Chapter 25
Alternating Currents

GOALS

When you have mastered the contents of this chapter, you will be able to achieve the following goals:

Definitions

Define each of the following terms and use it in an operational definition:

effective values of current and voltage	power factor
reactance	resonance
impedance	Q-factor

AC Circuits

Solve alternating-current problems involving resistance, inductance, and capacitance in a series circuit.

Phasor Diagrams

Draw phasor diagrams for alternating current circuits.

Transformer

Explain the operation of the transformer.

AC Measurements

Describe the use of alternating currents in physiological measurements.

PREREQUISITES

Before you begin this chapter you should have achieved the goals of Chapter 22, ***Basic Electrical Measurements***, and Chapter 24, ***Electromagnetic Induction***.

Chapter 25
Alternating Currents

25.1 Introduction

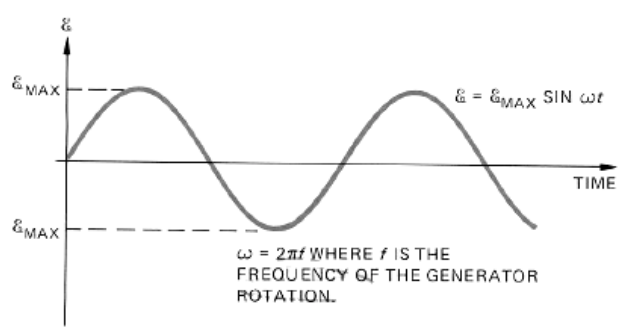
Most of your experience with electricity has probably been with alternating current (AC) circuits. Do you know the difference between AC and direct current (DC) electricity? At some time or another you have probably read the printing on the end of a light bulb. Usually a light bulb has printed on it that it is for use in a 120-volt AC circuit. What was the manufacturer trying to indicate? Could you use a 120-V DC bulb in your household sockets? What, if anything, would happen? Why is AC the dominant form of electrical energy in use? More than 90 percent of electrical energy is used as AC electricity.

If you owned an electric train in your childhood, you may recall that you used a transformer to reduce household voltage to the voltage required by the electric train. The transformer is a device that plays a most important role in the use of electrical energy. Do you know what its functions are in transmission of electric power? What is the role of the transformer in the coupling of two AC circuits? These are a few of the questions we will discuss in this chapter.

25.2 Nomenclature Used for Alternating Currents

In Chapter 24, we learned that a simple AC generator can be made by rotating a coil of conducting wire in a constant magnetic field. The voltage produced by such a generator has the form shown in Figure 25.1. This sinusoidal voltage *alternates* polarity + to -and produces an alternating current in a circuit connected to such an AC source. This alternating polarity is in contrast to the unindirectional nature of DC current.

FIGURE 25.1 Alternating emf from an AC generator.



In the United States the standard frequency in home and industrial use is 60 cycles/second (Hz). This means that there is a reversal of the direction of the current every 1/120 second. Radio broadcast frequencies are of the order of 10^6 Hz. Some microwave devices have frequencies the order of 10^{10} Hz. So the AC current in common use is of relatively low frequency.

If you have two equal resistors and in one there is an AC current of 1 ampere maximum and in the other a DC current of one ampere, would you expect the two resistors to produce the same heat? Let us consider the situation to see what we should expect. In the resistor with the DC current, the current is constant. In the resistor with AC current, the current varies from 0 to 1 A in one direction and the same in the opposite direction. You have learned that the heat produced depends upon the square of current for a given resistor. Hence, you would have been correct if you had said that more heat is produced by the 1-A DC current. For a varying current we would expect the heat produced to be proportional to the average value of the square of the current. The square root of this quantity is called *effective current*. That is, it is the current that would produce the same heating effect as one ampere of DC current. Let us find the relationship between the effective AC current and maximum AC current. The AC current at any instant of time can be expressed as a sinusoidal function of time,

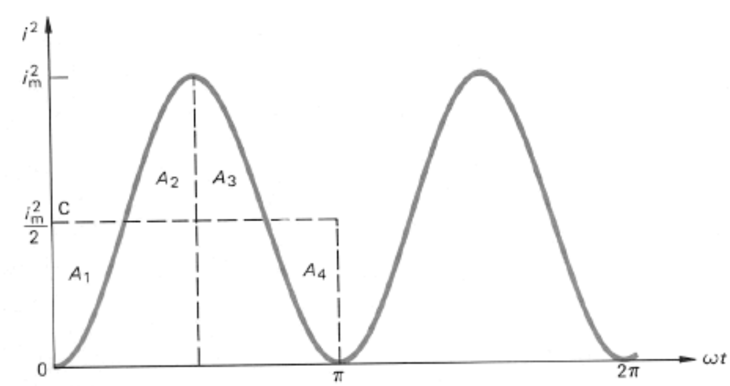
$$i = i_{max} \sin \omega t \tag{25.1}$$

where i_{max} is the maximum value of the current, ω is the angular frequency of the AC, and t is the time in seconds. The square of the current will be proportional to the power dissipated in the resistor; so

$$i^2 = i^2_{max} \sin^2 \omega t \tag{25.2}$$

We can draw a graph of the square of the current versus the time(Figure 25.2). The total heating effect will be proportional to the area under the curve. You see that the curve between π and 2π radians is exactly the same as that from 0 to π radians. So we will need to consider only half of the total cycle. The curve for the square of the DC current is a horizontal line represented by AB. The problem is then to find the altitude for the rectangle with the base of π which has the same areas under the $i^2_{max} \sin^2 \omega t$ curve. By inspection you might say that the area under AB is about equal to the area under the $i^2_{max} \sin^2 \omega t$ curve if $A_1 = A_2$ and $A_3 = A_4$. This is true if OC is equal to $i^2_{max}/2$. You can show this by measuring the two areas.

FIGURE 25.2
Graph of $(i_{max} \sin \omega t)^2$ as a function of ωt. The mean value of the current squared is the DC value that produces equal joule heating as the sinusoidal current over one period.



