Digital Lesson

Solving Linear Equations in One Variable

A **linear equation in one variable** is an equation which can be written in the form:

$$ax + b = c$$

for a, b, and c real numbers with $a \neq 0$.

Linear equations in one variable:

$$2x + 3 = 11$$

$2(x - 1) = 8$ can be rewritten $2x + (-2) = 8$.

$\dfrac{2}{3}x + 5 = x - 7$ can be rewritten $-\dfrac{1}{3}x + 5 = -7$.

Not linear equations in one variable:

$2x + 3y = 11$	$(x - 1)^2 = 8$	$\dfrac{2}{3x} + 5 = x - 7$
Two variables	x is squared.	Variable in the denominator

A **solution** of a linear equation in one variable is a real number which, when substituted for the variable in the equation, makes the equation true.

Example: Is 3 a solution of $2x + 3 = 11$?

$2x + 3 = 11$ Original equation

$2(3) + 3 = 11$ Substitute 3 for x.

$6 + 3 \neq 11$ False equation

3 is not a solution of $2x + 3 = 11$.

Example: Is 4 a solution of $2x + 3 = 11$?

$2x + 3 = 11$ Original equation

$2(4) + 3 = 11$ Substitute 4 for x.

$8 + 3 = 11$ True equation

4 is a solution of $2x + 3 = 11$.

Addition Property of Equations

If $a = b$, then $a + c = b + c$ and $a - c = b - c$.

That is, the same number can be added to or subtracted from each side of an equation without changing the solution of the equation.

Use these properties to solve linear equations.

Example: Solve $x - 5 = 12$.

$x - 5 = 12$	Original equation
$x - 5 + 5 = 12 + 5$	The solution is preserved when 5 is added to both sides of the equation.
$x = 17$	17 is the solution.
$17 - 5 = 12$	Check the answer.

Multiplication Property of Equations

If $a = b$ and $c \neq 0$, then $ac = bc$ and $\frac{a}{c} = \frac{b}{c}$.

That is, an equation can be multiplied or divided by the same nonzero real number without changing the solution of the equation.

Example: Solve $2x + 7 = 19$.

$2x + 7 = 19$	Original equation
$2x + 7 - 7 = 19 - 7$	The solution is preserved when 7 is subtracted from both sides.
$2x = 12$	Simplify both sides.
$\frac{1}{2}(2x) = \frac{1}{2}(12)$	The solution is preserved when each side is multiplied by $\frac{1}{2}$.
$x = 6$	6 is the solution.
$2(6) + 7 = 12 + 7 = 19$	Check the answer.

To solve a linear equation in one variable:

1. Simplify both sides of the equation.
2. Use the addition and subtraction properties to get all variable terms on the left-hand side and all constant terms on the right-hand side.
3. Simplify both sides of the equation.
4. Divide both sides of the equation by the coefficient of the variable.

Example: Solve $x + 1 = 3(x - 5)$.

$x + 1 = 3(x - 5)$	Original equation
$x + 1 = 3x - 15$	Simplify right-hand side.
$x = 3x - 16$	Subtract 1 from both sides.
$-2x = -16$	Subtract $3x$ from both sides.
$x = 8$	Divide both sides by –2.

The solution is 8.

Check the solution: $(8) + 1 = 3((8) - 5) \rightarrow 9 = 3(3)$ True

Example: Solve $3(x + 5) + 4 = 1 - 2(x + 6)$.

$3(x + 5) + 4 = 1 - 2(x + 6)$	Original equation
$3x + 15 + 4 = 1 - 2x - 12$	Simplify.
$3x + 19 = -2x - 11$	Simplify.
$3x = -2x - 30$	Subtract 19.
$5x = -30$	Add $2x$.
$x = -6$	Divide by 5.

The solution is -6.

$3(-6 + 5) + 4 = 1 - 2(-6 + 6)$	Check.
$3(-1) + 4 = 1 - 2(0)$	
$-3 + 4 = 1$	True

Equations with fractions can be simplified by multiplying both sides by a common denominator.

Example: Solve $\frac{1}{2}x+\frac{2}{3}=\frac{1}{3}(x+4)$

The lowest common denominator of all fractions in the equation is 6.

$$6\left(\frac{1}{2}x+\frac{2}{3}\right)=6\left(\frac{1}{3}(x+4)\right)$$ Multiply by 6.

$$3x+4=2x+8$$ Simplify.

$$3x=2x+4$$ Subtract 4.

$$x=4$$ Subtract $2x$.

$$\frac{1}{2}(4)+\frac{2}{3}=\frac{1}{3}((4)+4)$$ Check.

$$2+\frac{2}{3}=\frac{1}{3}(8)$$

$$\frac{8}{3}=\frac{8}{3}$$ True

Alice has a coin purse containing \$5.40 in dimes and quarters. There are 24 coins all together. How many dimes are in the coin purse?

Let the number of dimes in the coin purse = d.

Then the number of quarters = $24 - d$.

$$\boxed{10d + 25(24 - d) = 540}$$ Linear equation

$$10d + 600 - 25d = 540$$ Simplify left-hand side.

$$10d - 25d = -60$$ Subtract 600.

$$-15d = -60$$ Simplify right-hand side.

$$d = 4$$ Divide by -15.

There are 4 dimes in Alice's coin purse.

The sum of three consecutive integers is 54. What are the three integers?

Three *consecutive integers* can be represented as

$$n, n + 1, n + 2.$$

$\boxed{n + (n + 1) + (n + 2) = 54}$	Linear equation
$3n + 3 = 54$	Simplify left-hand side.
$3n = 51$	Subtract 3.
$n = 17$	Divide by 3.

The three consecutive integers are 17, 18, and 19.

$17 + 18 + 19 = 54.$	Check.