$

l $\dfrac{3}{x^2-x-6}-\dfrac{2}{x^2-2x-3}$

m $\dfrac{5}{x^2-3x-4}-\dfrac{3}{x^2-x-2}$

n $\dfrac{3}{2x^2+7x-4}-\dfrac{4}{3x^2+14x+8}$

o $\dfrac{2}{x^2-49}+\dfrac{4}{x^2-4x-21}$

p $\dfrac{4}{2x^2+x-1}-\dfrac{1}{x^2-1}$

q $\dfrac{x+1}{x^2+5x+6}+\dfrac{x-1}{x^2-9}$

r $\dfrac{x+3}{x^2-16}-\dfrac{x+2}{x^2-4x}$

s $\dfrac{2x}{5x^2-20}+\dfrac{x+1}{x^2+4x+4}$

t $\dfrac{5x+2}{2x^2-5x-3}+\dfrac{3x-1}{4x^2-1}$

Mathematical Terms 7

binomial
- An algebraic expression consisting of two terms.
  eg $2x + 4$, $3x - 2y$

coefficient
- The number that multiplies a pronumeral in an algebraic expression.
  eg In $3x - 5y$,
  – the coefficient of x is 3
  – the coefficient of y is -5

expand
- To remove grouping symbols by multiplying each term inside grouping symbols by the term or terms outside.

factorise
- To write an expression as a product of its factors.
- The reverse of expanding.

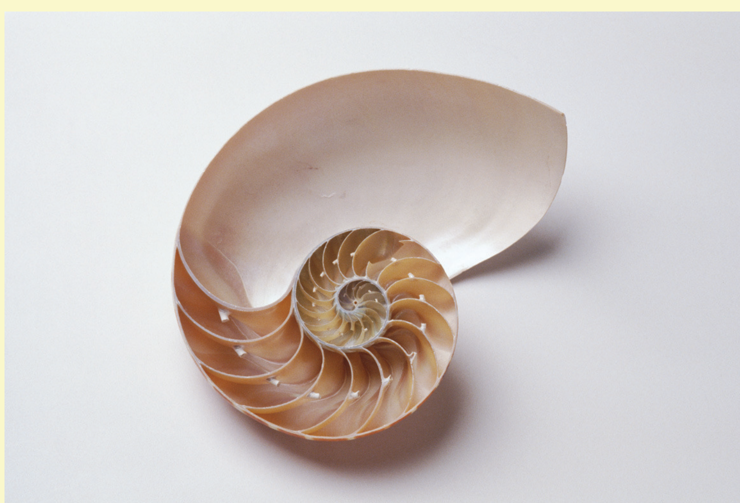
product
- The result of multiplying terms or expressions together.

quadratic trinomial
- Expressions such as $x^2 + 4x + 3$, which can be factorised as $(x + 3)(x + 1)$.
- The highest power of the variable is 2.

trinomial
- An algebraic expression consisting of three terms.

Mathematical terms 7

1 Factorising using common factors
2 Grouping in pairs
3 Factorising trinomials 1
4 Factorising trinomials 2
5 Mixed factorisations



- This spiral or helix is a mathematical shape.
- Discover how it can be drawn.
- Investigate its links to the golden rectangle.

Diagnostic Test 7: | Factorising Algebraic Expressions

- Each part of this test has similar items that test a particular skill.
- Errors made will indicate areas of weakness.
- Each weakness should be treated by going back to the section listed.