Let us use the symbol *I* for the effective current. We can see from Figure 25.2 that the effective current squared is equal to one-half of i_{max} squared,

$$I^2 = (1/2)\, i^2_{max} \tag{25.3}$$

It follows, taking the square root of both sides of Equation 25.3 that the effective current is equal to the maximum AC current divided by the square root of two,

$$I = i_{max} / \text{SQR RT [2]} = 0.707\, i_{max} \tag{25.4}$$

The effective current is called the root mean square current. Similarly we will use $\mathcal{E}$ to represent the effective emf of an AC source and V to represent the effective voltage drop in an AC circuit:

$$\mathcal{E} = \mathcal{E}_{rms} = \mathcal{E}_{max} / \textit{SQR RT [2]} = 0.707\, \mathcal{E}_{max} \tag{25.5}$$

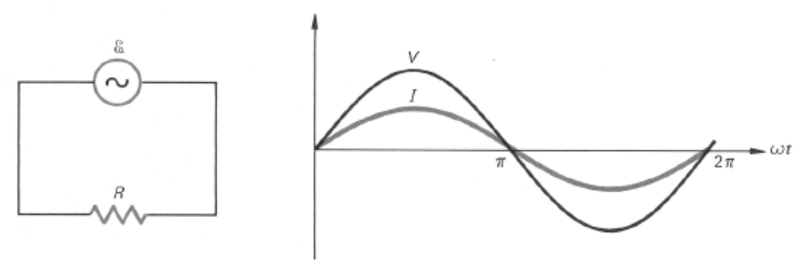
$$V = V_{rms} = V_{max} / \textit{SQR RT [2]} = 0.707\, V_{max} \tag{25.6}$$

The AC meters which you use measure the V_{rms} and I_{rms} values of an AC circuit. If the AC line in your home is said to be 110 V, that is the effective value of the voltage. The maximum or peak voltage would be 110 x SQR RT [2] or 155 V.

25.3 Phase Relations of Current and Voltage in AC Circuits

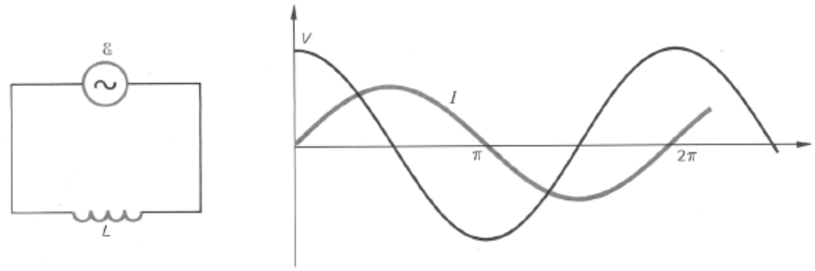
In an AC circuit containing only resistance, the instantaneous voltage and current are always in phase. This means they are both zero at the same time and reach their maximum value at the same time. This is shown in Figure 25.3.

FIGURE 25.3
The voltage $\mathcal{E}$ and current I are in phase in a pure resistance AC circuit as shown.



In Chapter 24 you learned that whenever the current is changing in an inductive coil, an emf is induced. This induced emf depends upon the induction of the coil and on the time rate of change of current. In a coil, continuously changing AC current produces an alternating emf from self-induction. According to Lenz's law this induced emf is opposite to the applied emf. If you neglect the resistance of the coil, the emf of the source will just be equal to the emf of self-induction in the coil. The induced emf will cause the current in the coil to lag behind the applied emf by one-quarter of a cycle. We say that the current in an inductor lags the voltage by 90°, or that the phase of the voltage across the inductor leads the phase of the current by 90°. See Figure 25.4.

FIGURE 25.4
The voltage leads the current by 90° in a pure inductance AC circuit as shown.



The inductance in the circuit not only causes the current to lag the emf but it also reduces the current to a smaller value than it would have if there were no inductance present. The voltage drop V across an inductance L is given by

$$V = I\omega L = 2\pi f LI \tag{25.7}$$

where f is the frequency in cycles per second and L is the inductance in the circuit in henries. It follows that the AC current in an inductance is given by the voltage drop across the inductance divided by $2\pi f$ times the inductance,

$$I = V/(2\ fL) \tag{25.8}$$

The factor $2\pi fL$ is called the inductive reactance of the circuit, represented by X_L, and is measured in ohms L is in henries and f is the frequency (in Hz),

$$X_L = 2\pi f L \tag{25.9}$$

An inductive element in an AC circuit acts as an inertia element and impedes the alternating current. We also note that this effect depends directly upon the frequency. Hence for high frequency an inductor exhibits a large inertia and thus greatly impedes a high-frequency alternating current.

If you connect a capacitor to an AC source the plates of the capacitor become charged alternatively positive and negative, according to the surge of charges back and forth in the connecting wires. So, even though there can be no constant DC current through a capacitor, we can say that there can be an alternating current through a capacitor. A capacitor does present an impedance to an alternating current which is called capacitive reactance. We shall represent the capacitive reactance by X_c. In a way analogous to the definition of capacitance as the ratio of voltage to charge, we shall define the capacitive reactance as the ratio of the voltage drop across the capacitor to the current through the capacitor,

$$X_c = V/I \tag{25.10}$$

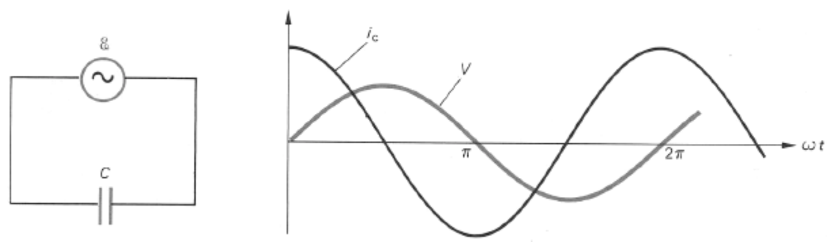
where

$$X_c = 1/(2\pi fC) \text{ ohms} \tag{25.11}$$

and C is the capacitance in farads and f is the frequency in cycles per second (Hz). Note that the capacitive reactance decreases as the frequency increases. A capacitor has infinite impedance for DC sources. The inertia effect of a capacitor in an AC circuit decreases as the frequency increases.

In a circuit with only a capacitor the current leads the voltage by one-quarter of a cycle (see Figure 25.5). We say that phase of the voltage across a capacitor lags the current by 90°.

FIGURE 25.5
The voltage lags the current by 90° in a pure capacitance AC circuit as shown.



25.4 AC Series Circuits

Let us consider a series circuit including a resistor and an inductor (see Figure 25.6). The source is 120 V, 60 cycle. Suppose you find that an AC voltmeter connected between the points *N* and *O* reads 90.0 V, connected between *O* and *P* reads 79.4 V and connected between *N* and *P* it reads 120 V. You see that the voltage drop across *NO* added to the voltage drop across *OP* is greater than 120 V. Is this possible? Can the sum of two voltage drops across series elements be greater than the applied voltage? The answer is yes.

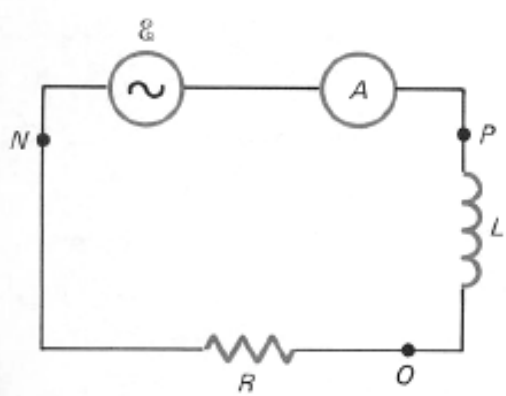


FIGURE 25.6
A resistor and inductor in series with an AC generator.

As we indicated in the previous section, the voltage drops across and the currents in various AC circuit elements may not be in phase with each other. To correctly add the voltage drops across AC circuit elements, these phase differences must be taken into account. To enable you to perform these additions correctly, we will introduce you to the graphical technique called *phasor diagrams*. The currents and voltages will be represented by vectors in the diagrams. In a series circuit the current is the same in all circuit elements; so we will choose the phasor of the current to be the *x*-axis (see Figure

25.7). Since voltage drop across a resistor is in phase with the current through it, the voltage drop across a resistor, V_R, in magnitude equal to IR, will also be a phasor along the x-axis.

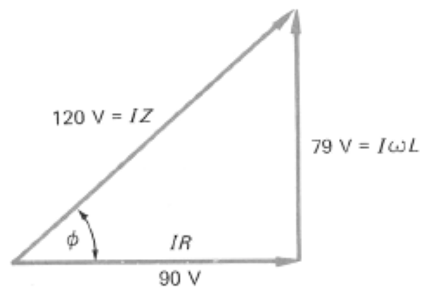


FIGURE 25.7
A phasor diagram for the circuit shown in Figure 25.6. The voltage leads the current for the resistance-inductance series circuit.

The voltage drop across an inductor leads the current by 90° so the voltage drop across the inductor V_L in magnitude equal to IX_L or IwL, is shown as a phasor at 90° to the x-axis and pointing in the positive y-direction. These two voltages can then be added using the rules of vector addition. The total applied voltage is represented by the hypotenuse of the right triangle formed by V_R and V_L. To preserve the simple structure of Ohm's law for use with AC circuits, we will define a quantity called the *total impedance* of the AC circuit, Z. The impedance Z is given by the number that must be multiplied by the current I to obtain the magnitude of the applied voltage. The impedance also has a phase angle, namely the angle between the current and the applied voltage. Remembering the form of multiplication for AC circuits involves magnitude and phase angle, we can write an AC circuit form of Ohm's law,

$$\mathcal{E} = \mathrm{IZ} \qquad (25.12)$$

where $\mathcal{E}$ is the applied emf, I is the current, and Z is the impedance. For a series circuit containing a resistor and an inductor we can show the Z is calculated by the following equation:

$$\mathrm{Z = SQR\ RT}\ [R^2 + X_L{}^2] = \mathrm{SQR\ RT}\ [R^2 + (\omega L)^2] \qquad (25.13)$$

The phase angle f is given by the angle whose tangent is the ratio of X_L to R,

$$\tan \phi = (X_L / R) = (\omega L\ / R) \qquad (25.14)$$

For a series circuit the ratio X_L/R is the same as the ratio V_L/V_R. From the Figure 25.7, we can calculate the phase angle for that example,

$$\tan \phi = X_L/R = V_L/V_R = 79.4\ /\ 90.0 = 0.882 \qquad (25.15)$$

so ϕ = 41.42°. For this example the phase angle represents the angle by which the applied voltage leads the current.

Let us now consider an AC circuit that contains only a resistor and a capacitor (see Figure 25.8).

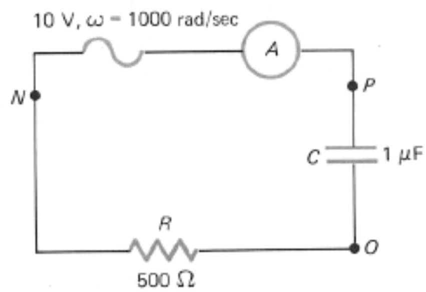


FIGURE 25.8
A resistor and capacitor in series with an AC generator.

Consider a 10-V source with $\omega = 1000$ rad/sec in series with 1μF capacitor and a 500Ω resistor. If you measure the voltage across the resistor (*NO*) and find it to be 4.47 V and across the capacitor *OP* and find it to be 8.94 V, how can you explain it? The two voltage drops seems to have a total greater than the 10 V of applied emf. We now return to our phasor diagram. Since we know that the voltage across a capacitor lags behind the current through it, we will indicate the voltage drop across the capacitor V_C, equal in magnitude to IX_C, by a phasor at 90° to the current through the resistor and to V_R pointing in the negative y-direction(see Figure 25.9). As before the total applied emf will be represented by the hypotenuse of the triangle but this time by one whose sides are V_R and V_C. We can again define an impedance and phase angle for the circuit,

$$Z = SQR\ RT\ [R^2 + X_C^{\ 2}] = SQR\ RT\ [R^2 + (1/\omega C)^2] \tag{25.16}$$

$$\tan \phi = (-X_C/R) = -(1/\omega RC) = -(V_C/V_R) \tag{25.17}$$

where the negative phase angle indicates that the voltage lags behind the current. For the circuit shown in Figure 25.8, we obtain the following results:

$V_R = 500\ I = 4.47\ V$

$X_C = (1/\omega C) = 1 / 1000 \times 1 \times 10^{-6} = 1000\ \Omega$

$V_C = IX_C = 8.94\ V$

$\tan \phi = -8.94 / 4.47 = -2.00\ \ \phi = -63.4^\circ$

which indicates that the applied 10 V lags behind the current by 63.4°.

Finally, let us consider a circuit consisting of a resistor, capacitor, and inductor in series with an AC generator (see Figure 25.10a). The current in such a circuit determines the total impedance of the circuit. From our discussion of inductance and capacitance, we know that these elements exhibit voltages 180° out of phase with each other, shown by vectors pointing in opposite y-directions on a phasor diagram as shown in Figure 25.10b. For this circuit we can find the applied voltage using the same procedures we used in the above two cases,

$$V_{applied} = SQR\ RT\ [V_R^2 + (V_L - V_C)^2] \tag{25.18}$$

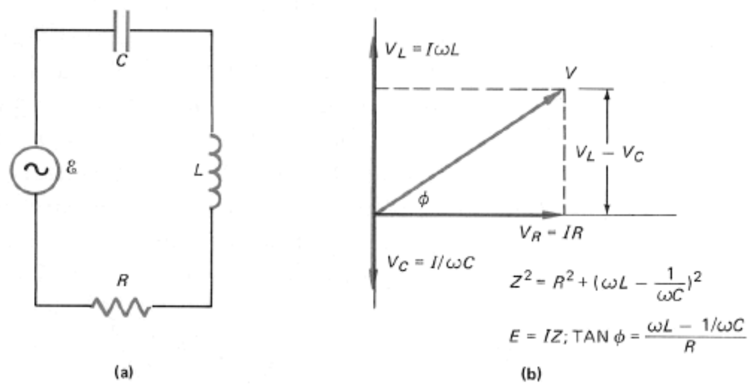


FIGURE 25.10
An AC generator connected in series with a resistor, inductor and capacitor. The phasor diagram for the circuit illustrates the resultant phase angle between the voltage and current in the circuit.