	Section
1 Factorise by taking out a common factor. **a** $3x - 12$ **b** $ax + ay$ **c** $-2x - 6$ **d** $ax + bx - cx$	7:01
2 Factorise by grouping the terms into pairs. **a** $ax + bx + 2a + 2b$ **b** $6m + 6n + am + an$ **c** $xy - x + y - 1$ **d** $ab + 4c + 4a + bc$	7:02
3 Factorise these 'differences of two squares'. **a** $x^2 - 25$ **b** $a^2 - x^2$ **c** $4 - m^2$ **d** $9x^2 - 1$	7:03
4 Factorise these trinomials. **a** $x^2 + 7x + 12$ **b** $x^2 - 5x + 6$ **c** $x^2 - 3x - 10$ **d** $x^2 + x - 20$	7:04
5 Factorise: **a** $2x^2 + 11x + 5$ **b** $3x^2 - 11x + 6$ **c** $4x^2 - x - 18$ **d** $6x^2 + 5x + 1$	7:05
6 Simplify, by first factorising where possible. **a** $\frac{6x + 12}{6}$ **b** $\frac{12a - 18}{14a - 21}$ **c** $\frac{x^2 + 5x}{ax + 5a}$ **d** $\frac{x^2 + 3x - 10}{x^2 - 4}$	7:07
7 Simplify: **a** $\frac{3x + 6}{4} \times \frac{8x}{x + 2}$ **b** $\frac{a^2 + 5a + 6}{a^2 - 9} \times \frac{a^2 - 1}{a^2 + 3a + 2}$ **c** $\frac{3m - 6}{m + 3} \div \frac{5m - 10}{3m + 9}$ **d** $\frac{x^2 - 3x - 10}{x^2 - x - 6} \div \frac{x^2 - 7x + 10}{x^2 - 4}$	7:07
8 Simplify: **a** $\frac{2}{x + 3} + \frac{1}{x - 1}$ **b** $\frac{1}{x(x + 2)} - \frac{1}{(x + 2)(x + 1)}$ **c** $\frac{5}{x^2 - 9} + \frac{3}{2x - 6}$ **d** $\frac{x}{x^2 + 7x + 12} - \frac{x + 2}{x^2 + 2x - 3}$	7:08

Chapter 7 | Revision Assignment

7A

1 Factorise the following expressions:

- **a** $a^2 + 9a + 20$
- **b** $2p - 4q$
- **c** $m^2 - 4m - 45$
- **d** $5x^3 + 10x^2 + x + 2$
- **e** $4x^2 - 1$
- **f** $x^2y - xy$
- **g** $6a^2 - 13a + 5$
- **h** $x^2 + x - 30$
- **i** $3a^2 - 4a - 15$
- **j** $xy + xz + py + pz$
- **k** $2x^2 + x - 1$
- **l** $x^3 - 3x^2 + 2x - 6$
- **m** $-5ab - 10a^2b^2$
- **n** $x^2 - y^2 + 2x - 2y$
- **o** $2 - 3x - 9x^2$

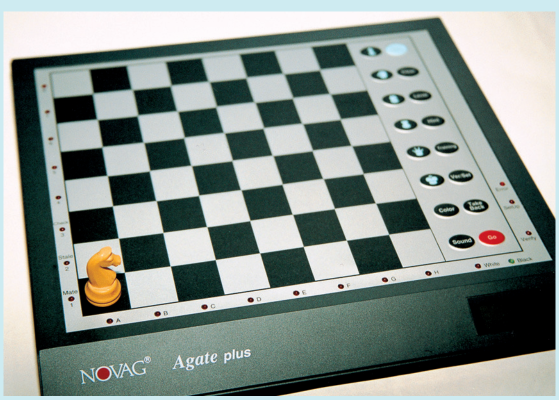
2 Factorise fully:

- **a** $2y^2 - 18$
- **b** $3r^2 + 9r - 84$
- **c** $4x^3 + 6x + 4x^2 + 6$
- **d** $2 - 18x^2$
- **e** $a^3 + a^2 - 72a$
- **f** $33 + 36a + 3a^2$
- **g** $(x - y)^2 + x - y$
- **h** $(x - 2)^2 - 4$

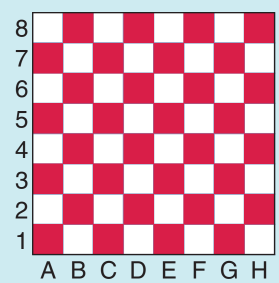
3 Simplify each of the following:

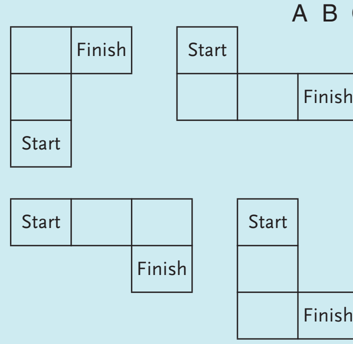
- **a** $\dfrac{x^2 + 9x - 36}{x^2 - 9}$
- **b** $\dfrac{20x^2 - 5}{2x^2 + 5x - 3}$
- **c** $\dfrac{3}{x + 2} + \dfrac{2}{x + 3}$
- **d** $\dfrac{x}{x - 1} - \dfrac{2x}{x - 2}$
- **e** $\dfrac{x^2 - 1}{5x} \times \dfrac{x^2 + x}{x^2 + 2 + 1}$
- **f** $\dfrac{x - 1}{x^2 - 4} \div \dfrac{x^2 - 4x + 3}{x^2 - x - 6}$
- **g** $\dfrac{x + 1}{x + 2} - \dfrac{x + 2}{x + 1}$
- **h** $\dfrac{2}{3x - 1} + \dfrac{1}{(3x - 1)^2}$
- **i** $\dfrac{4}{3 + 2x} - \dfrac{3}{2x + 3}$
- **j** $\dfrac{x^2 + 5x - 14}{5x^2 - 20} \times \dfrac{x^2 + 4x + 4}{x^2 - 49}$