We find that the impedance and phase angle are defined as before

$$Z = SQR\ RT\ [R^2 + (X_L - X_C)^2] = SQR\ RT\ [R^2 + ((\omega L) - (1/\omega C)^2))], \tag{25.19}$$

$$\tan \phi = (X_L - X_C)/R = ((\omega L) - (1/\omega C))/R \tag{25.20}$$

The power dissipated in the circuit is accounted for entirely by the resistor in the circuit. Ideal inductors and capacitors only store and exchange magnetic and electrical field energies. The power dissipated in the circuit is given by the product of the square of the current times the resistance,

$$P = I^2R = I^2Z \cos \phi \tag{25.21}$$

since the resistance is equal to the impedance times the cosine of the phase angle between the voltage and the current. This factor, $\cos \phi$, is known as the power factor of the circuit.

EXAMPLE

Given a 110-V (60-Hz) AC source in a circuit consisting of a 2.65 μF capacitor, a 0.534-H coil, and 600-Ω resistance(see Figure 25.11), find the current in the circuit, the phase angle between current and voltage (which leads?), the power factor, and power dissipated in the circuit.

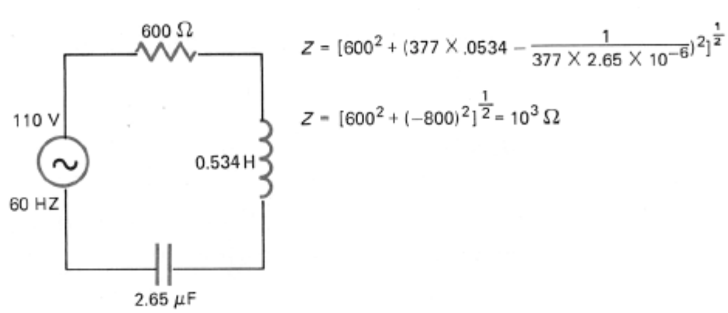


FIGURE 25.11
The series RLC AC circuit has an effective AC impedance that determines the magnitude of the current in the circuit.

$\omega = 2\pi f = 377$ rad / sec

$Z =$ SQR RT $[600^2 + [(377)(.534) - (10^6/(377)(2.65))]^2]$

$= 1000\ \Omega$

$I = (V/Z) = 0.11$ A

$\tan \phi = (-800/600) = -1.33$ $\phi = -53^o$;

The current leads the voltage by 53^o

power factor $= \cos \phi = 3/5 = 0.6$

power $= I^2R = (0.11)2(600) = 7.26$ watts

25.5 Resonance

We have noted that the voltages across a capacitor and inductor are exactly out of phase with each other. Consider the special case when these voltages are equal in magnitude in a series *RLC* circuit. By inspecting the equations for the impedance of an AC series circuit, Equation 25.19, we see that this condition will occur when X_C and X_L are equal,

$$\omega L = 1 / \omega C \tag{25.22}$$

This condition is called resonance. So the angular frequency for which resonance occurs is given by,

$$\omega_o = 1 / \text{SQR RT } [LC] \tag{25.23}$$

This is called the natural frequency of the circuit and is symbolized by ω_o.

As an example a radio station is broadcasting on a given frequency. This frequency is determined by the inductance and capacitance in its output circuit. In order for you to receive the signal from this station, you must tune the receiving circuit of your radio so that it has the same natural frequency as the broadcasting station. That is the product of *L* and *C* will have a fixed value. In order to do this one could theoretically change either *L* or *C*, or both. In practice the tuning of a radio receiver is done by changing only the capacitance.

Questions

1. What is the maximum current in a resonant circuit?

2. It should be noted that resonant circuits can exhibit voltages across *L* or *C* that are much larger than the AC source voltage. Does this violate the conservation of energy? Explain how this can be true.

25.6 Electrical Oscillations

In our study of mechanical vibratory motion we found that most objects can be set into vibration about their equilibrium points. If a slight displacement of the object from equilibrium is produced, then a restoring force may cause the object to oscillate until friction damps out the motion.

Electric oscillations can be set up in an electrical circuit if analogous conditions are met. Earlier in this chapter it was pointed out that an inductor acts as inertia in impeding the building up of the flow of charge in the circuit. The accumulation of charge in the capacitor plates produces a restoring force on the electrons. The resistance in the circuit produces heat and is analogous to friction in the mechanical system.

The frequency of the mechanical vibrations depended upon the inertia and the restoring force constant. In the electrical circuit we will see that the frequency of the electrical oscillations depend upon the inductance and capacitance in the circuit. In the circuit shown in Figure 25.12, a capacitor *C* and inductance *L* are connected in series with spark gap *G*. The spark gap *G* will have a high resistance until a spark jumps across it and a low resistance after the spark. The capacitor is charged until the potential across the gap is sufficiently high to produce a spark. The capacitor will then discharge. The effect of the inductor is to oppose the buildup of charge so the current does not stop at zero but the capacitor is charged in the opposite direction. It then discharges again the current reversing in the circuit. Originally, the energy of the system was stored in the charged capacitor. Upon complete discharge, the energy goes into the magnetic field of inductor and into heat produced by the resistance of the circuit. The circuit continues to oscillate until all of the energy originally stored in the capacitor has been converted to heat. If there is a source of energy of input to compensate for the heat losses through resistance, the system will continue to oscillate. The frequency of oscillation depends upon the values of *L* and *C* and corresponds to the natural frequency of the circuit, which occurs for $X_L = X_C$ and

$$\omega_o = 1\ / \text{SQR RT}\ [LC] \qquad (25.23)$$

the same equation as used to calculate the resonance frequency.

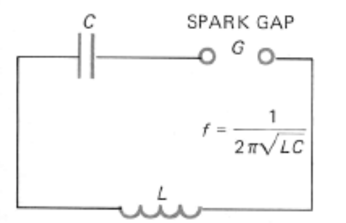


FIGURE 25.12
An AC resonant circuit results when the voltages across the inductor and capacitor are equal. Since these voltages are exactly out of phase, the effective impedance for such a circuit is equal to the total resistance in the circuit. The condition for resonance is $x_L = x_C$ or $f = 1/(2\pi\sqrt{LC})$. The current is a maximum for resonant conditions.

25.7 Q-Factor

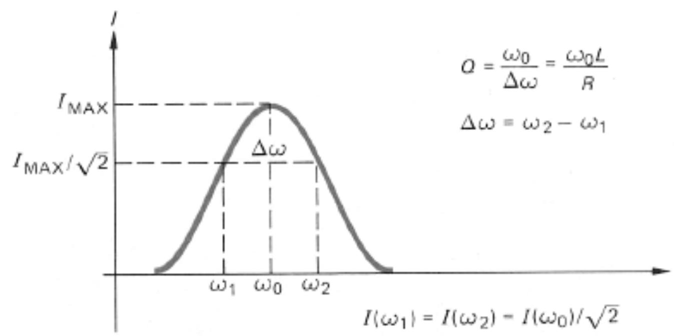
The *quality factor* of a resonance is the measure of its sharpness. The *Q-factor* of a resonant circuit is illustrated in Figure 25.13. If the current through an AC circuit is plotted as a function of the angular frequency ω, then the current is maximum at the natural resonance frequency ω_o. The height of the current peak compared to its width is a measure of the sharpness of resonance. The quality factor for an AC circuit is defined as the ratio of the reactance at resonance to the resistance,

$$Q = \omega_o L/R = \omega_o / \Delta\omega \tag{25.24}$$

where ω_o is the resonance frequency.

A high Q circuit has low resistance and low fractional energy loss per cycle. A radio receiver with a high Q tuning circuit will have good discrimination between radio signals of nearly the same frequency.

FIGURE 25.13
The current as a function of angular frequency ω shows a maximum at resonance $\omega_0 = 1/\sqrt{LC}$. The Q-factor measures the sharpness of the resonance; Q-factor $= \omega_0/\Delta\omega$.



EXAMPLE

It is desirable to make a resonant circuit for EEG waves at 50 Hz. Given an inductor of 2.4 H and 100 Ω resistance, find the capacitance necessary for resonance and the Q-factor for the circuit.

$\omega_o = 2\pi f_o = 2\pi(50) = 100\pi$ rad/sec $= 1$ / SQR RT $[LC]$

Thus,

$C = 1 / ((100\pi)^2 L) = 10^{-4} / [\pi^2(2.4)] = 4.15\ \mu\text{F}$

$Q = \omega_o L/R = (100\pi \times 2.4) / 100 = 7.6$

Questions

3. Show that the Q-factor is proportional to the ratio of the energy stored to the energy dissipated in the circuit per cycle?

25.8 The Transformer and Its Applications

The transformer is a most important AC device. In its simple form a transformer consists of two coils wound around a common ferromagnetic core. A changing current in one coil (the primary coil) produces a changing magnetic flux through the second (secondary) coil and thereby produces an emf in the secondary coil. There is also a back emf in the primary coil almost equal to the applied emf. This back emf, $\mathcal{E}_b$, equals the rate of total flux change,

$$\mathcal{E}_b = -N_p \Delta\phi/\Delta t \tag{25.25}$$

where N_p is the integral number of turns in the primary coil and $\Delta\phi/\Delta t$ is the time rate of change in magnetic flux through each turn.

The emf $\mathcal{E}_S$ induced in the secondary coil likewise is given by

$$\mathcal{E}_S = -\,N_S\,\Delta\phi/\Delta t \tag{25.26}$$

where N_S is the integral number of turns in the secondary. The ratio of the primary emf to the secondary emf is given by the ratio of Equations 25.25 and 25.26, since the primary coil emf is almost equal to the back emf, $\mathcal{E}_b$,

$$\mathcal{E}_P \;/\; \mathcal{E}_S = (-N_p\Delta\phi/\Delta t)/(-N_S\,\Delta\phi/\Delta t) = N_p/\;N_S \tag{25.27}$$

A transformer with $N_S > N_p$ is called a step-up transformer (stepping up voltage), while a transformer with $N_S < N_p$ is a step-down transformer. Transformers can be made with 99 percent efficiency. From conservation of energy, the power input should almost equal the power output.

power input ≈ power output

Thus,

$$I_p\,\mathcal{E}_P = I_s\,\mathcal{E}_S$$

$$I_s/\,I_p = \mathcal{E}_P\,/\,\mathcal{E}_S = N_p\,/\,N_s \tag{25.28}$$

Transformers make it possible to transmit AC electric power over high voltage (120,000 V) transmission lines, reducing the I^2R energy loss in the power line. When the power reaches the desired destination, the voltage can be stepped down (to 240 or 120 V) for use. A step-down transformer can be used to produce high currents at low voltages as is common in electric welders.

EXAMPLE

What is the maximum current available in a step-down transformer with $\mathcal{E}_P = 120$ V, $I_p = 5$A, and $\mathcal{E}_S = 6$V?

Since $\mathcal{E}_P\,I_p = \mathcal{E}_S\,I_S$,

$I_S = (120\text{ V} \times 5\text{ A})/6\text{ V} = 100\text{ A}$

Another important use of transformers is that of impedance matching. As in DC circuits, maximum power will be transferred from an AC source to a load of the same impedance magnitude with a 180° phase difference between their reactance components. This means the impedance of the load should equal the impedance of the source. If the impedance of a load Z_L equals the secondary impedance Z_S, then we can show that the load impedance is equal to a constant times the impedance of the primary,

$$Z_L = Z_S = \mathcal{E}_S / I_S = (N_s/N_p)^2 (\mathcal{E}_P / I_p) = (N_s/N_p)^2 Z_p \tag{25.29}$$

A transformer can be used to match impedance between sources and loads. A common example of this use of transformers is the coupling of audio-speaker systems (low impedance 8 or 16 Ω) with audio amplifiers (high impedance output).

EXAMPLE

It is desired to match an 8-Ω sound speaker to the audio amplifier from the ECG set up that has a 10,000 W output impedance. Find the turn ratio needed for the transformer to be used in this impedance match.

$8\ \Omega = 10^4 (N_s/N_p)^2$

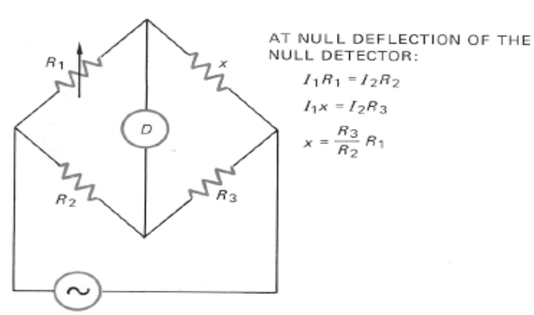
Therefore,

$N_p/N_s = 10^2 / \text{SQR RT } [8] = 36$

25.9 Alternating Current Applications

Direct-current measurements may cause polarization voltages in ionic conduction. These polarization voltages are generated by the charge separation produced by the electric field. To minimize the effects of polarization voltages, it is desirable to use AC measurements. One such measurement procedure involves the use of an AC impedance bridge, analogous to the DC Wheatstone bridge, to measure bioimpedances. For resistance measurements the equations are the same as those for the Wheatstone bridge. Such a bridge diagram is shown in Figure 25.14. Versatile AC bridges use reactive elements (capacitors and inductors) in the branches and are capable of measuring both resistive and reactive components of an unknown impedance.