Chapter Review Questions



- Which squares can the Knight in the photo move to? If the Knight was standing on the square C1 what squares could it move to? Give a sequence of squares showing how the Knight could move from A1 to B1 to C1 to . . . H1.
- Chess is played on an 8 by 8 square grid. Each square is named using a letter and a number. The Knight pictured is standing on the square A1.
- In Chess, a Knight can move 3 squares from its starting position to its finishing position. The squares must form an 'L' shape in any direction. Some possible moves are shown below.





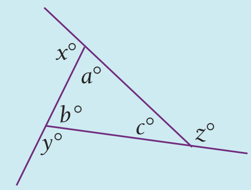
Chapter 7 | Working Mathematically

assignment 7B

1 Use ID Card 5 on page xvii to identify:
a 5 b 12 c 14 d 16 e 17 f 20 g 21 h 22 i 23 j 24

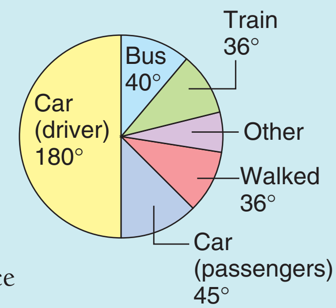
2 Use ID Card 6 on page xviii to identify:
a 4 b 12 c 13 d 14 e 15 f 17 g 21 h 22 i 23 j 24

3 If the exterior angles $x°$, $y°$ and $z°$ of a triangle are in the ratio $4:5:6$, what is the ratio of the interior angles $a°$, $b°$ and $c°$?



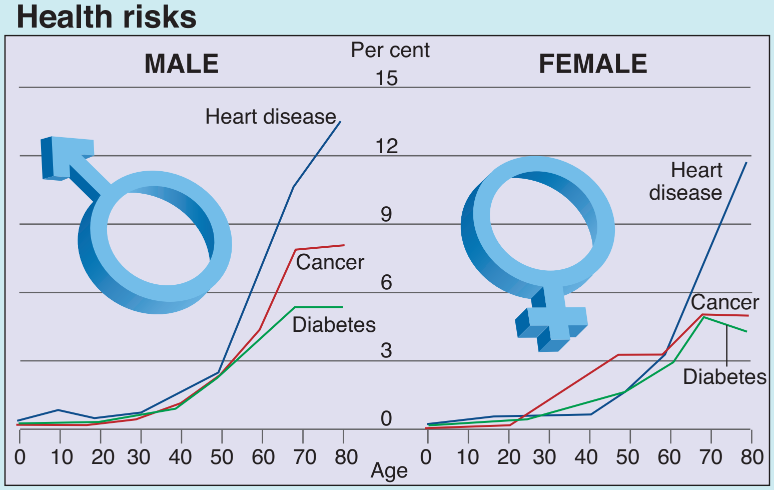
4 The average of five numbers is 11. A sixth number is added and the new average is 12. What is the sixth number?

5 This sector graph shows the method of travelling to work for all persons.
a What percentage of the workforce caught a train to work?
b What percentage of the workforce was driven to work?
c What is the size of the sector angle for 'other' means of transport? Do not use a protractor.
d What percentage of the workforce used a car to get to work?



6 a From the data in the graph below, who has the greater chance of having heart disease: a 60-year-old woman or a 60-year-old man?
b Who has the greater chance of having cancer: a 50-year-old woman or a 50-year-old man?
c Which of the three diseases reveals the greatest gender difference for the 20-to-50-year-old range?
d Would the number of 80-year-old men suffering from heart disease be greater or less than the number of 80-year-old women suffering from heart disease? Give a reason for your answer.

Health risks



Source: Australian Institute of Health and Welfare