FIGURE 25.14
An AC bridge circuit is analogous to the DC Wheatstone bridge. The AC bridge produces less heating in unknown resistors and avoids problems of polarization in liquid or ionic systems.



It has been found that electrode implantation can be aided by measuring the impedance between the inserted electrode and a reference electrode. For example, different tissue layers have different impedance values, and boundary layers are discerned by impedance changes as the electrode is moved into its desired position.

Alternating-current impedance measurements are also used for physiological measurements. Impedance plethysmography involves measuring changing impedance across the chest associated with breathing and the pulsating blood flow. (Plethysmography is the study of blood volume changes within an organ.)

Most of the electrical signals generated by the human body are AC in nature. Some of these signals, their frequency, and amplitudes are shown in Table 25.1. (It should be noted that while these signals are periodic they are not sinusoidal.)

TABLE 25.1
Frequency and Amplitudes of Some Electrical Signals Generated in the Human Body

Signal	Frequency (Hz)	Amplitudes (order of magnitude)
EEG α	8–13	20 μV
EEG β	14–50	10 μV
EEG δ	0.5–4	50 μV
EEG θ	5–7	50 μV
ECG beats/min	1–1.5	1 mV
Eye blink potentials	1–3	0.5 mV

Electrical hazards are associated with leakage currents of electrical equipment. These currents result from pathways to ground that are not intended. Most frequently this path results from capacitive coupling between the high voltage side of the power line and ground. (One wire of the powerline is grounded. The third wire of modern equipment grounds the case of the equipment.) Maximum leakage currents of electrical equipment are subject to federal regulations and should be less than 1 mA. For capacitive leakage (a pair of conducting wires have such capacitive coupling), the effective impedance for a pathway is given by $Z_c = 1/\omega C$ for 1 mA current at 120 V

$$Z = 120 \text{ V}/10^{-3} \text{ A} = 1.2 \times 10^5 \ \Omega = 1/\omega C$$

If $\omega = 2\pi\,(60)$, then $C = 0.22\ \mu F$.

This is a rather large "leakage" capacitance, a more typical value might be 10^{-9} F or 1000 picofarads.

Currents as small as 20 μA have been known to cause fibrillation when internal electrical contacts (such as implanted electrodes or catheters) are used. A current of this size would result for a capacitance leakage given by

$$Z = 120 \text{ V}/(20 \times 10^{-6} \text{ A}) = 6 \times 10^6 \ \Omega = 1/\omega C$$

If $\omega = 2\pi\,(60)$, then $C = 440 \times 10^{-12}$ F.

This is a small leakage capacitance and points out the care that must be taken when internal electrical contacts are present.

Another possible electrical hazard results when ground connections break or equipment is improperly grounded. In these cases, a human body may provide the pathway to ground when the instrument or piece of equipment "floats" at a potential above ground. This commonly occurs when three-wire equipment is used without proper ground connection. In these cases, the subject provides resistive coupling to ground. The value of this resistive coupling is determined by the nature of the electrical contacts to the high voltage side and to the ground side of the AC power. Moisture at the contact points reduces the resistance and increases danger of electrical shock.

ENRICHMENT

25.10 Calculation of Effective Current

The instantaneous power dissipated in a resistor in an AC circuit is $P = i^2R = (I_0 \sin\omega t)^2R$.

We want to determine the average power dissipated. This can be found by averaging over a period t as follows:

$$P_{\text{ave}} = \frac{1}{\tau}\int_0^\tau i^2 R\, dt = \frac{1}{\tau}\int_0^\tau I_0^2 R \sin^2 \omega t\, dt$$

$$P_{\text{ave}} = \frac{I_0 2R}{\tau}\int_0^\tau \left(\frac{1-\cos 2\omega t}{2}\right)dt$$

$$P_{\text{ave}} = \frac{I_0 2R}{2\tau}\left(t - \frac{\sin 2\omega t}{2\omega}\right)\Bigg|_0^\tau \quad \text{where } \tau = \frac{2\pi}{\omega}$$

$$P_{\text{ave}} = \frac{I_0^2 R}{2} = I^2 R$$

So

$$I = \frac{I_0}{\sqrt{2}}$$

Similarly, using the instantaneous voltage $P = V^2/R$, again the average power is

$$P_{\text{ave}} = \frac{1}{\tau}\int_0^\tau \left(\frac{V_0^2 \sin^2 \omega t}{R}\right) dt = \left[\int_0^\tau \left(\frac{1-\cos 2\omega t}{2}\right)dt\right]\frac{V_0^2}{R\tau}$$

$$P_{\text{ave}} = \frac{V_0^2\tau}{R\tau 2} = \frac{V_0^2}{2R} = \frac{V^2}{R}$$

Thus,

$$V = \frac{V_0}{\sqrt{2}}$$

and

$$P_{\text{ave}} = VI = \frac{V_0 I_0}{2}$$

The effective values of the current and voltage in an AC circuit are also referred to as the root-mean-square (rms) values. Most AC meters are calibrated to give rms values. These rms values produce the same heating as equal dc values of voltage and current.

ENRICHMENT

25.11 Calculation of Reactances

We can determine the reactance of a capacitor and an inductor as follows: The voltage across a capacitor is defined as charge (coulombs) divided by capacitance.

$$\mathscr{V}_C = \frac{Q}{C} = \int_0^t \frac{I\,dt}{C} = \int_0^t \frac{I_0}{C} \sin \omega t\, dt = \frac{I_0 \cos \omega t}{\omega C}$$

We see that the voltage across a capacitor is 90° out of phase with the AC current, that is, in this case the voltage lags the current. Also, if we define reactance such that

$$\mathscr{V}_C = X_C\, I_0 \cos \omega t$$

then $X_C = 1/\omega C$. The voltage across an inductor (based on Faraday's law of induction) is

$$\mathscr{V}_L = -L \frac{di}{dt}$$

$$\mathscr{V}_L = -L \frac{d}{dt} (I_0 \sin \omega t) = -\omega L I_0 \cos \omega t$$

$$= \omega L I_0 (-\cos \omega t) = I_0 X_L\, (-\cos \omega t)$$

Again we see the voltage is 90° out of phase with the AC current; in this case the voltage leads the current. The inductive reactance XL equalswL.

SUMMARY

Use these questions to evaluate how well you have achieved the goals of this chapter. The answers are given at the end of this summary with the number of the section where you can find related content material.

Definitions

1. The values for AC current and AC voltage that result in the same average power per cycles as the DC values are called the __________ or _________ values of current and voltage and they are equal to ___________ times the peak current and voltage values respectively.

2. The sharpness of the ___________ of an AC circuit is measured by the ___________ which is equal to the ratio of the inductive ___________ to the resistance when the frequency is equal towo, the ____________ frequency.

3. For an AC circuit the ratio of the voltage to the current is called the ____________ which has both a __________ and a ____________.

4. In general the product of voltage and current for an AC circuit is not equal to the power, but that product must be reduced by a ___________ factor, equal to ____________, where f is the angle between the AC current and the AC voltage.

AC Circuits

5. A coil of wire has a resistance of 30.0 W and an inductance of 0.100 H. Find

    a. its inductive reactance if connected to a 60-cycle line

    b. its impedance

    c. What would the current be if the coil were connected to a 120-V DC line?

    d. What would the current be if it were to a 120-V AC, 60-cycle line?

6. A 120-W rheostat and a 15 mF capacitor are connected in a series circuit to 120-V, 60-cycle emf.

    a. What is the reactance of the capacitor?

    b. What is the total impedence of the circuit?

    c. What is the current through the circuit?

    d. What is the voltage drop across each circuit element?

Phasor Diagrams

7. Draw a phasor diagram for problems 5 and 6 above and determine the phase angle for each.

Transformer

8. If a 110-V line is connected to the primary of a step-up transformer, it delivers 2 amps on the secondary coil. The ratio of turns on the two windings is 25. Assume no losses in the transformer. Find

    a. the secondary voltage

    b. the primary current

AC Measurements

9. Electrical equipment may be hazardous because of ___________.

10. List at least three periodic electrical signals generated by the human body, and give their typical frequencies and amplitudes.

Answers

1. effective, rms, 0.707 (Section 25.2)
2. resonance, Q-factor, reactance, resonance (Section 25.5 and 25.7)
3. impedance, magnitude, phase angle (Section 25.4)
4. power, cos ϕ (Section 25.4)
5. a. 37.7 Ω; b. 48.2 Ω; c. 4 A; d. 2.49 A (Sections 25.3 and 25.4)
6. a. 177 Ω; b. 214 Ω; c. 0.561 A; d. V_C = 99.3 V, V_R = 67.3 V (Section 25.3 and 25.4)
7. prob. 5, ϕ = 51.5°; prob. 6, ϕ = -55.9° (Sections 25.4)
8. a. 2750 V; b. 50 A (Section 25.8)
9. leakage currents (Section 25.9)
10. ECQ, ~ 1 Hz, ~ 1 mV; EEG α, ~ 10 Hz, ~ 20 μV; EEGθ, ~ 5 Hz, ~ 50μV (Section 25.9)

ALGORITHMIC PROBLEMS

Listed below are the important equations from this chapter. The problems following the equations will help you learn to translate words into equations and to solve-single concept problems.

Equations

$i = i_{max} \sin \omega t$ (25.1)

$I = i_{max}$ / SQR RT [2] = 0.707 i_{max} (25.4)

$V = V_{rms} = V_{max}$/SQR RT [2] = 0.707 V_{max} (25.6)

$X_L = 2\pi f L$ (25.9)

$X_c = 1/(2\pi f C)$ ohms (25.11)

Z = SQR RT [$R^2 + (X_L - X_C)^2$] = SQR RT [$R^2 + ((\omega L) - (1/\omega C)^2)$)], (25.19)

$\tan \phi = (X_L - X_C)/R = ((\omega L) - (1/\omega C))/R$ (25.20)

$P = I^2R = I^2Z \cos \phi$ (25.21)

ω_o = 1 / SQR RT [LC] (25.23)

$Q = \omega_o L/R = \omega_o / \Delta\omega$ (25.24)

$I_s / I_p = \mathcal{E}_P / \mathcal{E}_S = N_p / N_s$ (25.28)

Problems

1. If the effective voltage to an electric stove is 208 V, what is the peak voltage?

2. Compare the inductive reactance of a 1-H inductance on a 25-cycle source and a 60-cycle source.

3. Compare the capacitive reactance of a 2 μF capacitor connected to a 25-cycle source and a 60-cycle source.

4. What is the impedance of a circuit which has a resistance of 30 Ω and an inductive reactance of 40 Ω?

5. What is the phase angle for a circuit that has an inductance reactance of 30 Ω, a capacitive reactance of 20 Ω and 20 Ω resistance?

6. What is the capacitance needed in a circuit to produce resonance in a 60-cycle circuit having an inductance of 1 H?

7. What is the ratio of primary turns to secondary turns in a transformer which is designed to operate a 6-V bell system when connected to a 114-V line?

Answers

1. 293 V

2. $X_L(f = 60) = 2.4\ X_L(f = 25)$

3. $X_c(f = 25) = 2.4\ X_c(f = 60)$

4. 50 Ω

5. $\phi = 26.6°$, voltage leads current

6. 7.04 μF

7. 19:1

EXERCISES

These exercises are designed to help you apply the ideas of a section to physical situations. When appropriate, the numerical answer is given in brackets at the end of the exercise.

Section 25.2

1. What does a 120-V, 60-cycle source mean? What peak voltage must insulation stand for this source? What is its effective voltage? What is its average voltage? Compare it with a 120-V DC source. [170 V, 120 V, 0 V]

Section 25.3

2. What is the reactance of a 2.00-μF capacitance at a frequency of 1, 60, 440, 10^6 Hz? What does this indicate? [$7.96 \times 10^4\ \Omega$, $1.3 \times 10^3\ \Omega$, 181 Ω, $7.96 \times 10^{-2}\ \Omega$]

3. What is the reactance of a 2-H inductor at frequency of 1, 60, 440, 10^6? What does this indicate? [1.26×10^1, 7.54, $\times 10^2$, 5.53×10^3,1.26×10^7]

Section 25.4

4. What is the total impedance at 60 cycles of the resistor, capacitor and inductor shown in Figure 25.15? [1290 Ω]

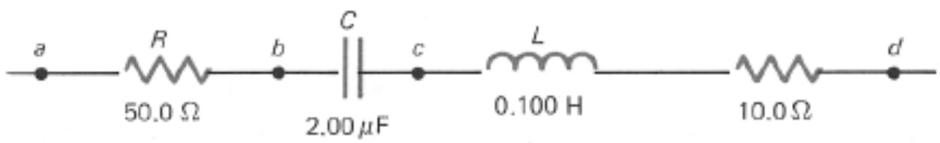


FIGURE 25.15
Exercises 4, 5, and 6.

5. In Figure 25.15, what is the current, and what are the voltages V_{ab}, V_{bc}, V_{cd},V_{ac}, and V_{bd}, if a 120-V, 60-cycle source is connected across ad? [0.093 A; $V_{ab} = 4.65$ V; $V_{bc} = 123$ V; $V_{cd} = 3.63$ V; $V_{ac} = 123$ V; $V_{bd} = 119$ V]

6. Using data from problems 4 and 5, what is the power loss for the entire circuit and for each component? What is the phase angle? [$P_{tot} = 0.52$ watt; $P_R = 0.43$ watt; $P_c = 0$; $P_L = 0.09$ watt; phase angle = 89.96°]

7. A resistance of 100 Ω, an inductance of 75.0 mH, and a capacitor of 4.0 μF are connected in series with a generator (100 volts at 2500 rad/sec). Find

   a. the current in the circuit

   b. the voltage across each circuit component

   c. the power dissipated in the circuit

   d. the phase angle between the current and the voltage in the circuit.

   Draw an appropriate phasor diagram.

   [a. 0.75 A; b. $V_R = V_C = 75$ V; $V_L = 141$ V; c. 56 watt; d. 41.2°]

8. An AC series circuit has R = 300 Ω, L = 0.90 H, and C = 2.0μF with a generator of 50 V and ω = 1000 rad/sec. Find

   a. the current in the circuit

   b. the voltage across each of R, L, and C

   c. the phase angle between voltage and current in the circuit.

   Draw a phasor diagram.

   d. the power dissipated in the circuit

   [a. 0.10 A; b. 30 V, 90 V, 50 V; c. 53°; d. 3 watts]

Section 25.5

9. Assume your radio has an inductance of 18 mH in its receiving circuit. To what capacitance must you turn your radio dial to receive your favorite radio programs broadcast at 1490 kilocycles? [0.63 $\mu\mu$F]

Section 25.7

10. Assume the receiving circuit of your radio has a resistance of 1800 Ω. What is the Q-factor of the circuit? What is its ratio of response to the signal from a nearby station at 1540 kc when you have it adjusted to receive your favorite 1490-kc program? Do you judge this to be a high-quality or low-quality radio? [14.9, S_{1540}/S_{1490} = 1/2, low quality]

Section 25.8

11. A step-up transformer has a turns ratio of 200:1, and 100 V are applied to the primary side of this transformer.

    a. Find the secondary output voltage.

    b. If the secondary current is 100 mA, find the primary current.

    c. Find the power output of the transformer. [a. 2 x 10^4 V; b. 20 A; c. 2 x 10^3 watts]

12. It is desired to operate a 4-V lamp by using a transformer on a 120-V supply. Find the ratio of the secondary current to the primary current in the operation of this transformer. [30]

13. The internal resistance of an AC source is 100 Ω. Find the turns ratio of a transformer that could be used to match this source to a 25-W load with maximum power transferred to the load. [2]

Section 25.9

14. A patient undergoes fibrillation while being catheterized. A current of 5 x 10^{-5} A is produced by leakage potential of 120 V (60 Hz) through capacitive coupling. Find the value of this capacitance. [1.1 x 10^{-9} F]

15. One potential hazard of a microwave oven is its capacitive coupled leakage current. For an oven operating at 100 MHz (1 MHz = 10^6 Hz) at 120 V, find the leakage current if the capacitive coupling is 10^{-12} F. [7.5×10^{-2} A]

PROBLEMS

These problems may involve more than one physical concept. When appropriate, the answer is given in brackets at the end of the problem.

16. A rectangular coil of 100 turns, which has dimensions of 10 cm by 15 cm, rotates at 300 rpm about an axis through midpoint of the short sides. The axis of rotation is perpendicular to the direction of the magnetic field of strength 0.5 Wb/m^2. Plot the induced emf for two complete revolutions of the coil. Choose a position for $t = 0$. [$\mathcal{E} = 7.5\,\pi \sin 10\pi t$]

17. Assume you have available a 50-Ω resistor, an inductor with resistance of 10 Ω and inductance of 0.10 H, a 1.0-μF capacitor, and two sources of electric energy, a DC source of 120 V and an AC source 60-cycle 120 V. What currents would you get if you connected two of these components in series with a source? [Answer see table]

	RC	*RL*	*LC*
DC	0.0 A	2.0 A	0.0 A
AC	0.045 A	1.7 A	0.046 A

18. A 120-V, 60-cycle source is dangerous. It has been estimated that the maximum safe current is 15 mA. At this frequency it is thought that a current of 70 mA for one second could be lethal, that is, it could produce ventricular fibrillation. What is the impedance of the body for these currents? How much energy would be expended in the electrocution? [8000 Ω; 1700 Ω, 8.4 J]

19. Given a resistance of 100 Ω, an inductor of 250 mH, and a capacitor of 1.00 μF in a series with a 10.0-V variable-frequency generator, find

    a. the resonant frequency for the circuit

    b. the voltage across each of *R*, *L*, and *C* at resonance

    c. the power supplied by the generator at resonance

    d. the *Q*-factor of the circuit.

    [a. 318 Hz; b. 10 V, 50 V, 50 V; c. 1 watt; d. 5]

20. A circuit consists of a 500 Ω resistor, an inductor of 100 mH, and a variable capacitor connected in series with a 100-V generator operating at 100 Hz. Find

    a. the value of C that produces resonance in the circuit

    b. the voltage across L, R, and C at resonance

    c. the Q-factor for this circuit

    d. the power dissipated in the circuit.

    [a. 25.3 μF; b. 12.6 V, 100 V, 12.6 V; c. 0.126; d. 20 watt]

21. Inductive coupling is much less common in leakage currents than capacitive coupling, but two adjacent conductors 10 m in length have inductive coupling as well as capacitive coupling. Find the leakage current for a 10^{-5} H leakage inductance with a 1.20 mV (60 Hz) leakage potential. (Is the neglect of resistance of the inductance justified? $R_e = 1.6 \times 10^{-3}\ \Omega/\mathrm{m}$) [$3.18 \times 10^{-4}$ A]

22. For an inductive leakage of 1.00×10^{-5} H in series with a capacitive leakage of 1.00×10^{-9} F,

    a. find the resonant frequency

    b. If 10.0 V leakage voltage results at this resonant frequency, find the leakage current if the resistance (due to dielectric loss) is 1.00×10^{7} Ω.

    c. Find the voltage across the capacitance and inductance in this problem.

    [a. 1.6×10^{6} Hz; b. 10^{-6} A; c. 10^{-4} V]

23. Find the power loss in a transmission line whose resistance is 2 Ω, if 50 kilowatts are delivered by the line

    a. at 50,000 V

    b. at 5000 V

    c. What kind of transformer would you need at the input end if the voltage at the generator is 2500 V?

    [a. 2 watts, b. 200 watts; c. for 50 kv, $N_p/N_s = 1/20$, for 5 kv, 1/2]